

# Affective cognition: Exploring lay theories of emotion



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## ABSTRACT

Humans skillfully reason about others' emotions, a phenomenon we term *affective cognition*. Despite its importance, few formal, quantitative theories have described the mechanisms supporting this phenomenon. We propose that affective cognition involves applying domain-general reasoning processes to domain-specific content knowledge. Observers' knowledge about emotions is represented in rich and coherent lay theories, which comprise consistent relationships between situations, emotions, and behaviors. Observers utilize this knowledge in deciphering social agents' behavior and signals (e.g., facial expressions), in a manner similar to rational inference in other domains. We construct a computational model of a lay theory of emotion, drawing on tools from Bayesian statistics, and test this model across four experiments in which observers drew inferences about others' emotions in a simple gambling paradigm. This work makes two main contributions. First, the model accurately captures observers' flexible but consistent reasoning about the ways that events and others' emotional responses to those events relate to each other. Second, our work models the problem of *emotional cue integration*—reasoning about others' emotion from multiple emotional cues—as rational inference via Bayes' rule, and we show that this model tightly tracks human observers' empirical judgments. Our results reveal a deep structural relationship between affective cognition and other forms of inference, and suggest wide-ranging applications to basic psychological theory and psychiatry.

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## 1. Introduction

It is easy to predict that people generally react positively to some events (winning the lottery) and negatively to others (losing their job). Conversely, one can infer, upon encountering a crying friend, that it is more likely he has just experienced a negative, not positive, event. These inferences are examples of reasoning about another's emotions: a vital and nearly ubiquitous human skill. This ability to reason about emotions supports countless social behaviors, from maintaining healthy relationships to scheming for political power. Although it is possible that some features of emotional life carries on with minimal influence from cognition, reasoning about others' emotions is clearly an aspect of cognition. We propose terming this phenomenon *affective cognition*—the collection of cognitive processes that involve *reasoning about emotion*.

For decades, scientists have examined how people manage to make complex and accurate attributions about others' psychological states (e.g., Gilbert, 1998; Tomasello, Carpenter, Call, Behne, & Moll, 2005; Zaki & Ochsner, 2011). Much of this work converges

on the idea that individuals have *lay theories* about how others react to the world around them (Flavell, 1999; Gopnik & Wellman, 1992; Heider, 1958; Leslie, Friedman, & German, 2004; Pinker, 1999). Lay theories—sometimes called intuitive theories or folk theories—comprise structured knowledge about the world (Gopnik & Meltzoff, 1997; Murphy & Medin, 1985; Wellman & Gelman, 1992). They provide an abstract framework for reasoning, and enable both explanations of past occurrences and predictions of future events. In that sense, lay theories are similar to scientific theories—both types of theories are coherent descriptions of how the world works. Just as a scientist uses a scientific theory to describe the world, a lay *observer* uses a lay theory to make sense of the world. For instance, people often conclude that if Sally was in another room and did not see Andy switch her ball from the basket to the box, then Sally would return to the room thinking that her ball was still in the basket: Sally holds a *false belief*, where her beliefs about the situation differs from reality (Baron-Cohen, Leslie, & Frith, 1985). In existing models, this understanding of others' internal states is understood as a *theory* that can be used flexibly and consistently to reason about other minds. In this paper, we propose a model of how people likewise reason about others' emotions using structured lay theories that allow complex inferences.

