LECTURE 10 – MONTE CARLO INTEGRATION

INTRACTABLE POSTERIORS

- MULTI PARAMETERS $\theta_1 \dots \theta_n$
  - $f(x) = \int_{\theta_1} \cdots \int_{\theta_n} f(\underline{x}|\underline{\theta})\,p(\underline{\theta})\,d\theta_1 \dots d\theta_n$
  - $\pi(\theta_1|\underline{x}, \cancel{\theta_{(-1)}}) = \int_{\theta_2} \cdots \int_{\theta_n} \pi(\underline{\theta}|\underline{x})\,d\theta_2 \dots d\theta_n$

  TYPO! → θ_1

- SINGLE PARAMETER NON-CONJUGATE PRIOR →

$$x_1, \dots, x_n \overset{iid}{\sim} N(\mu, \sigma^2)$$

$$\mu \sim \log N(\phi, \tau^2)$$

- $\pi(\mu|\underline{x}) \propto \underbrace{\exp\left\{-\frac{\sum(x_i-\mu)^2}{2\sigma^2}\right\}\frac{1}{\mu}\exp\left\{\frac{(\log\mu-\phi)^2}{2\tau^2}\right\}}_{A}$?
- $\pi(\mu|x) = \dfrac{A}{\int_{-\infty}^{+\infty} A\,d\mu}$ DIFFICULT, POSSIBLY INTRACTABLE!

NOTE

If θ^i ARE INDEPENDENT SAMPLES FROM $\pi(\theta|x)$

$$\Rightarrow \frac{1}{n}\sum^{n} \theta^i \xrightarrow{p} E_\pi(\theta) \text{ as } n \to \infty$$

[LAW OF LARGE NUMBERS]

MONTE CARLO ERROR (see SLIDES)