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[1]

Chapter 1

Introduction

It was first believed that the universe was made up entirely of matter. We were wrong. Then we though that the universe was made up of matter and radiation. We were wrong. Then we thought that the universe was made up of matter, radiation and dark matter. We were wrong. Now we believe that the universe is made up of radiation, matter, dark matter and dark energy...

Chapter 2

Theory

In this chapter we will introduce the relevant background knowledge. This will include a summary of GR and the equation that can be derived from it. A GR description of Dark matter and neutrinos, the problem with neutrinos in GR. Neutrinos as dark matter and the neutrino sound speed.
Additionally we will of cause be using the that unity values, e.g.

$$\hbar = c = 1. \tag{2.1}$$

2.1 Einstein Notation

We will use Einstein notation in this thesis. Its very simple, we ignore summation signs and just use indices. When we have $x_\mu x^\mu$, it means

$$x_\mu x^\mu = \sum_{\mu=0}^{3} x_\mu x^\mu \tag{2.2}$$

A summation happens when we have and upper and a lower indices that are the same.

2.2 General relativity

The fundamental equation in General relativity is the space-time interval,

$$ds^2 = g_{\nu\mu} dx^\nu dx^\mu, \tag{2.3}$$

where $g_{\nu\mu}$ is the metric of a given system (the geometric description of that system), $dx = (dt, dx, dy, dz)$ is any change/movement in space and time. The metric is what is really the most important part, as it is through this that we related changes/perturbations in space-time to how the system behaves.
For the governing the behavior of a given system we have the Einstein field equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2.4}$$

Where G is the gravitational constant, $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor.

This equation might not seem to tell us much, but that is only until one looks at the Einstein tensor itself,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \tag{2.5}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and Ricci scalar ($R = g^{\mu\nu}R_{\mu\nu}$) respectively.
To explain the Ricci tensor we need to introduce the Riemann tensor,

$$R^{\rho}_{\alpha\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\alpha} - \partial_\nu \Gamma^{\rho}_{\mu\alpha} + \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\alpha} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\alpha}, \tag{2.6}$$

where the gammas are the Christoffel symbols. Christoffel symbols describe the curvature of a space with non-Euclidean geometry and are given as

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\beta\mu} - \partial_\eta g_{\mu\nu}). \tag{2.7}$$

The Ricci tensor is a specific case of the Riemann tensor where $R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu}$. Whit all the extra background eq. (2.4) becomes a whole lot more compact. More relevant for us is that eq. (2.4) can be used to derive several of the equations that we will use later.

2.3 The Friedmann equation And Einstein-De-Sitter

Using the flat version of the Friedmann–Lemaître–Robertson–Walker metric,

$$ds^2 = -c^2dt^2 + a(t)^2[dr^2/(1-kr^2) + r^2d^2 + r^2sin^2()d^2] \tag{2.8}$$

When it comes to describing the components of the universe an equation of fundamental importance is of cause the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}, \tag{2.9}$$

where G is the gravitational constant, ρ is the mass density, a is the scala factor, κ describes the curvature of the universe and R_0 is something about the universe. As the goal of this project what amount other things to get a semi analytical expression of δ_ν during matter domination, we choose to focus on the Einstein-De-Sitter universe[1]. The no curvature part means that we set $\kappa = 0$ and so eq. (2.9) reduces to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}. \tag{2.10}$$

Additionally it means that we may set scalar factor as being proportional to

$$a \propto t^{2/3} \tag{2.11}$$

and from that the Hubble parameter as

$$H(a) = \frac{2}{3}t^{-1}. \tag{2.12}$$

AS one can hopefully see this seriously reduces the complexity of eq. (2.13)

[1]A flat universe where there is only matter, which is a fairly good approximation for matter domination

2.4 Gauges

Just like in Electrodynamics we use different gauges in cosmology. The two that we need to discuss are the Conformal Newtonian gauge, and the Synchronous Gauge. (Check if there are others, just in case).

Conformal Newtonian

Synchronous

2.5 Dark Matter

2.6 Neutrinos

It is not possible to analytically describe neutrinos in GR. The simple reason for this is that the GR description of GR has infinite terms.

2.7 Dark-Matter as Neutrinos with sound speed

Neutrinos are dark matter. For one that is not versed in cosmology this might seem ridicules but think about it. THe "simple" requiremetn for a parrticle to be dark matter is that is does not interact with photons. And this is the case for neutrinos, which is why is so complicated to detect them. THis means that one approach to desribing dark matter, is to decribe them as amssive neutrinos.

Although we will write a propper section on dark matter for now we will talk a bit a bout a certain equation. THat being an equation for dark matter as neutrinos with sound speed

$$\ddot{\delta} + 2H(a)\dot{\delta} + \frac{c_s^2 k^2}{a^2}\delta - 4\pi G\rho_0 \sum_i \epsilon_i \delta_i, \tag{2.13}$$

where δ is how disturbed things are from being homogeneus, $H(a)$ the HUbble paramater, c_s is the sound speed, k is the wave numbers, δ_i is the perturbation for a given type of particle in the universe and ϵ_i is the fraction of the universe that is that particle.

2.8 The power Spectrum

2.9 The Bi-Spectrum

Chapter 3

Perturbation Theory

I will do the following in this chapter. i will derive the first order cold dark matter perturbation, then the second order perturbation in lambda cdm. Then we will go to Einstein deSitter and do it again. Then we will do it for neutrinos, first order and hopefully second order.

To begin, we note that we closely follow the derivation for first and second order cold dark matter perturbation in [3]. We repeat it here for clarity.

We begin with the equations that describe the density contrast δ and divergence of peculiar velocity flow θ equations,

$$\dot{\delta} = -\partial_j \left[(1+\delta)\partial_j \nabla^{-2}\theta \right] \tag{3.1}$$

$$\dot{\theta} = \mathcal{H}\theta - \partial_i \partial_j \nabla^{-2}\theta \partial_j \partial_i \nabla^{-2}\theta - \partial_j \nabla^{-2}\theta \partial_j \theta - \frac{3}{2}\frac{H_0^2 \Omega_m}{a}\delta, \tag{3.2}$$

in conformal time. For the uninitiated δ is the density perturbation dark matter in the cosmos relative to the homogeneous background. θ and thus peculiar velocity flow represents the movement of dark matter that is not just Hubble flow[1]. For a derivation of these see[3].

