

or as this (find out which),

$$\delta_\nu = \quad (4.26)$$

$$\left(\frac{3}{10}a - \frac{1}{5}K\right)\delta_{cdm,0}a + \cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\left(c_1 - \frac{4}{5}\delta_{cdm,0}K^2\text{Ci}\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\right) \quad (4.27)$$

$$-2\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\left(c_2 + \frac{4}{5}\delta_{cdm,0}K^2\text{Si}\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\right) \quad (4.28)$$

$$\delta_\nu \simeq c_1 \cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) - \left(\frac{2}{5}\delta_{cdm,0}K^2\pi + c_2\right) \sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \quad (4.29)$$

$$\delta_\nu \simeq c_1 \cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) - \left(\frac{2}{5}\delta_{0,2}K^2\pi + c_2\right) \sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \quad (4.30)$$

Ansatz: For early time

$$\delta_\nu \simeq A \cos\left(\frac{2\sqrt{K}}{\sqrt{a}} + \Phi\right) \quad (4.31)$$

$$= A \cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \cos(\Phi) - A \sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \sin(\Phi) \quad (4.32)$$

$$\Phi = 0 \quad (4.33)$$

$$c_1 = A \cos(\Phi) \quad c_2 = A \sin(\Phi) - \frac{2}{5}\delta_{cdm,0}K^2\pi \quad (4.34)$$

$$c_1 = A \quad c_2 = -\frac{2}{5}\delta_{cdm,0}K^2\pi \quad (4.35)$$

$$\dot{\delta}_\nu = \frac{2\sqrt{K}}{2\sqrt{a^3}} \sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) = \sqrt{\frac{K}{a^3}} \sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \quad (4.36)$$

$$A = \sqrt{\delta_\nu^2 + \left(\frac{\dot{\delta}_\nu}{\frac{\sqrt{K}}{\sqrt{a^3}}}\right)^2} = \sqrt{\delta_\nu^2 + \left(\frac{\dot{\delta}_\nu \sqrt{a^3}}{\sqrt{K}}\right)^2} \Big|_{a=a_0} \quad (4.37)$$

4.2 second order

Dette er nok forkert. (det er forresten i conformal time, tror jeg).

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} + 2\mathcal{H}\theta^{(2)} + 2k^2 c_s(a)^2 \left(\delta^{(1)} \delta^{(1)} + \frac{1}{2} \delta^{(2)} + (\partial_j \delta^{(1)}) \partial_j \nabla^{-2} \delta^{(1)} \right) \quad (4.38)$$

$$= 2\theta^{(1)}\theta^{(1)} + 2(\partial_i \partial_j \nabla^{-2} \theta^{(1)})^2 + 4(\partial_j \nabla^{-2} \theta^{(1)}) (\partial_j \theta^{(1)}) \quad (4.39)$$

$$+ 3H_0^2 \frac{\Omega_m}{a} \left(\delta^{(1)} \delta_{cdm}^{(1)} + \frac{1}{2} \delta_{cdm}^{(2)} + (\partial_j \delta^{(1)}) (\partial_j \nabla^{-2} \delta_{cdm}^{(1)}) \right) \quad (4.40)$$

nyt forsøg

først har vi vores startligninger

$$\dot{\delta}^{(1)} = -\theta^{(1)} \quad \dot{\theta}^{(1)} = -\mathcal{H}\theta^{(1)} - \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(1)} - c_s^2 \nabla^2 \delta^{(1)} \quad (4.41)$$

$$\dot{\delta}_\nu^{(2)} = -2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \delta^{(1)} - 2\delta^{(1)} \theta^{(1)} - \theta^{(2)} \quad (4.42)$$

$$\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(2)} - 2\left(\partial_i \partial_j \nabla^{-2} \theta^{(1)} \partial_i \partial_j \theta^{(1)}\right) - 2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \theta^{(1)} \quad (4.43)$$

$$- \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(2)} - c_s^2 \nabla^2 \delta^{(2)} \quad (4.44)$$

