SimTech

Bosch Student Competition

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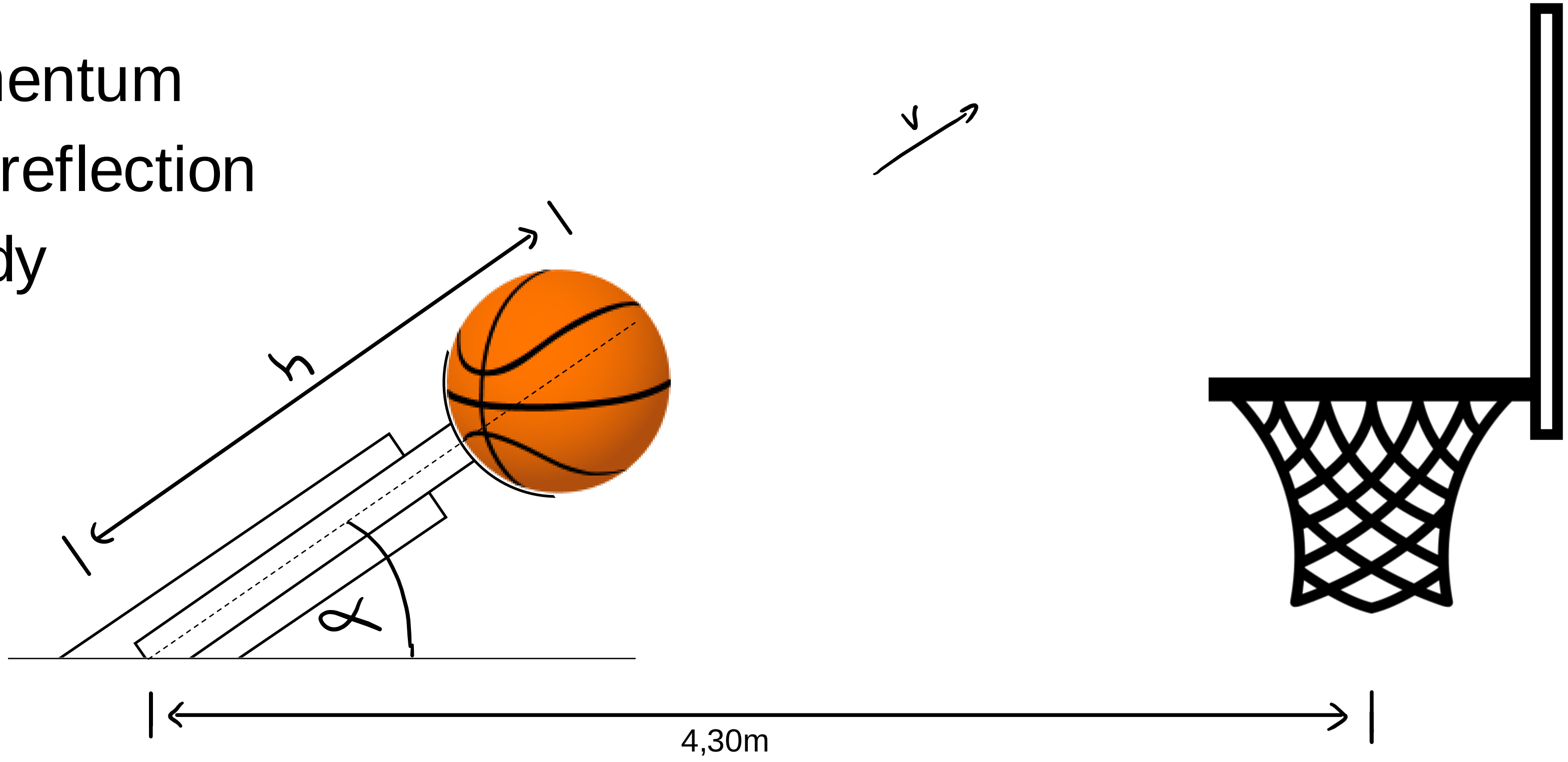
BOSCH

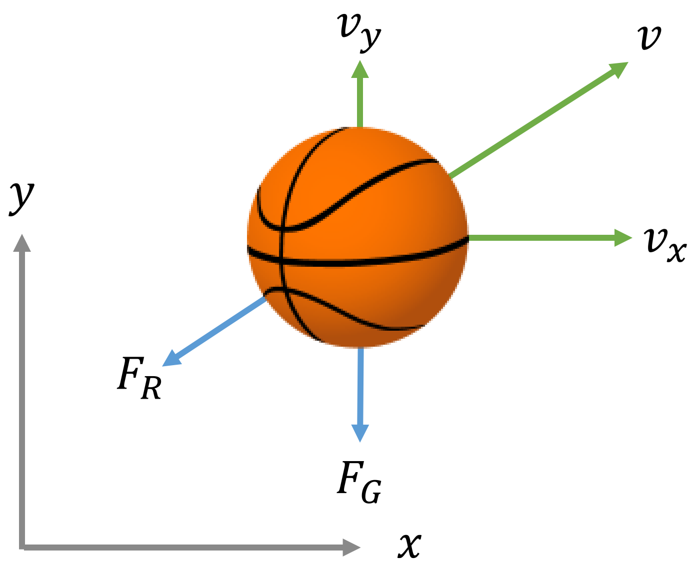
Problem Statement

- Robot throwing a ball into a basket
- Find optimal parameters with tolerances
- Assumptions:
  1. No angular momentum
  2. Perfectly elastic reflection
  3. Ball is a rigid body



