ME 418/518 Data-Based Control

Midterm Exam

Deadline – 18 November 2025, 11.59pm.

Using large language models or any AI assistant or code-generation tools is strictly prohibited, unless they are permitted in the problem statement. You must solve the questions and write your report entirely on your own. Collaboration with other students is not allowed. Submit your solutions as a single pdf file, where the related code must be in the appendix. Also submit your code to Moodle as a single zip file. Name your MATLAB m files using the problem number. Your code must be error-free and create the required plots. Make sure that you add comments to your code explaining each step of the solution.

Please sign the following statement:

I confirm that I completed this exam completely on my own, without any use of large language models, AI assistants, or code-generation tools.

Name: Erinç Ada Ceylan
Date: 18.11.2025
Signature:

Problem 1 (25pts). Consider the plant transfer function $\frac{Y(s)}{U(s)} = \frac{5}{s^2+1.2s+5}$. The sampling interval is T=0.01s. The experiment will last 40 seconds, therefore you will obtain N=4000 samples. The control input will be fed to the plant using ZOH. The sensor dynamics is given as $G_s(s) = \frac{1}{0.15s+1}$. Sensor bias is -0.25. Measurement noise is zero-mean Gaussian with s.d. $\sigma = 0.05$ (added after sampling). Insert 25 random spikes to the sampled output. Use a PRBS input u[k] with range [-1,1], with dwell time 0.2 seconds.

For all the questions below, show your code in the appendix of the report, and submit to Moodle.

3.1. Create u[k] and u(t) (using ZOH). Create y[k]. Clearly show these signals in separate figures in your report.

3.2. For this problem, you will clean your data. <u>At the end, show raw and cleaned data in the same plot in your report.</u>

a) First detect, then replace the spikes in y[k] by interpolation. Explain your spike detection method.

b) Detect the bias (you can use zero input for a while for this) and remove it.

c) Filter the noise using a moving average. Explain your method clearly, including the window size. (use the same filter for the input when you identify models in the next problem)

Problem 2 (25pts). In this problem, you will create an FIR model using the cleaned data in the previous problem. You can use MATLAB functions except direct FIR functions. Do not forget to apply to the inputs the same filter you used for the output in the previous problem.

The Model: $\hat{y}[k] = \sum_{1}^{n_b} b_i u[k-i] = \Phi[k]^T b, \;\; \Phi[k] = [u[k],\, u[k-1], .., \; u[k-n_b]]^T$

a) Split the data into two sets: Training (first 35 seconds) and Validation (last 15 seconds). Estimate the model parameters for $n_b = 40$ and $n_b = 120$.

b) Using the validation data generate $\hat{y}[k]$ for each model and plot $\hat{y}[k]$ and the measured $y[k]$ on the same plot. Notice that you will create two plots, one for each model.

c) Using the validation data, report RMSE and fit percentage for both models.

Problem 3 (25pts). Using the same input-output data and the data split, obtain an ARX model. Try 3 different pairs (n_a, n_b), where n_a is number of output history and n_b is the number of input history, and determine the best pair based on the FIT percentage.

Problem 4 (25pts). Consider the following plant dynamics

$x_{k+1} = 0.9x_k + 2u_k, \;\; y_k = x_k.$

We know that the actuator has the following saturation limits: $|u| < 0.55, \;\; |\Delta u| < 0.3$

a) Design an MPC controller to track a step reference of magnitude 2. You are free to choose your prediction and control horizons. Assuming an 0.05 sampling interval, simulate the system for 6 seconds and show the tracking and control signal curves in separate figures. (Do not use MPC toolbox in MATLAB. You can use "quadprog".)

b) Add measurement noise to your system and repeat "a)". You are free to choose the noise structure. Make sure that the noise amplitude you choose makes the noise visible in the plots. Discuss the results by comparing them with "a)".

c) Repeat "b)" with the same noise structure but this time the reference is a sine wave given as $r[k] = \sin\left(\frac{0.1\pi}{3}k\right)$.

ME 418/518 Data-Based Control
Midterm Exam Report

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November 18, 2025

Problem 1 – Data Generation and Cleaning

A PRBS input with dwell time of 0.2 seconds was generated and applied to the continuous-time plant using a zero-order hold. The experiment duration was 40 seconds, producing 4000 samples. The plant output was passed through the given sensor dynamics, and bias, Gaussian noise, and 25 random spikes were added. The raw signals exhibit clear noise and spikes. Spike detection was performed using a Median Absolute Deviation (MAD)–based thresholding. Detected spikes were replaced via linear interpolation. Sensor bias was estimated from the initial zero-input segment and removed. A moving-average filter with window size $M = 5$ was applied to both $u[k]$ and $y[k]$ to reduce noise without distorting the dynamics.

Figures included show:

- PRBS input $u[k]$ and ZOH signal $u(t)$,
- Raw output $y[k]$ with bias, noise, and spikes,
- Final cleaned output after spike removal, bias correction, and filtering.

Problem 2 – FIR Model Identification

The cleaned dataset was split into a 25-second training set and a 15-second validation set. Two FIR models were identified with $n_b = 40$ and $n_b = 120$. The smaller model ($n_b = 40$) fails to capture system dynamics and produces poor validation performance. Increasing the model order to $n_b = 120$ significantly improves the prediction quality.

Validation metrics obtained:

- RMSE improves from 0.6163 to 0.2726,
- Fit improves from 3.61% to 57.37%.

Both validation plots are included in the report.

Problem 3 – ARX Model Identification

Using the same training/validation split, three ARX models were tested with different (n_a, n_b) values:

$$(1, 40),\ (2, 40),\ (2, 80)$$

All ARX models perform significantly better than FIR due to the inclusion of output history. The best validation fit was obtained with:

$$(n_a,\ n_b) = (2,\ 80)$$

achieving approximately 97% fit.

Validation plots for all three structures are included.

