

# Lab 5

Erin Canada

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This dataset records the salary of  $n = 263$  Major League Baseball players during the 1987 season as well as  $q = 19$  statistics associated with the performance of each player during the previous season. Specifically, the dataset contains observations from the following variables:

. AtBat: Number of times at bat in 1986 . Hits: Number of hits in 1986 . HmRun: Number of home runs in 1986 . Runs: Number of runs in 1986 . RBI: Number of runs batted in in 1986 . Walks: Number of walks in 1986 . Years: Number of years in the major leagues . CAtBat: Number of times at bat during his career . CHits: Number of hits during his career . CHmRun: Number of home runs during his career . CRuns: Number of runs during his career . CRBI: Number of runs batted in during his career . CWalks: Number of walks during his career . League: A categorical variable with levels A (for American) and N (for National) indicating the player's league at the end of 1986 . Division: A factor with levels E (for East) and W (for West) indicating the player's division at the end of 1986 . PutOuts: Number of put outs in 1986 . Assists: Number of assists in 1986 . Errors: Number of errors in 1986 . Salary: 1987 annual salary on opening day in thousands of dollars . NewLeague: A factor with levels A and N indicating the player's league at the beginning of 1987

Interest lies in developing a model that relates a player's annual salary to their previous performance. Your job in this Lab is to investigate several such models. Where computation is required, you must perform the calculations in both R and Python (unless otherwise indicated).

```
getwd()
```

```
## [1] "C:/Users/Erin Canada/Documents/USF/Fall 2018/Regression/Lab"
```

```
library(car)
```

```
## Loading required package: carData
```

```
hitter <- read.csv("hitters.csv")
```

```
head(hitter)
```

```
##   AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1   315   81     7   24  38   39   14   3449   835    69   321  414
## 2   479  130    18   66  72   76    3   1624   457    63   224  266
## 3   496  141    20   65  78   37   11   5628  1575   225   828  838
## 4   321   87    10   39  42   30    2    396   101    12    48   46
## 5   594  169     4   74  51   35   11   4408  1133    19   501  336
## 6   185   37     1   23   8   21    2    214    42     1    30    9
##   CWalks League Division PutOuts Assists Errors Salary NewLeague
## 1    375      N        W      632     43    10  475.0          N
## 2    263      A        W      880     82    14  480.0          A
## 3    354      N        E      200     11     3  500.0          N
## 4     33      N        E      805     40     4   91.5          N
## 5    194      A        W      282    421    25  750.0          A
## 6     24      N        E       76    127     7   70.0          A
```

(a) Calculate the variance inflation factor (VIF) for each of the explanatory variables. Comment on whether multicollinearity appears to be an issue. If it is, identify the three explanatory variables that are most seriously affected by the issue.

```
model <- lm(Salary ~ ., data = hitter)

vif(model)

##      AtBat      Hits      HmRun      Runs      RBI      Walks
## 22.944366 30.281255  7.758668 15.246418 11.921715  4.148712
##      Years      CAtBat      CHits      CHmRun      CRuns      CRBI
##  9.313280 251.561160 502.954289 46.488462 162.520810 131.965858
##      CWalks      League      Division      PutOuts      Assists      Errors
## 19.744105  4.134115  1.075398  1.236317  2.709341  2.214543
## NewLeague
##  4.099063

collinearity <- c(vif(model))

multi <- sort(collinearity)
multi

##      Division      PutOuts      Errors      Assists      NewLeague      League
##  1.075398  1.236317  2.214543  2.709341  4.099063  4.134115
##      Walks      HmRun      Years      RBI      Runs      CWalks
##  4.148712  7.758668  9.313280 11.921715 15.246418 19.744105
##      AtBat      Hits      CHmRun      CRBI      CRuns      CAtBat
## 22.944366 30.281255 46.488462 131.965858 162.520810 251.561160
##      CHits
## 502.954289

tail(multi,3)

##      CRuns      CAtBat      CHits
## 162.5208 251.5612 502.9543
```

It seems as though multicollinearity appears to be an issue. Three explanatory variables seriously affected by this issue are CHits, CAtBat, and CRuns. Each of these explanatory variables have a variance inflation factor of well above a range of 5 or 10, which indicates multicollinearity.