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Within the realm of social cognition, lay theories comprise knowledge about how people's behavior and mental states relate to each other, and allow observers to reason about invisible but important factors such as others' personalities and traits (Chiu, Hong, & Dweck, 1997; Heider, 1958; Jones & Nisbett, 1971; Ross, 1977; Ross & Nisbett, 1991), beliefs and attitudes (Kelley & Michela, 1980), and intentions (Jones & Davis, 1965; Kelley, 1973; Malle & Knobe, 1997). Crucially, lay theories allow social inference to be described by more general principles of reasoning. For example, Kelley (1973)'s Covariational Principle describes how observers use statistical co-variations in observed behavior to determine whether a person's behavior reflects a feature of that person (e.g., their preferences or personality) or a feature of the situation in which they find themselves. There are many similar instances of lay-theory based social cognition: Fig. 1 lists just several such examples, such as how lay theories of personality (e.g., Chiu et al., 1997), race (e.g., Jayaratne et al., 2006), and "theories of mind" (e.g., Gopnik & Wellman, 1992) inform judgments and inferences—not necessarily made consciously—about traits and mental states. Although lay theories in different domains contain vastly different *domain-specific* content knowledge, the same common principles of reasoning—for example, statistical co-variation, deduction, and induction—are *domain-general*, and can be applied to these lay theories to enable social cognitive capabilities such as inferences about traits or mental states.

Lay theories can be formalized using Bayesian statistics using *ideal observer models* (Geisler, 2003). This approach has been used successfully to model a wide range of phenomena in vision, memory, decision-making (Geisler, 1989; Liu, Knill, & Kersten, 1995; Shiffrin & Steyvers, 1997; Weiss, Simoncelli, & Adelson, 2002), and, more recently, social cognition (e.g., Baker, Saxe, & Tenenbaum, 2009). An ideal observer analysis describes the optimal conclusions an observer would make given (i) the observed evidence and (ii) the observer's assumptions about the world. Ideal observer models describe reasoning without making claims as to the mechanism or process by which human observers draw these conclusions (cf. Marr, 1982), and provide precise, quantitative hypotheses through which to explore human cognition.

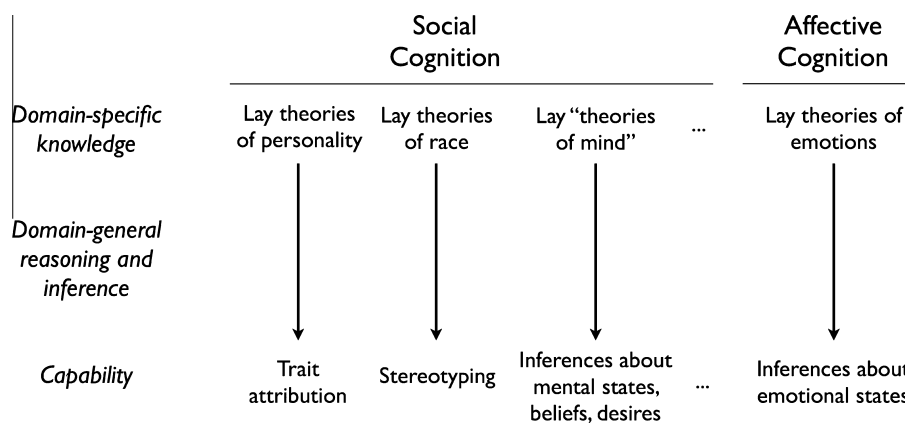
We propose that affective cognition, too, can be understood as reasoning with a lay theory: that is, affective cognition comprises domain-general cognitive processes applied to domain-specific knowledge about emotions (Fig. 1). Domain-specific knowledge comprises the observers' lay theory of emotion, and includes, for example, beliefs about what emotions are, how they are caused, and how people behave in response to emotions. We propose that this complex knowledge can be captured in a causal model, and

that observers use domain-general reasoning and inference processes to draw conclusions from this knowledge, similar to those used in perception and other domains. We make these ideas precise below by constructing an ideal observer model of emotional reasoning: we describing the domain-specific knowledge in a statistical causal model, and the domain-general reasoning as an application of Bayesian inference.

