

```

close all hidden; clear; clc;
rng(418);                                % seed for reproducibility,
% for this hw changing seed influences FIT drastically

```

## 1.a Synthetic Data

```

Ts    = 0.02;
N    = 3000;
t    = (0:N-1)'*Ts;

% Discrete-time plant

A    = [0.5 0.0;
       0.0 0.2];
B    = [1; 3];
C    = [1 -0.6];
D    = 0;

% PRBS input signal in [-1,1], dwell = 10

dwell = 10;
levels = sign(randn(ceil(N/dwell),1));      % ±1 levels
u     = repelem(levels, dwell);
u     = u(1:N); u = u(:);                  % trim, column vector

% Simulate the system

x     = zeros(2,N);
for k = 1:N-1
    x(:,k+1) = A*x(:,k) + B*u(k);
end
y0    = (C*x).' + D*u;                      % noise free output
sigma = 0.03;
y     = y0 + sigma*randn(N,1);                % noisy output

% Split into estimation and validation datasets

N_est = round(15/Ts);                      % first 15 s for estimation
u_est = u(1:N_est);   y_est  = y(1:N_est);   y0_est = y0(1:N_est);
u_val = u(N_est+1:end); y_val  = y(N_est+1:end); y0_val = y0(N_est+1:end);
t_est = t(1:N_est);   t_val  = t(N_est+1:end);

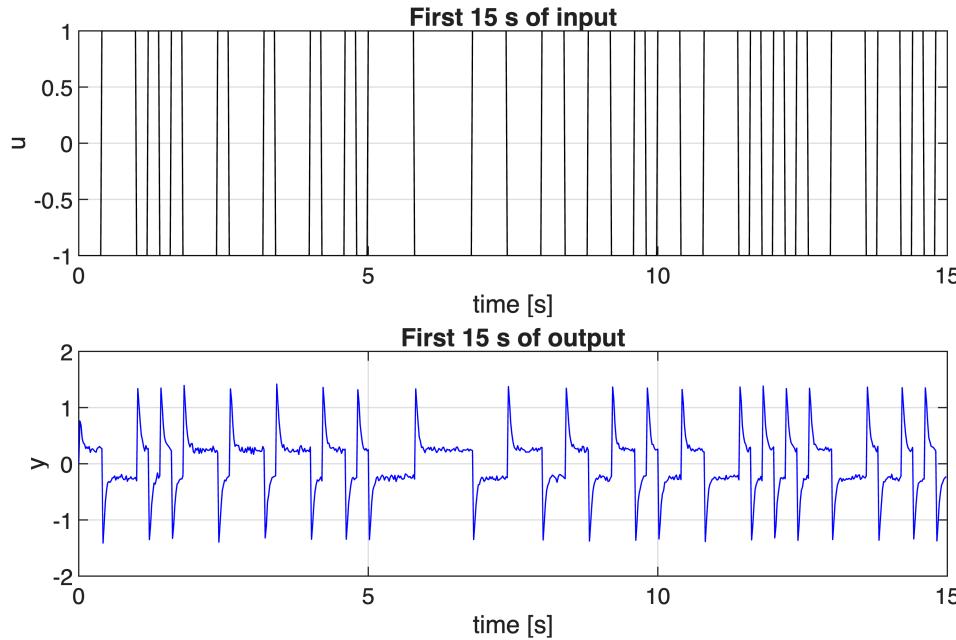
% Plot the first 15 seconds

```

```

figure;
subplot(2,1,1); plot(t_est,u_est,'k'); grid on; xlabel('time [s]');
ylabel('u');
title('First 15 s of input');
subplot(2,1,2); plot(t_est,y_est,'b'); grid on; xlabel('time [s]');
ylabel('y');
title('First 15 s of output');