$$\ddot{\delta}^{(2)} = -2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \delta^{(1)} - 2\partial_j \nabla^{-2} \dot{\theta}^{(1)} \partial_j \dot{\delta}^{(1)} - 2\dot{\delta}^{(1)} \theta^{(1)} - 2\delta^{(1)} \dot{\theta}^{(1)} - \dot{\theta}^{(2)} \quad (4.45)$$

$$\ddot{\delta}_\nu^{(2)} = -2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \delta^{(1)} - 2\partial_j \nabla^{-2} \dot{\theta}^{(1)} \partial_j \dot{\delta}^{(1)} - 2\dot{\delta}^{(1)} \theta^{(1)} - 2\delta^{(1)} \dot{\theta}^{(1)} \quad (4.46)$$

$$+ \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(2)} + c_s^2 \nabla^2 \delta^{(2)} + 2\left(\partial_i \partial_j \nabla^{-2} \theta^{(1)} \partial_i \partial_j \theta^{(1)}\right) + 2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \theta^{(1)} + \mathcal{H}\theta^{(2)} \quad (4.47)$$

$$\ddot{\delta}^{(2)} = -2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \delta^{(1)} - 2\partial_j \nabla^{-2} \dot{\theta}^{(1)} \partial_j \dot{\delta}^{(1)} - 2\dot{\delta}^{(1)} \theta^{(1)} - 2\delta^{(1)} \dot{\theta}^{(1)} \quad (4.48)$$

$$+ \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(2)} + c_s^2 \nabla^2 \delta^{(2)} + 2\left(\partial_i \partial_j \nabla^{-2} \theta^{(1)} \partial_i \partial_j \theta^{(1)}\right) + 2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \theta^{(1)} \quad (4.49)$$

$$+ \mathcal{H}(-2\partial_j \nabla^{-2} \theta^{(1)} \partial_j \delta^{(1)} - 2\delta^{(1)} \theta^{(1)} - \dot{\delta}_\nu^{(2)}) \quad (4.50)$$

$$(4.51)$$

$$\ddot{\delta}^{(2)} = 2\partial_j \nabla^{-2} \delta^{(1)} \partial_j \delta^{(1)} + 2\partial_j \nabla^{-2} (\mathcal{H}\delta^{(1)} - \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(1)} - c_s^2 \nabla^2 \delta^{(1)})^{(1)} \partial_j \dot{\delta}^{(1)} \quad (4.52)$$

$$+ 2\dot{\delta}^{(1)} \dot{\delta}^{(1)} - 2\delta^{(1)} (\mathcal{H}\delta^{(1)} - \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(1)} - c_s^2 \nabla^2 \delta^{(1)}) \quad (4.53)$$

$$+ \frac{3}{2} \frac{H_0^2 \Omega_M}{a} \delta_M^{(2)} + c_s^2 \nabla^2 \delta^{(2)} + 2\left(\partial_i \partial_j \nabla^{-2} \delta^{(1)} \partial_i \partial_j \dot{\delta}^{(1)}\right) + 2\partial_j \nabla^{-2} \dot{\delta}^{(1)} \partial_j \dot{\delta}^{(1)} \quad (4.54)$$

$$+ \mathcal{H}(2\partial_j \nabla^{-2} \dot{\delta}^{(1)} \partial_j \delta^{(1)} + 2\delta^{(1)} \dot{\delta}^{(1)} - \dot{\delta}^{(2)}) \quad (4.55)$$

bedre opskrivning

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} = \frac{3}{2} \frac{H_0^2}{a} \left(2\partial_j \nabla^{-2} \delta_M^{(1)} \partial_j \delta^{(1)} + 2\delta^{(1)} \delta_M^{(1)} + \delta_M^{(2)} \right) \quad (4.56)$$

$$+ c_s^2 \left(2\partial_j \delta^{(1)} \partial_j \delta^{(1)} + 2\delta^{(1)} \nabla^2 \delta^{(1)} + \nabla^2 \delta^{(2)} \right) \quad (4.57)$$