Modeling

- Ball throw with air resistance (drag equation)
- $F_G = m_{\text{ball}} g$
- $F_R = k v^2$ where $k = \frac{1}{2} \rho c_w \pi r_{\text{ball}}^2$, $v = \sqrt{v_x^2 + v_y^2}$

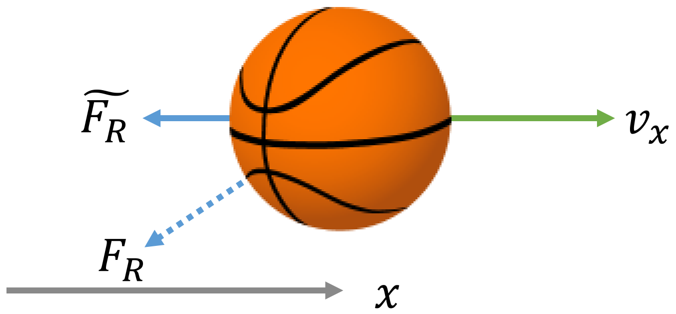


- ODE system: $\dot{\xi} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{v}_x \\ \dot{v}_y \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ -\frac{k}{m_{\text{ball}}} v_x \sqrt{v_x^2 + v_y^2} \\ -g - \frac{k}{m_{\text{ball}}} v_y \sqrt{v_x^2 + v_y^2} \end{pmatrix}$, $\xi(0) = \begin{pmatrix} h \cos \alpha \\ h \sin \alpha \\ v_0 \cos \alpha \\ v_0 \sin \alpha \end{pmatrix}$

Modeling

- First approach: Solve numerically using a Runge-Kutta method
  → not efficient enough in a multi-query setting
- Instead: Decouple the ODE system and solve it analytically

- $\dot{v}_x = -\frac{k}{m_{\text{ball}}} v_x \sqrt{v_x^2 + v_y^2} \approx -\frac{k}{m_{\text{ball}}} v_x^2$



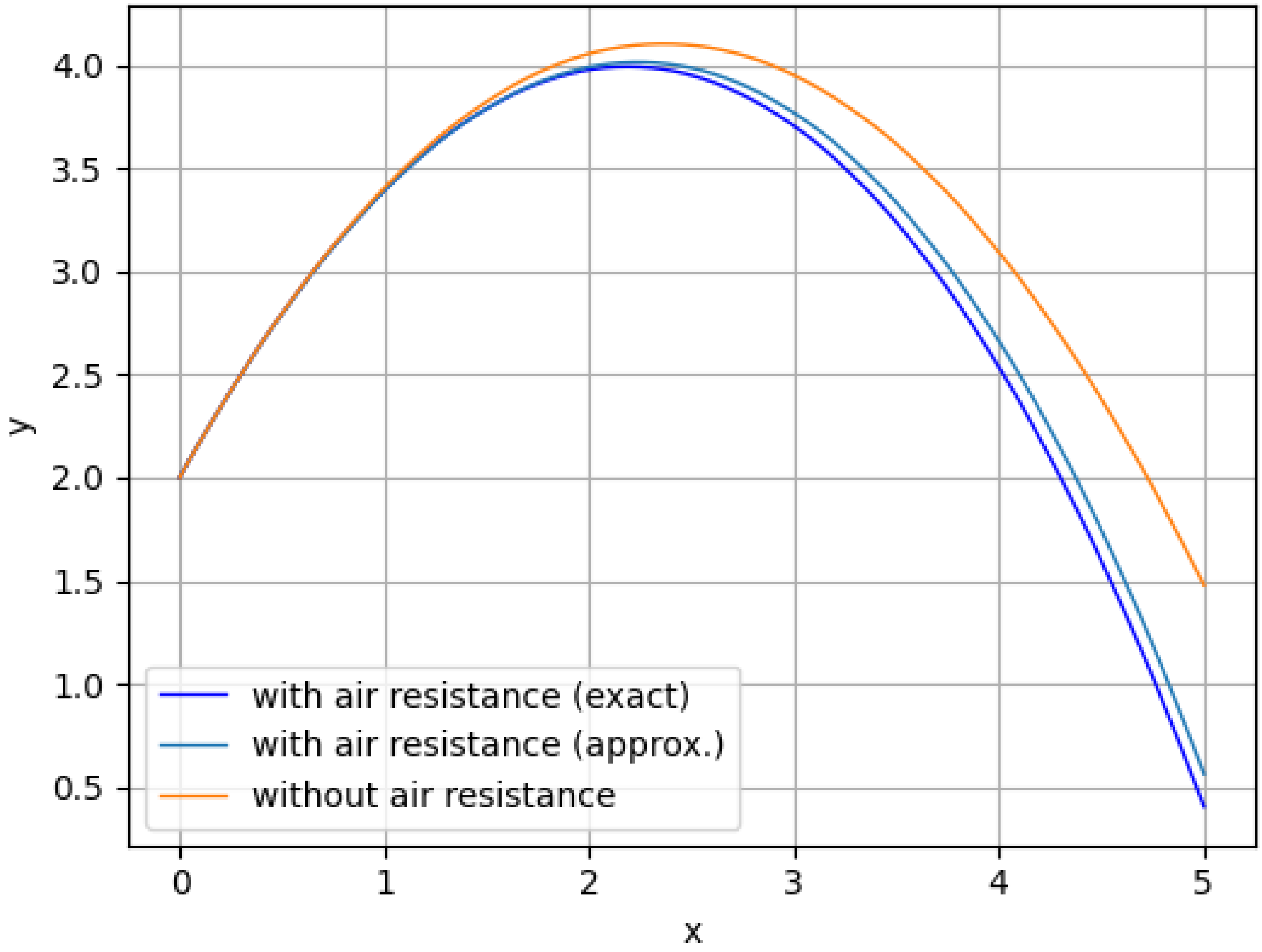
- $\dot{v}_y = -g - \frac{k}{m_{\text{ball}}} v_y \sqrt{v_x^2 + v_y^2} \approx \begin{cases} -g - \frac{k}{m_{\text{ball}}} v_y^2, & t < t_{\text{peak}} \\ -g + \frac{k}{m_{\text{ball}}} v_y^2, & t \geq t_{\text{peak}} \end{cases}$ → upward stage / → downward stage