```



## 1.b Input suitability

```

% i=j=30: check full row rank of Uf and [Up;Uf].
% The input must be exciting (PE) for correct system identification.
% PRBS is theoretically PE, but it may not excite all frequencies equally
% well.
% A multisine input would provide stronger broadband excitation (tried for
% input, got %66 FIT).

i = 30; j = 30;
Ns = N_est - i - j + 1;
assert(Ns>0, 'Ns<=0; increase N_est or reduce i/j.');

% Hankel matrices for input/output blocks

Up = zeros(i,Ns); Uf = zeros(j,Ns); Yf = zeros(j,Ns);
for r = 1:i, Up(r,:) = u_est(r : r+Ns-1); end % past input
for r = 1:j, Uf(r,:) = u_est(i+r : i+r+Ns-1); end % future input
for r = 1:j, Yf(r,:) = y_est(i+r : i+r+Ns-1); end % future output

% Rank checks

```

```

thr = @(s1,s) 1e-8*max(s1,1);
sUf = svd(Uf, 'econ'); rankUf = sum(sUf > thr(sUf(1), sUf));
sW = svd([Up;Uf], 'econ'); rankW = sum(sW > thr(sW(1), sW));
fprintf('\nPE check: rank(Uf)=%d (target j=%d) | rank([Up;Uf])=%d (target i+j=%d)\n', ...
    rankUf, j, rankW, i+j);

```

PE check: rank(Uf)=30 (target j=30) | rank([Up;Uf])=60 (target i+j=60)

```

% Note:
% PRBS fails to sufficiently excite certain frequency bands (%33 FIT),
% rank(Uf) or rank([Up;Uf]) may drop below the target.
% In such cases, a multisine input (superposition of sine waves)
% can provide better excitation.

```

## 1.c Hankel blocks

```

% Pre-computed at 1.b
% Up (i×Ns): past inputs
% Uf (j×Ns): future inputs
% Yf (j×Ns): future outputs
% Ns = N_est - i - j + 1 → number of shifted data windows.

```

## 1.d Input Projection

```

W      = [Up;Uf];
[Qw,~] = qr(W.',0); % thin QR of W^T
P      = eye(Ns) - Qw*Qw.'; % orthogonal complement
projector
Yfp    = Yf * P; % input-projected output

```

## 1.e SVD of Yf\*P

```

[Uyy,Syy,~] = svd(Yfp, 'econ');
sv      = diag(Syy);
n       = 2; % known true order (A is 2×2)
fprintf('SVD: chosen order n=%d', n, n);

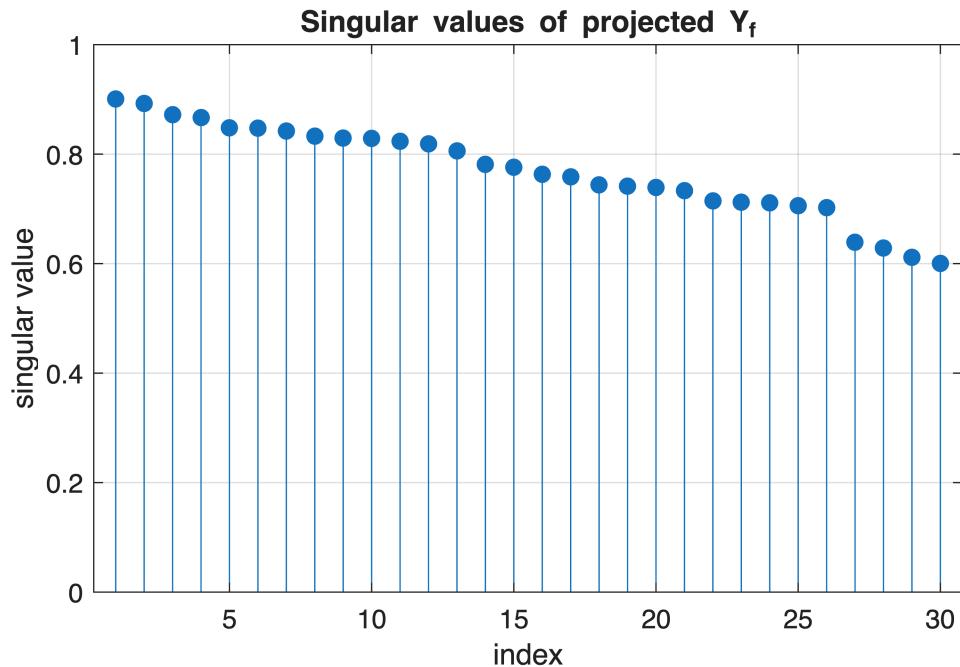
```