3.1 First order dark matter

We start off by taking the first order expansion of δ and θ in eq. (3.1) and eq. (3.2), this gives us

$$\dot{\delta}^{(1)} = -\theta^{(1)} \tag{3.3}$$

$$\dot{\theta}^{(1)} = -\mathcal{H}\theta^{(1)} - \frac{3}{2}\frac{H_0^2 \Omega_M}{a}\delta^{(1)}, \tag{3.4}$$

these equations of motion combined gives us

$$\ddot{\delta}^{(1)} + \mathcal{H}\dot{\delta}^{(1)} = \frac{3}{2}\frac{H_0^2 \Omega_M}{a}\delta^{(1)}. \tag{3.5}$$

[1]Hubble flow being the general flowiness of the universe as a consequence of expansion

δ and θ are time and space dependent, but as our equations are linear[2] we can split the dependencies as $\delta^{(1)}(\tau, x) = D(\tau, x)\tilde{\delta}(x)$, we can insert this into eq. (3.5) and remove the space dependence,

$$\ddot{D} + \mathcal{H}\dot{D} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}D = 0. \tag{3.6}$$

So we see that we have a nice differential equation for time component of $\delta^{(1)}$. As for $\theta^{(1)}$ we can reexpress it as $\theta^{(1)} = -\frac{\dot{D}}{D}\delta^{(1)}$ and we remind ourselves that we are in conformal time. We then go to the second order.

3.2 Second order Dark Matter

We now take the second order expansion of δ and θ

$$\begin{aligned}\delta &= \delta^{(1)} + \frac{1}{2}\delta^{(2)}\\ \theta &= \theta^{(1)} + \frac{1}{2}\theta^{(2)}.\end{aligned}$$

with these expansions, eq. (3.1), eq. (3.2), eq. (3.3) and eq. (3.4) and we get two new equations of motions that aren't as simple,

$$\dot{\delta}^{(2)} = -2\partial_j\nabla^{-2}\theta^{(1)}\partial_j\delta^{(1)} - 2\delta^{(1)}\theta^{(1)} - \theta^{(2)} \tag{3.7}$$

$$\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(2)} - 2\Big(\partial_j\partial_i\nabla^{-2}\theta^{(1)}\partial_i\partial_j\nabla^{-2}\theta^{(1)}\Big) - 2\partial_j\nabla^{-2}\theta^{(1)}\partial_j\theta - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)}. \tag{3.8}$$

Using the equations for θ and δ we can rewrite and combine eq. (3.7) and eq. (3.8) into a second order differential equation given as

$$\begin{aligned}\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} =& 2\Big(\mathcal{H}\dot{D}D + \ddot{D}D + \dot{D}^2\Big)\Big[\partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta} + \tilde{\delta}^2\Big] + \frac{3}{2}\frac{H_0^2\Omega_m}{a}\delta^{(2)}\\ &+ 2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_i\partial_j\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big].\end{aligned} \tag{3.9}$$

As we can clearly see this is a rather complicated equation with no clear solution. We refer to [3] for the step by step approach to solving this. To show the solution to eq. (3.9) we need to quickly introduce two things. The first is b_- which is part of a solution to $\delta^{(2)}$ and satisfies the equation

$$\Big(\mathcal{H}\dot{D}D + \ddot{D}D + 2\dot{D}^2\Big)b_- + \Big(4D\dot{D} + \mathcal{H}D^2\Big)\dot{b}_- + 2D^2\ddot{b}_- = \frac{3}{2}H_0^2\Omega_m\frac{D^2}{a}. \tag{3.10}$$

The second thing is the growth function$F \equiv D^2 b_-$, which the equation

$$\ddot{F} + \mathcal{H}\dot{F} = \frac{3}{2}\frac{H_0^2\Omega_M}{a}\Big(F + D^2\Big). \tag{3.11}$$

With that we can finally give the solution to the cold dark matter density perturbation as [3]

$$\delta^{(2)} = 2\partial_j\nabla^{-2}\delta^{(1)}\partial_j\delta^{(1)} + \Big(1 + \frac{F}{D^2}\Big)\delta^{(1)}\delta^{(1)} + \Big(1 - \frac{F}{D^2}\Big)\partial_i\partial_j\nabla^{-2}\delta^{(1)}\partial_i\partial_j\nabla^{-2}\delta^{(1)} \tag{3.12}$$

[2] tjek hvad du mener med det

With all these derivatives and Laplace operators we Laplace transform to get a more compact version [3]

$$\frac{1}{2}\delta^{(2)}(k) = \mathcal{C}_k = \left\{ \mathcal{K}_N(k_1, k_2, k)\delta^{(1)}(k_1)\delta^{(1)}(k_2) \right\}, \tag{3.13}$$

$$\mathcal{K}_N(k_1, k_2, k) \equiv (\beta_N - \alpha_N) + \frac{\beta_N}{2}\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \left(\frac{k_2}{k_1} + \frac{k_1}{k_2} \right) + \alpha_N \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \right)^2, \tag{3.14}$$

$$\alpha_N = \frac{7 - 3v}{14}, \quad \beta_N = 1, \quad v = 7F/3D^2, \tag{3.15}$$

where $\mathcal{C}_k$ is the convolution integral,

$$\mathcal{C}_k\{f(\boldsymbol{k}_1, \boldsymbol{k}_2)\} \equiv \int \frac{d\boldsymbol{k}_1\boldsymbol{k}_2}{(2\pi)^3} f(\boldsymbol{k}_1, \boldsymbol{k}_2)\delta^D(\boldsymbol{k} - \boldsymbol{k}_1 - \boldsymbol{k}_2) \tag{3.16}$$

We are now done with the Λ_{cdm} dark matter we now repeat this using Einstein deSitter.

3.3 First order Dark Matter in Einstein deSitter

The main principle of Einstein desitter is explained earlier that $\Omega_M = 1$, which among other things reduces the ¿Hubble parameter to

$$\mathcal{H} \tag{3.17}$$

NOTE til digselv. Tjek lige alt det der emd hubble paramteren i conformal vs. fysisk tid om der skal et ekstra a med, det tror jeg nemlig, men jeg er lidt i tvivl.

With the cold dark matter done, we now move on to the neutrinos.

3.4 First order neutrinos

We now turn to the massive neutrinos. Now technically speaking we can't actually solve the equations for massive neutrinos in GR[3]. This is why we use the sound speed (vi skal have et afsnit om hvorfor det er legit at gøre). In addition we also take into account that neutrinos aren't a very strong source of gravity and so these neutrinos will not source from themselves but from the cold dark matter. Using this we rewrite eq. (3.1) and eq. (3.2) and get,

$$\dot{\delta}_\nu = -\partial_j \left[(1 + \delta)\partial_j \nabla^{-2}\theta_\nu \right] \tag{3.18}$$

$$\dot{\theta}_\nu = \mathcal{H}\theta_\nu - \partial_i\partial_j\nabla^{-2}\theta_\nu\partial_j\partial_i\nabla^{-2}\theta_\nu - \partial_j\nabla^{-2}\theta_\nu\partial_j\theta_\nu - \frac{3}{2}\frac{H_0^2\Omega_m}{a}\delta_{\text{cdm}} + c_s^2\delta_\nu, \tag{3.19}$$