Modeling

Analytical solution of the decoupled ODE system:

- $x(t) = \frac{m_{\text{ball}}}{k} \ln\left(\frac{k v_0 \cos\alpha}{m_{\text{ball}}} t + 1\right)$
- $y(t) = \begin{cases} h + \frac{m_{\text{ball}}}{k}\left(\ln\left(\cos\left(\sqrt{\frac{kg}{m_{\text{ball}}}} t - c\right)\right) - \ln(\cos c)\right), & t < t_{\text{peak}} \\ h + \frac{m_{\text{ball}}}{k}\left(-\ln\left(\cosh\left(\sqrt{\frac{kg}{m_{\text{ball}}}} t - c\right)\right) - \ln(\cos c)\right), & t \geq t_{\text{peak}} \end{cases}$
- where $t_{\text{peak}} = \sqrt{\frac{m_{\text{ball}}}{kg}}\, c\,,\quad c = \arctan\left(\sqrt{\frac{k}{m_{\text{ball}} g}}\right) v_0 \sin\alpha$

Modeling



4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
y
0
1
2
3
4
5
x
with air resistance (exact)
with air resistance (approx.)
without air resistance

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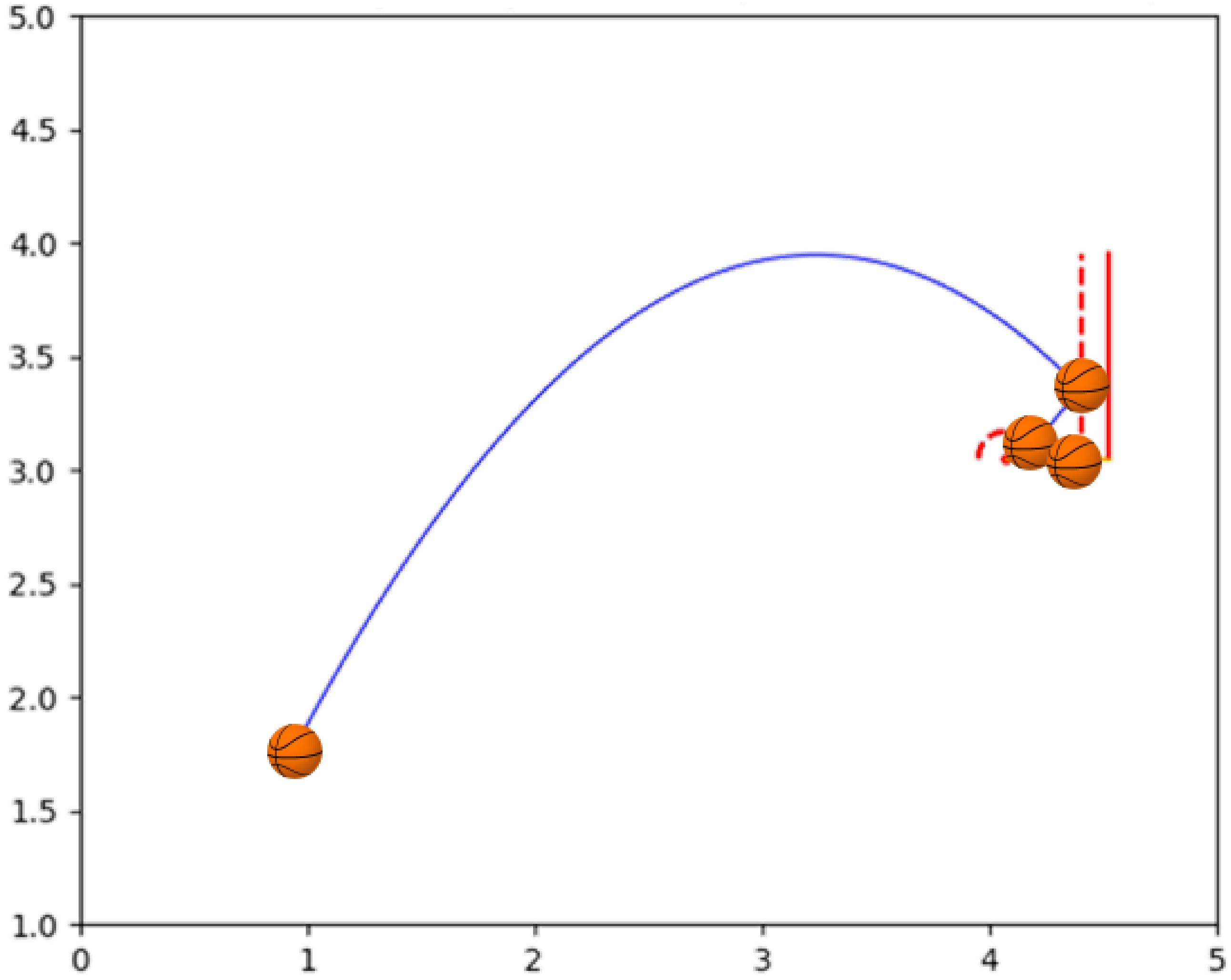
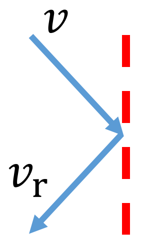
Modeling



