

# LECTURE 10 - MONTE CARLO INTEGRATION

INTRACTABLE  
POSTERIOR

- MULT. PARAMETERS  
 $\theta_1, \dots, \theta_n$

- SINGLE PARAMETER  
NON-CONJUGATE PRIOR  $\rightarrow$

$$\bullet f(x) = \int_{\theta_1} \dots \int_{\theta_n} f(x|\underline{\theta}) p(\underline{\theta}) d\theta_1 \dots d\theta_n$$

$$\bullet \pi(\theta_1 | x_1, \theta_2, \dots, \theta_n) = \int_{\theta_2} \dots \int_{\theta_n} \pi(\underline{\theta} | x) d\theta_2 \dots d\theta_n$$

TYPO!  $\rightarrow$

$$x_1, \dots, x_n \stackrel{i.i.d.}{\sim} N(\mu, \sigma^2)$$

$$\mu \sim \log N(\mu, \tau^2)$$

$$\bullet \pi(\mu | x) \propto \exp\left\{-\frac{\sum (x_i - \mu)^2}{2\sigma^2}\right\} \frac{1}{M} \exp\left\{\frac{(\log M - \mu)^2}{2\tau^2}\right\} ?$$

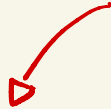
$$\bullet \pi(\mu | x) = \frac{A}{\int_{-\infty}^{\infty} A d\mu}$$

DIFF. CULT,  
POSSIBLY  
INTRACTABLE!

## NOTE

If  $\theta^i$  ARE INDEPENDENT SAMPLES FROM  $\pi(\theta|x)$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n \theta^i \xrightarrow{p} E_{\pi}(\theta) \text{ as } \underline{n \rightarrow \infty} \quad \left[ \begin{array}{l} \text{LAW OF} \\ \text{LARGE NUMBERS} \end{array} \right]$$



MONTE CARLO ERROR (see SLIDES)