### 1.1. Attributing emotional reactions

How does an observer infer that agents (the targets of affective cognition) who spill a cup of coffee, miss the bus, or fall off a bicycle, likely feel similar (negative) emotions? One problem that any model of affective cognition must deal with is the combinatorial explosion of outcomes and emotional states that people can experience. It would be both inefficient and impractical for observers to store or retrieve knowledge about the likely affective consequences of every possible situation. We hypothesize that people circumvent this complexity by evaluating situations based on a smaller number of "active psychological ingredients" those situations contain. For instance, many emotion-inducing situations share key common features (e.g., the attainment or nonattainment of goals) that consistently produce particular emotions (Barrett, Mesquita, Ochsner, & Gross, 2007; Ellsworth & Scherer, 2003). An individual in a situation can take advantage of this commonality by *appraising* the situation along a small number of relevant appraisal dimensions: that is, reducing a situation to a low-dimensional set of emotion-relevant features (Ortony, Clore, & Collins, 1988; Schachter & Singer, 1962; Scherer, Schorr, & Johnstone, 2001; Smith & Ellsworth, 1985; Smith & Lazarus, 1993).

We propose that observers similarly reduce others' experience to a small number of emotionally relevant features when engaging in affective cognition. The examples above—spilling coffee, missing the bus, and falling off a bicycle—could all be associated, for instance, with unexpectedly losing something (e.g. coffee, time, and health). Note that the features relevant to the person's actual emotions (identified by appraisal theories) may not be identical to the features used by the observer (which are part of the observer's lay theory). The latter is our focus when studying affective cognition. Thus, we will first elucidate the situation features relevant for attributing emotion to another person. We operationalize this in Experiment 1 by studying a simple family of scenarios—a gambling game—and considering a variety of features such as amount of money won, prediction error (the amount won relative to the expected value of the wheel), and distance from a better or worse outcome.



**Fig. 1.** Lay theories within social cognition comprise domain-specific knowledge about behavior and mental states. Inferences about traits and beliefs occur when observers apply domain-general reasoning processes to these lay theories. In an analogous fashion, we propose that affective cognition is domain-general reasoning over domain-specific knowledge in a lay theory of emotions.

## 1.2. Reasoning from emotional reactions

A lay theory should support multiple inferences that are coherently related to each other: we can reason from a cause to its effects, but also back from an effect to its cause, and so on. For example, we can intuit that missing one's bus makes one feel sad, and we can also reason, with some uncertainty, that the frowning person waiting forlornly at a bus stop might have just missed their bus. If affective cognition derives from a lay theory, then it should allow observers to both infer unseen emotions based on events, and also to infer the type of event that a social target has experienced based on that person's emotions. In the framework of statistical causal models, these two types of inference—from emotions to outcomes and from outcomes to emotions—should be related using the rules of probability. In Experiment 2, we explicitly test this proposal: do people reason flexibly back and forth between emotions and the outcomes that cause them? Do forward and reverse inferences cohere as predicted by Bayesian inference?

## 1.3. Integrating sources of emotional evidence

Domain-general reasoning should also explain more complex affective cognition. For instance, observers often encounter multiple cues about a person's emotions: They might witness another person's situation, but also the expression on the person's face, their body posture, or what they said. Sometimes these cues even conflict—for instance, when an Olympic athlete cries after winning the gold medal. This seems to be a pair of cues that individually suggest conflicting valence. A comprehensive theory of affective cognition should address how observers translate this deluge of different information types into an inference, a process we call *emotional cue integration* (Zaki, 2013).

Prior work suggests two very different approaches that observers might take to emotional cue integration. On the one hand, the facial dominance hypothesis holds that facial expressions universally broadcast information about emotion to external observers (Darwin, 1872; Ekman, Friesen, & Ellsworth, 1982; Smith, Cottrell, Gosselin, & Schyns, 2005; Tomkins, 1962; for more extensive reviews, see Matsumoto, Keltner, Shiota, O'Sullivan, & Frank, 2008; Russell, Bachorowski, & Fernández-Dols, 2003). This suggests that observers should draw primarily on facial cues in determining social agents' emotions (Buck, 1994; Nakamura, Buck, & Kenny, 1990; Wallbott, 1988; Watson, 1972). On the other hand, contextual cues often appear to drive affective cognition even when paired with facial expressions. For instance, observers often rely on written descriptions of a situation (Carroll & Russell, 1996; Goodenough & Tinker, 1931) body postures (Aviezer, Trope, & Todorov, 2012; Aviezer et al., 2008; Mondloch, 2012; Mondloch, Horner, & Mian, 2013; Van den Stock, Righart, & de Gelder, 2007), background scenery (Barrett & Kensinger, 2010; Barrett, Mesquita, & Gendron, 2011; Lindquist, Barrett, Bliss-Moreau, & Russell, 2006), and cultural norms (Masuda et al., 2008) when deciding how agents feel.