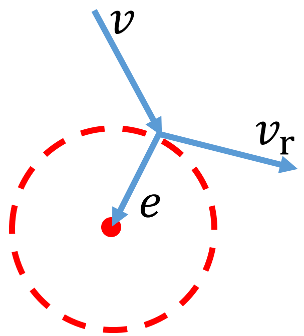
- Only few points are relevant:
  - Intersection with backboard
  - Intersection with ring zone
  - Intersection with basket level
- At bounce: Invoke simulation with new initial conditions recursively

Modeling

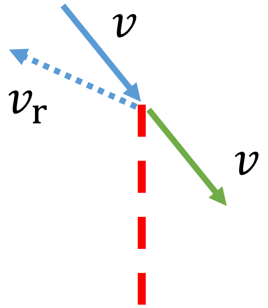
- Reflection at the backboard: $v_{\mathrm{r}} = \begin{pmatrix} v_{\mathrm{r},x} \\ v_{\mathrm{r},y} \end{pmatrix} = \begin{pmatrix} -v_x \\ v_y \end{pmatrix}$



- Reflection at the ring zone: $v_{\mathrm{r}} = v - 2\,\frac{v \cdot e}{e \cdot e}\,e$



- Reflections on top of the backboard are neglected



Uncertainties

- Mapping $H: (h, \alpha, v_0, r_{\text{ball}}, m_{\text{ball}}) \mapsto \begin{cases} 1, \text{if ball goes in} \\ 0, \text{if ball passes} \end{cases}$
- Assume uniform distribution:
  - $h \sim \mathcal{U}(h_{\text{opt}} - 15\text{cm}, h_{\text{opt}} + 15\text{cm})$
  - $\alpha \sim \mathcal{U}(\alpha_{\text{opt}} - 5°, \alpha_{\text{opt}} + 5°)$
  - $v_0 \sim v_{0,\text{opt}} \cdot \mathcal{U}(1 - 0.05, 1 + 0.05)$
  - $r_{\text{ball}} \sim \mathcal{U}(r_{\text{ball,ref}} - 15\text{mm}, r_{\text{ball,ref}} + 15\text{mm})$
  - $m_{\text{ball}} \sim \mathcal{U}(m_{\text{ball,ref}} - 41\text{g}, m_{\text{ball,ref}} + 41\text{g})$
- Hit rate: $\mathbb{E}(H)$

Uncertainties

- Monte Carlo experiment:

