Lab 1: The Apollo Missions
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## Introduction:

In this report I will be discussing several physics related concepts that are important to the Saturn V rocket's mission into space. First I will inform you about the gravitational potential that the Earth creates in the space around it, and I will show you two plots to help you visualize it better. Next I will discuss the same concepts but now taking into account the effect the moon has on the system. Then I will try and show how the moon and earth create a gravitational field that will affect the rocket as it moves through space. And finally I will discuss how the altitude of the rocket is affected by the amount of fuel carried initially by the rocket and the rate at which it burns when being ejected from the rear. All graphics were created using python, more specifically imported function libraries numpy, matplotlib, and the math library. All are tools for calculating and visualizing data.

## Part II: The Gravitational Potential of the Earth and the Earth Moon System

The gravitational potential is an important concept to understand when visualizing and calculating the flight of the rocket. Before I discuss the potential I would like to quickly familiarize you with work, which is the change in energy in an object by way of a force acting on that object. In simple terms the gravitational potential is the work, based only on the mass of the earth, it would take to move an object from infinitely far away, where the potential is zero, to a point in space some distance r meters away from the center of the earth. The gravitational potential is denoted by the equation:

$$\Phi(r) \ = - \frac{GM}{r}$$

G (6.67 x $10^{-11} m^3/kg/s^2$) is the universal gravitational constant, it is basically the inherent pulling force an object creates based on its mass

M (5.9 x $10^{24} kg$) is the mass of the Earth

R (m) is the distance away from the Earth's center being measured.

So being farther away from the earth would mean you have a lower gravitational potential. As seen below in figure 1 and 2.

Figure 1:

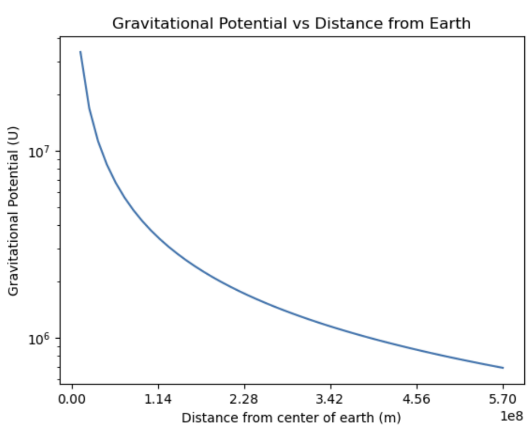
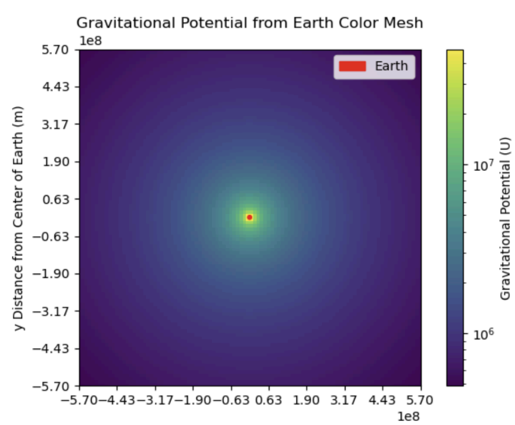



Figure 2:




Now you may have figured that if the earth having some mass will create a gravitational potential then the moon will also do the same. These two gravitational potentials can be combined into one single field. As seen in figures 3 and 4 below.

Figure 3:

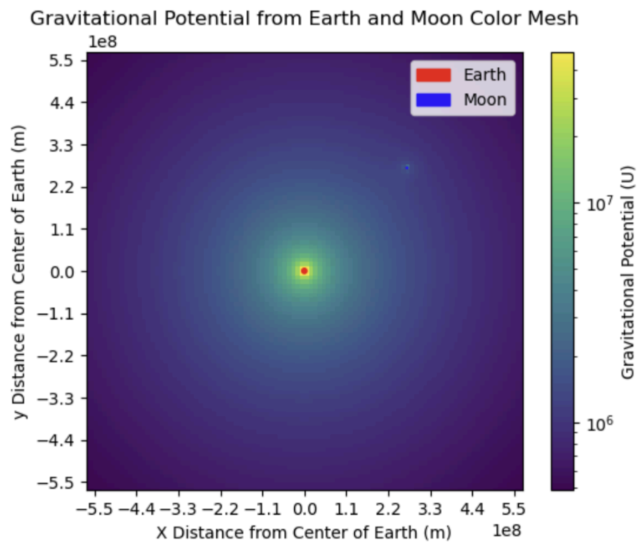
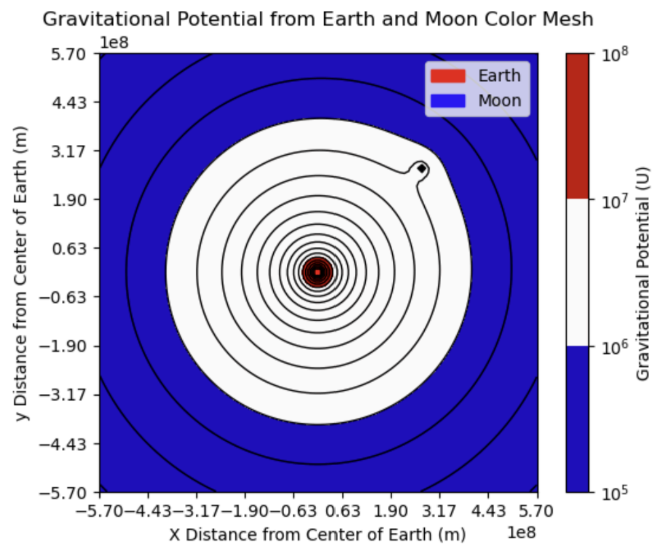