[3]The problem arises as a consequence of the fact that you need and infinite amount of terms to correctly express the neutrinos properly[2] (tjek med marBerthinger og Thomas)

(bemærk at der muligvis $-\nabla^2 c_s^2$ og ikke bare c_s^2).
Taking the first order of δ_ν,θ_ν and δ_{cdm} we get

$$\dot{\delta}_\nu^{(1)} = -\theta_\nu^{(1)} \tag{3.20}$$

$$\dot{\theta}_\nu^{(1)} = -\mathcal{H}\theta_\nu^{(1)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta_{cdm}^{(1)} + c_s^2\delta_\nu^{(1)} \tag{3.21}$$

$$\ddot{\delta}_\nu^{(1)} + \mathcal{H}\dot{\delta}_\nu^{(1)} + c_s^2\delta_\nu = \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta_{\mathrm{cdm}}^{(1)}. \tag{3.22}$$

Chapter 4

From numerical til analytical

Our numerical version for the differential equation for the δ_ν is given as

$$\ddot{\delta_\nu} = -4/3\dot{\delta_\nu}/t - c^2k^2(\frac{t_0}{t})^{8/3} + 2/3\delta_{cdm,0}/a_0\frac{1}{t_0^{2/3}t^{4/3}} \tag{4.1}$$

A goal of this project was to get a semi or completely analytical solution to this. It was also clearly the most time consuming part of the project, but in the end we did succeed. A general solution to this equation is given as

$$\delta_\nu = \frac{\delta_{cdm,0}}{a_0}\left(\frac{t}{t_0}\right)^{2/3} + c_1\cos(\frac{3ckt_0^{4/3}}{t^{1/3}}) - c_2\sin(\frac{3ckt_0^{4/3}}{t^{1/3}}) \tag{4.2}$$

$$+ 9c^2k^2\delta_{cdm,0}/a_0t_0^2\big(\cos(\frac{3ckt_0^{4/3}}{t^{1/3}})\mathrm{Ci}(\frac{3ckt_0^{4/3}}{t^{1/3}}) \tag{4.3}$$

$$+ \sin(\frac{3ckt_0^{4/3}}{t^{1/3}})\mathrm{Si}(\frac{3ckt_0^{4/3}}{t^{1/3}})\big) \tag{4.4}$$

$$\delta_\nu = \frac{\delta_{cdm,0}}{a_0}a + c_1\cos(3c_s(a)kt_0\sqrt{a}) - c_2\sin(3c_s(a)kt_0\sqrt{a}) \tag{4.5}$$

$$+ 9\frac{c^2k^2\delta_{cdm,0}}{a_0}t_0^2\Bigg(\cos(3c_s(a)kt_0\sqrt{a})\mathrm{Ci}(3c_s(a)kt_0\sqrt{a}) \tag{4.6}$$

$$+ \sin(3c_s(a)kt_0\sqrt{a})\mathrm{Si}(3c_s(a)kt_0\sqrt{a})\Bigg) \tag{4.7}$$

$$\delta_\nu = \frac{\delta_{cdm,0}}{a_0}a + c_1\cos\left(\frac{2c_s(a)\sqrt{a}}{H_0}\right) - c_2\sin\left(\frac{2c_s(a)\sqrt{a}}{H_0}\right) \tag{4.8}$$

$$+ 9\frac{c^2k^2\delta_{cdm,0}}{a_0}t_0^2\Bigg(\cos\left(\frac{2c_s(a)\sqrt{a}}{H_0}\right)\mathrm{Ci}\left(\frac{2c_s(a)\sqrt{a}}{H_0}\right) \tag{4.9}$$

$$+ \sin\left(\frac{2c_s(a)\sqrt{a}}{H_0}\right)\mathrm{Si}\left(\frac{2c_s(a)\sqrt{a}}{H_0}\right)\Bigg) \tag{4.10}$$

(where t_0 is current time), Finding the proper values for c_1 and c_2 proved quit challenging. But we did succeed. The key lies in realizing that for early time before matter domination, the cdm sourcing term (the first term) is negligible. So we expanded around $t = 0$ giving

$$\delta_\nu = c_1 \cos(\frac{3ckt_0^{4/3}}{t^{1/3}}) - (2\pi M - c_2)\sin(\frac{3ckt_0^{4/3}}{t^{1/3}}), \tag{4.11}$$

where $M = \frac{9}{4}c^2k^2t_0^2\delta_{cdm,0}/a0$ is a collection of constants. We then claim, rightfully so, that δ_ν may be written as,

$$\delta_\nu = A\cos(\frac{3ckt_0^{4/3}}{t^{1/3}} + \Phi) \tag{4.12}$$

$$\dot{\delta_\nu} = -Ack(t_0/t)^{4/3}\sin(\frac{3ckt_0^{4/3}}{t^{1/3}} + \Phi) \tag{4.13}$$

Choosing for simplicity the phase $\Phi = 0$ we then have that

$$c_1 = A \; c_2 = 2\pi M \tag{4.14}$$

And

$$A = \sqrt{\delta_\nu^2 + \left(\frac{\dot{\delta_\nu}}{ck(t_0/t)^{4/3}}\right)^2} \tag{4.15}$$

We then pick a δ_ν and $\dot{\delta_\nu}$ value at the initial time value of our simulations. Plucking this into eq. (4.2) we then have an equation that describes the behaviour of massive neutrinos in a matter dominated universe. That takes into account the mass of the neutrinos and the wavenumber.

$$\ddot{\delta}(a) + \frac{3}{2}\frac{1}{a}\dot{\delta}(a) + \frac{K}{a^3}\delta(a) - \frac{3}{2}\frac{1}{a^2}a\frac{\delta_0}{a_0} = 0 \tag{4.16}$$

$$\ddot{\delta}(a) + \frac{3}{2}\frac{1}{a}\dot{\delta}(a) + \frac{K}{a^3}\delta(a) - \frac{3}{2}\frac{1}{a}\frac{\delta_0}{a_0} = 0 \tag{4.17}$$

$$a\frac{\delta_0}{a_0} + \cos(2\sqrt{\frac{K}{a}})(c_1 + 4\frac{\delta_0}{a_0}KCi(2\sqrt{\frac{K}{a}})) + \sin(2\sqrt{\frac{K}{a}})(-c_2 + 4\frac{\delta_0}{a_0}KSi(2\sqrt{\frac{K}{a}})) \tag{4.18}$$

$$\simeq \tag{4.19}$$

$$c_1\cos(2\sqrt{\frac{K}{a}}) + (2\frac{\delta_0}{a_0}\pi K - c_2)\sin(2\sqrt{\frac{K}{a}}) \tag{4.20}$$

$$c_2 = 2\pi k\frac{\delta_0}{a_0} \qquad c_1 = A \tag{4.21}$$

$$A = \sqrt{\delta_\nu^2 + \left(\dot{\delta_\nu}\sqrt{\frac{a^3}{K}}\right)^2} \tag{4.22}$$