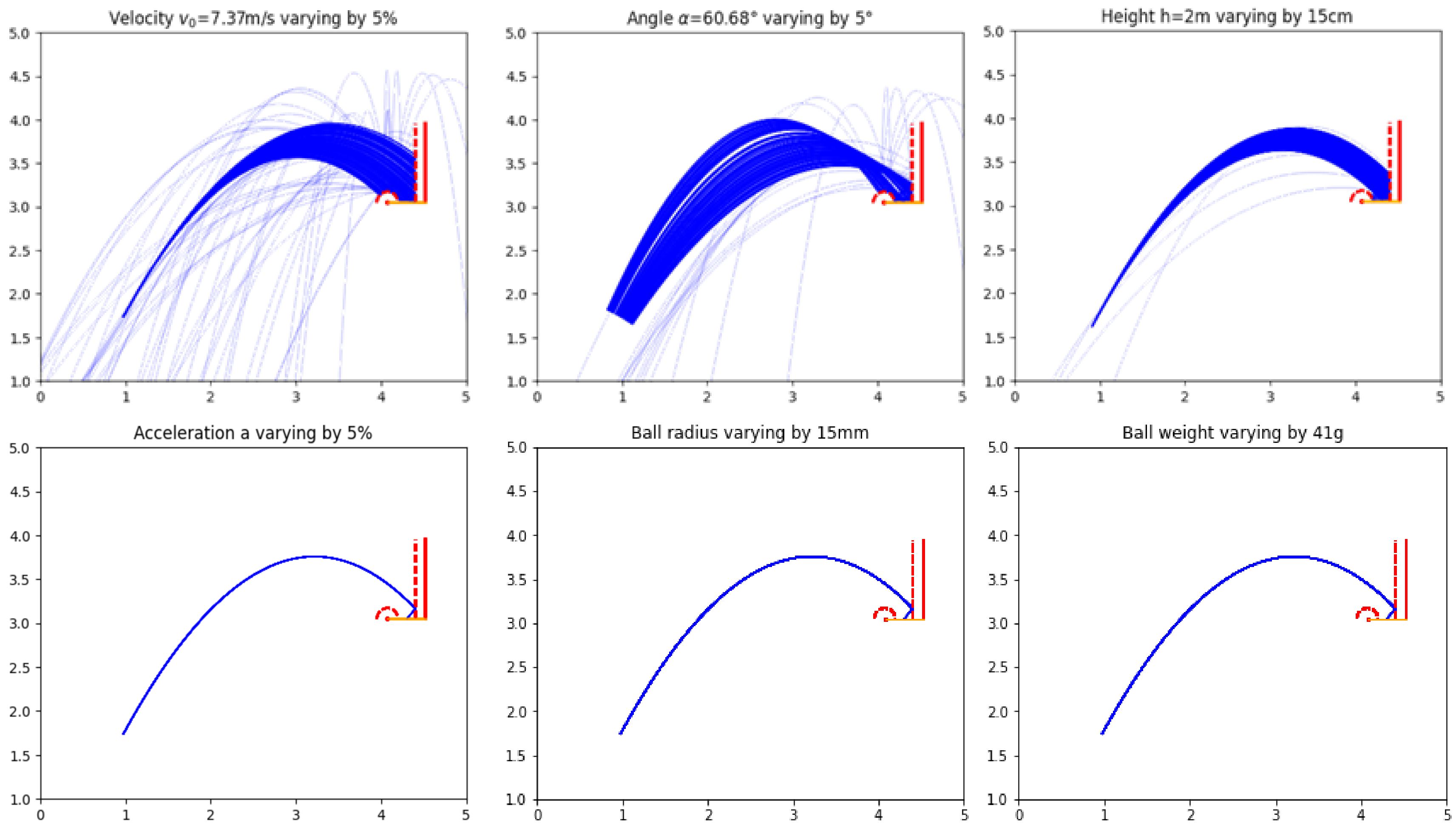
$$\mathbb{E}(H) \approx \langle H \rangle = \frac{1}{N} \sum_{i=1}^{N} H\left(h_i\ , \alpha_i, v_{0,i}, r_{\text{ball},i}, m_{\text{ball},i}\right)$$

