BF FOR MODEL SELECTION

$$M_0 : \underline{\theta} = \{\alpha, \sigma^2\} \qquad\qquad H_0 : \beta = 0$$
$$\Longrightarrow$$
$$M_1 : \underline{\theta} = \{\alpha, \beta, \sigma^2\} \qquad\qquad H_1 : \beta \neq 0$$

$$B_{01} = \frac{\pi(M_0 \mid x)}{\pi(M_1 \mid x)} \cdot \frac{p(M_1)}{p(M_0)}$$

NOTE $p(M)$ IS A JOINT PRIOR $p(M, \theta)$ WHICH CAN BE FACTORISED IN $p(M_j) \cdot p(\theta \mid M_j)$

THEORETICAL REMARKS

$$\pi(\underline{\theta}, M_j \mid x) \propto f(x \mid \underline{\theta}, M_j)\, p(\underline{\theta} \mid M_j)\, p(M_j)$$

$\pi(M_j \mid x)$ IS "SIMPLY" THE MARGINAL POSTERIOR DISTRIBUTION

$$\pi(M_j \mid x) \propto \int f(x \mid \underline{\theta}, M_j)\, p(\underline{\theta} \mid M_j)\, p(M_j)\, d\underline{\theta}$$
$$= p(M_j) \int f(x \mid \underline{\theta}, M_j)\, p(\underline{\theta} \mid M_j)\, d\underline{\theta} = p(M_j)\, f(x \mid M_j)$$

PROBLEMS

$\boxed{A}$ $\int f(x|\underline{\theta}, M_j) P(\underline{\theta}|M_j)\, d\underline{\theta} = f(x|M_j)$

OFTEN ANALYTICALLY INTRACTABLE

$\boxed{B}$ WITH MANY COMPETING MODELS BFs AND POSTERIOR MODEL PROBABILITIES ARE COMPUTATIONALLY CHALLENGING

1 SIMPLE MC APPROACH

TARGET $\hat{\pi}(M_j|x) \propto \hat{f}(x|M_j)\, P(M_j)$

$$f(x|M_j) = E_{\theta}\left[f(x|\underline{\theta}, M_j)\right]$$

$\Rightarrow$ $\hat{f}(x|M_j)$ BY DRAWING K SAMPLES FROM THE PRIOR OF THE PARAMETERS IN MODEL j AND THEN USE THEM IN THE MC ESTIMATE

$$\hat{f}(x|M_j) = \frac{1}{K}\sum^{K} f(x|\underline{\theta}^k, M_j)$$

ONCE WE HAVE $\hat{\pi}(M_j|x)$, THEN $\hat{\pi}(M_j|x) \to \pi(M_j|x)$ AS $K \to \infty$

- REPEAT THIS PROCESS FOR ALL 2^J MODELS AND RENORMALISE THE ESTIMATES

$$\hat{\pi}(M_j \mid x) = \frac{\hat{f}(x \mid M_j)\, p(M_j)}{\sum_{j=1}^{J} \hat{f}(x \mid M_j)\, p(M_j)}$$

HOWEVER...

THE ESTIMATE $\hat{f}(x \mid M_j)$ ARE GENERALLY VERY UNSTABLE AS A RESULT OF THE PARAMETERS BEING DRAWN FROM THE PRIOR!

ALTERNATIVE APPROACHES

- IMPORTANCE SAMPLING (NEXT WEEK)
- REVERSIBLE JUMP MCMC
- BAYESIAN VARIABLE SELECTION
- MODEL COMPARISON CRITERIA (e.g. DIC & WAIC) ← NEXT LECTURE!