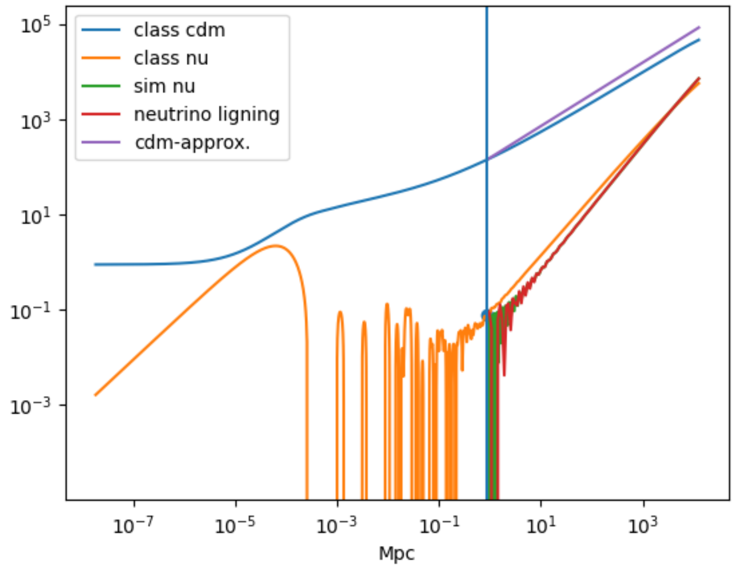

Figure 4.1: Caption

4.1 For science

Higher orders of δ_{cdm} have more complex dependencies on a. It is therefor of interest to see what we get if we set $\delta_{cdm} \propto a^2$. This gives us[1]

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2c^2}{H_0^2a^3}\delta - \frac{3}{2}\delta_{0,2} = 0, \tag{4.23}$$

with a as the time variable.

The solution to this equation is given either as this

$$\delta_\nu(a) = \cos\left(\frac{2c_s(a)k\sqrt{a}}{H_0}\right)\left(5H_0^2c_1 - 4c^4k^4\delta_0\mathrm{Ci}\left(\frac{2c_s(a)k\sqrt{a}}{H_0}\right)\right) - \tag{4.24}$$

$$\sin\left(\frac{2c_s(a)k}{H_0}\sqrt{a}\right)\left(5H_0^2c_2 + 4c^4k^4\delta_0\mathrm{Si}\left(\frac{2c_S(a)k}{H_0}\sqrt{a}\right)\right) \tag{4.25}$$

[1]Derivation in Appendix

or as this (find out which),

$$\delta_\nu = \tag{4.26}$$

$$\Big(\frac{3}{10}a - \frac{1}{5}K\Big)\delta_{cdm,0}a + \cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\Big(c_1 - \frac{4}{5}\delta_{cdm,0}K^2\mathrm{Ci}\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\Big) \tag{4.27}$$

$$-2\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\Big(c_2 + \frac{4}{5}\delta_{cdm,0}K^2\mathrm{Si}\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\Big) \tag{4.28}$$

$$\delta_\nu \simeq c_1\cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) - \left(\frac{2}{5}\delta_{cdm,0}K^2\pi + c_2\right)\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \tag{4.29}$$

$$\delta_\nu \simeq c_1\cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) - \left(\frac{2}{5}\delta_{0,2}K^2\pi + c_2\right)\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \tag{4.30}$$

Ansatz:For early time

$$\delta_\nu \simeq A\cos\left(\frac{2\sqrt{K}}{\sqrt{a}} + \Phi\right) \tag{4.31}$$

$$= A\cos\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\cos\left(\Phi\right) - A\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right)\sin\left(\Phi\right) \tag{4.32}$$

$$\Phi = 0 \tag{4.33}$$

$$c_1 = A\cos\left(\Phi\right) \quad c_2 = A\sin\left(\Phi\right) - \frac{2}{5}\delta_{cdm,0}K^2\pi \tag{4.34}$$

$$c_1 = A \quad c_2 = -\frac{2}{5}\delta_{cdm,0}K^2\pi \tag{4.35}$$

$$\dot{\delta_\nu} = \frac{2\sqrt{K}}{2\sqrt{a^3}}\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) = \sqrt{\frac{K}{a^3}}\sin\left(\frac{2\sqrt{K}}{\sqrt{a}}\right) \tag{4.36}$$

$$A = \sqrt{\delta_\nu^2 + \left(\frac{\dot{\delta_\nu}}{\frac{\sqrt{K}}{\sqrt{a^3}}}\right)^2} = \sqrt{\delta_\nu^2 + \left(\frac{\dot{\delta_\nu}\sqrt{a^3}}{\sqrt{K}}\right)^2}\Bigg|_{a=a_0} \tag{4.37}$$

4.2 second order

Dette er nok forkert. (det er forresten i conformal time, tror jeg).

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} + 2\mathcal{H}\theta^{(2)} + 2k^2c_s(a)^2\Big(\delta^{(1)}\delta^{(1)} + \frac{1}{2}\delta^{(2)} + (\partial_j\delta^{(1)})\partial_j\nabla^{-2}\delta^{(1)}\Big) \tag{4.38}$$

$$= 2\theta^{(1)}\theta^{(1)} + 2(\partial_i\partial_j\nabla^{-2}\theta^{(1)})^2 + 4(\partial_j\nabla^{-2}\theta^{(1)})(\partial_j\theta^{(1)}) \tag{4.39}$$

$$+ 3H_0^2\frac{\Omega_m}{a}\Big(\delta^{(1)}\delta^{(1)}_{cdm} + \frac{1}{2}\delta^{(2)}_{cdm} + (\partial_j\delta^{(1)})(\partial_j\nabla^{-2}\delta^{(1)}_{cdm})\Big) \tag{4.40}$$

nyt forsøg

først har vi vores startligninger

$$\dot{\delta}^{(1)} = -\theta^{(1)} \quad \dot{\theta}^{(1)} = -\mathcal{H}\theta^{(1)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta_M^{(1)} - c_s^2\nabla^2\delta^{(1)} \tag{4.41}$$

$$\dot{\delta}_\nu^{(2)} = -2\partial_j\nabla^{-2}\theta^{(1)}\partial_j\delta^{(1)} - 2\delta^{(1)}\theta^{(1)} - \theta^{(2)} \tag{4.42}$$

$$\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(2)} - 2\Big(\partial_i\partial_j\nabla^{-2}\theta^{(1)}\partial_i\partial_j\theta^{(1)}\Big) - 2\partial_j\nabla^{-2}\theta^{(1)}\partial_j\theta^{(1)} \tag{4.43}$$