Of course, both facial expressions and contextual cues influence affective cognition. It is also clear that neither type of cue ubiquitously “wins out,” or dominates inferences about others' emotions. An affective cognition approach suggests that observers should solve emotional cue integration using domain-general inference processes. There are many other settings—such as binocular vision (Knill, 2007) and multisensory perception (Alais & Burr, 2004; Shams, Kamitani, & Shimojo, 2000; Welch & Warren, 1980)—that require people to combine multiple cues into coherent representations. These ideal observer models assume that observers combine cues in an optimal manner given their prior knowledge and uncertainty. In such models, sensory cue integration is modeled as

Bayesian inference (for recent reviews, see de Gelder & Bertelson, 2003; Ernst & Bühlhoff, 2004; Kersten, Mamassian, & Yuille, 2004).

Our framework yields an approach to emotional cue integration that is analogous to cue integration in object perception: a rational information integration process. Observers weigh available cues to an agent's emotion (e.g., the agent's facial expression, or the context the agent is in) and combine them using statistical principles of Bayesian inference. This prediction naturally falls out of our claim that affective cognition resembles other types of theory-driven inference, with domain-specific content knowledge: the lay theory of emotion describes the statistical and causal relations between emotion and each cue; joint reasoning over this structure is described by domain-general inference processes.

We empirically test the predictions of this approach in Experiments 3 and 4. We aim to both extend the scope of our lay theory model and resolve the current debate in emotion perception by predicting how different cues are weighted as observers make inferences about emotion.

## 1.4. Overview

We first describe the components of our model and how it can be used to compute inferences about others' emotions. We formalize this model in the language of Bayesian modeling (Goodman & Tenenbaum, 2014; Goodman, Ullman, & Tenenbaum, 2011; Griffiths, Kemp, & Tenenbaum, 2008). Specifically, we focus on the ways that observers draw inferences about agents' emotions based on the situations and outcomes those agents experience. In all our experiments, we restricted the types of situations that agents experience to a simple gambling game. Although this paradigm does not capture many nuances of everyday affective cognition, its simplicity allowed us to quantitatively manipulate features of the situation and isolate situational features that best track affective cognition.

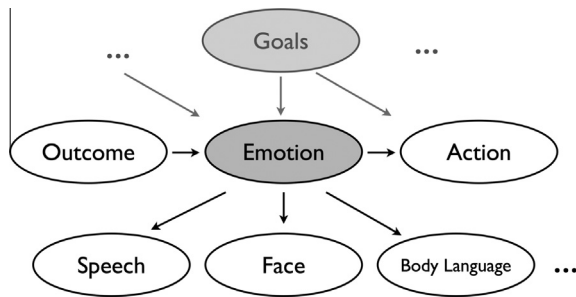
Experiment 1 sheds light on the process of inferring an agent's emotions given a situation, identifying a set of emotion-relevant situation features that observers rely on to understand others' affect. Experiment 2 tests the *flexibility* of emotional lay theories, by testing whether they also track observers' reasoning about the outcomes that agents' likely encountered based on their emotions; our model's predictions tightly track human judgments.

We then expand the set of observable evidence that our model considers, and describe how our model computes inferences from multiple cues—emotional cue integration. Experiment 3 tests the model against human judgments of emotions from both situation outcomes and facial expressions; Experiment 4 replicates this with situation outcomes and verbal utterances. In particular, we show that the Bayesian model predicts human judgments accurately, outperforming the baseline single cue dominance (e.g. facial or context dominance) models. Together, the results support the claim that reasoning about emotion represents a coherent set of inferences over a lay theory, similar to reasoning in other domains of psychology.