Problem 4 – Model Predictive Control

The discrete-time plant

$$x_{k+1} = 0.9x_k + 2u_k, \qquad y_k = x_k$$

was controlled using MPC with constraints

$$|u_k| \leq 0.55, \qquad |\Delta u_k| \leq 0.30.$$

Prediction and control horizons were chosen as $N_p = 8$, $N_c = 4$. Quadratic programs were solved using `quadprog`.

(a) Step Reference Tracking

The MPC controller successfully tracks the reference of magnitude 2 within one second. The control signal respects the saturation and rate constraints. Plots of the tracking performance and control input are included.

(b) Tracking with Measurement Noise

Zero-mean Gaussian measurement noise was added. The tracking performance remains stable, but the control signal becomes more active due to noisy feedback, as expected. Plots of the noisy output and corresponding control input are included.

(c) Sine-Wave Reference Tracking

The same noise structure was applied while tracking

$$r[k] = \sin\left(\frac{0.1\pi}{3}k\right).$$

The controller successfully follows the sinusoidal reference while satisfying constraints. Tracking, control input, and predicted outputs are included.

Appendix

The following appendix contains:

- All MATLAB code for Problems 1–4,
- All figures required in the exam,

```
close all hidden;
clearvars -except;
clc;

rng(418);                           %seed for reproducibility
```

PROBLEM 1 (25 Pts.)

1.1 Data generation

```
Tsampling = 0.01;
Tfinal = 40;
t = 0:Tsampling:(Tfinal - Tsampling);     %to produce exactly N=4000 samples
N = numel(t);

Tzero = 2;                                % 2 seconds zero input
(segmentation for bias estimation)
Nz = round(Tzero / Tsampling);            % number of zero samples
```

PRBS input

```
dwell = 0.2;
L = round(dwell / Tsampling);
nLev = ceil((N - Nz) / L);                %PRBS only after zero segment

%first producing levels
lev = zeros(nLev, 1);
for k = 1:nLev
    if rand > 0.5
        lev(k) = 1;
    else
        lev(k) = -1;
    end
end

%then dwelling them for the input
u_prbs = zeros(N - Nz, 1);
idx = 1;
for k = 1:nLev
    for j = 1:L
        if idx > (N - Nz)
            break;
        end
        u_prbs(idx) = lev(k);
        idx = idx + 1;
    end
end

u = [zeros(Nz,1); u_prbs];                 % zero input + PRBS
```

```
u_t = u;                                        % ZOH-held input
```

Plant & Sensor

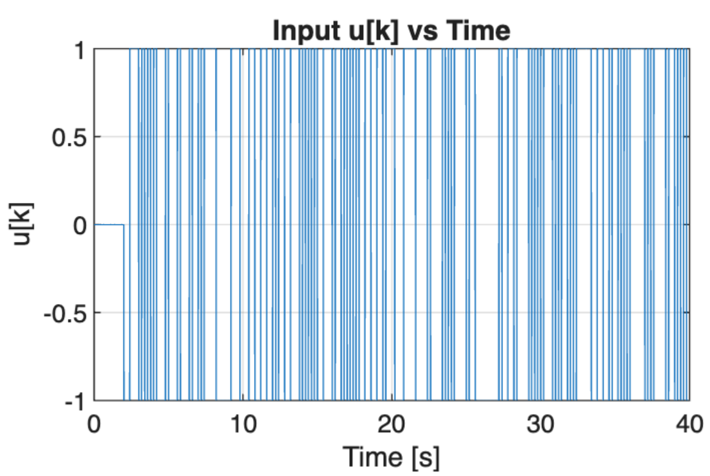
```
s  = tf('s');
Gp = 5/(s^2 + 1.2*s + 5);                                            %plant
transfer function
Gs = 1/(0.15*s + 1);                                                 %sensor
dynamics
y_plant = lsim(Gp, u, t, 'zoh');                                     %defining
"zoh" at lsim for both plant and the sensor
y_sens  = lsim(Gs, y_plant, t, 'zoh');
```

Bias, noise, spikes

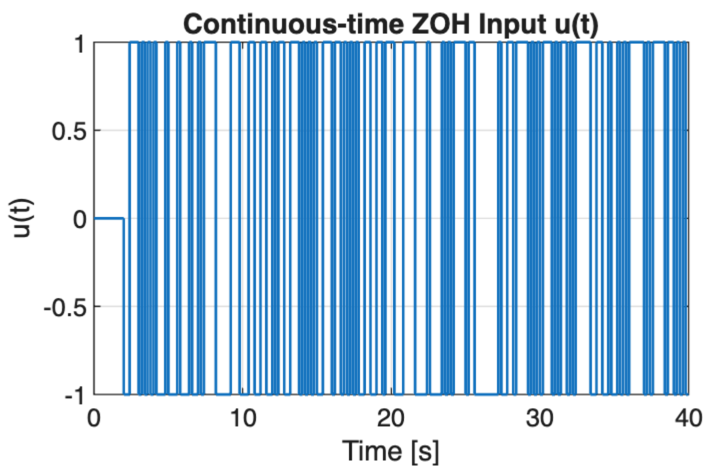
```
bias  = -0.25;                                                        %sensor bias
sigma = 0.05;                                                         %zero-mean
gaussian for measurement noise
y_raw = y_sens + bias +
sigma*randn(size(y_sens));
idx_spike = randsample((Nz+2):(N-1), 25);                             %spikes
only in PRBS region
y_raw(idx_spike) = y_raw(idx_spike) + 0.5 * sign(randn(numel(idx_spike),
1));    %adding twenty five 0.5 magnitude spikes
```