$$-\frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta_M^{(2)} - c_s^2\nabla^2\delta^{(2)} \tag{4.44}$$

bedre opskrivning

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} = \frac{3}{2}\frac{H_0^2}{a}\Big(2\partial_j\nabla^{-2}\delta_M^{(1)}\partial_j\delta^{(1)} + 2\delta^{(1)}\delta_M^{(1)} + \delta_M^{(2)}\Big) \tag{4.45}$$

$$+c_s^2\Big(2\partial_j\delta^{(1)}\partial_j\delta^{(1)} + 2\delta^{(1)}\nabla^2\delta^{(1)} + \nabla^2\delta^{(2)}\Big) \tag{4.46}$$

$$4\partial_j\nabla^{-2}\dot{\delta}^{(1)}\partial_j\dot{\delta}^{(1)} + 2\dot{\delta}^{(1)}\dot{\delta}^{(1)} + 2\partial_i\partial_j\nabla^{-2}\dot{\delta}^{(1)}\partial_i\partial_j\nabla^{-2}\dot{\delta}^{(1)} \tag{4.47}$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} = \frac{3}{2}\frac{H_0^2}{a}\Big(2\partial_j\nabla^{-2}\delta_M^{(1)}\partial_j\delta^{(1)} + 2\delta^{(1)}\delta_M^{(1)} + \delta_M^{(2)}\Big) \tag{4.48}$$

$$+4\partial_j\nabla^{-2}\dot{\delta}^{(1)}\partial_j\dot{\delta}^{(1)} + 2\dot{\delta}^{(1)}\dot{\delta}^{(1)} + 2\partial_i\partial_j\nabla^{-2}\dot{\delta}^{(1)}\partial_i\partial_j\nabla^{-2}\dot{\delta}^{(1)} \tag{4.49}$$

$$a = \frac{1}{4}H_0^2\tau^2 \quad \delta^{(1)} = D\tilde{\delta} \quad \delta_M = a\tilde{\delta}_M \quad \mathcal{H} = H_0/\sqrt{a} = \frac{2}{\tau} \tag{4.50}$$

$$\tag{4.51}$$

Vi indsætter og ser hvad der sker,

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - c_s^2\nabla^2\delta^{(2)} - \frac{6}{\tau^2}\delta_M^{(2)} = \frac{6}{\tau^2}\Big(2\partial_j\nabla^{-2}a\tilde{\delta}_M^{(1)}\partial_j D\tilde{\delta}^{(1)} + 2D\tilde{\delta}^{(1)}a\tilde{\delta}_M^{(1)}\Big) \tag{4.52}$$

$$+4\dot{D}^2\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\dot{D}^2\tilde{\delta}^{(1)}\tilde{\delta}^{(1)} + 2\dot{D}^2\partial_i\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\partial_i\partial_j\nabla^{-2}\tilde{\delta}^{(1)} \tag{4.53}$$

$$+c_s^2\Big(2\partial_j D\tilde{\delta}^{(1)}\partial_j D\tilde{\delta}^{(1)} + 2D\tilde{\delta}^{(1)}\nabla^2 D\tilde{\delta}^{(1)}\Big) \tag{4.54}$$

reducere

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - c_s^2\nabla^2\delta^{(2)} - \frac{6}{\tau^2}\delta_M^{(2)} = aD\frac{6}{\tau^2}\Big(2\partial_j\nabla^{-2}\tilde{\delta}_M^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)}\tilde{\delta}_M^{(1)}\Big) \tag{4.55}$$

$$+4\dot{D}^2\Big(\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)}\tilde{\delta}^{(1)} + 2\partial_i\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\partial_i\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\Big) \tag{4.56}$$

$$+c_s^2D^2\Big(2\partial_j\tilde{\delta}^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)}\nabla^2\tilde{\delta}^{(1)}\Big) \tag{4.57}$$

$$\ddot{\delta}^{(2)} + \frac{2}{\tau}\dot{\delta}^{(2)} - c_s^2\nabla^2\delta^{(2)} - \frac{6}{\tau^2}\delta_M^{(2)} = D\frac{3}{2}H_0^2\Big(2\partial_j\nabla^{-2}\tilde{\delta}_M^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)}\tilde{\delta}_M^{(1)}\Big) \tag{4.58}$$

$$+4\dot{D}^2\Big(\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)}\tilde{\delta}^{(1)} + 2\partial_i\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\partial_i\partial_j\nabla^{-2}\tilde{\delta}^{(1)}\Big) \tag{4.59}$$

$$+c_s^2D^2\Big(2\partial_j\tilde{\delta}^{(1)}\partial_j\tilde{\delta}^{(1)} + 2\tilde{\delta}^{(1)}\nabla^2\tilde{\delta}^{(1)}\Big) \tag{4.60}$$

Chapter 5

Simulations

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Chapter 6

Appendix

$$\dot{\delta} = -\theta \tag{6.1}$$
$$\dot{\theta} = aH(a)\theta - \frac{3}{2}H_0^2\frac{\Omega_M}{a}\delta \tag{6.2}$$

Siden vi bruger Einstein-deSitter kan vi antage at

$$\Omega_M = 1 \tag{6.3}$$

Vi differnetiere med hensyn til conformal time så vi skifter til a

$$\dot{\delta}a^2H(a) = -\theta \tag{6.4}$$
$$\dot{\theta}a^2H(a) = aH(a)\theta - \frac{3}{2}H_0^2\frac{\Omega_M}{a}\delta \tag{6.5}$$

Så differentiere vi delta

$$\ddot{\delta}a^2H(a) + 2a\dot{\delta}H(a) + \dot{\delta}a^2\dot{H(a)} + \dot{\theta} = 0 \tag{6.6}$$
$$\ddot{\delta}a^2H(a) + 2a\dot{\delta}H(a) + \dot{\delta}a^2\dot{H(a)} + (\frac{1}{a^2H(a)})(+aH(a)\theta - \frac{3}{2}H_0^2\frac{1}{a}\delta) = 0 \tag{6.7}$$
$$\ddot{\delta}a^2H(a) + \dot{\delta}(2aH(a) + a^2\dot{H(a)} + aH(a)) - \frac{3H_0^2}{a^3H(a)}\delta = 0 \tag{6.8}$$

Since we are in deSitter H(a) reduces to

$$H(a)^2 = H_0^2a^{-3} \tag{6.9}$$
$$\ddot{\delta}\sqrt{a}H_0 + \dot{\delta}\left(2H_0\frac{1}{\sqrt{a}} - \frac{3}{2}H_0\frac{1}{\sqrt{a}} + H_0\frac{1}{\sqrt{a}}\right) - \frac{3H_0}{2a^{3/2}}\delta = 0 \tag{6.10}$$
$$\ddot{\delta}\sqrt{a} + \dot{\delta}\frac{3}{2}\frac{1}{\sqrt{a}} - \frac{3}{2a^{3/2}}\delta = 0 \tag{6.11}$$
$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} - \frac{3}{2a^2}\delta = 0 \tag{6.12}$$