Finally, we describe some limitations of our model, motivate future work, and discuss the implications of an affective cognition approach for emotion theory, lay theories in other domains, and real-world applications.

## 2. Exploring the flexible reasoning between outcomes and emotions

Our model of a lay theory of emotion is shown schematically in Fig. 2. An **observer** (i.e. the reasoner) uses this lay theory to reason about an **agent** (i.e. the target of the reasoning). There are emotion-eliciting situations in the world, and the **outcomes** of



**Fig. 2.** Model of a lay theory that an observer could use during affective cognition. Using the notation of Bayesian networks, we represent variables as circles, and causal relations from a causal variable to its effect as arrows. Shaded variables represent unobservable variables. Although causal flows are unidirectional, as indicated by the arrows, information can flow the other way, as when making inferences about upstream causes from downstream outcomes. In this model, observers believe that situation **outcomes** cause an agent to feel an **emotion**, which then causes certain behavior such as **speech**, **facial expressions**, **body language** or posture, and importantly, **actions** that potentially result in new **outcomes** and a new emotion cycle. From the observable variables—which we call “cues”—we can infer the agent’s *latent*, or unobservable, **emotion**. Other mental states could be added to this model. One such extension includes the agent’s motivational states or **goals**, which would interact with the outcome of a situation to produce emotions; such goals would also influence the actions taken.

these situations, interacting with other mental states such as **goals**, cause an agent to feel **emotions**. The agent’s emotions in turn produce external **cues** including **facial expressions**, **body language**, and **speech**, as well as further **actions**. All of these variables, except mental states such as **emotion** and **goals**, are potentially observable variables; in particular, **emotion** is a latent variable that is unobservable because it is an internal state of the agent.

Each of these directed causal relationships can be represented as a probability distribution. For example, we can write different levels of happiness and anger given the *outcome* of winning the lottery as  $P(\text{happy}|\text{won lottery})$  and  $P(\text{angry}|\text{won lottery})$ . In our model, we represent the relationship between general outcomes **o** and emotions **e** as  $P(\mathbf{e}|\mathbf{o})$ . Similarly, the causal relationship between an agent’s emotions and his resultant facial expressions **f** can be written as  $P(\mathbf{f}|\mathbf{e})$ , and so forth.

As we discussed above, it would be impractical for observers to store the affective consequences of every possible situation outcome (i.e.,  $P(\mathbf{e}|\mathbf{o})$  for every possible outcome **o**). We hypothesize that observers reduce the multitude of possible outcomes into a low-dimensional set of emotion-relevant features via, for example, appraisal processes (e.g., Ortony et al., 1988). One potentially important outcome feature is value with respect to the agent’s goals. Indeed, a key characteristic of emotion concepts, as compared to non-emotion concepts, is their inherent relation to a psychological value system (Osgood, Suci, & Tannenbaum, 1957), that is, representations of events’ positive or negative affective valence (Clore et al., 2001; Frijda, 1988). As economists and psychologists have long known, people assess the value of events relative to their expectations: winning \$100 is exciting to the person who expected \$50 but disappointing to the person who expected \$200 (Carver & Scheier, 2004; Kahneman & Tversky, 1979). Deviations from an individual’s expectation are commonly termed *prediction errors*, and prediction errors in turn are robustly associated with the experience of positive and negative affect (e.g., Knutson, Taylor, Kaufman, Peterson, & Glover, 2005). Responses to prediction errors are not all equal, however; individuals tend to respond more strongly to negative prediction errors, as compared to positive prediction errors of equal magnitude, a property commonly referred to as *loss aversion* (Kahneman & Tversky, 1984). These value computations are basic and intuitively seem linked to emotion concepts, and we propose that they form an integral part of

observers’ lay theory of emotion. Thus, we hypothesize that reward, prediction error, and loss aversion constitute key outcome features that observers will use to theorize about others’ emotions, and facilitate affective cognition. Other, less “rational” features likely also influence affective cognition. Here we consider one such factor: the distance from a better (or worse) outcome, or how close one came to achieving a better outcome. In Experiment 1 we explore the situation features that parameterize  $P(\mathbf{e}|\mathbf{o})$  in a simple gambling scenario.