Plots

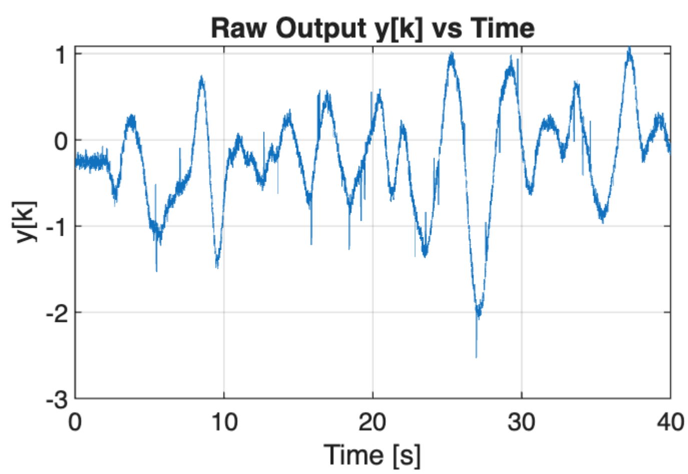
```
figure;
plot(t, u); grid on; xlabel('Time [s]'); ylabel('u[k]');    %u[k] input
plotting
title('Input u[k] vs Time');
```




```


figure;
stairs(t, u_t, 'LineWidth', 1); grid on;                         %u(t) ZOH plot
xlabel('Time [s]'); ylabel('u(t)');
title('Continuous-time ZOH Input u(t)');
```




```
figure;
plot(t, y_raw); grid on; xlabel('Time [s]'); ylabel('y[k]'); %raw y[k]
plotting with bias, noise, spikes
title('Raw Output y[k] vs Time');
```



1.2 Data cleaning

(a) spike

We detect spikes using the Median Absolute Deviation method,a any sample that deviates more than 4*mean_abs_dev from the median is considered a spike.

```
y = y_raw;
spikes = false(size(y));
med = median(y);
mad_val = median(abs(y - med));

for i = (Nz+1):N                          % do not detect spikes inside
the zero-input segment
    if abs(y(i) - med) > 4 * mad_val
        spikes(i) = true;
    end
```

```
end

% replace spikes by NaN (marking them for interpolation)
y(spikes) = NaN;

% linear interpolation
for i = 2:N-1
    if isnan(y(i))
        y(i) = (y(i-1) + y(i+1)) / 2;
    end
end
```

(b) Bias

```
bias_val = mean(y(1:Nz));                         %proper bias estimation
y = y - bias_val;
fprintf('Removed bias: %.4f\n', bias_val);
```

```
Removed bias: -0.2530
```

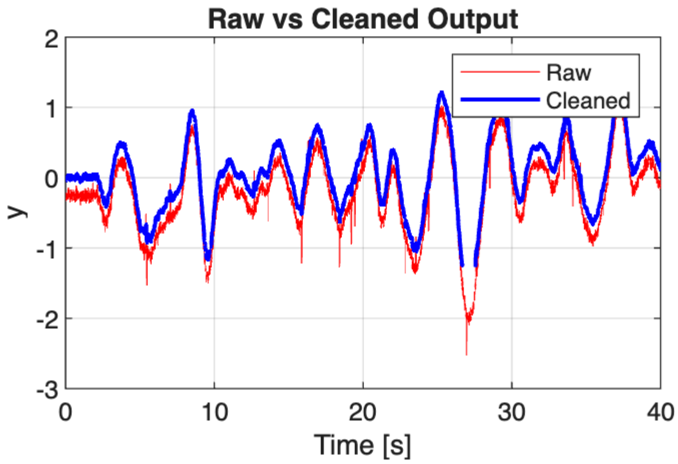
(c) Moving average

To reduce measurement noise, we apply a moving average filter. For each sample, movmean replaces y[k] with the average of the previous M samples. We use M = 5, which smooths the noise without distorting the dynamics. The same filter is also applied to the input signal because filtered input will be used in the next problem.

```
M = 5;                                        % number of steps used
y_clean = movmean(y, M);                      % used built-in func. instead of
hardcoding
u_filter = movmean(u, M);
```

Raw and Clean Plot

```
figure;
plot(t, y_raw, 'r'); hold on;
plot(t, y_clean, 'b', 'LineWidth', 1.5);
xlabel('Time [s]'); ylabel('y');
legend('Raw','Cleaned');
title('Raw vs Cleaned Output');
grid on;
```



PROBLEM 2 (25 Pts.)

```
% Consequent NAN's may occure (which our hardcoded linear interpolation is
not available to overcome).
% Because spike detection is hardcoded (not with the built-in MATLAB
function) this minor error is inevitable.
% Remove remaining NaNs from filtered signals, at spike interpolation (with
built-in function)

u_filter = fillmissing(u_filter, 'linear');
y_clean  = fillmissing(y_clean, 'linear');
```

(a) Split data into training (first 25 s) and validation (last 15 s) , estimate FIR parameters for nb = 40 and nb = 120

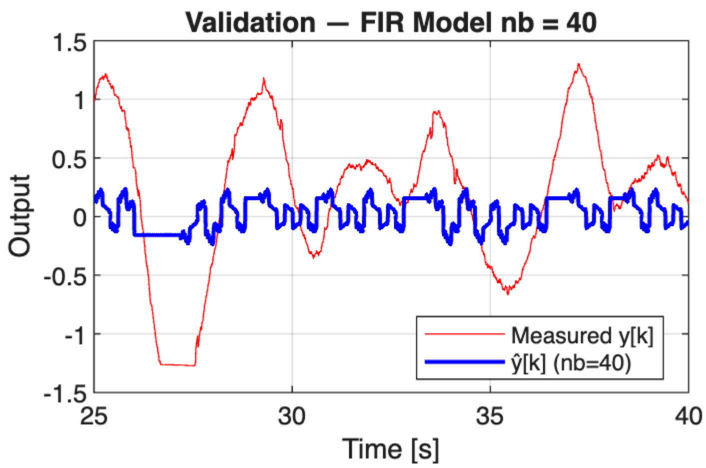
```
% Split data
train_end = round(25 / Tsampling);  % 25 seconds - 2500 sample
train_u = u_filter(1:train_end);
train_y = y_clean(1:train_end);

val_u   = u_filter(train_end+1 : end);
val_y   = y_clean(train_end+1 : end);
t_val   = t(train_end+1 : end);


%nb = 40
nb1 = 40;
Ntr = length(train_u);

Phi40 = zeros(Ntr-nb1, nb1);
y40   = zeros(Ntr-nb1, 1);

for k = nb1+1:Ntr
    for i = 1:nb1
        Phi40(k-nb1, i) = train_u(k - i);
    end
```

```
    y40(k-nb1) = train_y(k);
end

b40 = Phi40 \ y40;


%nb = 120
nb2 = 120;
Phi120 = zeros(Ntr-nb2, nb2);
y120   = zeros(Ntr-nb2, 1);

for k = nb2+1:Ntr
    for i = 1:nb2
        Phi120(k-nb2, i) = train_u(k - i);
    end
    y120(k-nb2) = train_y(k);
end

b120 = Phi120 \ y120;
```