$$4\partial_j \nabla^{-2} \dot{\delta}^{(1)} \partial_j \dot{\delta}^{(1)} + 2\dot{\delta}^{(1)} \dot{\delta}^{(1)} + 2\partial_i \partial_j \nabla^{-2} \dot{\delta}^{(1)} \partial_i \partial_j \nabla^{-2} \dot{\delta}^{(1)} \quad (4.58)$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} = \frac{3}{2} \frac{H_0^2}{a} \left(2\partial_j \nabla^{-2} \delta_M^{(1)} \partial_j \delta^{(1)} + 2\delta^{(1)} \delta_M^{(1)} + \delta_M^{(2)} \right) \quad (4.59)$$

$$+ 4\partial_j \nabla^{-2} \dot{\delta}^{(1)} \partial_j \dot{\delta}^{(1)} + 2\dot{\delta}^{(1)} \dot{\delta}^{(1)} + 2\partial_i \partial_j \nabla^{-2} \dot{\delta}^{(1)} \partial_i \partial_j \nabla^{-2} \dot{\delta}^{(1)} \quad (4.60)$$

$$a = \frac{1}{4} H_0^2 \tau^2 \quad \delta^{(1)} = D\tilde{\delta} \quad \delta_M = a\tilde{\delta}_M \quad \mathcal{H} = H_0/\sqrt{a} = \frac{2}{\tau} \quad (4.61)$$

$$(4.62)$$

Vi indsætter og ser hvad der sker,

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - c_s^2 \nabla^2 \delta^{(2)} - \frac{6}{\tau^2} \delta_M^{(2)} = \frac{6}{\tau^2} \left(2\partial_j \nabla^{-2} a \tilde{\delta}_M^{(1)} \partial_j D \tilde{\delta}^{(1)} + 2D \tilde{\delta}^{(1)} a \tilde{\delta}_M^{(1)} \right) \quad (4.63)$$

$$+ 4\dot{D}^2 \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\dot{D}^2 \tilde{\delta}^{(1)} \tilde{\delta}^{(1)} + 2\dot{D}^2 \partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \quad (4.64)$$

$$+ c_s^2 \left(2\partial_j D \tilde{\delta}^{(1)} \partial_j D \tilde{\delta}^{(1)} + 2D \tilde{\delta}^{(1)} \nabla^2 D \tilde{\delta}^{(1)} \right) \quad (4.65)$$

reducere

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - c_s^2 \nabla^2 \delta^{(2)} - \frac{6}{\tau^2} \delta_M^{(2)} = aD \frac{6}{\tau^2} \left(2\partial_j \nabla^{-2} \tilde{\delta}_M^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \tilde{\delta}_M^{(1)} \right) \quad (4.66)$$

$$+ 4\dot{D}^2 \left(\partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \tilde{\delta}^{(1)} + 2\partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \right) \quad (4.67)$$

$$+ c_s^2 D^2 \left(2\partial_j \tilde{\delta}^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \nabla^2 \tilde{\delta}^{(1)} \right) \quad (4.68)$$

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - c_s^2 \nabla^2 \delta^{(2)} - \frac{6}{\tau^2} \delta_M^{(2)} = D \frac{3}{2} H_0^2 \left(2\partial_j \nabla^{-2} \tilde{\delta}_M^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \tilde{\delta}_M^{(1)} \right) \quad (4.69)$$

$$+ 4\dot{D}^2 \left(\partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \tilde{\delta}^{(1)} + 2\partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \right) \quad (4.70)$$

$$+ c_s^2 D^2 \left(2\partial_j \tilde{\delta}^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \nabla^2 \tilde{\delta}^{(1)} \right) \quad (4.71)$$

Citat fra Steen “anden ordens termer for neutrino bliver først relevant når neutrinoerne er stoppet med at oscillere og derfor kan vi tillade os at se bort fra lydhastigheds termer.”

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - \frac{6}{\tau^2} \delta_M^{(2)} = D \frac{3}{2} H_0^2 \left(2\partial_j \nabla^{-2} \tilde{\delta}_M^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \tilde{\delta}_M^{(1)} \right) \quad (4.72)$$

$$+ 4\dot{D}^2 \left(\partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_j \tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)} \tilde{\delta}^{(1)} + 2\partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \partial_i \partial_j \nabla^{-2} \tilde{\delta}^{(1)} \right) \quad (4.73)$$