(b) Using the all-possible-subsets approach, find the model that best fits the observed data. This procedure may be automated using the `regsubsets()` function in R, but you must explain in your own words how this algorithm identifies the 'best' model. Note that you do not need to perform this task in Python.

```
# Fit all possible models and for a given number of explanatory variables, find
# the best model (in terms of R^2)
library(leaps)
all_oss <- regsubsets(Salary ~ ., data = hitter, nvmax = 19)
all_oss_summ <- summary(all_oss)
all_oss_summ

## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = hitter, nvmax = 19)
```

```

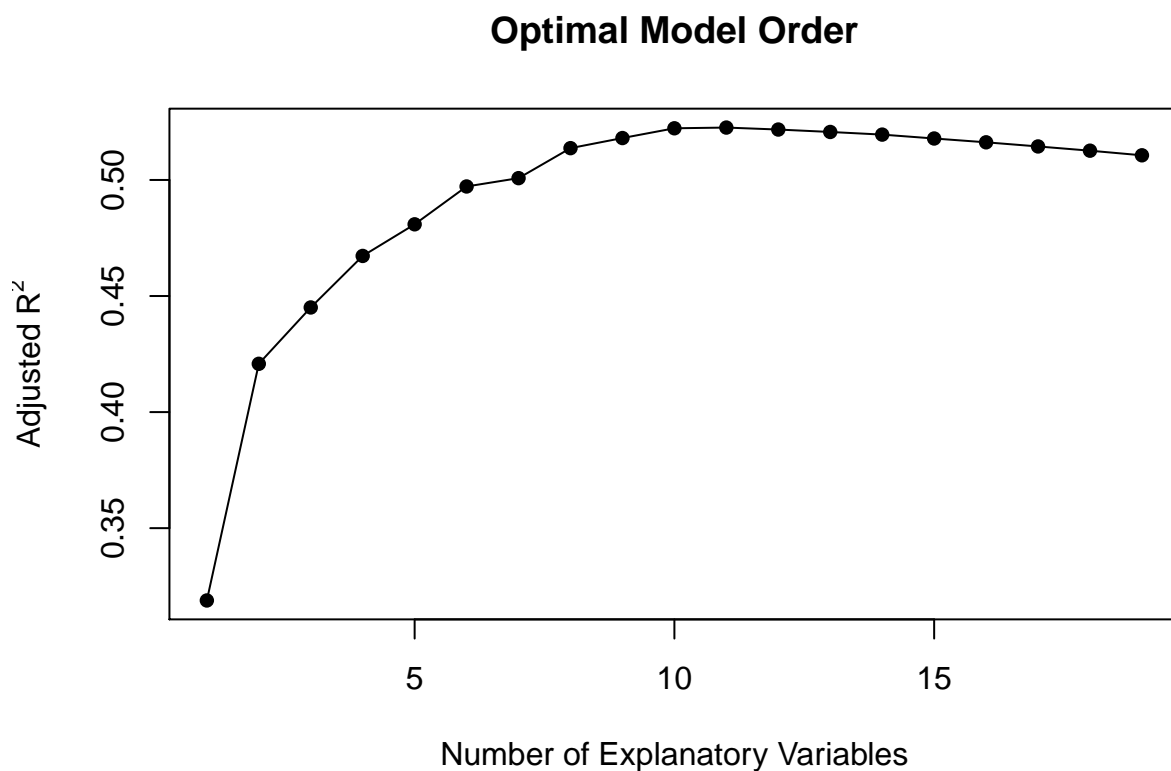
## 19 Variables (and intercept)
##           Forced in Forced out
## AtBat      FALSE      FALSE
## Hits       FALSE      FALSE
## HmRun      FALSE      FALSE
## Runs       FALSE      FALSE
## RBI        FALSE      FALSE
## Walks      FALSE      FALSE
## Years      FALSE      FALSE
## CAtBat     FALSE      FALSE
## CHits      FALSE      FALSE
## CHmRun     FALSE      FALSE
## CRuns      FALSE      FALSE
## CRBI       FALSE      FALSE
## CWalks     FALSE      FALSE
## LeagueN    FALSE      FALSE
## DivisionW  FALSE      FALSE
## PutOuts    FALSE      FALSE
## Assists    FALSE      FALSE
## Errors     FALSE      FALSE
## NewLeagueN FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: exhaustive
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1 ( 1 ) " " " " " " " " " " " " " " " " " "
## 2 ( 1 ) " " "*" " " " " " " " " " " " " " "
## 3 ( 1 ) " " "*" " " " " " " " " " " " " " "
## 4 ( 1 ) " " "*" " " " " " " " " " " " " " "
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## 6 ( 1 ) "*" "*" " " " " " " "*" " " " " " " "
## 7 ( 1 ) " " "*" " " " " " " "*" " " "*" "*" " "
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## 12 ( 1 ) "*" "*" " " "*" " " "*" " " "*" " " "*"
## 13 ( 1 ) "*" "*" " " "*" " " "*" " " "*" " " "*"
## 14 ( 1 ) "*" "*" "*" "*" " " "*" " " "*" " " "*"
## 15 ( 1 ) "*" "*" "*" "*" " " "*" " " "*" " " "*"
## 16 ( 1 ) "*" "*" "*" "*" "*" "*" " " "*" "*" " " "*"
## 17 ( 1 ) "*" "*" "*" "*" "*" "*" " " "*" "*" " " "*"
## 18 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" "*" " " "*"
## 19 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" "*" "*" "*"
##           CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1 ( 1 ) "*" " " " " " " " " " " " "
## 2 ( 1 ) "*" " " " " " " " " " " " "
## 3 ( 1 ) "*" " " " " " " "*" " " " " "
## 4 ( 1 ) "*" " " " " "*" "*" " " " " "
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## 6 ( 1 ) "*" " " " " "*" "*" " " " " "
## 7 ( 1 ) " " " " " " "*" "*" " " " " "
## 8 ( 1 ) " " "*" " " " "*" "*" " " " " "
## 9 ( 1 ) "*" "*" " " " "*" "*" " " " " "
## 10 ( 1 ) "*" "*" " " "*" "*" "*" " " " "

