# ACT Combinatorics

Eric Ingram

2024-03-20

erimingram.github.io

## Combinatorics Overview

1. Fundamental Counting
2. Permutations
3. Combinations
4. How to do with a Calculator
5. Practice Questions
6. Practice Questions Answers

## Fundamental Counting

Combinatorics is the subfield of mathematics related to counting. It is an advanced topic, but we need to know the basic fundamentals of counting combinations of items or ways to do things for the ACT.

We will start with what we call the fundamental counting principle. This says if there are $n$ ways of doing one thing and $m$ ways of (independently) doing another thing after that, there are $n * m$ ways to do both things.

This can be extended to more dimensions. If there are $n$ ways of doing one thing, $m$ ways of doing another thing after that, and $p$ ways of doing a third thing after the second, then there are $n * m * p$ ways to perform all three.

So, for a simple example, imagine you are going to a restaurant and you have 3 choices for a starter, 5 choices for a main course, and 2 choices for dessert. How many different meals can you order? According to the fundamental counting principle, the total number of different meals you can order is $3 * 5 * 2 = 30$.

## Permutations

Permutations describe the arrangement of objects in a specific order. Imagine you have a set of three books: A, B, and C. If you want to know in how many different ways you can arrange these books on a shelf, you are thinking about permutations.

With the three books, the permutations would be ABC, ACB, BAC, BCA, CAB, and CBA. So, there are 6 different ways to arrange these three books.

When we have a lot of objects, we clearly can't list them all out. This is where the math comes in. If you have $n$ distinct objects, the number of different ways to arrange them is $n!$. Remember that $n! = (n) * (n-1) * (n-2) * ... * 1$. So, $5! = 5 * 4 * 3 * 2 * 1 = 120$. Also, remember that $0! = 1$.

So, for example, if we have 4 books (A, B, C, D), the number of permutations is $4! = 24$. You can clearly see for that for the 3 book example from earlier, $3! = 6$.

Now, we need to get a bit more advanced. We need to do permutations of a subset. So, say we want to only arrange a subset of the entire set; For example, selecting 3 books out of 5 and arranging them.

We have a new, fancy formula for this scenario. The number of ways to arrange $r$ objects out of a set of $n$ objects is given by $P(n,r) = \frac{n!}{(n-r)!}$. Just to check your understanding, if we choose $r = n$, so permutations of the entire set, we just end up getting $n!$ which is what we did earlier.

But now, let's use the selecting 3 books out of 5 and arranging them example. In this case, $r = 3$ and $n = 5$. So, $P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$. So, there are 60 different arrangements.

## Combinations

Combinations are very similar to permutations, but there is a key difference. In Permutations, order mattered. In combinations, order will not matter. Let's show this in an example.

Imagine you have three books: A, B, and C. You want to know how many ways you can pick two books to take on a trip.

For permutations, there are 6 ways to pick 2 books out of 3: AB, BA, AC, CA, BC, CB. We can see this with the formula $P(3,2) = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{6}{1} = 6$. However, who cares about picking AB versus BA in this scenario. With AB and BA you are still choosing the same two books to take on the trip!

For this scenario, combinations are more appropriate. When the order doesn't matter, there are only 3 ways to pick 2 books out of 3: AB, AC, BC.

The combination formula is the same as the permutation formula with one slight modification: the addition of a $r!$ in the denominator. $C(n,r) = \frac{n!}{r!(n-r)!}$.

## How to do with a Graphing Calculator

Here, I've given instructions on how to access combinations and permutations on the 4 most common graphing calculators. You can type it out by hand, but why do that when you have a calculator?

### TI-84

Press MATH and then press the left arrows twice. You should be in the PROB menu. You should then see nPr and nCr. nPr is for permutations and nCr is for combinations. Select the one you want to use and then just plug in n and r. Then, click ENTER to evaluate it. You should also see ! in the PROB menu if you wish to use that.

### TI-Nspire

Press menu and then go to Probability. You can then see Factorial, Permutations, Combinations. For Permutations and Combinations, after you select it, enter n then a comma and then enter r. Then, click enter to evaluate it.

### HP Prime

Click the toolbox. Go to Probability. You can then see Factorial, Permutations, Combinations. For Permutations and Combinations, after you select it, enter n then a comma and then enter r. Then, click Enter to evaluate it.

### Casio fx-9750GII

Click OPTN Click F6. Click F3 to open the PROB menu. You can see ! under F1, nPr under F2, and nCr under F3. For Permutations and Combinations, type in n, then select nPr or nCr (resulting in N or C showing up on the screen), then type in r. Then, click EXE to evaluate it.

## Practice Questions

Here are a few practice questions to help you understand the concepts. These are **not** ACT questions. They are meant to help you understand which formulas to pick. I have an ACT problem set for this topic which you can ask me for if you want. Answers to each problem will be provided at the end. Questions 6 and 7 are more difficult than questions 1-5.

