EA overview

Based on Baeck- Schwefel 1993

- General outlines
- Evolution strategies
- Evolutionary programming
- Genetic algorithms
- Simulated annealing
- Road map of EC
- Why and when to use EC?

Components of an Evolutionary Algorithm

- $f : \mathbb{R}^n \to \mathbb{R}$ objective function to be optimized
- $\bar{x} \in \mathbb{R}^n$ an object variable vector
- I the space of individuals
- $a \in I$ an individual
- $\Phi : I \to \mathbb{R}$ the fitness function
- $\mu \geq 1$ size of the (parent) population
- $\lambda \geq 1$ number of offspring created in one cycle
- $P(t) = \{\bar{a}_1(1), \ldots, \bar{a}_\mu(t)\}$ population at generation t
- $r_{\Theta_r} : I^\mu \to I^\lambda$ recombination operator
- $m_{\Theta_m} : I^\lambda \to I^\lambda$ mutation operator
- $s_{\Theta_s} : (I^\lambda \cup I^{\mu+\lambda}) \to I^\mu$ selection operator
- $\iota : I^\mu \to \{true,\ false\}$ termination criterion

Θ_r, Θ_m and Θ_s are control parameters of r, m and s respectively

Outline of an Evolutionary Algorithm

$t := 0;$

initialize $P(0) := \{\bar{a}_{\mathbf{1}}(0), \ldots, \bar{a}_{\boldsymbol{\mu}}(0)\} \in I^{\mu};$

evaluate $P(0) : \{\Phi(\bar{a}_{\mathbf{1}}(0)), \ldots, \Phi(\bar{a}_{\boldsymbol{\mu}}(0))\};$

while $(\iota(P(t)) \neq$ **true**$)$

{

  recombine: $P'(t) := r_{\boldsymbol{\Theta r}}(P(t));$

  mutate: $P''(t) := m_{\boldsymbol{\Theta m}}(P'(t));$

  evaluate $P''(t) : \Phi(\bar{a}_{\mathbf{1}}''(0)), \ldots, \Phi(\bar{a}_{\boldsymbol{\mu}}''(0))\};$

  select: $P(t + 1) := s_{\boldsymbol{\Theta s}}(P''(t) \cup Q);$

  $t := t + 1;$

}

where $Q \in \{0, P(t)\}$

Evolution Strategies (1)

Representation (most general case)

$\overline{a} = (\overline{x}, \overline{\sigma}, \overline{\alpha})$, where

- $x_i, i \in \{1, \ldots, n\}$, are object variables
- $\sigma_i, i \in \{1, \ldots, n\}$, are the mutation stepsizes,

  that is the standard deviations $\sigma_i^2 = c_{ii}$
- $\alpha_i, j \in \{1, \ldots, \frac{n \cdot (n-1)}{2}\}$, are rotation angles

  that is the covariances

  $\alpha_j \in \{c_{km} \mid k \in \{1, \ldots, n-1\}, m \in \{k+1, \ldots, n\}\}$

where $c_{\mathbf{km}}$ (k, m $\in$ {1, … , n}) are the elements of the covariance matrix belonging to the generalized n-dimensional normal distribution with expectation vector 0

Evolution Strategies (2)

Fitness function: $\Phi(\bar{a}) = f(\bar{x})$

Mutation (most general case)

$$\sigma_i' = \sigma_i \cdot \exp(\tau \cdot N(0,1) + \tau \cdot N_i(0,1))$$

$$\alpha_j' = \alpha_j + \beta \cdot N_j(0,1)$$

$$\bar{x}' = \bar{x} + \bar{N}(\bar{0}, \bar{\sigma}', \bar{\alpha}')$$

Recombination (most general case)

$$x_i' = \begin{cases} x_{S,i} & \text{without recombination} \\ x_{S,i} \text{ or } x_{T,i} & \text{discrete recombination} \\ x_{S,i} + \chi \cdot (x_{T,i} - x_{S,i}) & \text{intermediate recombination} \\ x_{S_i,i} \text{ or } x_{T_i,i} & \text{global*, discrete} \\ x_{S_i,i} + \chi_i \cdot (x_{T_i,i} - x_{S_i,i}) & \text{global*, intermediate} \end{cases}$$

* S, T ∈ {1, … , μ} are redrawn for each i anew.