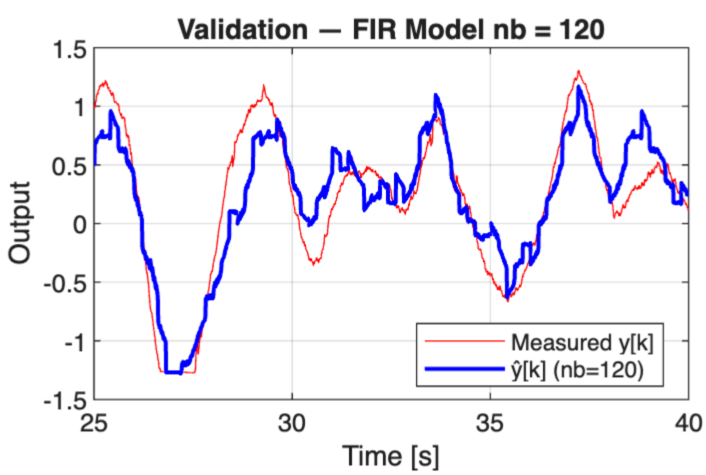
(b) Prediction on validation set

```
% Full length filtered input from problem 1
Nu = length(u_filter);

% Prediction nb = 40
yhat40_full = zeros(Nu,1);
for k = 1:Nu
    s = 0;
    for i = 1:nb1
        if (k-i) >= 1
            s = s + b40(i) * u_filter(k-i);
        end
    end
    yhat40_full(k) = s;
end

% Prediction nb = 120
yhat120_full = zeros(Nu,1);
for k = 1:Nu
    s = 0;
    for i = 1:nb2
        if (k-i) >= 1
            s = s + b120(i) * u_filter(k-i);
        end
    end
    yhat120_full(k) = s;
end

% Extract validation portion
```

```
yhat40  = yhat40_full(train_end+1:end);
yhat120 = yhat120_full(train_end+1:end);

% Plot nb = 40
figure;
plot(t_val, val_y, 'r'); hold on;
plot(t_val, yhat40, 'b', 'LineWidth', 1.5);
xlabel('Time [s]'); ylabel('Output');
legend('Measured y[k]', 'ŷ[k] (nb=40)', 'Location', 'best');
title('Validation — FIR Model nb = 40');
grid on;
```




```

% Plot nb = 120
figure;
plot(t_val, val_y, 'r'); hold on;
plot(t_val, yhat120, 'b', 'LineWidth', 1.5);
xlabel('Time [s]'); ylabel('Output');
legend('Measured y[k]', 'ŷ[k] (nb=120)', 'Location', 'best');
title('Validation — FIR Model nb = 120');
grid on;
```



(c) RMSE and Fit

```
rmse40  = sqrt(mean((val_y - yhat40 ).^2));
rmse120 = sqrt(mean((val_y - yhat120).^2));

fit40  = (1 - norm(val_y - yhat40 )/norm(val_y - mean(val_y))) * 100;
fit120 = (1 - norm(val_y - yhat120)/norm(val_y - mean(val_y))) * 100;

fprintf('\n VALIDATION METRICS \n');
```

```
 VALIDATION METRICS
```

```
fprintf('nb =  40 → RMSE = %.4f, Fit = %.2f %%\n', rmse40,  fit40);
```

```
nb =  40 → RMSE = 0.6163, Fit = 3.61 %
```

```
fprintf('nb = 120 → RMSE = %.4f, Fit = %.2f %%\n', rmse120, fit120);
```

```
nb = 120 → RMSE = 0.2726, Fit = 57.37 %
```

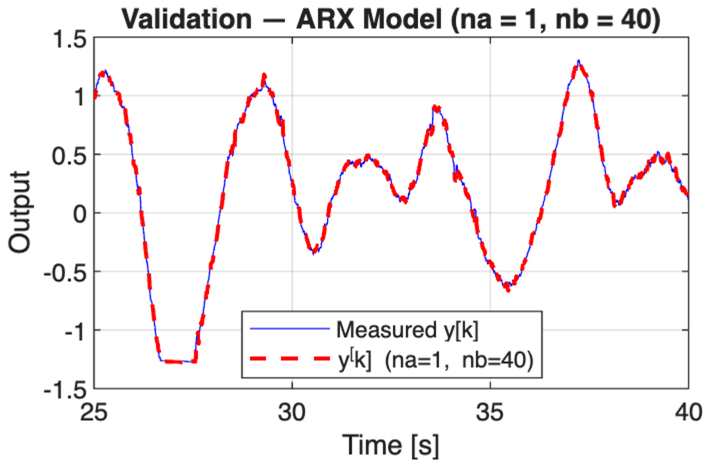
PROBLEM 3 (25 Pts.)