```

```
## 11 ( 1 ) "*" "*" "*" "*" "*" "*" " " " "
## 12 ( 1 ) "*" "*" "*" "*" "*" "*" " " " "
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## 16 ( 1 ) "*" "*" "*" "*" "*" "*" "*" " "
## 17 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*"
## 18 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*"
## 19 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*"

```

```
# Plot the Adjusted R2 for each of these
plot(all_poss_summ$adjr2, type = "l", xlab = "Number of Explanatory Variables",
      ylab = bquote("Adjusted R2"), main = "Optimal Model Order")
points(all_poss_summ$adjr2, pch = 16)
```

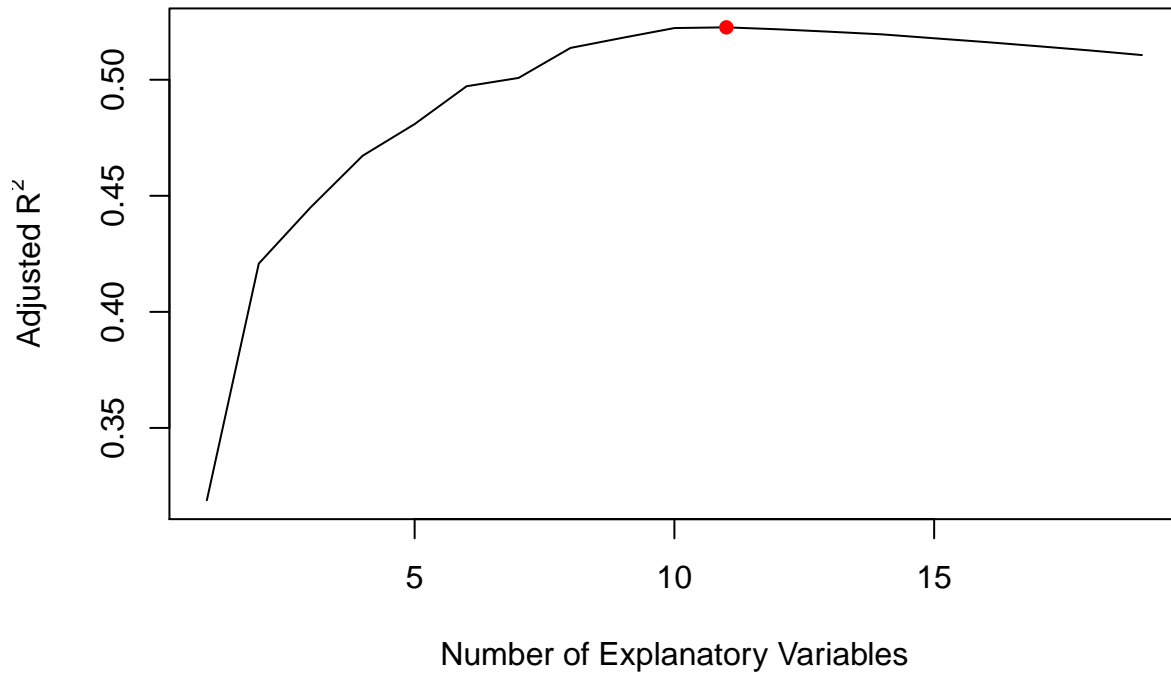


```
# Find the optimal number of explanatory variables
max_idx <- which.max(all_poss_summ$adjr2)
max_idx
```

```
## [1] 11
```

```
plot(all_poss_summ$adjr2, type = "l", xlab = "Number of Explanatory Variables",
      ylab = bquote("Adjusted R2"), main = "Optimal Model Order")
points(x = max_idx, y = all_poss_summ$adjr2[max_idx], pch = 16, col = "red")
```