Although the lay theory we posit is composed of directed causal relationships—signaled by arrows in Fig. 2—people can also draw “reverse inferences” about causes based on the effects they observe (for instance, a wet front lawn offers evidence for the inference that it has previously rained.). In addition to reasoning about emotions **e** given outcomes **o**, observers can also draw inferences about the *posterior probability* of different outcomes **o** having occurred, given the emotion **e**. This posterior probability is written as  $P(\mathbf{o}|\mathbf{e})$  and is specified by Bayes’ Rule:

$$P(\mathbf{o}|\mathbf{e}) = \frac{P(\mathbf{e}|\mathbf{o})P(\mathbf{o})}{P(\mathbf{e})} \quad (1)$$

where  $P(\mathbf{e})$  and  $P(\mathbf{o})$  represent the prior probabilities of emotion **e** and outcome **o** occurring, respectively. By way of example, imagine that you walk into your friend’s room and find her crying uncontrollably; you want to find the outcome that made her sad—likely the **o** with the highest  $P(\mathbf{o}|\text{sad})$ —so you start considering possible candidate outcomes using your knowledge of your friend. She is a conscientious and hardworking student, and so if she fails a class exam, she would be sad, i.e.,  $P(\text{sad}|\text{fail exam})$  is high. But, you also recall that she is no longer taking classes, and so the prior probability of failing an exam is small, i.e.,  $P(\text{fail exam})$  is low. By combining those two pieces of information, you can infer that your friend probably did not fail an exam, i.e.,  $P(\text{fail exam}|\text{sad})$  is low, and so you can move on and consider other possible outcomes. In Experiment 2 we consider whether people’s judgments from emotions to outcomes are predicted by the model of  $P(\mathbf{e}|\mathbf{o})$  identified in Experiment 1 together with Bayes’ rule.

### 2.1. Experiment 1: Emotion from outcomes<sup>1</sup>

Our first goal is to understand how  $P(\mathbf{e}|\mathbf{o})$  relates to the features of a situation and outcome. We explore this in a simple gambling domain (Fig. 3) where we can parametrically vary a variety of outcome features, allowing us to understand the quantitative relationship between potential features and attributed emotions.

#### 2.1.1. Participants

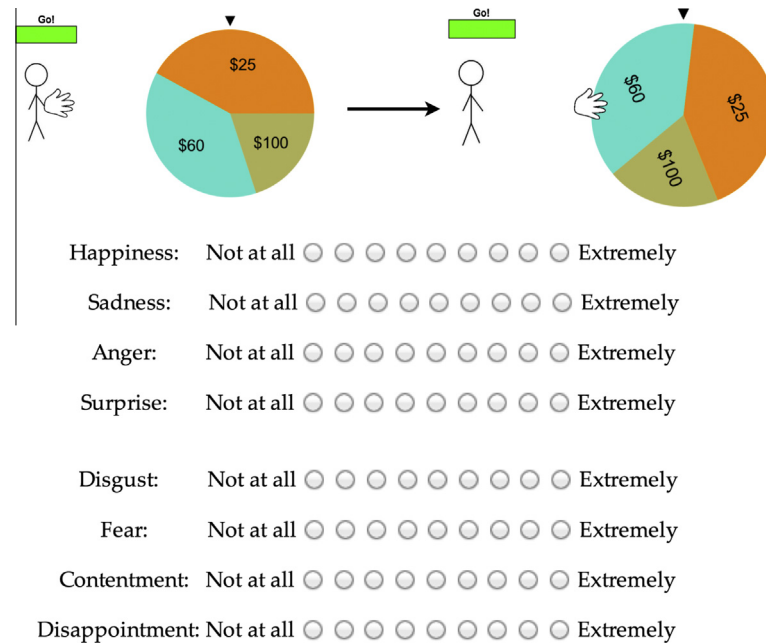
We recruited one hundred participants through Amazon’s Mechanical Turk and paid them for completing the experiment. All experiments reported in this paper were conducted according to guidelines approved by the Institutional Review Board at Stanford University.

