LECTURE 7 - INTERVAL ESTIMATES

- RECAP CONFIDENCE INTERVALS (FREQUENTIST)

$$P[a(T) < \theta < b(T)] = 1-\alpha$$

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- θ = PARAMETER (FIXED, UNKNOWN)
- T = ESTIMATOR (R.V.)
- $1-\alpha$ = FIXED (BY THE INVESTIGATOR)

IS A RANDOM INTERVAL

DEFINE $1-\alpha$:

~~A~~ $1-\alpha$ IS THE PROBABILITY THAT θ IS BETWEEN $a(t)$ AND $b(t)$

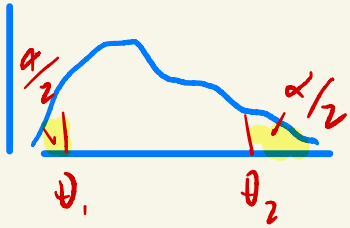
~~B~~ $1-\alpha$ IS THE PROBABILITY THAT THE INTERVAL $[a(t), b(t)]$ INCLUDES θ

=> $1-\alpha$ IS THE PROBABILITY THAT THE OBSERVED INTERVAL BELONGS TO THE GROUP OF INTERVALS WHICH INCLUDE θ

- THERE ARE 2 COMMON & UNIQUE CREDIBLE INTERVALS

[1] SYMMETRIC CI

$$P_\pi(\theta \le \theta_1) = \int_{-\infty}^{\theta_1} \pi(\theta|x)\,d\theta = \frac{\alpha}{2} = \int_{\theta_2}^{\infty} \pi(\theta|x)\,d\theta = P_\pi(\theta \ge \theta_2)$$



"gnorm" function in R

→ NOTE: SYMMETRIC CI IS NOT NECESSARILY THE NARROWEST

[2] HIGHEST POSTERIOR DENSITY INTERVAL (HPDI)

- HPDI DENOTED BY A REGION R IS SUCH THAT

a) R IS A $100(1-\alpha)\%$ CI

b) $\forall\, \theta_i \in R$ AND $\theta_j \notin R$ THEN $\pi(\theta_i|x) \ge \pi(\theta_j|x)$

... Symmetric

... HPDi

$\pi(\theta_i|x) > \pi(\theta_j|x)$ for some θ_i & θ_j

$\Rightarrow$ Symmetric is NOT HPDi !