ME 418/518 – Data-Based Control

Problem Set 1

Deadline – October 17th, 2025

Employment of LLMs is strictly prohibited

Problem 1 (25). Consider the continuous time, LTI state space representation of a single-input single-output (SISO) system given below:

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t) + Du(t),$$

where,

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad D = 0$$

The system is sampled with a zero-order hold (ZOH) every $T = 0.1$ seconds.

a) Calculate the exact discrete-time model (F, G, C, D) under ZOH, using the formulas given in the lecture. (Do not use any software, show your hand calculations)

b) Calculate the discrete-time poles. What are their relations with the continuous-time poles?

Problem 2 (5). Assume that the bandwidth of your closed loop system is expected to be 5Hz. How would you choose your sampling interval? Explain the consequences of very fast and very slow sampling rates.

Problem 3 (30). You will run an experiment in MATLAB, produce input output pair, $u[k], y[k]$, and clean the raw data.

<u>- You can use MATLAB functions, toolboxes etc. for this problem.</u>

<u>- Provide your MATLAB code in your answer sheet and comment your code explaining which part of the code completes which task given in the problem.</u>

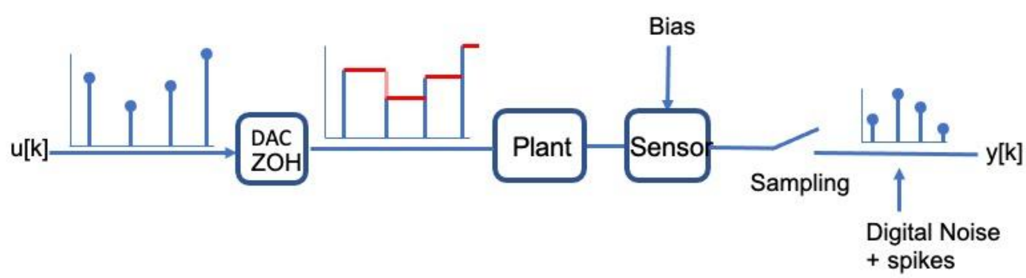
<u>- Your code must be running without error. It should produce all the plots required in the problem.</u>

The transfer function for the physical plant is given as

$$G(s) = \frac{4}{s^2 + 2s + 4}$$

Use a sampling interval of $T = 0.02$ seconds. The duration of the experiment will be 60 seconds, so you will have 3000 samples ($N = 3000$). Use ZOH on the input. Assume that the sensor has a dynamics of $G(s) = 1/(0.1s + 1)$. The sensor has a bias of +0.3. We also add a Gaussian noise with s.d. $\sigma = 0.05$ (after sampling). Randomly place 30 "spikes". The input *u* is Pseudo-random binary sequence (PRBS) with range [-0.8, 0.8] with dwell time 0.2 seconds (10 samples/level).

3.1. Simulate the experiment in Matlab/Simulink. First you will create u[k] based on above descriptions, then use ZOH to create the continuous signal to be fed to the physical plant. Then add the sensor dynamics and bias. Sample the biased continuous output, add the noise and the spikes with the specifications given above to obtain the raw output data y[k].



Plot u[k] and y[k] v.s. time (not v.s. sample index) in two separate plots.

3.2. Clean the data using the following:
a) Replace the spikes by linear interpolation.

b) Remove the bias.

c) Reduce the effect of the noise using a moving average (report the number of steps you used)

(apply the same moving average to the input to the input **later in the next problem** when you

create the FIR model)

Plot the raw and cleaned data sequences on the same plot.

Problem 4 (40). (Use MATLAB with basic functions) In this problem you will estimate an FIR model of the plant+sensor dynamics given the input-output data you generated in Problem 2. Use preprocessed signals ($u_f[k]$:input after applying the same moving average filter you used on the output, and $y_c[k]$: the cleaned output data).

Since there is no delay, $n_d = 0$.

Model: $\hat{y}[k] = \sum_{1}^{n_b} b_i u_f[k-i] = \Phi\,[k]^T b, \quad \Phi[k] = \left[u_f[k-1], u_f[k-2], \dots, u_f[k-n_b]\right]^T$

a) Split the data into training and validation sets. (Use the first 42 seconds as training and the rest as validation.) Using the training data, find b_i values, which are the parameters of the model, using $n_b = 30$ and $n_b = 100$.

b) Compare the model outputs with the validation data set: Use the two identified models to create $\hat{y}[k]$ for the last 18 seconds and plot $\hat{y}[k]$ and $y[k]$.

c) Using the validation data set, compare the root mean square error (RMSE) and fit percentages of the two models.

Fit percentage = $\left(1 - \frac{||y-\hat{y}||}{||y-\bar{y}||}\right) x100$

d) Discuss: What is the settling time for the given plant+sensor dynamics? Given the sampling interval of T=0.02 seconds, how many samples are required for the impulse response of the plant to converge and reach to the steady state value? Based on your answers, what is an ideal n_b?

e) Bonus: Create your model using the ideal n_b value you determined in part “d” and calculate the RMSE and the fit percentage. Compare your results with part “c”.