For σ's and α's the same mechanism

Evolution Strategies (3)

Selection:

Deterministic, selecting the μ best ($1 \leq \mu < \lambda$) out of

- the set of λ offspring individuals : (μ , λ)-selection
- the union of parents and offspring: $(\mu + \lambda)$-selection

Evolution Strategies (4)

Outline of an Evolutionary Strategy:

t := 0;

initialize $P(0) := P(0) := \{\bar{a}_1(0), \dots, \bar{a}_\mu(0)\} \in I^\mu$;

where $I = \mathbb{R}^{n+w}$

and $\bar{a}_k = (x_i, \sigma_i, \alpha_j)$

$\forall i \in \{1, \dots, n\}, \forall j \in \{1, \dots, n \cdot (1 - n) / 2\}$;

evaluate $P(0): \{\Phi(\bar{a}_1(0)), \dots, \Phi(\bar{a}_\mu(0))\}$;

where $\Phi(\bar{a}_k(0)) = f(\bar{x}_k(0))$;

while ($\iota(P(t)) \neq$ **true**)

{

recombine: $\bar{a}_k'(t) := r'(P(t))$ $\forall k \in \{1, \dots, \lambda\}$;

mutate: $\bar{a}_k''(t) := m_{\{\tau, \tau', \beta\}}'(\bar{a}_k'(t))$ $\forall k \in \{1, \dots, \lambda\}$;

evaluate: $P''(t) := \{\bar{a}_1''(t), \dots, \bar{a}_\lambda''(t)\}$: $\{\Phi(\bar{a}_1''(t)), \dots, \Phi(\bar{a}_\lambda''(t))\}$

where $\Phi(\bar{a}_k''(0)) = f(\bar{x}_k''(0))$;

select: $P(t + 1) :=$ **if** (μ, λ)-selection **then** $s(\mu, \lambda)(P''(t))$;

else $(\mu + \lambda)(P(t) \cup P''(t))$;

t := t + 1;

}

Evolutionary Programming (1)

Representation

- Standard EP: $\bar{a} = \bar{x}$
- Meta-EP: $\bar{a} = (\bar{x}, \bar{\sigma})$

Fitness function

scaling objective function values $f(\bar{x})$ to positive values and possibly imposing some random alteration

$\Phi(\bar{a}) = \delta(f(\bar{x}), \kappa)$

where

- $\alpha_i : \mathbb{R} \times S \rightarrow \mathbb{R}^+$ denotes the scaling function and S is an additional set of parameters,
- κ is some random alteration factor

Evolutionary Programming (2)

Mutation

- Standard EP: most common way is $x_i' = x_i + \sqrt{\Phi(\overline{x})} \cdot N_i(0,1)$
- Meta-EP: $x_i' = x_i + \sigma_i \cdot N_i(0,1)$

  $\sigma_i' = \sigma_i + \alpha \cdot \sigma_i \cdot N_i(0,1)$

where the parameter α ensures that v_i tends to remain positive

Recombination: None

Selection

stochastic q-tournament (μ + μ) style selection, sorting individuals by their score w, where

$$w_i = \sum_{j=1}^{q} \begin{cases} 1 & \text{if } \Phi(\overline{a}_i) \leq \Phi(\overline{a}_{\chi_j}) \\ 0 & \text{otherwise} \end{cases}$$

where $\chi_j \in \{1, \dots, 2\mu\}$ is a uniform integer random variable, sampled anew for each comparison

Evolutionary Programming (3)

Outline of an Evolutionary Strategy:

t := 0;

initialize $P(0) := P(0) := \{\bar{a}_{\mathbf{1}}(0), \ldots, \bar{a}_{\boldsymbol{\mu}}(0)\} \in I^{\mu}$;

where $I = \mathbb{R}^{n} \times \mathbb{R}^{+n}$

and $\bar{a}_{\mathbf{k}} = (x_{\mathbf{i}}, v_{\mathbf{i}})$ $\forall i \in \{1, \ldots, n\}$;

evaluate $P(0)$: $\{\Phi(\bar{a}_{\mathbf{1}}(0)), \ldots, \Phi(\bar{a}_{\boldsymbol{\mu}}(0))\}$

where $\Phi(\bar{a}_{\mathbf{k}}(0)) = \delta(f(\bar{x}_{\mathbf{k}}(0)), \kappa_{\mathbf{k}})$;

while ($\iota(P(t)) \neq$ **true**)