Figure 4:




## Part III: The Gravitational Force on the Earth-Moon System

As you probably know any object with mass has a gravitational force that it exerts on all other objects. For this pull to be noticeable the object must have a very high mass, like the earth or moon. Newton's third law tells us that for every action there is an equal and opposite reaction, meaning that if an object with some mass exerts a force on another mass gravitationally, the second mass will exert the same force back.

The equation for the force ($F_{12}$) (that points in the direction r ) from an object M (kg) on another mass m (kg) is denoted by:

$$F_{12} = -G\frac{Mm}{r^2}$$

Where G (6.67 x $10^{-11} m^3/kg/s^2$) is again the universal gravitational constant

M (kg) is the mass of object 1

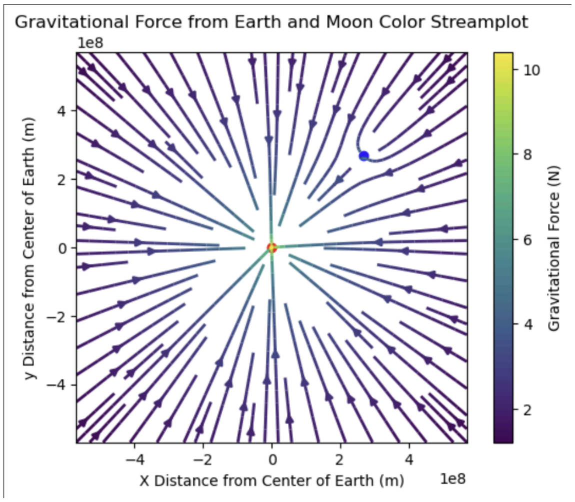
m (kg) is the mass of object 2

r (meters) is the distance between the two

This is important for the flight of a rocket because these forces will impact how it moves through space. To visualize this better I have created a streamplot, which shows the magnitude (denoted in color) of the force and its direction (denoted by the arrow).

This can be seen in figure 5.

Figure 5:




## Part IV: Projected Performance of the Saturn V Stage 1

The projected performance of the Saturn Rocket can be measured by taking into account its momentum. Momentum is calculated by taking an object's mass and multiplying it by its velocity. The rocket operates under the conservation of momentum, meaning that if it has some mass and is moving at some velocity it must be propelling some other mass with a velocity but in the negative direction. This applies by way of the rocket's fuel, which is propelled backwards to move the ship forwards.

The rocket's velocity at a time t (second) can be determined using Tsiolkovsky's equation

$$V(t) = v_e ln(\frac{m_0}{m(t)}) - gt$$

$v_e$ is the rockets fuel exit velocity (2.4 x $10^3$ m/s)

$m_0$ is the initial "wet" mass (fuel + rocket parts + payload) (2.8 x $10^6$ kg)

m(t) is the mass of the rocket at time t, meaning the initial mass mf (7.5 x $10^5$) minus the burn rate m (1.3 x $10^4$ kg/s)

g is the gravitational acceleration ($9.8\ m/s^2$) And t is the time since launch in seconds

By taking the integral of this equation we can find the total distance the rocket will cover in this initial stage.

## Conclusion:

In summation of our calculations they are accurate but not as accurate as they could be. We treated the earth as a point mass with uniform density which is not true, and neglecting any other factors that impacted the velocity, like air drag and friction from the air in the atmosphere. Using our simpler equation and the quantities we estimate the rocket should travel 74,093 meters and by using the equation for the total fuel it should burn for around 157.7 seconds. The first stage was tested to travel 74,000 meters over 160 seconds of fuel burn time. Our estimate for the distance just using the rocket equation could be an overestimate due to neglecting factors like air drag, which pushes against the rocket slowing it down.