#### 2.1.2. Procedures

Participants played the role of observers and watched characters play a simple gambling game. These characters each spun a wheel and won an amount of money depending on where the wheel landed (Fig. 3). On each trial, one character spun one wheel. We pre-generated a total of 18 wheels. Each wheel comprised three possible outcomes, all of which were non-negative. We systematically de-correlated the probabilities of the outcomes and the values of the outcomes of each wheel, allowing for separate

<sup>1</sup> Materials, data, and code can be found at: <http://www.github.com/desmond-ong/affCog>





**Fig. 3.** Sample screenshots from Experiment 1. Participants see a character spin a wheel that lands, in this case, on \$60. Participants then attribute emotions to the character on eight separate Likert scales.

calculation of reward amount and expected value. We used a standard definition of expected value: the averaged reward that the character expected to win. For each “wedge”, or sector of the wheel, we take the reward amount on that sector, and multiply it with the notational probability—based on the size—of that sector. The expected value is the sum of these products over all the sectors. The final correlation of the amount won with the expected value was 0.3. The 18 wheels with 3 outcomes each resulted in (after discarding one to make a number that could be evenly divided by total trials) 50 scenarios, where each scenario corresponds to a particular outcome on a particular wheel. On each trial, the experimental program selected the scenario (the particular outcome to be won) randomly from the total set of 50 scenarios, and not proportional to the sector size within a wheel.<sup>2</sup> This was a design choice to ensure that small sectors (with low notational probability) were equally represented in the data. This fact was not obvious to the participants, and we assume that participants treated the wheel as being fairly spun. The exact position *within* the sector where the spinner lands—for example, whether the spinner lands in the center of the sector, or near the edge of the sector, as in the example shown in Fig. 3—was a real number in the interval (0, 1) drawn from a uniform distribution.

Each participant completed 10 trials. On each trial, they saw a stick figure character with a randomized male name (e.g. “Bob”) spin a wheel. The names of the characters were randomized on every trial. After the result of the spin, participants had to rate how the character feels, on a set of 9 point Likert scales. There were eight different Likert scales for each of eight emotion adjectives: *happy*, *sad*, *angry*, *surprised*, *fearful*, *disgusted*, *content*, and *disappointed*. The first six were the classic “basic emotions” described by Ekman and colleagues (e.g., Ekman et al., 1982). We added “content” and “disappointed” to capture emotion concepts related to counterfactual comparisons with outcomes that could have, but did not occur (Gilovich & Medvec, 1995; Sweeney & Vohs, 2012).

<sup>2</sup> The alternative, which we did not do, would be to randomly choose a wheel with all its possible outcomes from a set of all possible wheels, and then uniformly choose where it lands; each sector would then be sampled proportional to its notational probability.

### 2.1.3. Forward regression model

We expected that affective cognition should depend on specific situation features—especially those related to value computation—that often predict an agent’s emotions. As such, we chose several *a priori* features based on previous work in decision theory as model regressors to predict observers’ affective judgments. It is worth reiterating here that our prediction is not that these features *actually* affect the ways that agents feel; that point has been made by decades of economic and psychological research. Rather, our prediction here is that observers spontaneously rely on these features when inferring how others feel—in essence applying sophisticated lay theories to affective cognition.