{

recombine: $\bar{a}_{\mathbf{k}}'(t) := m_{\{\alpha\}}'(\bar{a}_{\mathbf{k}}(t))$ $\forall k \in \{1, \ldots, \mu\}$;

evaluate: $P''(t) := \{\bar{a}_{\mathbf{1}}'(t), \ldots, \bar{a}_{\boldsymbol{\mu}}'(t)\}$: $\{\Phi(\bar{a}_{\mathbf{1}}'(t)), \ldots, \Phi(\bar{a}_{\boldsymbol{\mu}}'(t))\}$

where $\Phi(\bar{a}_{\mathbf{k}}'(t)) = \delta(f(\bar{x}_{\mathbf{k}}'(t)), \kappa_{\mathbf{k}})$

select: $P(t + 1) := s_{\{q\}}(P(t) \cup P'(t))$;

t := t + 1;

}

Genetic Algorithms (1)

Representation

Bit strings of fixed length ℓ, i.e. I = $\{0, 1\}^{\ell}$.

For a continuous objective function:

$$f : \prod_{i=1}^{n} [u_i, v_i] \rightarrow \mathbb{R}, \text{ with } u_i < v_i$$

- $\ell = n \cdot \ell_x$,
- $\overline{a} = (a_{11} \dots a_{n\ell_x}) \in \{0,1\}^{n \cdot \ell_x} = I$ and
- a typical decoding is $\Gamma = \Gamma^1 \times \dots \times \Gamma^n$, where

$$\Gamma^i(a_{i1} \dots a_{i\ell_x}) = u_i + \frac{v_i - u_i}{2^{\ell_x}} \left(\sum_{j=1}^{\ell_x} a_{ij} 2^{j-1} \right)$$

Fitness function

$\Phi(\bar{a}) = \delta(f(\Gamma(\bar{a})))$,

Where δ is a scaling function assuring positive fitness values and best individual ~ largest fitness.

E.g. linear scaling with a scaling window of w generations.

$$\delta(f(\Gamma(\overline{a})), P(t-w)) = \max\{ f(\Gamma(\overline{a}_j)) \mid \overline{a}_j \in P(t-w)\} - f(\Gamma(\overline{a}))$$

Genetic Algorithms (2)

Mutation

Bit flips with probability p_m per bit, that is

$$s_i' = \begin{cases} s_i, \chi_i > p_m \\ 1 - s_i, \chi_i \le p_m \end{cases}$$

where p_m is an external parameter, the mutation rate, χ_i is drawn from a uniform distribution

Recombination

One-point crossover: if $\bar{s} = (s_1, \ldots, s_\ell), \bar{v} = (v_1, \ldots, v_\ell)$ are selected as would-be parents, crossover takes place with probability p_c, called the crossover rate.

$$\bar{s}' = (s_1 \ldots s_{\chi-1}, s_\chi, v_{\chi+1}, \ldots, v_\ell)$$

$$\bar{v}' = (v_1 \ldots v_{\chi-1}, v_\chi, s_{\chi+1}, \ldots, s_\ell)$$

Where $\chi \in \{1, \ldots, \ell - 1\}$ denotes a uniform random variable, the crossover point.

Genetic Algorithms (3)

Selection

Fitness proportional selection: probability of being selected is

$$p_s(\overline{a}_i) = \frac{\Phi(\overline{a}_i)}{\sum_{j=1}^{\mu} \Phi(\overline{a}_j)}$$

Genetic Algorithms (4)

Outline of an Genetic Algorithm:

t := 0;

initialize $P(0) := P(0) := \{\bar{a}_{\mathbf{1}}(0), \ldots, \bar{a}_{\boldsymbol{\mu}}(0)\} \in I^{\boldsymbol{\mu}}$;

where $I = \{0,1\}^{\ell}$

evaluate $P(0): \{\Phi(\bar{a}_{\mathbf{1}}(0)), \ldots, \Phi(\bar{a}_{\boldsymbol{\mu}}(0))\}$

where $\Phi(\bar{a}_{\mathbf{k}}(0)) = \delta(f(\Gamma(\bar{a}_{\mathbf{k}}(0))), P(0))$;

while ($\iota(P(t)) \neq$ **true**)