## Optimal Model Order



```
all_poss_summ$which[max_idx,]
```

```
## (Intercept)      AtBat      Hits      HmRun      Runs      RBI
##          TRUE         TRUE         TRUE      FALSE      FALSE      FALSE
##          Walks      Years      CatBat      CHits      CHmRun      CRuns
##          TRUE         FALSE      TRUE      FALSE      FALSE      TRUE
##          CRBI      CWalks      LeagueN      DivisionW      PutOuts      Assists
##          TRUE         TRUE         TRUE         TRUE         TRUE         TRUE
##          Errors      NewLeagueN
##          FALSE         FALSE
```

```
m_all <- lm(Salary ~ AtBat + Hits + Walks + CatBat + CRuns + CRBI
            + CWalks + League + Division + PutOuts + Assists, data = hitter)
```

The all possible subsets compares each of the possible models' adjusted R square values. This approach is done in two stages. For a given number of explanatory variables, we choose the best model using  $R^2$  which yields  $q + 1$  optimal models. Next, these models are compared to one another using the adjusted R square which incorporates a penalty for including too many explanatory variables. In this case, we compare each of the possible models to one another and chose the best one. The best model using the all possible subsets includes the variables AtBat, Hits, Walks, CatBat, CRuns, CRBI, Walks, League, Division, PutOuts, and Assists. According to this approach, these variables will create the best model to use in predicting future salaries.

(c) Using the forward-stepwise-selection approach, find the model that best fits the observed data. This procedure may be automated using the `stepAIC()` function in R, but you must explain in your own words how this algorithm identifies the ‘best’ model. Note that you do not need to perform this task in Python.

```
library(MASS)
sml <- lm(Salary ~ 1, data = hitter)
lrg <- lm(Salary ~ ., data = hitter)

# Forward
stepAIC(object = sml, scope = list(upper = lrg, lower = sml), direction = "forward", trace = 0)

##
## Call:
## lm(formula = Salary ~ CRBI + Hits + PutOuts + Division + AtBat +
##     Walks + CWalks + CRuns + CAtBat + Assists, data = hitter)
##
## Coefficients:
## (Intercept)      CRBI      Hits    PutOuts  DivisionW
##    162.5354    0.7743    6.9180    0.2974   -112.3801
##      AtBat      Walks    CWalks      CRuns      CAtBat
##   -2.1687    5.7732   -0.8308    1.4082   -0.1301
##    Assists
##     0.2832

m_f <- stepAIC(object = sml, scope = list(upper = lrg, lower = sml), direction = "forward", trace = 0)
```

In the forward model selection using AIC, each important variable will be added until there are no more important variables remaining that would improve the model, however once a variable has been added, it cannot be removed. The best model using this approach would be CRBI, Hits, PutOuts, Division, AtBat, Walks, CWalks, CRuns, CAtBat, and Assists.