SVD: chosen order n=2SVD: chosen order n=2

```

figure; stem(1:min(30,numel(sv)), sv(1:min(30,numel(sv))), 'filled');
grid on; xlabel('index'); ylabel('singular value');
title('Singular values of projected Y_f');

```



### 1.f Extended Observability and State Sequence

```

Un    = Uyy(:,1:n);
Sn    = Syy(1:n,1:n);
Ohat = Un * sqrt(Sn);                                % extended observability matrix

[~,~,Vn] = svd(Yfp, 'econ');
Vn    = Vn(:,1:n);
Xhat = sqrt(Sn) * Vn.';                            % estimated state sequence

```

### 1.g Estimated $C^\wedge$ and $A^\wedge$

```

p      = size(C,1);                                % number of outputs
Chat   = Ohat(1:p,:);
Otop   = Ohat(1:end-p,:);
Obot   = Ohat(p+1:end,:);
Ahat   = Otop \ Obot;                             % least squares A^w
fprintf('eig(Ahat) = %s\n', mat2str(eig(Ahat).',4));

eig(Ahat) = [0.6628+0.7285i 0.6628-0.7285i]

```

### 1.h State Propagation and Averaging

```
% For each column l of Yf, the estimated state at time t_l = i+l is
Xhat(:,l).
```

```

% Each future prediction is propagated by Ahat, and all overlapping
estimates
% are averaged for smoother state trajectories.

nA      = n;
x_sum = zeros(nA, N_est);          % accumulated state estimates per time index
x_cnt = zeros(1,   N_est);         % number of estimates contributing to each
time index

% Precompute A^d

Apow = cell(j,1);
Apow{1} = eye(nA);
for d = 2:j
    Apow{d} = Apow{d-1} * Ahat;
end

% Sliding window accumulation

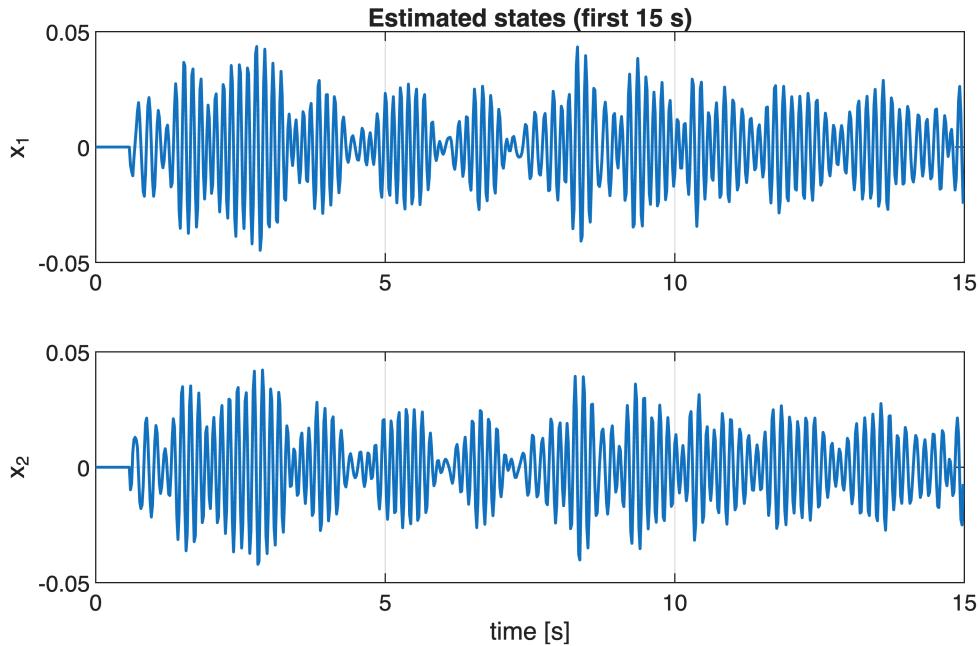
for l = 1:Ns
    tl = i + l;
    x0l = Xhat(:,l);
    for d = 0:(j-1)
        k = tl + d;
        if k <= N_est
            x_sum(:,k) = x_sum(:,k) + Apow{d+1} * x0l;
            x_cnt(k)   = x_cnt(k) + 1;
        end
    end
end

x_cnt(x_cnt==0) = 1;
xbar = x_sum ./ x_cnt;           % avg.

% Plot

figure;
subplot(2,1,1);
plot(t_est, xbar(1,1:N_est), 'LineWidth', 1.1);
grid on; ylabel('x_1');
title('Estimated states (first 15 s)');
subplot(2,1,2);
plot(t_est, xbar(2,1:N_est), 'LineWidth', 1.1);
grid on; xlabel('time [s]'); ylabel('x_2');

