Lab 2 Report: Mine Crafting

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Introduction: We will be dropping a projectile in order to determine the depth of the mine located at the earth's equator. A number of factors have to be taken into consideration, including the time it takes for the projectile to hit the bottom, the force of gravity on the projectile, the drag force on the projectile, the coriolis force, and the density of Earth. All these factors will be explained individually throughout the report. I will also be comparing our calculations of the projectile falling on Earth to if the mine was located on the moon.

The mass of the projectile is not important as we know it will fall at the same rate regardless of mass. For simplicity all calculations have been done assuming that the mass weighs 1 kilogram. I have defined a few important constants below that I will be referring to throughout the report.

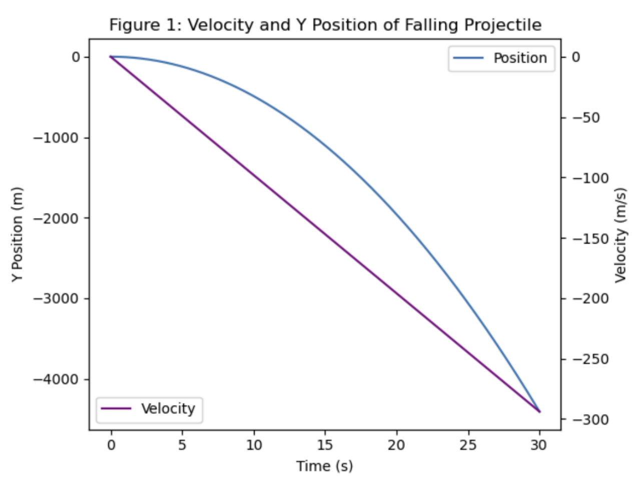
Mass of Earth: $5.972\times10^{24} kg$ Radius of Earth: $6378.1\ km$

Mass of Moon: $7.35\times10^{22} kg$ Radius of Moon: $1738.1\ km$

Universal Gravitational Constant (G): $6.6743\times10^{-11} m^3/kg/s^3$

Earth's Rotation Rate at the Equator (Ω): 7.272×10^{2}

Calculation of Fall Time: Before we get into the calculations made I'd like to quickly discuss the theoretical fall time using simple kinematic equations. Knowing that the 1 kg projectile has no initial velocity we can calculate the time (t) it takes for it to fall a distance (d) of 4000 meters using the equation under uniform gravity (g): $t = \sqrt{\frac{2d}{g}}$ Using this equation we calculated a fall time of 28.6 seconds. Now to access the fall time differently, we will use a simple couple of first order differential equations. The first equation is the rate of change of the vertical position (y) is equal to the velocity (v). And the second is the rate of change of the velocity is equal to the acceleration of gravity plus the drag force. The drag force is equal to the drag coefficient alpha (α) multiplied by the absolute value of the velocity to the power of the speed dependence gamma (γ). This pair of equations can be seen belo $\frac{dy}{dt} = v$ $\frac{dv}{dt} = -g + \alpha|v|^{\gamma}$ For our first calculation we will disregard the drag force and not yet consider a changing gravity. Using these two equations we can utilize python's *solve_ivp* function. In simple terms it will solve both equations for some period of time. We can also use the built in event detection of *solve_ivp* to calculate when the projectile hits the bottom, or when the vertical position (y) is equal to 0. Using this function we calculated a fall time of 28.6 seconds, confirming our theoretical fall time. A graph of the velocity and position can be seen to the left.



Now we will factor in an Earth of uniform density which will cause the gravitational force to slightly decrease as the projectile gets deeper into earth's surface. To do this we will have a variable gravitational force g(y) that depends on the height y, the initial gravitational force g_o, and the radius of the earth R_e. This will then be factored into our second differential equation replacing g, and can be described as: $g(y)= g_o(\frac{y}{R_e})$

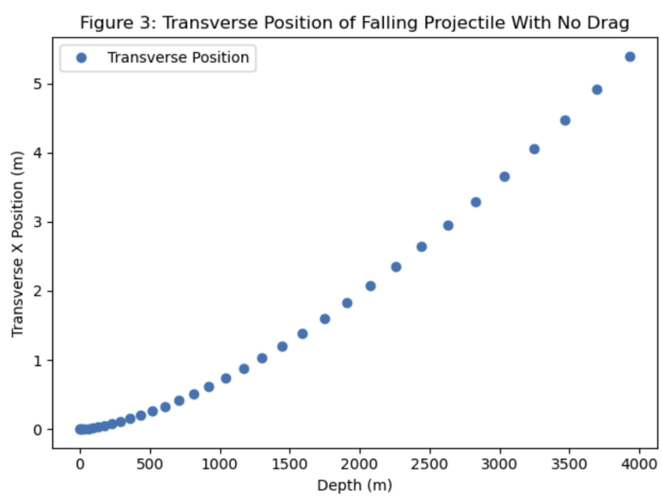
Now we can reuse our *solve_ivp* function and calculate the fall time. This yields a result of 28.6 seconds. We will also need to consider the drag force that will push against the projectile as it falls. In order to properly calculate the speed and position of the projectile we need to use the second part of our second differential equation that we previously discarded. We will use a gamma γ of 2 and need to calculate the drag coefficient α based on the terminal velocity. Eventually the drag force, which is proportional to the speed will reach a point where it balances the force of gravity causing the projectile to stay at its terminal velocity. We assumed the terminal velocity to be $50\frac{m}{s}$, to achieve this we can again use *solve_ivp* and eventually arrive at a drag coefficient of .00391. Will will then use the same function and test for the crash time. Doing this results in a fall time of 83.5 seconds.