(d) Using the backward-stepwise-selection approach, find the model that best fits the observed data. This procedure may be automated using the `stepAIC()` function in R, but you must explain in your own words how this algorithm identifies the ‘best’ model. Note that you do not need to perform this task in Python.

```
# Backward
stepAIC(object = lrg, scope = list(upper = lrg, lower = sml), direction = "backward", trace = 0)

##
## Call:
## lm(formula = Salary ~ AtBat + Hits + Walks + CAtBat + CRuns +
##     CRBI + CWalks + Division + PutOuts + Assists, data = hitter)
##
## Coefficients:
## (Intercept)      AtBat      Hits      Walks      CAtBat
##    162.5354   -2.1687    6.9180    5.7732   -0.1301
##      CRuns      CRBI      CWalks  DivisionW    PutOuts
##    1.4082    0.7743   -0.8308   -112.3801    0.2974
##    Assists
##     0.2832
```

```
m_b <- stepAIC(object = lrg, scope = list(upper = lrg, lower = sml), direction = "backward", trace = 0)
```

In a sense, backward selection is similar to forward selection except, you start with the full model and remove variables that are unimportant one at a time. However, once these variables are removed, they cannot be added back. The best model using this approach is AtBats, Hits, Walks, CAtBat, CRuns, CRBI, CWalks, Division, PutOuts, and Assists.

(e) Using the hybrid-stepwise-selection approach, find the model that best fits the observed data. This procedure may be automated using the `stepAIC()` function in R, but you must explain in your own words how this algorithm identifies the ‘best’ model. Note that you do not need to perform this task in Python.

```
# Hybrid
stepAIC(object = sml, scope = list(upper = lrg, lower = sml), direction = "both", trace = 0)

##
## Call:
## lm(formula = Salary ~ CRBI + Hits + PutOuts + Division + AtBat +
##     Walks + CWalks + CRuns + CAtBat + Assists, data = hitter)
##
## Coefficients:
## (Intercept)      CRBI      Hits    PutOuts  DivisionW
##    162.5354    0.7743    6.9180    0.2974   -112.3801
##      AtBat      Walks    CWalks     CRuns     CAtBat
##   -2.1687    5.7732   -0.8308    1.4082    -0.1301
##    Assists
##     0.2832

m_h <- stepAIC(object = sml, scope = list(upper = lrg, lower = sml), direction = "both", trace = 0)
```

The Hybrid-stepwise selection approach does a combination of both forward and backward model selection. It starts by fitting the intercept only model and then considers adding the most influential variable. If a variable is added, it is then considered to add another variable or remove the least influential variable, repeating for each stage to improve the model. This is continued until there are no variables that can be added or removed to improve the model. However, variables are never stuck in or out of the model. Using this approach the best model uses the variables CRBI, Hits, PutOuts, Division, AtBat, Walks, CWalks, CRuns, CAtBat, and Assists.

(f) In this part you will compare the predictive performance of four models:

- i. The full model with all 19 explanatory variables.
- ii. The optimal model identified in part (b).
- iii. The best model from parts (c)-(e) (i.e., the best stepwise-selection model).
- iv. The model that is considered optimal with respect to the Bayesian Information Criterion (BIC) which contains the variables AtBat, Hits, Walks, CRBI, Division and PutOuts.

## Randomly split the observed data into a training set (containing roughly 80% of all of the data) and a held-out test set (containing roughly 20% of all of the data). Calculate the predictive root-mean-square error (RMSE) for each of the four models. Which model appears to be most appropriate? Justify why this model is most appropriate.

```

rmse <- function(data,model){
  n <- dim(data)[1]
  trn <- sample(x = c(rep(TRUE, round(0.8*n)), rep(FALSE, n-round(0.8*n))), size = n, replace = FALSE)
  train <- data[trn,]
  tst <- !trn
  test <- data[tst,]

  pred <- predict(object = model , newdata = test)
  result<- sqrt(mean((test$Salary - pred)^2))
  return(result)
}

l <- c()

#i The full model with all 19 explanatory variables.
full <- lm(Salary ~ ., data = hitter)
rmse_full <- rmse(hitter,full)

## ii. The optimal model identified in part (b).
rmse_all <- rmse(hitter,m_all)
l[1] <- rmse_all

## iii. The best model from parts (c)-(e) (i.e., the best stepwise-selection model).
rmse_f <- rmse(hitter,m_f)
rmse_b <- rmse(hitter,m_b)
rmse_h <- rmse(hitter,m_h)

## iv. The model that is considered optimal with respect to the Bayesian
#Information Criterion (BIC) which contains the variables AtBat, Hits, Walks, CRBI, Division and PutOut.
m_bic <- lm(Salary ~ AtBat + Hits + Walks + CRBI + Division + PutOuts, data = hitter)
BIC(m_bic)

## [1] 3817.785

rmse_bic <- rmse(hitter,m_bic)

compare.rmse <- data.frame("Full" = rmse_full,"All"=rmse_all,
                           "Forward"=rmse_f,"Backward"=rmse_b,"Hybrid"=rmse_h,"Bic"=rmse_bic)
rownames(compare.rmse) <- "Cross-Fold"