```



### 1.i Estimated $\mathbf{B}^{\wedge}$ and $\mathbf{D}^{\wedge}$

```

nA = size(Ahat,1);
L  = max(4*nA, 30);                      % horizon length
L  = min(L, N_est-1);                      % safety limit

% Precompute powers of A
Apow_t = cell(L,1); Apow_t{1} = eye(nA);
for tpow = 2:L, Apow_t{tpow} = Apow_t{tpow-1} * Ahat; end

Apow_k = cell(N_est,1); Apow_k{1} = eye(nA);
for k = 2:N_est, Apow_k{k} = Apow_k{k-1} * Ahat; end

% regression
K      = N_est - L;
Phi    = zeros(K, nA + 1 + nA);      % [B(n×1) , D(1×1) , x0(n×1)]
yy     = y_est(L+1:N_est);

for k = (L+1):N_est
    rowB = zeros(1,nA);
    for tconv = 0:(L-1)
        rowB = rowB + u_est(k-1-tconv) * (Chat * Apow_t{tconv+1});
    end
    colD = u_est(k);                  % D term
end

```

```

rowX0 = Chat * Apow_k{k};           % x0 term
idx = k - L;
Phi(idx, 1:nA)                      = rowB;
Phi(idx, nA+1)                      = colD;
Phi(idx, nA+2 : nA+1+nA)            = rowX0;
end

theta = Phi \ yy;                   % LS solution
Bhat = theta(1:nA);
Dhat = theta(nA+1);
x0hat = theta(nA+2:end);
fprintf('Bhat = %s | Dhat = %.4f\n', mat2str(Bhat.',4), Dhat);

Bhat = [-0.1917 0.2361] | Dhat = -0.3743

```

## 1.j FIT Evaluation (Estimation and Validation)

```

FIT = @(ytrue,yh) 100*(1 - norm(ytrue - yh)/norm(ytrue - mean(ytrue)));

% Estimation

x_est = zeros(nA, N_est);
x_est(:,1) = x0hat;
yhat_est = zeros(N_est,1);
for k = 1:N_est-1
    yhat_est(k) = Chat*x_est(:,k) + Dhat*u_est(k);
    x_est(:,k+1) = Ahat*x_est(:,k) + Bhat*u_est(k);
end
yhat_est(N_est) = Chat*x_est(:,N_est) + Dhat*u_est(N_est);

fit_est_y0 = FIT(y0_est, yhat_est);
fit_est_y = FIT(y_est, yhat_est);
fprintf('Estimation FIT: vs y0 = %.2f% | vs noisy y = %.2f%\n',
fit_est_y0, fit_est_y);

```

Estimation FIT: vs y0 = 35.15% | vs noisy y = 34.76%

```

% Validation

N_val = numel(y_val);
h = min(40, N_val);
Phi_x0 = zeros(h, nA); gamma = zeros(h,1);
for k = 1:h
    Akm1 = eye(nA);
    for p = 1:k-1, Akm1 = Akm1 * Ahat; end
    Phi_x0(k,:) = Chat * Akm1;

```

```

s = Dhat*u_val(k);
for tau = 0:k-2
    Aexp = eye(nA);
    for q = 1:(k-2-tau), Aexp = Aexp * Ahat; end
    s = s + (Chat * Aexp * Bhat) * u_val(tau+1);
end
gamma(k) = s;
end
x0_val = Phi_x0 \ (y_val(1:h) - gamma);