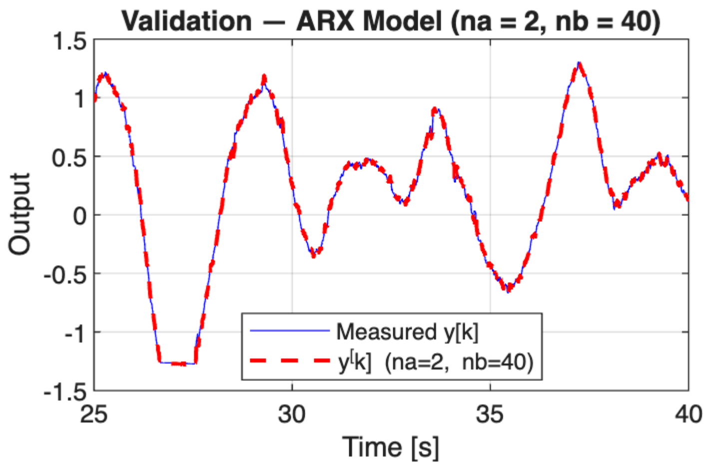
```
% Using the same data from problem 2

arx_pairs = [1 40;      % Model 1: short output history, same nb as FIR for
comparision
             2 40;      % Model 2: more poles, same nb
             2 80];     % Model 3: more poles and longer input history

numModels = 3
```

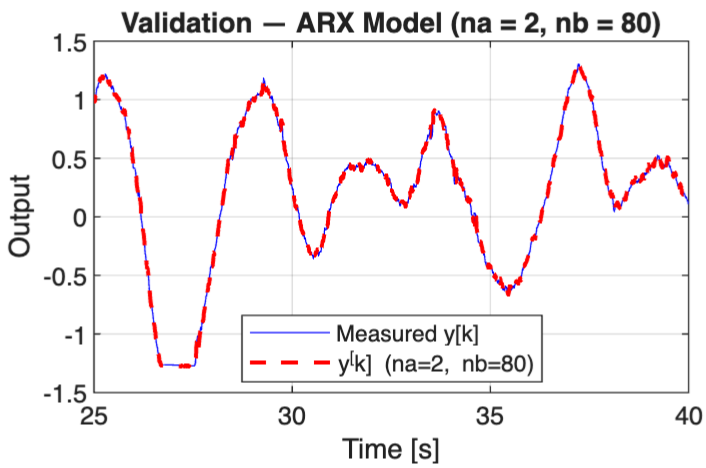
```
numModels =
3
```

```
rmse_arx  = zeros(numModels, 1);
fit_arx   = zeros(numModels, 1);

Nu_total  = length(u_filter);
Ntr       = length(train_u);    % number of training samples

for m = 1:numModels

    na = arx_pairs(m, 1);       % number of past outputs
    nb = arx_pairs(m, 2);       % number of past inputs

    % First usable index computation (need at least n_a past outputs and nb
past inputs)
    k0  = max(na, nb) + 1;
    Ktr = Ntr - k0 + 1;         % number of training rrows

    % Build regression matrix and output vector y_arx on training data
    Phi   = zeros(Ktr, na + nb);
    y_arx = zeros(Ktr, 1);
```

```matlab
    for k = k0:Ntr
        % past outputs
        Phi(k-k0+1, 1:na) = -train_y(k-1:-1:k-na).';

        % past inputs
        Phi(k-k0+1, na+1:end) = train_u(k-1:-1:k-nb).';

        % current output
        y_arx(k-k0+1) = train_y(k);
    end

    % Least-squares estimate (theta)
    theta = Phi \ y_arx;
    a_arx = theta(1:na);
    b_arx = theta(na+1:end);

    % One step ahead prediction, just like in the lecture sldies
    yhat_full = zeros(Nu_total, 1);

    for k = k0:Nu_total
        y_past = y_clean(k-1:-1:k-na);
        u_past = u_filter(k-1:-1:k-nb);

        % y^[k]
        yhat_full(k) = -a_arx.' * y_past + b_arx.' * u_past;
    end

    % Take only validation part
    yhat_val = yhat_full(train_end+1:end);

    % Compute RMSE and FIT on validation data
    rmse_arx(m) = sqrt(mean((val_y - yhat_val).^2));
    fit_arx(m)  = (1 - norm(val_y - yhat_val) / norm(val_y - mean(val_y)))
* 100;

    % plot for each na,nb pair
    figure;
    plot(t_val, val_y, 'b'); hold on;
    plot(t_val, yhat_val, 'r--', 'LineWidth', 1.5);

    xlabel('Time [s]'); ylabel('Output');
    legend('Measured y[k]', ...
           sprintf('y^[k] (na=%d, nb=%d)', na, nb), ...
           'Location', 'best');
    title(sprintf('Validation - ARX Model (na = %d, nb = %d)', na, nb));
    grid on;

end
```








```
% Compare models
[bestFit, bestIdx] = max(fit_arx);
best_na = arx_pairs(bestIdx, 1);
best_nb = arx_pairs(bestIdx, 2);

fprintf('\n ARX Model Comparision (validation set)\n');
```

```
ARX Model Comparision (validation set)
```

```
for m = 1:numModels
```

```
    fprintf('(na,nb) = (%2d, %3d)   RMSE = %.4f, Fit = %.2f %%\n', ...
        arx_pairs(m,1), arx_pairs(m,2), rmse_arx(m), fit_arx(m));
end
```

```
(na,nb) = ( 1,  40)   RMSE = 0.0194, Fit = 96.97 %
(na,nb) = ( 2,  40)   RMSE = 0.0189, Fit = 97.05 %
(na,nb) = ( 2,  80)   RMSE = 0.0189, Fit = 97.05 %
```

```
fprintf('Best pair based on validation FIT: (na, nb) = (%d, %d), Fit = %.2f
%%\n', ...
        best_na, best_nb, bestFit);
```

```
Best pair based on validation FIT: (na, nb) = (2, 80), Fit = 97.05 %
```

PROBLEM 4 (25 Pts.)