- Considered alternatives:
  - Error propagation, Interval arithmetic

    → Too complex due to high non-linearity introduced by bounces

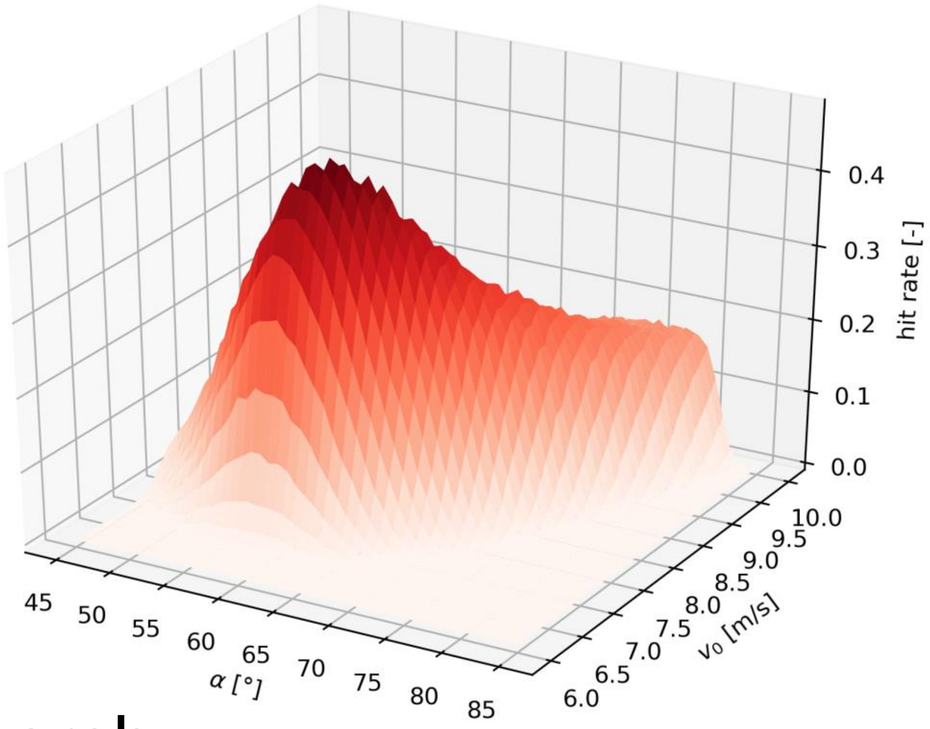
Uncertainties

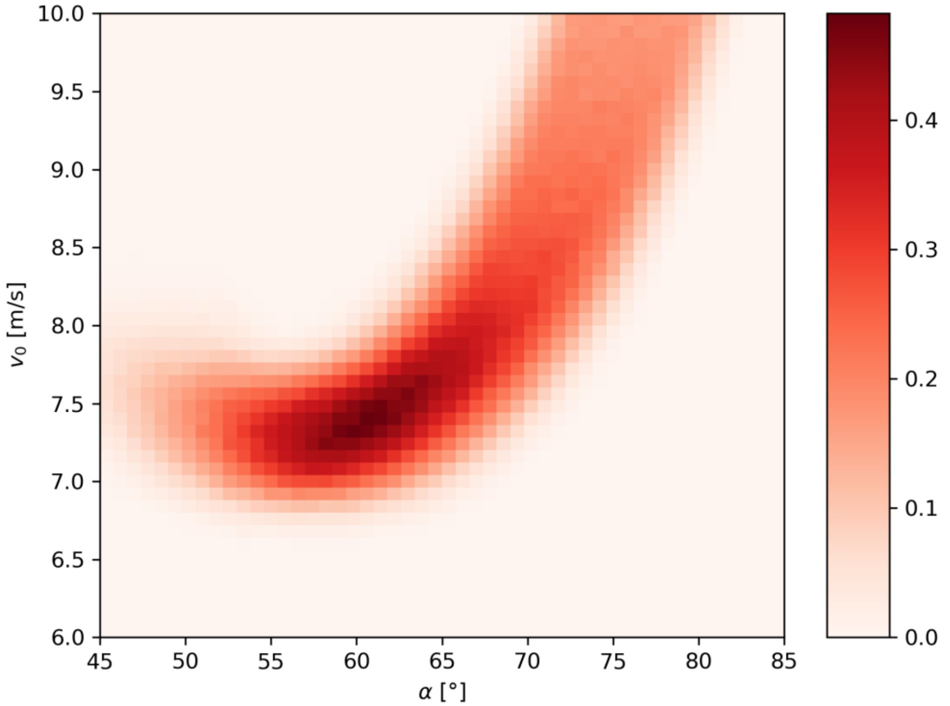
- Fix uncertainties
- Only free one
- Variables have different influence
- Better with air resistance



Optimization

- Optimization landscape (using 10000 samples, $h = 2.0\mathrm{m}$):



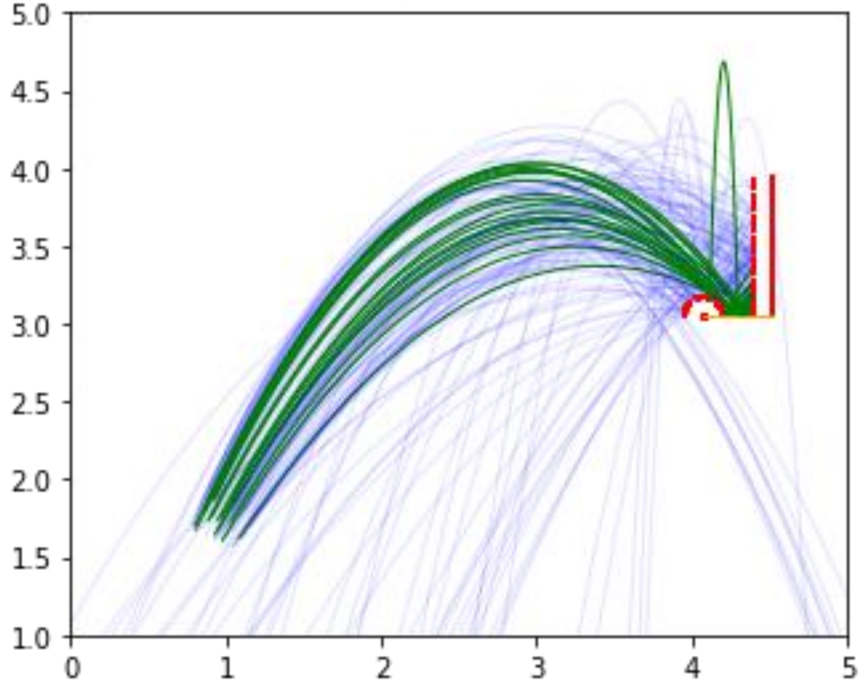


- Grid search
  - Refine twice around maximum
  - Second step did not change result