Så for cdm

$$\dot{\delta} = -\theta \tag{6.13}$$

$$\dot{\theta} = aH(a)\theta + k^2 c_S(a)^2 - \frac{3}{2}H_0^2\frac{\Omega_M}{a}\delta \tag{6.14}$$

Siden vi bruger Einstein-deSitter kan vi antage at

$$\Omega_M = 1 \tag{6.15}$$

Vi differnetiere med hensyn til conformal time så vi skifter til a

$$\dot{\delta}a^2 H(a) = -\theta \tag{6.16}$$

$$\dot{\theta}a^2 H(a) = aH(a)\theta + k^2 c_S(a)^2 - \frac{3}{2}H_0^2\frac{\Omega_M}{a}\delta \tag{6.17}$$

Så differentiere vi delta

$$\ddot{\delta}a^2 H(a) + 2a\dot{\delta}H(a) + \dot{\delta}a^2 \dot{H(a)} + \dot{\theta} = 0 \tag{6.18}$$

$$\ddot{\delta}a^2 H(a) + 2a\dot{\delta}H(a) + \dot{\delta}a^2 \dot{H(a)} + (\frac{1}{a^2 H(a)})(+aH(a)\theta + k^2 c_S(a)^2 - \frac{3}{2}H_0^2\frac{1}{a}\delta_{cdm}) = 0 \tag{6.19}$$

$$\ddot{\delta}a^2 H(a) + \dot{\delta}(2aH(a) + a^2\dot{H(a)} + aH(a)) + k^2\frac{c_S(a)^2}{a^2 H(a)} - \frac{3H_0^2}{a^3 H(a)}\delta_{cdm} = 0 \tag{6.20}$$

Since we are in deSitter H(a) reduces to

$$H(a)^2 = H_0^2 a^{-3} \tag{6.21}$$

$$\ddot{\delta}\sqrt{a}H_0 + \dot{\delta}\left(2H_0\frac{1}{\sqrt{a}} - \frac{3}{2}H_0\frac{1}{\sqrt{a}} + H_0\frac{1}{\sqrt{a}}\right) + \frac{c_S(a)^2}{H_0\sqrt{a}} - \frac{3H_0}{2a^{3/2}}\delta_{cdm} = 0 \tag{6.22}$$

$$\ddot{\delta}\sqrt{a} + \dot{\delta}\frac{3}{2}\frac{1}{\sqrt{a}} + \frac{k^2 c_s(a)^2}{H_0^2\sqrt{a}} - \frac{3}{2a^{3/2}}\delta_{cdm} = 0 \tag{6.23}$$

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2 c_s(a)^2}{H_0^2 a} - \frac{3}{2a^2}\delta_{cdm} = 0 \tag{6.24}$$

Assuming $\delta_{cdm} = a\delta_0$

$$\delta_{cdm} = a\delta_0 \tag{6.25}$$

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2 c_s(a)^2}{H_0^2 a} - \frac{3}{2a^2}\delta_{cdm} = 0 \tag{6.26}$$

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2 c_s(a)^2}{H_0^2 a} - \frac{3}{2a}\delta_0 = 0 \tag{6.27}$$

the solution to this is given as (solved in wolfram cloud)
Jeg har fucket op med noget $\sqrt{H_0}$

$$\delta_\nu(a) = \tag{6.28}$$

$$\delta_{cdm,0}a + \cos\left(\frac{2c_s(a)k\sqrt{a}}{\sqrt{H_0}}\right)\left(c_1 + \delta_{cdm,0}\left(\frac{2c_s(a)ka}{H_0}\right)^2 \mathrm{Ci}\left(\frac{2c_s(a)k\sqrt{a}}{H_0}\right)\right) \tag{6.29}$$

$$- \sin\left(\frac{2c_s(a)k\sqrt{a}}{\sqrt{H_0}}\right)\left(c_1 - \delta_{cdm,0}\left(\frac{2c_s(a)ka}{H_0}\right)^2 \mathrm{Si}\left(\frac{2c_s(a)k\sqrt{a}}{H_0}\right)\right) \tag{6.30}$$

$$\delta_\nu \simeq c_1 \cos\left(2\frac{c_s(a)k}{\sqrt{aH_0}}\right) + (2\pi\delta_{cdm,0}\frac{c_s(a)^2 a^2 k^2}{H_0} - c_2)\sin\left(2\frac{c_s(a)k}{H_0}\right) \tag{6.31}$$

Something that is of the utmost importance is that this is not the $H_0 \simeq 69$km/sec/Mpc but the deSitter $H_0 = \frac{2}{3}\frac{1}{t_0}$ where t_0 is current time

or we could assume $\delta_{cdm} = \frac{\delta_0}{a_0}a^2$ (jeg tror at der er en fejl her, det skal måske snare være $\delta_{cdm} = \frac{\delta_0}{a_0^2}a^2$

$$\delta_{cdm} = a^2\delta_{0,2} \tag{6.32}$$

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2 c_s(a)^2}{H_0^2 a} - \frac{3}{2a^2}\delta_{cdm} = 0 \tag{6.33}$$

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2 c_s(a)^2}{H_0^2 a}\delta - \frac{3}{2}\delta_{0,2} = 0 \tag{6.34}$$

$$\ddot{\delta} + \frac{3}{2}\dot{\delta}\frac{1}{a} + \frac{k^2 c^2}{H_0^2 a^3}\delta - \frac{3}{2}\delta_{0,2} = 0 \tag{6.35}$$

6.1 Udledning for første og anden ordens cdm perturbationer

$$\dot{\delta} = -\partial_j\Big[(1+\delta)\partial_j\nabla^{-2}\theta\Big] \tag{6.36}$$

$$\dot{\theta} = -\mathcal{H}\theta - \partial_i\partial_j\nabla^{-2}\theta\partial_j\partial_i\nabla^{-2}\theta - \partial_j\nabla^{-2}\theta\partial_j\theta - \nabla^{-2}\Phi_E \tag{6.37}$$

første orden

$$\dot{\delta}^{(1)} = -\partial_j\Big[(1+\delta^{(1)})\partial_j\nabla^{-2}\theta^{(1)}\Big] = -\theta^{(1)} \tag{6.38}$$

$$\dot{\theta}^{(1)} = -\mathcal{H}\theta^{(1)} - \partial_i\partial_j\nabla^{-2}\theta^{(1)}\partial_j\partial_i\nabla^{-2}\theta^{(1)} - \partial_j\nabla^{-2}\theta^{(1)}\partial_j\theta^{(1)} - \nabla^{-2}\Phi_E \tag{6.39}$$

$$= -\mathcal{H}\theta^{(1)} - \nabla^{-2}\Phi_E \tag{6.40}$$

$$= -\mathcal{H}\theta^{(1)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(1)} \tag{6.41}$$

$$\ddot{\delta}^{(1)} = -\dot{\theta}^{(1)} = -(-\mathcal{H}\theta^{(1)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(1)}) = \mathcal{H}\theta^{(1)} + \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(1)} \tag{6.42}$$

$$= -\mathcal{H}\dot{\delta}^{(1)} + \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(1)} \tag{6.43}$$

$$\ddot{\delta}^{(1)} + \mathcal{H}\dot{\delta}^{(1)} = \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(1)} \tag{6.44}$$

$$\ddot{D} + \mathcal{H}\dot{D} = \frac{3}{2}\frac{H_0^2\Omega_M}{a}D \tag{6.45}$$