{

recombine: $\bar{a}_{\mathbf{k}}'(t) := r_{\{pc\}}'(P(t))$ $\forall k \in \{1, \ldots, \mu\}$;

mutate: $\bar{a}_{\mathbf{k}}''(t) := m_{\{pm\}}'(\bar{a}_{\mathbf{k}}'(t))$ $\forall k \in \{1, \ldots, \mu\}$;

evaluate: $P''(t) := \{\bar{a}_{\mathbf{1}}''(t), \ldots, \bar{a}_{\boldsymbol{\mu}}''(t)\}$: $\{\Phi(\bar{a}_{\mathbf{1}}'(t)), \ldots, \Phi(\bar{a}_{\boldsymbol{\mu}}''(t))\}$

where $\Phi(\bar{a}_{\mathbf{k}}''(t)) = \delta(f(\Gamma(\bar{a}_{\mathbf{k}}''(0))), P(t - w))$;

select: $P(t + 1) := s(P''(t))$;

where $p_{\mathbf{s}}(\bar{a}_{\mathbf{k}}''(t)) = \Phi(\bar{a}_{\mathbf{k}}''(t)) / \Sigma_{\mathbf{j=1}}^{\boldsymbol{\mu}} \Phi(\bar{a}_{\mathbf{j}}''(t))$

t := t + 1;

}

Comparative overview of EA'S

	ES	EP	GA
Repr.	Real	Real	Binary
Self-adapt.	Standard dev's and covariances	Variances (in meta-EP)	None
Fitness	Objective function	Scaled obj. function	Scaled obj. function
Mutation	Main op.	Only op.	Background
Recomb.	Different, important for self-adaptation	None	Main op.
Select.	Deterministic, extinctive	Probab., extinctive	Probab., preservative

Simulated Annealing (1)

Outline of a Local Search procedure

procedure *local-search*:

{

initialize(i_{start});

repeat

generate($j \in S_i$);

if (f(j) < f(i)) **then** i := j;

until (f(j) ≥ f(i)) **for all** $j \in S_i$;

}

Simulated Annealing (2)

Main drawback: gets easily stuck in local optima Possible cures:

1) several restarts with new $i_{\mathbf{start}}$
2) sophisticated neighborhood structure
3) accept j even if it is worse than i

Option 3) by analogy from condensed matter physics, where state transitions lead to minimal energy level

$$P[\text{accept } j] = \exp\left(\frac{E_i - E_j}{K_b \cdot T}\right)$$

This is called the Metropolis criterion.

Simulated Annealing (3)

Outline of simulated annealing:

procedure *simulated-annealing*:

{

initialize(i_{start}; c_0; L_0);

k := 0; i := i_{start};

repeat

for ℓ := 1 **to** L_k **do** {

generate ($j \in S_i$);

if (f(j) < f(i)) **then** i := j;

else if $\exp\left(\frac{f(i) - f(j)}{c_k}\right) > random[0,1)$ **then** i := j;

}

k := k + 1;

calculate-length(L_k);

calculate-control(c_k);

until *stopcriterion*;

}

Simulated Annealing (4)

Remarks:

