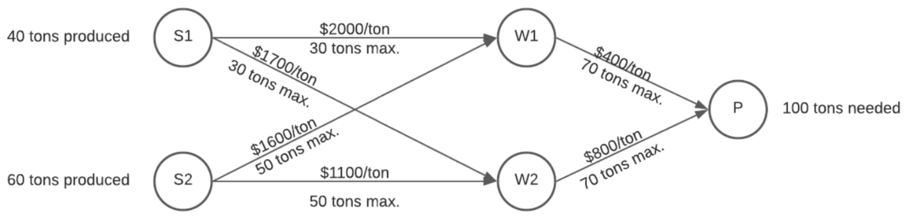
IE 310 - Group Assignment 2

1. Tony's Chocolonely is currently working with two suppliers producing cocoa. The cocoa produced is transferred from the suppliers to either of two warehouses. When needed, it then is shipped on to the chocolate production plant of Tony's. The diagram below demonstrates the distribution network, where S1 and S2 are the two suppliers, W1 and W2 are the two warehouses, and P is the chocolate production plant. The diagram also indicates the quarterly amounts produced at the cocoa suppliers and needed at the chocolate plant, as well as the transferring cost and maximum amount that can be transferred per quarter through each transfer roads.



Management now wants to decide on the most economical plan for transferring the cocoa from the suppliers to the chocolate plant through the distribution network given.

(a) Formulate a linear programming model for this problem.

(b) Solve the model formulated in part (a) by simplex method.

2. Convert the following problem to LP standard form.
(Hint: $|a| \leq b$ is equivalent to $a \leq b$ and $a \leq b$.)

$$\min \; z = max\{5x_1, 7|x_2 - 10|, 2|x_1 - 2x_3|\}$$

s.t.

$$\begin{aligned} & x_1 + x_2 + x_3 = 50 \\ & x_1 + 3x_2 - 2x_3 \geq 10 \\ & -3x_1 + 2x_2 - x_3 \leq 22 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

3. Solve the following LP problem using simplex method:

$$\min \; z = -\frac{3}{4}x_4 + 20x_5 - \frac{1}{2}x_6 + 6x_7$$

s.t.

$$\begin{array}{lllllll} x_1 & & & + \frac{1}{4}x_4 & - 8x_5 & - x_6 & + 9x_7 = 0 \\ & x_2 & & + \frac{1}{4}x_4 & - 12x_5 & - \frac{1}{2}x_6 & + 3x_7 = 0 \\ & & x_3 & & & + 6x_6 & = 1 \end{array}$$

$$x_i \geq 0 \; for \; i = 1, ..., 7$$

4. Solve the following problem by the big-M method:

$$\begin{aligned}
\min\ \ & z = 3x_1 + 2x_2 + 4x_3 + 8x_4 \\
\text{s.t.}\ \ & \\
& x_1 - 2x_2 + 3x_3 + 6x_4 \geq 8 \\
& -2x_1 + 5x_2 + 3x_3 - 5x_4 \leq 3 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

5. In this programming assignment, you are going to write a program which solves a given set of linear equations. As you may have already noticed, linear algebra is the premise of linear optimization and we frequently need to solve system of linear equations. Gaussian elimination or Gauss-Jordan method can be used to solve such systems. Both methods take advantage of three basic elementary row operations listed below:

   (a) Interchanging two rows.

   (b) Multiplying a row by a nonzero constant.

   (c) Adding a multiple of a row to another row

   Suppose that a system of linear equations is given as $Ax = b$, where A is a nxn matrix, b is a column vector with n entries and x is a column vector with n variables. As you may recall, there are three possibilities about the solution of such a system:

   (a) If $rank(A) = n$, then $Ax = b$ has a unique solution.

   (b) If $rank(A) = rank(A|b) < n$, then $Ax = b$ has infinitely many solutions.

   (c) If $rank(A) < rank(A|b)$, then $Ax = b$ has no solutions.

   You should write a computer program in C, C++ or Python which first identifies from the cases above and then, performs the following operations:

   (a) If $rank(A) = n$, then you should find the unique solution and additionally, you should also find A^{-1}.

   (b) If $rank(A) = rank(A|b) < n$, then you should give an arbitrary solution.

   (c) If $rank(A) < rank(A|b)$, then you should only state that the problem has no solution at all.

   You need to test your algorithm with the three problem instances attached. The first number in .txt files gives the number n. Then, there is a n x $(n + 1)$ matrix which represents the augmented matrix $[A|b]$. You can safely assume that the data is given correctly. Do not change the input format.

   Include text files into your program by using commands like "open" instead of giving numbers into the program.

Run all three instances in your source code instead of changing the file name and running the program for each instance (i.e. write a function for operations and call it for each instances in main).

You are encouraged **not to use any linear algebra libraries** available! If you use, you will be graded partially.

You need to hand in a report which contains your description of the method and outputs of the problem for each instance (you may include screenshots or type your outputs in your report.) Check below for the output format.

Note: If you have difficulty in remembering the details of Gauss-Jordan method, refer to the lecture notes or any elementary linear algebra book.

Output formats:
i. "Unique solution: x y z ...
Inverted A: a b c...

d e f...

g h i...

... "

j. "Arbitrary variables: p r...
Arbitrary solution: p q r ..."

k. "Inconsistent problem"

Questions	Points
1	15
2	15
3	15
4	15
5	40

NOTE:

- Include your source code into your .zip file.
- Submit your folders via Moodle page until due time.