So that is the first order differential equation for cdm density perturbations. Now for the second order part.

second order cdm

$$\delta = \delta^{(1)} + \frac{1}{2}\delta^{(2)} \tag{6.46}$$

$$\theta = \theta^{(1)} + \frac{1}{2}\theta^{(2)} \tag{6.47}$$

$$\dot{\delta} = -\partial_j\Big[(1+\delta^{(1)}+\frac{1}{2}\delta^{(2)})\partial_j\nabla^{-2}(\theta^{(1)}+\frac{1}{2}\theta^{(2)})\Big] \tag{6.48}$$

$$\dot{\theta} = -\mathcal{H}(\theta^{(1)}+\frac{1}{2}\theta^{(2)}) - \partial_i\partial_j\nabla^{-2}(\theta^{(1)}+\frac{1}{2}\theta^{(2)})\partial_j\partial_i\nabla^{-2}(\theta^{(1)}+\frac{1}{2}\theta^{(2)}) \tag{6.49}$$

$$-\partial_j\nabla^{-2}(\theta^{(1)}+\frac{1}{2}\theta^{(2)})\partial_j(\theta^{(1)}+\frac{1}{2}\theta^{(2)}) - \nabla^{-2}\Phi_E \tag{6.50}$$

$$\dot{\theta}^{(1)} + \frac{1}{2}\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(1)} - \frac{1}{2}\mathcal{H}\delta^{(2)} - \partial_i\partial_j\nabla^{-2}\theta^{(1)}\partial_j\partial_i\nabla^{-2}\theta^{(1)} - \partial_j\nabla^{-2}\theta^{(1)}\partial_j\theta^{(1)} \tag{6.51}$$

$$-\frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(1)} - \frac{3}{4}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.52}$$

$$\dot{\delta}^{(1)} + \frac{1}{2}\dot{\delta}^{(2)} = -(\theta^{(1)}+\frac{1}{2}\theta^{(2)}) - \partial_j\delta^{(1)}\partial_j\nabla^{-2}\theta^{(1)} - \delta^{(1)}\theta^{(1)} \tag{6.53}$$

using the first order equations we can reduce this to

$$\frac{1}{2}\dot{\theta}^{(2)} = -\frac{1}{2}\mathcal{H}\theta^{(2)} - \partial_i\partial_j\nabla^{-2}\theta^{(1)}\partial_j\partial_i\nabla^{-2}\theta^{(1)} - \partial_j\nabla^{-2}\theta^{(1)}\partial_j\theta^{(1)} - \frac{3}{4}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.54}$$

$$\frac{1}{2}\dot{\delta}^{(2)} = -\frac{1}{2}\theta^{(2)} - \partial_j\delta^{(1)}\partial_j\nabla^{-2}\theta^{(1)} - \delta^{(1)}\theta^{(1)} \tag{6.55}$$

$$\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(2)} - 2\partial_i\partial_j\nabla^{-2}\theta^{(1)}\partial_j\partial_i\nabla^{-2}\theta^{(1)} - 2\partial_j\nabla^{-2}\theta^{(1)}\partial_j\theta^{(1)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.56}$$

$$\dot{\delta}^{(2)} = -\theta^{(2)} - 2\partial_j\delta^{(1)}\partial_j\nabla^{-2}\theta^{(1)} - 2\delta^{(1)}\theta^{(1)} \tag{6.57}$$

then we introduce the product ansatz for the first order equations that

$$\delta^{(1)}(x,\tau) = D(\tau)\tilde{\delta}(x) \tag{6.58}$$

$$\theta^{(1)}(x,\tau) = -\dot{D}\tilde{\delta} = -\frac{\dot{D}}{D}\delta^{(1)} \tag{6.59}$$

$$\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(2)} - 2\partial_i\partial_j\nabla^{-2}\dot{D}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\dot{D}\tilde{\delta} - 2\partial_j\nabla^{-2}\dot{D}\tilde{\delta}\partial_j\dot{D}\tilde{\delta} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.60}$$

$$= -\mathcal{H}\theta^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} - 2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big] \tag{6.61}$$

$$\dot{\delta}^{(2)} = -\theta^{(2)} + 2\partial_j D\tilde{\delta}\partial_j\nabla^{-2}\dot{D}\tilde{\delta} + 2D\tilde{\delta}\dot{D}\tilde{\delta} \tag{6.62}$$

$$= -\theta^{(2)} + 2\dot{D}D\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] \tag{6.63}$$

$$\dot{\theta}^{(2)} = -\mathcal{H}\theta^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} - 2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big] \tag{6.64}$$

$$\dot{\delta}^{(2)} = -\theta^{(2)} + 2\dot{D}D\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] \tag{6.65}$$

$$\ddot{\delta}^{(2)} = -\dot{\theta}^{(2)} + 2\Big[\ddot{D}D + \dot{D}^2\Big]\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] \tag{6.66}$$

$$\ddot{\delta}^{(2)} = \mathcal{H}\theta^{(2)} + \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} + 2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big] \tag{6.67}$$

$$+ 2\Big[\ddot{D}D + \dot{D}^2\Big]\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] \tag{6.68}$$

$$\ddot{\delta}^{(2)} = \mathcal{H}(-\dot{\delta}^{(2)} + 2\dot{D}D\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big]) + 2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big] \tag{6.69}$$

$$+ 2\Big[\ddot{D}D + \dot{D}^2\Big]\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] + \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.70}$$

$$= -\mathcal{H}\dot{\delta}^{(2)} + 2\mathcal{H}\dot{D}D\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] + 2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big] \tag{6.71}$$

$$+ 2\Big[\ddot{D}D + \dot{D}^2\Big]\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] + \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.72}$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} = +2\dot{D}^2\Big[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_j\partial_i\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\Big] + \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} \tag{6.73}$$

$$+ 2\Big[\ddot{D}D + \dot{D}^2 + \mathcal{H}\dot{D}D\Big]\Big[\partial_j\tilde{\delta}\partial_j\nabla^{-2}\tilde{\delta} + \tilde{\delta}^2\Big] \tag{6.74}$$