```

(g) As in part (f), you must compare the predictive performance of the same four models, but here you must determine the predictive accuracy (predictive RMSE) by using 10-Fold Cross Validation. Which model appears to be most appropriate? Justify why this model is most appropriate.

```

library(boot)

##
## Attaching package: 'boot'
## The following object is masked from 'package:car':
##

```



```
##      logit
mg_full <- glm(Salary ~ ., data = hitter)

mg_ap <- glm(Salary ~ AtBat + Hits + Walks + CRuns+ CRBI + CWalks +
             League + Division + PutOuts + Assists, data = hitter)# 6 multicollinear
mg_f <- glm(Salary ~ CRBI + Hits + PutOuts + Division + AtBat + Walks
            + CWalks + CRuns + CAtBat + Assists, data = hitter)#6
mg_b<- glm(Salary ~ AtBat + Hits + Walks + CAtBat + CRuns + CRBI +
            CWalks + Division + PutOuts + Assists, data = hitter)#6
mg_h <- glm(Salary ~ CRBI + Hits + PutOuts + Division + AtBat + Walks
            + CWalks + CRuns + CAtBat + Assists, data = hitter)#6
mg_bic<- glm(Salary ~ AtBat + Hits + Walks + CRBI + Division + PutOuts, data = hitter)#3

rmse.k <- function(model){
  result <- sqrt((cv.glm(hitter, model, K = 10)$delta)[1])
  return(result)
}

## i. The full model with all 19 explanatory variables.
rmse.k_full <- rmse.k(mg_full)

## ii. The optimal model identified in part (b).
rmse.k_ap <- rmse.k(mg_ap)

## iii. The best model from parts (c)-(e) (i.e., the best stepwise-selection model).
rmse.k_f <- rmse.k(mg_f)
rmse.k_b <- rmse.k(mg_b)
rmse.k_h <- rmse.k(mg_h)

## iv. The model that is considered optimal with respect to the
#Bayesian Information Criterion (BIC) which contains the variables
#AtBat, Hits, Walks, CRBI, Division and PutOuts.
rmse.k_bic <- rmse.k(mg_bic)

k.rmse <- c(rmse.k_full,rmse.k_ap,rmse.k_f,rmse.k_b,rmse.k_h,rmse.k_bic)
compare.rmse <- rbind(compare.rmse,k.rmse)
rownames(compare.rmse) <- c("Cross-Fold",'K-Fold')
compare.rmse
```

```
##           Full      All Forward Backward  Hybrid      Bic
## Cross-Fold 226.7866 285.3646 319.7999 308.5801 262.8926 264.6210
## K-Fold    340.3170 323.9113 326.4622 326.5075 329.9707 328.5902
```

##(h) Given the estimates of predictive accuracy from parts (f) and (g) indicate which estimates you believe to be more accurate. In other words, indicate which validation approach (i.e., cross validation vs. k-fold cross validation) you believe will most accurately estimate the predictive capability of a model. Briefly explain your rationale.

Given the parts from (f) and (g), it is obvious to see that the K-Fold cross validation is more accurate. This is because RMSE is used to measure how close a predicted value is to the response. With Cross-Fold validation, it is highly variable due to the specific observations when selecting our test set, overall changing the test error. The K-Fold cross validation which combats this with testing k estimates of the test set, stabilizing our test error and then we can get a more precise value with our model to overall better predict.

**(i) Accounting for all of the analyses you've performed (i.e., multicollinearity, goodness-of-fit, and predictive accuracy), which model would you be most comfortable using? Briefly justify your choice. [Note: I'm not looking for a right or wrong answer here; I want to see that you can sensibly and eloquently justify your choice].**

Accounting for all of the analyses I have performed, the model I would be most comfortable using would be the model using the BIC because it has the least amount of variables accounting for just as much as the other models and it has less multicollinear variables. It also does not contain any of the three worst multicollinear variables which could seriously effect the model, however, we should still be careful when including any variables that have multicollinearity.