Problem setup

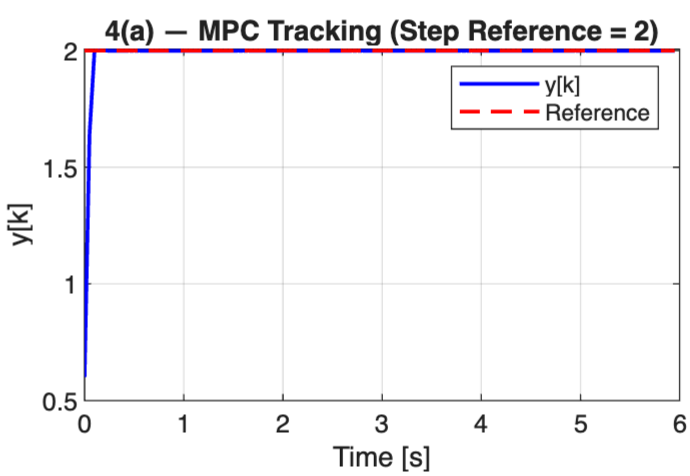
```
% Plant dynamics: x[k+1] = 0.9 x[k] + 2 u[k],  y[k]   = x[k]
A = 0.9;
B = 2.0;
C = 1.0;
T = 0.05;                 % sample time
Tf = 6;                   % simulation time
Nsim = round(Tf/T);


Np = 8;       % MPC prediction horizon
Nc = 4;       % MPC control horizon

Q = 1;        % Cost weights
R = 0.1;      % Cost weights


u_max  = 0.55; % Constraints
du_max = 0.30; % Constraints


F = zeros(Np,1); % F vector
Ap = A;
for i = 1:Np
    F(i) = C * Ap;
    Ap = Ap * A;
end

H = zeros(Np, Nc); % H matrix
for i = 1:Np
    for j = 1:Nc
        if j <= i
            Aexp = 1;
            for k = 1:(i-j)
                Aexp = Aexp * A;
```

```
            end
            H(i,j) = C * (Aexp * B);
        end
    end
end

% S matrix
S = tril(ones(Nc));

% Phi matrix
Phi = H * S;

Qbar = Q * eye(Np);
Rbar = R * eye(Nc);

options = optimoptions('quadprog','Display','off');
```

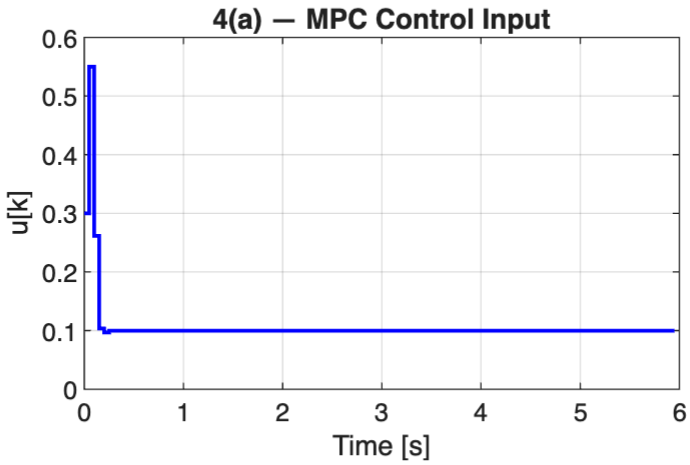
(a) Design MPC, reference is step with magnitude 2

```
r_step = 2;

x = 0;          % state
u_prev = 0;     % previous control

% For 4(d): store first predicted output y_hat[k+1|k]
yhat_hist_a = zeros(Nsim,1);
yhat_hist_b = zeros(Nsim,1);
yhat_hist_c = zeros(Nsim,1);

u_hist_a = zeros(Nsim,1);
ref_hist_a = zeros(Nsim,1);

for k = 1:Nsim

    % Reference over prediction horizon
    r_vec = r_step * ones(Np,1);

    % b_k
    u_vec = u_prev * ones(Nc,1);
    b_k = F*x + H*u_vec;

    % QP
    Hqp = Phi.' * Qbar * Phi + Rbar;
    fqp = Phi.' * Qbar * (b_k - r_vec);

    % Constraints
    A_rate = [ eye(Nc); -eye(Nc) ];
    b_rate = [ du_max*ones(Nc,1); du_max*ones(Nc,1) ];

    A_u = [ S; -S ];
```

```
    b_u = [ u_max*ones(Nc,1) - u_vec;
            u_max*ones(Nc,1) + u_vec ];

    Aineq = [A_rate; A_u];
    bineq = [b_rate; b_u];

    % Solve
    dU = quadprog(Hqp, fqp, Aineq, bineq, [], [], [], [], [], options);

    % Predicted output horizon for case (a)
    Ypred_a = b_k + Phi*dU;
    yhat_hist_a(k) = Ypred_a(1);     % one-step ahead prediction

    du = dU(1);
    u = u_prev + du;

    % Plant update
    x = A*x + B*u;
    y = x;

    % Log
    y_hist_a(k) = y;
    u_hist_a(k) = u;
    ref_hist_a(k) = r_step;

    u_prev = u;
end

t_mpc = (0:Nsim-1)*T;

figure;
plot(t_mpc, y_hist_a,'b','LineWidth',1.4); hold on;
plot(t_mpc, ref_hist_a,'r--','LineWidth',1.4);
xlabel('Time [s]'); ylabel('y[k]');
title('4(a) - MPC Tracking (Step Reference = 2)');
legend('y[k]','Reference'); grid on;
```



