MAS5050 Mathematical Methods for Statistics

Assignment 1

This assignment has five questions, of equal weight.

1. (a) Find the sum of the even numbers between 100 and 200, inclusive.
   (b) Express the recurring decimal $0.407407407\ldots$ as a fraction in its lowest terms.
   (c) Find the value of k such that
   $$\sum_{j=1}^{\infty} kp^j = 1,$$
   in terms of p.

2. Let
   $$a_n = \frac{n+4}{n(n+1)(n+2)}.$$
   Use partial fractions and telescoping series techniques to find $\sum_{j=1}^{n} a_j$ and hence $\sum_{j=1}^{\infty} a_j$.

3. Let $f(x) = x^3$. Calculate $\frac{f(x)-f(2)}{x-2}$ for a range of values of x approaching 2 from above and repeat for a range of values of x approaching 2 from below. Do the values appear to be converging to a limit as $x \to 2$?

   Compare your answers with the derivative of $f(x)$ evaluated at $x = 2$.

4. Differentiate the following functions of x:
   (a) $x^2 \sin x$
   (b) $\frac{x^2+1}{x+4}$
   (c) $\frac{\cos^2 x+1}{\cos x+4}$

5. Calculate the first four derivatives of $\cos x$, and hence calculate the Taylor series of $\cos x$ about $x = \pi$ as far as the x^4 term.