Optimal Values

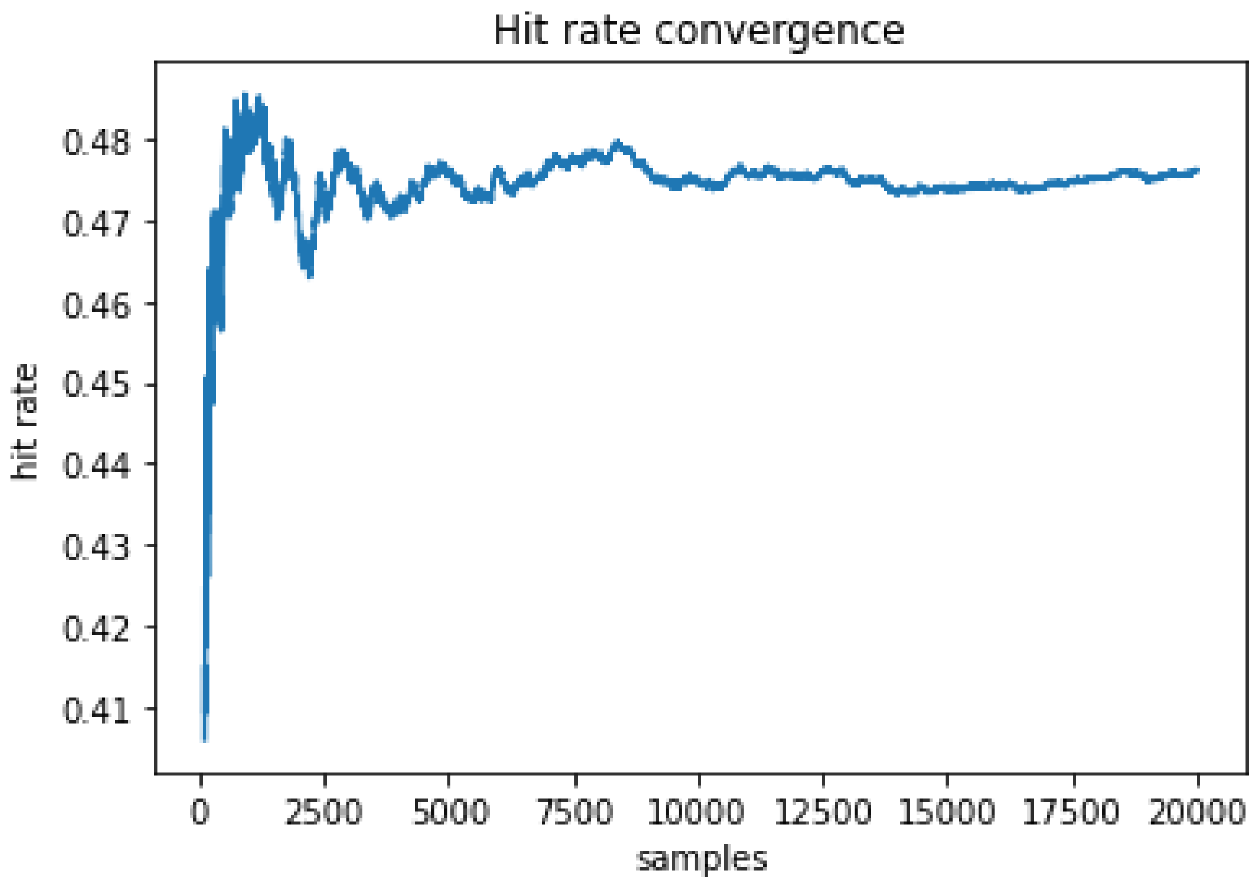
h_{opt}	2.0m
α_{opt}	$60.68°$
$v_{0,\text{opt}}$	7.37ms^{-1}

100 throws with optimal parameters (with uncertainties)