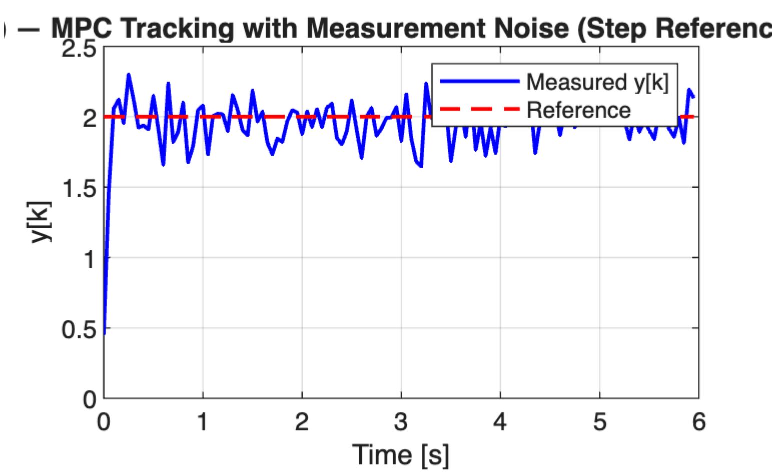
```
figure;
stairs(t_mpc, u_hist_a,'b','LineWidth',1.4);
xlabel('Time [s]'); ylabel('u[k]');
title('4(a) — MPC Control Input');
grid on;
```



(b) Part a controller with measurement noise

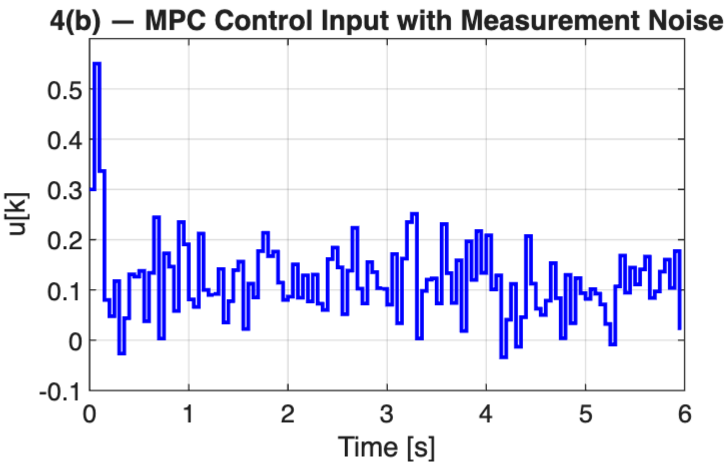
```
noise_amp = 0.15;   % Visible amplitude

x = 0;
u_prev = 0;

y_hist_b = zeros(Nsim,1);
u_hist_b = zeros(Nsim,1);
ref_hist_b = zeros(Nsim,1);

for k = 1:Nsim

    r_vec = r_step * ones(Np,1);

    u_vec = u_prev * ones(Nc,1);
    b_k = F*x + H*u_vec;

    Hqp = Phi.'*Qbar*Phi + Rbar;
    fqp = Phi.'*Qbar*(b_k - r_vec);

    A_rate = [ eye(Nc); -eye(Nc) ];
    b_rate = [ du_max*ones(Nc,1); du_max*ones(Nc,1) ];

    A_u = [ S; -S ];
    b_u = [ u_max*ones(Nc,1) - u_vec;
            u_max*ones(Nc,1) + u_vec ];

    Aineq = [A_rate; A_u];
```

```
    bineq = [b_rate; b_u];

    dU = quadprog(Hqp, fqp, Aineq, bineq, [], [], [], [], [], options);

    % Predicted output horizon for case (b)
    Ypred_b = b_k + Phi*dU;
    yhat_hist_b(k) = Ypred_b(1);


    du = dU(1);
    u = u_prev + du;

    % True plant
    x_true = A*x + B*u;

    % Measurement
    y_meas = x_true + noise_amp*randn;

    % The controller uses y_meas as state estimate
    x = y_meas;

    y_hist_b(k) = y_meas;
    u_hist_b(k) = u;
    ref_hist_b(k) = r_step;

    u_prev = u;
end

figure;
plot(t_mpc, y_hist_b,'b','LineWidth',1.4); hold on;
plot(t_mpc, ref_hist_b,'r--','LineWidth',1.4);
xlabel('Time [s]'); ylabel('y[k]');
title('4(b) — MPC Tracking with Measurement Noise (Step Reference = 2)');
legend('Measured y[k]','Reference'); grid on;
```




```
figure;
```

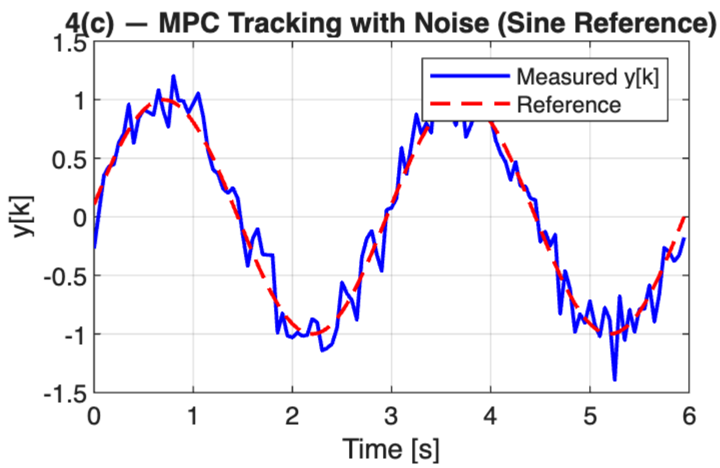
```
stairs(t_mpc, u_hist_b,'b','LineWidth',1.4);
xlabel('Time [s]'); ylabel('u[k]');
title('4(b) — MPC Control Input with Measurement Noise');
grid on;
```