Then we do this

$$B_1(x) \equiv 2\partial_j \nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}, \quad B_2 = 2\tilde{\delta}^2 \quad B_3 = 2\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_i\partial_j\nabla^{-2}\tilde{\delta} \tag{6.75}$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} = +\dot{D}^2\Big[B_3 + B_1\Big] + \Big[\ddot{D}D + \dot{D}^2 + \mathcal{H}\dot{D}D\Big]\Big[B_1 + B_2\Big] + 0 \tag{6.76}$$

$$0 = \Big[\ddot{D}D + \mathcal{H}\dot{D}D\Big](B_3 - B_3) \tag{6.77}$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} = \Big[\ddot{D}D + 2\dot{D}^2 + \mathcal{H}\dot{D}D\Big]\Big[B_1 + \frac{B_2 + B_3}{2}\Big] \tag{6.78}$$

$$+ \Big[\ddot{D}D + \mathcal{H}\dot{D}D\Big]\frac{B_2 - B_3}{2} \tag{6.79}$$

$$\tag{6.80}$$

Artiklens antagelse "ansatz" er at denne omskrivning betyder at man kan få en løsning af denne form

$$\delta^{(2)} = D^2\Big(b_1 B_1 + b_+\frac{B_2 + B_3}{2} + b_-\frac{B_2 - B_3}{2}\Big) \tag{6.81}$$

$$= D^2\Big(b_1 B_1 + b_+ B_+ + b_- B_-\Big) \tag{6.82}$$

$$\ddot{D} + \mathcal{H}\dot{D} = \frac{3}{2}\frac{H_0^2\Omega_M}{a}D \tag{6.83}$$

$$\delta^{(2)} = D^2\sum b_i B_i \tag{6.84}$$

$$\dot{\delta}^{(2)} = 2D\dot{D}\sum b_i B_i + D^2\sum \dot{b}_i B_i = \sum(2D\dot{D}b_i + D^2\dot{b}_i)B_i \tag{6.85}$$

$$\ddot{\delta}^{(2)} = 2D\ddot{D}\sum b_i B_i + 2\dot{D}^2\sum b_i B_i + 2D\dot{D}\sum \dot{b}_i B_i + 2D\dot{D}\sum \dot{b}_i B_i + D^2\sum \ddot{b}_i B_i \tag{6.86}$$

$$\ddot{\delta}^{(2)} = \sum\Big(b_i(2D\ddot{D} + 2\dot{D}^2) + \dot{b}_i 4D\dot{D} + D^2\ddot{b}_i\Big)B_i \tag{6.87}$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} = \sum\Big(b_i(2D\ddot{D} + 2\dot{D}^2) + \dot{b}_i 4D\dot{D} + D^2\ddot{b}_i\Big)B_i \tag{6.88}$$

$$+ \mathcal{H}\sum(2D\dot{D}b_i + D^2\dot{b}_i)B_i - \frac{3}{2}\frac{H_0^2\Omega_M}{a}D^2\sum b_i B_i \tag{6.89}$$

$$= \sum\Big(b_i(2D\ddot{D} + 2\dot{D}^2) + \dot{b}_i 4D\dot{D} + D^2\ddot{b}_i\Big)B_i + \mathcal{H}\sum(2D\dot{D}b_i + D^2\dot{b}_i)B_i \tag{6.90}$$

$$- (\ddot{D} + \mathcal{H}\dot{D})D\sum b_i B_i \tag{6.91}$$

$$\ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} = \tag{6.92}$$

$$\sum\left(\left(\mathcal{H}D\dot{D} + D\ddot{D} + 2\dot{D}^2\right)b_i + \left(4D\dot{D} + \mathcal{H}D^2\right)\dot{b}_i + \left(D^2\right)\ddot{b}_i\right)B_i \tag{6.93}$$

$$b_1 = 1 \quad B_+ = 1 \quad \ddot{\delta}^{(2)} + \mathcal{H}\dot{\delta}^{(2)} - \frac{3}{2}\frac{H_0^2\Omega_M}{a}\delta^{(2)} = \tag{6.94}$$

$$\left(\left(\mathcal{H}D\dot{D} + D\ddot{D} + 2\dot{D}^2\right)b_- + \left(4D\dot{D} + \mathcal{H}D^2\right)\dot{b}_- + \left(D^2\right)\ddot{b}_-\right)B_- + \left(\mathcal{H}D\dot{D} + D\ddot{D} + 2\dot{D}^2\right)B_1 \tag{6.95}$$

$$+\left(\mathcal{H}D\dot{D} + D\ddot{D} + 2\dot{D}^2\right)B_+ \tag{6.96}$$

$$\delta^{(2)} = D^2(B_1 + B_+ + b_-B_-) \quad \dot{\delta}^{(2)} = 2D\dot{D}(B_1 + B_+ + b_-B_-) + 2D^2\dot{b}_-B_- \tag{6.97}$$

$$\ddot{\delta}^{(2)} = 2D\ddot{D}(B_1 + B_+ + b_-B_-) + 2\dot{D}^2(B_1 + B_+ + b_-B_-) + 2D\dot{D}(\dot{b}_-B_-) + 2D^2\dot{b}_-B_- \tag{6.98}$$

man sætter HS af ligning 2.17 og 2.19 fra artiklen lig hinanden.
bla. bla. bla. jeg har vist det på nogel stykker påapir. Nu tiil E.dS.

$$\dot{\delta}^{(1)} = -\theta^{(1)} \tag{6.99}$$

$$\dot{\theta}^{(1)} = -\mathcal{H}\theta^{(1)} - \frac{3}{2}\frac{H_0^2}{a}\delta^{(1)} \tag{6.100}$$

$$\ddot{\delta}^{(1)} + \mathcal{H}\dot{\delta}^{(1)} = \frac{3}{2}\frac{H_0^2}{a}\delta^{(1)} \tag{6.101}$$

$$\ddot{\delta}^{(1)} + H_0 a^{-1/2}\dot{\delta}^{(1)} = \frac{3}{2}\frac{H_0^2}{a}\delta^{(1)} \tag{6.102}$$

$$a\ddot{\delta}^{(1)} + a^{1/2}H_0\dot{\delta}^{(1)} = \frac{3}{2}H_0^2\delta^{(1)} \tag{6.103}$$

$$\delta = D\tilde{\delta} \quad \theta^{(1)} = -\dot{D}\tilde{\delta} = -\frac{\dot{D}}{D}\delta^{(1)} \tag{6.104}$$

$$a\ddot{D} + a^{1/2}H_0\dot{D} = \frac{3}{2}H_0^2 D \tag{6.105}$$

$$\tag{6.106}$$