Optimal Values

- Final hit rate: 47.4%



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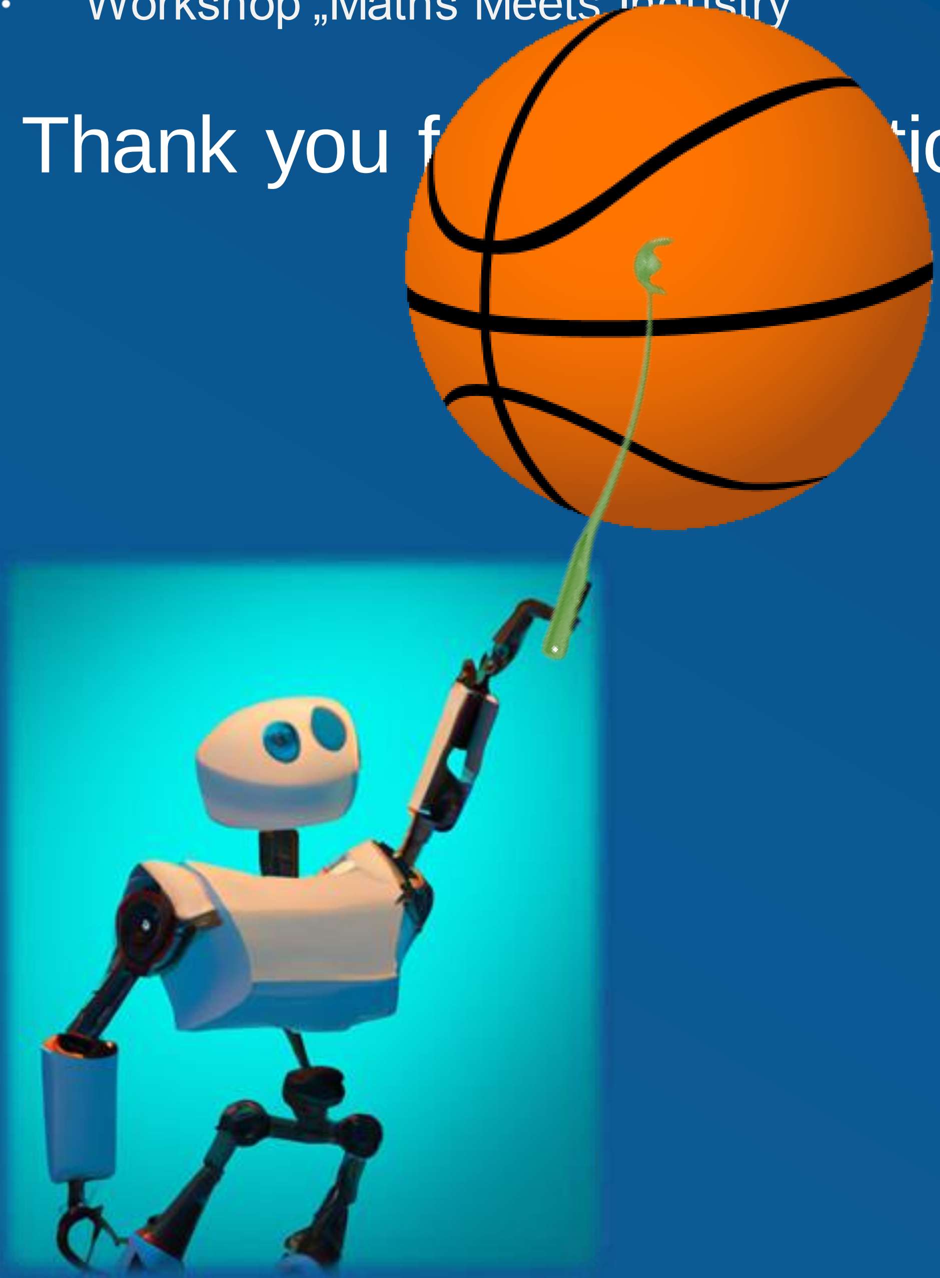
Thank you f… tion!



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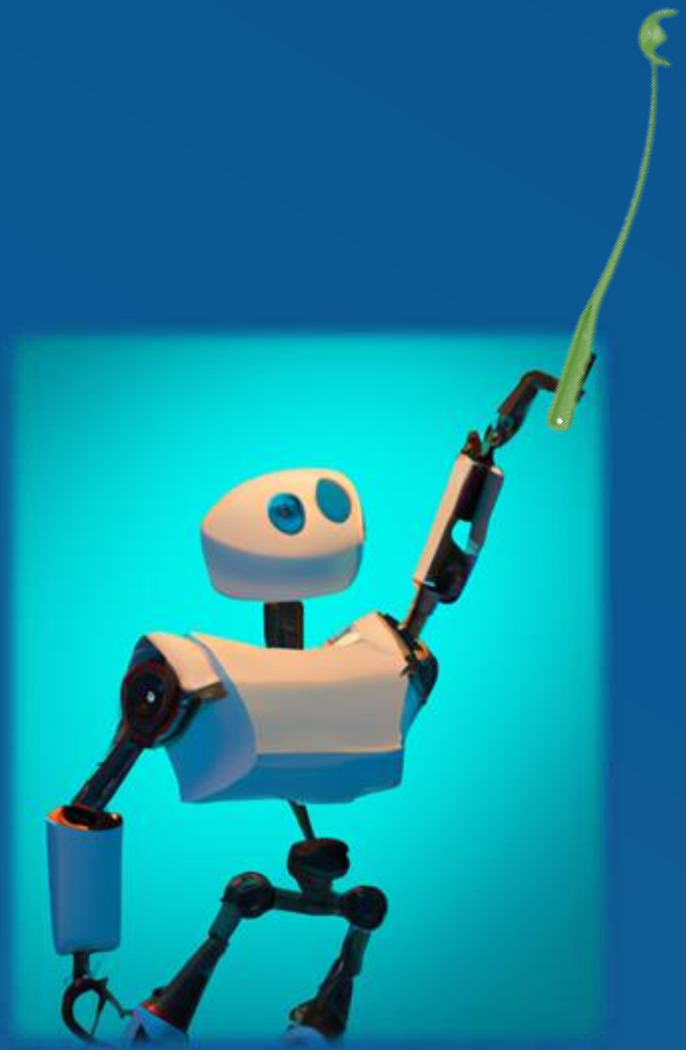
Thank you for your attention!



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