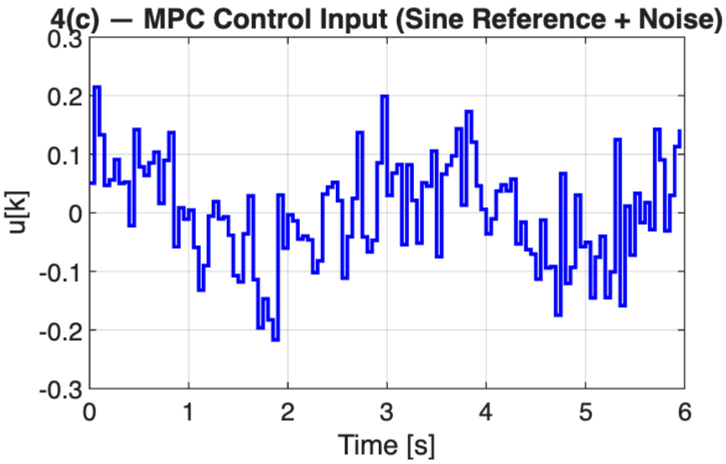
(c) Same construct at part b but reference is a sine wave r[k] = sin(0.1*pi/3 * k)

```
x = 0;
u_prev = 0;

y_hist_c = zeros(Nsim,1);
u_hist_c = zeros(Nsim,1);
ref_hist_c = zeros(Nsim,1);

for k = 1:Nsim

    % Sine reference Np steps ahead
    ref_horizon = sin( (0.1*pi/3) * (k : k+Np-1) ).';

    u_vec = u_prev * ones(Nc,1);
    b_k = F*x + H*u_vec;

    Hqp = Phi.'*Qbar*Phi + Rbar;
    fqp = Phi.'*Qbar*(b_k - ref_horizon);

    A_rate = [ eye(Nc); -eye(Nc) ];
    b_rate = [ du_max*ones(Nc,1); du_max*ones(Nc,1) ];

    A_u = [ S; -S ];
    b_u = [ u_max*ones(Nc,1) - u_vec;
            u_max*ones(Nc,1) + u_vec ];

    Aineq = [A_rate; A_u];
    bineq = [b_rate; b_u];

    dU = quadprog(Hqp, fqp, Aineq, bineq, [], [], [], [], [], options);
```

```
    % Predicted output horizon for case (c)
    Ypred_c = b_k + Phi*dU;
    yhat_hist_c(k) = Ypred_c(1);

    du = dU(1);
    u = u_prev + du;

    x_true = A*x + B*u;
    y_meas = x_true + noise_amp*randn;

    x = y_meas;

    y_hist_c(k) = y_meas;
    u_hist_c(k) = u;
    ref_hist_c(k) = ref_horizon(1);

    u_prev = u;
end

figure;
plot(t_mpc, y_hist_c,'b','LineWidth',1.4); hold on;
plot(t_mpc, ref_hist_c,'r--','LineWidth',1.4);
xlabel('Time [s]'); ylabel('y[k]');
title('4(c) — MPC Tracking with Noise (Sine Reference)');
legend('Measured y[k]','Reference'); grid on;
```

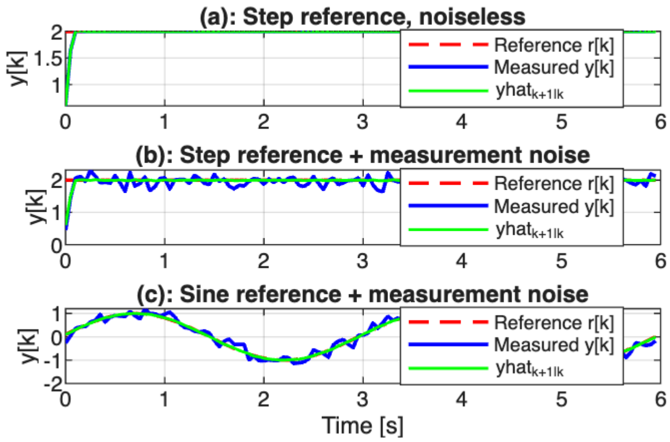



```
figure;
stairs(t_mpc, u_hist_c,'b','LineWidth',1.4);
xlabel('Time [s]'); ylabel('u[k]');
title('4(c) — MPC Control Input (Sine Reference + Noise)');
grid on;
```



(d) Reference tracking vs predicted output

```
figure;

subplot(3,1,1);
plot(t_mpc, ref_hist_a, 'r--', 'LineWidth', 1.3); hold on;
plot(t_mpc, y_hist_a,   'b',   'LineWidth', 1.4);
plot(t_mpc, yhat_hist_a,'g',   'LineWidth', 1.1);
grid on;
title('(a): Step reference, noiseless');
ylabel('y[k]');
legend('Reference r[k]', 'Measured y[k]', 'yhat_{k+1|k}', 'Location',
'best');

subplot(3,1,2);
plot(t_mpc, ref_hist_b, 'r--', 'LineWidth', 1.3); hold on;
plot(t_mpc, y_hist_b,   'b',   'LineWidth', 1.4);
plot(t_mpc, yhat_hist_b,'g',   'LineWidth', 1.1);
grid on;
title('(b): Step reference + measurement noise');
ylabel('y[k]');
legend('Reference r[k]', 'Measured y[k]', 'yhat_{k+1|k}', 'Location',
'best');

subplot(3,1,3);
plot(t_mpc, ref_hist_c, 'r--', 'LineWidth', 1.3); hold on;
plot(t_mpc, y_hist_c,   'b',   'LineWidth', 1.4);
plot(t_mpc, yhat_hist_c,'g',   'LineWidth', 1.1);
grid on;
title('(c): Sine reference + measurement noise');
xlabel('Time [s]');
ylabel('y[k]');
legend('Reference r[k]', 'Measured y[k]', 'yhat_{k+1|k}', 'Location',
'best');
```


(a): Step reference, noiseless
(b): Step reference + measurement noise
(c): Sine reference + measurement noise
Reference r[k]
Measured y[k]
yhat$_{k+1|k}$
y[k]
Time [s]