ME-418
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ME-PHYS

1) $\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Cx(t) + Du(t)$

$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = [1 \quad 1], \quad D = 0$

$T = 0.1s$, ZOH

a) $F = e^{AT}$, $G = \int_0^T e^{A\sigma} d\sigma B$, $G = A^{-1}(e^{AT} - I)B$
(if A has inverse)

$F = e^{AT} = \mathcal{L}^{-1}(sI - A)^{-1}$

$sI - A = \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix} \Rightarrow (sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$

$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}, \quad \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t}$

$F = e^{AT} = \begin{bmatrix} e^{-T} & 0 \\ 0 & e^{-2T} \end{bmatrix} \xrightarrow{T=0.1s} \boxed{F \approx \begin{bmatrix} 0.905 & 0 \\ 0 & 0.819 \end{bmatrix}}$

$A^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1/2 \end{bmatrix}, \quad e^{AT} - I = \begin{bmatrix} e^{-T} - 1 & 0 \\ 0 & e^{-2T} - 1 \end{bmatrix}$

$G = \begin{bmatrix} -1 & 0 \\ 0 & -1/2 \end{bmatrix} \begin{bmatrix} e^{-T} - 1 & 0 \\ 0 & e^{-2T} - 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 - e^{-T} \\ \frac{1}{2}(1 - e^{-2T}) \end{bmatrix}$

$\boxed{G \approx \begin{bmatrix} 0.0952 \\ 0.0906 \end{bmatrix}}$

$x[k+1] = Fx[k] + Gu[k]$

$y[k] = Cx[k] + Du[k]$

$F = \begin{bmatrix} 0.905 & 0 \\ 0 & 0.819 \end{bmatrix}$

$G = \begin{bmatrix} 0.0952 \\ 0.0906 \end{bmatrix}$

$C = [1 \quad 1], \quad D = 0$

b) λ_d, discrete poles are eigenvalues of F.

$\lambda_d = e^{\lambda_c T}$ (relationship with cont. poles)

$\lambda_c \Rightarrow \det(sI - A) = \det\left(\begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}\right) = 0$

$\Rightarrow \lambda_{c,1} = -1$, $\lambda_{c,2} = -2$

$\lambda_d = e^{\lambda_d T}$, $\boxed{\lambda_{d,1}} = e^{-1(0.1)} = \boxed{0.904837}$

$\boxed{\lambda_{d,2}} = e^{-2(0.1)} = \boxed{0.818731}$ from cont. poles rel.

$\det(zI - F) = \det\left(\begin{bmatrix} z-e^{-0.1} & 0 \\ 0 & z-e^{-0.2} \end{bmatrix}\right) = 0$

$\det(zI - F) = (z - e^{-0.1})(z - e^{-0.2}) = 0$

$\boxed{\lambda_{d,1} = e^{-0.1} = 0.904837}$, $\boxed{\lambda_{d,2} = e^{-0.2} = 0.818731}$ from eigenval's of F $\rightarrow$ equivalent

2) If the closed-loop bandwith is 5 hz, the sampling freq. should be 10-20 times faster than that bandwith. (Therotical min. is (Nyquist) 2-times)

$$f_s = 50 - 100\ Hz \Rightarrow T = 0.01 - 0.02\,s$$

If sampling is too slow:

- Causes aliasing and phase delay, which destabilizes the closed loop
- The discrete model no longer represents the true dynamics

If sampling is too fast:

- Increases noise sensitivity and computational load without much performance gain.

A good practise of T would be 0.01s (100 Hz) if meas. sys. is available of capacity.

1) d) For the plant $G_p(s) = \dfrac{4}{s^2+2s+4}$

Standart second-order form $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\omega_n = 2$ rad/s $\qquad 2\xi\omega_n = 2$

$\xi = 0.5$

The %2 settling time of a second order sys.

$$t_s \approx \frac{4}{\xi\omega_n} = \frac{4}{0.5\times 2} = 4s$$

Sensor has $\left[\, G(s) = \dfrac{1}{0.1s+1} \,\right]$ small time const.

$\tau = 0.1s$. Overall settling time is dominated by the plant.

with a sampling interval $T=0.02s$

$$N_s \approx \frac{t_s}{T} \approx \frac{4}{0.02} = 200 \text{ samples}$$

Therefore, for an input to die out, around $\boxed{200 \approx n_b}$, or FIR horizon is ideal.

One can scan between 150 - 250 interval to find a further finer n_b value (due to noise, bias etc. n_b may fluctuate.)

at (e) we find this value.