**Question 1**

10 cars are in a race. In how many ways can three cars finish in first, second and third place?

**Question 2**

Josh has a summer reading list of eight books. He has to read five of the books before the end of the summer. In how many ways can he read five of the eight books?

**Question 3**

Daphne wants to pick three pieces of candy to take to school. If there are 9 pieces of candy to choose from, how many ways can she pick the three pieces?

**Question 4**

The school wind ensemble is planning to play four pieces at their next concert. How many different programs are possible?

**Question 5**

The Student Council has 24 members. An election is held to choose a president, vice-president and secretary. In how many ways can the three officers be chosen?

**Question 6**

A serial number for a video game consists of a letter followed by four digits and ends with two letters. Neither letters nor numbers can be repeated. How many different serial numbers are possible?

**Question 7**

A class consists of 11 boys and 16 girls. In how many ways can two boys and two girls be chosen to participate in an in-class activity?

## Practice Questions Answers

### Question 1

First, identify that the order matters. We are saying that 1st is different than 2nd which is different than 3rd. Therefore, this is a permutation. We have 10 total cars, so $n = 10$. We are interested in a subset of 3 cars, so $r = 3$. Then, we can do $P(10, 3) = 720$.

Alternatively, we can do this with the fundamental counting principle. Initially, all 10 drivers could've come first. Then, since a driver came first, only 9 drivers could've come second. Then, since a driver came first and a driver came second, only 8 drivers could've come third. So, $10 * 9 * 8 = 720$.

So, there are 720 different ways for these 10 cars to in 1st, 2nd, and 3rd.

### Question 2

First, identify that order does not matter. What matters are the books that are read, not the order in which he read them. Therefore, this is a combination. We have 8 total books and we care about a subset of 5 books. So, $n = 8$ and $r = 5$ which leads to $C(8, 5) = 56$.

So, there are 56 ways for Josh to choose 5 books to read.

### Question 3

It doesn't matter which order the candy is chosen. What is important is the pieces chosen, not the sequence in which they are selected. So, this is a combination. $n = 9$ and $r = 3$, so $C(9, 3) = 84$

There are 84 different ways for Daphne to pick her candy.

### Question 4

This is a permutation since they are arranging the pieces in order to make the program. Hence, order matters. $n = 4$ and $r = 4$, so $P(4, 4) = 24$.

You can forgo the permutation formula here and just say $n! = 4! = 24$.

Using the fundamental counting principle, you first have 4 pieces that can be played first. Then, since a piece had to be played first, there are 3 pieces that can be played 2nd. Since pieces had to be played first and second, there are 2 pieces that can be played 3rd. Since a piece had to be played first, second, and third, there is only one piece that can be played 4th. So, $4 * 3 * 2 * 1 = 4! = 24$.

There are 24 different ways to arrange the 4 pieces on the program.

### Question 5

This is just like the race problem. The order matters since president, vice president, and secretary are distinct positions. So, this is a permutation. $n = 24$ and $r = 3$, so $P(24, 3) = 12144$.

You can also use the fundamental counting principle. There are 24 possible presidents, then after you've chosen a president there are 23 possible vice presidents, then after you've chosen a president and vice president there are 22 possible secretaries. $24 * 23 * 22 = 12144$

So, there are 12,144 different ways to select a president, vice president, and secretary for the student council.

### Question 6

A serial number for a video game consists of a letter followed by four digits and ends with two letters. Neither letters nor numbers can be repeated. How many different serial numbers are possible?

This question is more complicated. This is a permutation because order matters in a serial number. However, we can't use the permutation formula because we have letters. The formula can't account for this.

Therefore, we have to use the fundamental counting principle. There are first 26 letters that can take up the first spot in the serial number. Then, there are 10 possible digits (0-9) for the second spot. Then, the third spot has 9 possible digits since we can't repeat letters or numbers. We then have 8 possible digits and then 7 possible digits for the next two spots. For the same reason, we then have 25 possible letters and then 24 possible letters. So, $26 * 10 * 9 * 8 * 7 * 25 * 24 = 78624000$.

So, there are 78,624,000 possible serial numbers for the video game.

**Question 7**

A class consists of 11 boys and 16 girls. In how many ways can two boys and two girls be chosen to participate in an in-class activity?

This question is also more complicated. This is a combination since the order the boys and girls are chosen is not important.

However, here we have two different populations we are drawing from. Hence, we have to calculate the combinations separately.

For the boys, we get $C(11, 2) = 55$. For the girls, we get $C(16, 2) = 120$.

To get the total number of ways, use the fundamental counting principle: $55 * 120 = 6600$.

So, there are 6600 ways to choose two boys and two girls to participate.