As can be seen in our three different fall times, the addition of linear gravity causes the fall time to increase slightly, while factoring in linear gravity and drag causes the fall time to almost triple.

Feasibility of depth measurement approach:

Now we need to consider the side to side, or transverse, position of the projectile as it falls. Now there is no force acting horizontally on the projectile as it falls, but since it is falling for a considerable time the earth will rotate enough so that it appears that the projectile has moved in the x direction with a x velocity v_x. This is the Coriolis force and as well as moving the projectile side to side it will also cause the projectile to fall faster in the downward direction, v_y. The Coriolis force then has two components, in the x direction, F_{Cx} and y direction F_{Cy} , each can be described as,

$F_{Cx} = 2\Omega v_y$ $\qquad$ $F_{Cy} = 2\Omega v_x$



Again where omega is the earth's rotation rate at the equator. Now we will need to augment our pair of differential equations to include these forces. We will need to add a pair of differential equations where the rate of change in the x direction is equal to the velocity v_x, and the rate of change of v_x is equal to the force in the x direction F_{Cx}. We will also need to add the equation for F_{Cy} to our equation for the rate of change for the velocity in the y direction, v_y. We will again use *solve_ivp* and can trace the x position as a function of depth.

Like above we will do this both with and without drag present. The transverse x position can be seen as a function of depth in the graph to the right. According to solve_ivp over a depth of 4000 meters the projectile will travel 5.5 meters. This is greater than the assumed width of 5 meters of the shaft meaning that the projectile will collide with the wall before the bottom. It does so at 27. 6 seconds. This is also true for the case with drag. Again we run the *solve_ivp* with drag, and calculate a final transverse position of the projectile to be 23.3 meters. It collides with the 5 meter wall at 40. 5 seconds. Drag makes a significant difference and makes the projectile hit the wall earlier in its total fall time. In both cases the projectile moves too far in the x direction to hit the bottom, meaning I would not recommend using this technique for depth measurements.

Calculation of crossing times for homogeneous and non-homogeneous earth

As we know the density of the Earth is not uniform and should not be treated as such. This non-uniform density will cause a varying gravitational force depending on how deep the projectile is. In simple terms the density of earth based on the position from the center of the earth (r) can be expressed as a constant p_n times 1 minus the position over the radius of earth R_e to the n power. p_n varies based on n. $p(r) = p_n(1 - \frac{r}{R_e})^n$ We will test 4 different n values, 0, 1, 2, and 9. In order to show this we will be calculating the times it takes for the projectile to fall through a hypothetical shaft that goes across the diameter of the Earth at the equator. We can use our equation for p(r) in an integral to find the mass underneath a radius of r to calculate the gravitational force that pulls the projectile as it falls. We can then again use the equations for gravity in our differential equations and calculate the time it takes for the projectile to traverse the diameter. We will be disregarding the drag force and coriolis force.

For an n of 0 it takes 2534.5 seconds, for an n of 1 it takes 2193.8, for an n of 2 it takes 2070.3 seconds, and for an n of 9 it takes 1887.7 seconds. As can be seen the value of n has an effect on the fall time, as n increases the fall time decreases. Contrasting when n is equal to 0 versus when it is equal to 9 we can see that increasing the value of n by 9 decreases fall time by roughly 25%.

Now considering the same case but on the moon we can find a relationship between fall time and density. Solving for fall time using our earlier kinematic equation and factoring in the density p we can determine that fall time is proportional to the square root of 1 over the density. I would also like to compare the orbital period of the projectile circling the earth, with its centripetal acceleration perfectly balanced with the force of gravity, to the crossing time. In this case the orbital period can be calculated to be 5069.4 seconds and the crossing time, without drag or a Coriolis force, to be 2515.5 seconds. The crossing time is roughly half the orbital period.

Conclusion:
In summary of our findings we found that without drag and a Coriolis force the projectile will fall to the bottom of the mine in 28.6 seconds. With a drag force it will take 83.5 seconds. When considering the Coriolis force we found that with and without drag the projectile will move too far in the transverse direction and will hit the sides of the shaft before it reaches the bottom. We also found that when considering density In order to make our calculations more accurate we would need to assume a non circular earth, which would affect the gravitational force and the density.