1. The 'temperature' c_k is decreased over time.

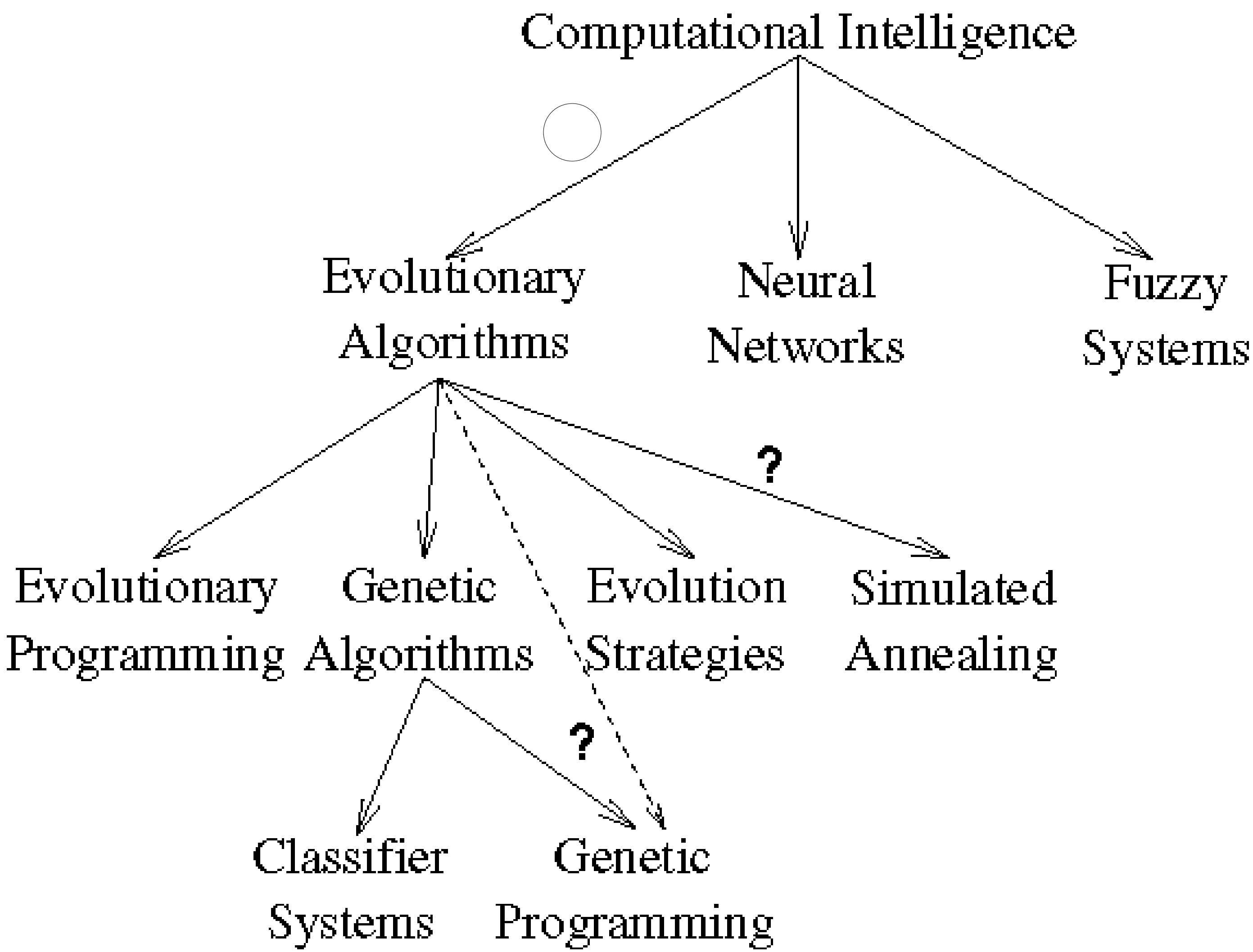
2. As c_k decreases, selection becomes more and more elitist.

3. The neighborhood S_i is mostly defined 'operationally', i.e. as the set of points generated with one modification (mutation) operator from i.

Simulated Annealing can be seen as an EA

- using a (1+1) selection strategy together with a
- a specific, time dependent selection regime
- where the representation and the mutation operator are not specified (left as problem dependent component)

Roadmap of EC



Why & when use evolutionary computing?

- **No in-depth mathematical understanding** of the problems needed
- Can **solve “out of range ” problems** (that cannot be solved by analytical mathematical techniques), for instance: many variables, many local optima, moving goal posts
- They are **extremely robust**; they cope well with noisy, inaccurate and incomplete data
- They are relatively **cheap and quick to implement**
- They are **easily hybridised**; they combine very productively with other techniques such as greedy methods, heuristics, simulated annealing and neural networks

Why & when use evolutionary computing?

- Extremely **adaptable**; changed priorities can be incorporated simply by changing weightings in the fitness function
- They are **modular and therefore portable**; because the evolutionary mechanism is separate from the problem representation they can be transferred from problem to problem
- They provide an extremely open and flexible approach to design, **allowing** arbitrary **constraints**, simultaneous **multiple objectives** and the **mixing of continuous and discrete parameters**
- Unlike many other methods, when evolutionary algorithms are **implemented on parallel computers** they make very efficient use of the available power