```

% Validation free run sim

```

xv = zeros(nA,N_val); yhat_val = zeros(N_val,1);
xv(:,1) = x0_val;
for k = 1:N_val-1
    yhat_val(k) = Chat*xv(:,k) + Dhat*u_val(k);
    xv(:,k+1) = Ahat*xv(:,k) + Bhat*u_val(k);
end
yhat_val(N_val) = Chat*xv(:,N_val) + Dhat*u_val(N_val);

fit_val_y0 = FIT(y0_val, yhat_val);
fit_val_y = FIT(y_val, yhat_val);
fprintf('Validation FIT: vs y0 = %.2f%% | vs noisy y = %.2f%%\n',
fit_val_y0, fit_val_y);

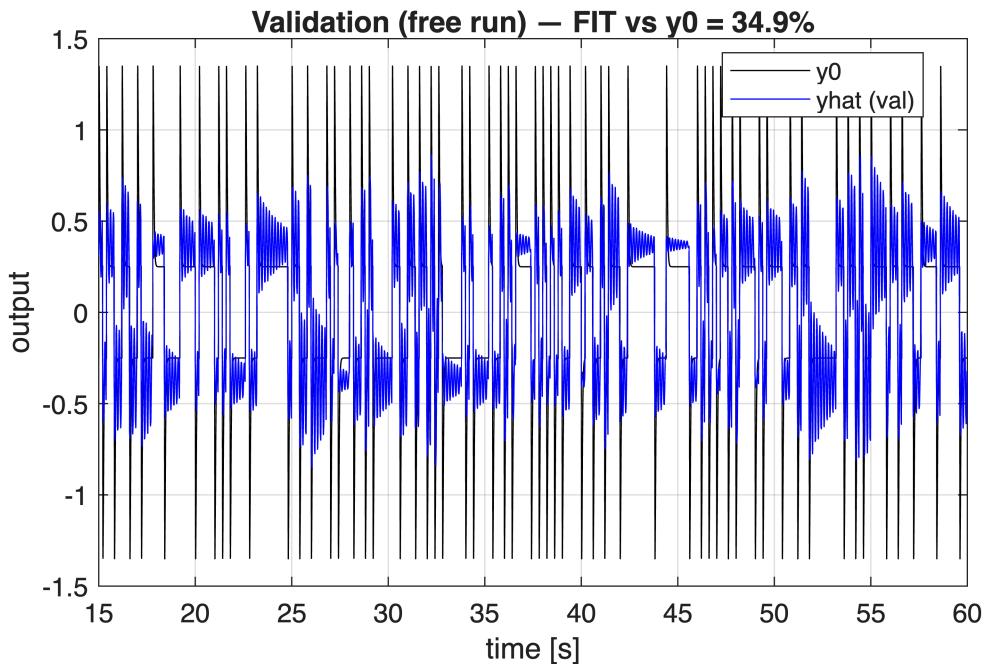
```

Validation FIT: vs y0 = 34.90% | vs noisy y = 34.79%

```

figure;
plot(t_val, y0_val,'k-', 'DisplayName','y0'); hold on;
plot(t_val, yhat_val,'b-', 'DisplayName','yhat (val)'); grid on;
legend('Location','best'); xlabel('time [s]'); ylabel('output');
title(sprintf('Validation (free run) - FIT vs y0 = %.1f%%', fit_val_y0));

```



### 1.k Comparison with n4sid (same estimation data)

```

data_est = iddata(y_est, u_est, Ts);
sys_n4  = n4sid(data_est, 2, ...
                  'Feedthrough', false, ...
                  'DisturbanceModel','none', ...
                  'Focus','simulation');

yhat_n4_est = lsim(sys_n4, u_est, t_est);
yhat_n4_val = lsim(sys_n4, u_val, t_val);

fit_n4_est  = FIT(y_est, yhat_n4_est);
fit_n4_val  = FIT(y_val, yhat_n4_val);
fprintf('Manual N4SID: Est %.2f% | Val %.2f%\nMATLAB n4sid: Est %.2f% | 
Val %.2f%\n',...
       fit_est_y, fit_val_y, fit_n4_est, fit_n4_val);

```

Manual N4SID: Est 34.76% | Val 34.79%  
MATLAB n4sid: Est 94.04% | Val 92.99%