We predicted that reward, prediction error, and loss aversion form key features of affective cognition. We operationalized these as regressors representing the amount won by the agent in a trial (“win”), the prediction error (“PE”; the difference between what the agent won and the expected value of the wheel), and the absolute value of the prediction error (“|PE|”), respectively. Using both PE and |PE| in the same model allows the coefficient on PE to differ when PE is positive and negative, modeling loss aversion.<sup>3</sup>

We additionally evaluated several other *a priori* plausible regressors (none of which survived model selection below). People compare their results with salient possible counterfactuals (e.g. what they “could have” won; Medvec, Madey, & Gilovich, 1995). To examine whether observers weigh such comparisons during affective cognition, we computed—for each gamble—a score representing *regret* (the difference between how much the agent won and the *maximum* he could have won<sup>4</sup>; Loomes & Sugden, 1982) and *relief* (the difference between how much the agent won and the *minimum*

<sup>3</sup> Note that researchers model loss aversion as the difference between the coefficient of PE when PE is negative and when PE is positive, often using a piecewise function. It is often modeled using the piecewise equations:  $y = a \text{ PE}$ , for  $\text{PE} > 0$ , and  $y = b \text{ PE}$ , for  $\text{PE} < 0$ . We chose to adopt a mathematically equivalent formulation using both PE and |PE| as regressors in the same model:  $y = c \text{ PE} + d |\text{PE}|$  across all values of PE. We can easily show that  $a = c + d$ , and  $b = c - d$ .

<sup>4</sup> Note that this ‘game-theoretic’ definition of regret (over outcomes given a specified choice) is slightly distinct from most psychological definitions that involve regret over *choices*. We chose not to call this variable disappointment (usually contrasted with regret), as that was one of the emotions we measured.

he could have won). Finally, we drew on previous work on “luck” (e.g. Teigen, 1996), where observers tend to attribute more “luck” to an agent who won when the chances of winning were low, even after controlling for expected payoffs. To model whether observers account for this when attributing other emotions, we included a regressor to account for the probability of winning, i.e., the size of the sector that the wheel landed on. Since a probability is bounded in  $[0,1]$  and the other regressors took on much larger domains of values (e.g. win varied from 0 to 100), we used a logarithm to transform the probability to make the values comparable to other regressors (“logWinProb”).

The final regressor we included was a “near-miss” term to model agents’ affective reactions to outcomes as a function of their distance from other outcomes. Such counterfactual reasoning often affects emotion inference. For instance, people reliably judge someone to feel worse after missing a flight by 5 min, as compared to 30 min (Kahneman & Tversky, 1982). Near-misses “hurt” more when the actual outcome is close (e.g., in time) to an alternative, better outcome. In our paradigm, since outcomes are determined by how much the wheel spins, “closeness” can be operationalized in terms of the angular distance between (i) where the wheel landed (the actual outcome; as defined by the pointer) and (ii) the boundary between the actual outcome and the closest sector. We defined a normalized distance, which ranged from 0 to 0.5, with 0 being at the boundary edge, and 0.5 indicating the exact center of the current sector. Near-misses have much greater impact at smaller distances, so we took a reciprocal transform<sup>5</sup> ( $1/x$ ) to introduce a non-linearity that favors smaller distances. Finally, we scaled this term by the difference in payment amounts from the current sector to the next-nearest sector, to weigh the near-miss distance by the difference in utility in the two payoffs.

In total, we tested seven outcome variables: *win*, *PE*,  $|PE|$ , *regret*, *relief*, *logWinProb* and *nearMiss*. We fit mixed-models predicting each emotion using these regressors as fixed effects, and added a random intercept by subject. We performed model selection by conducting backward stepwise regressions to choose the optimal subset of regressors that predicted a majority of observers’ ratings of the agents’ emotions. This was done using the *step* function in the *R* package *lmerTest*. Subsequently, we used the optimal subset of regressors as fixed effects with the same random effect structure. Full details of the model selection and results are given in Appendix A.

#### 2.1.4. Results

Model selection (in Appendix A) revealed that participants’ emotion ratings were significantly predicted only by three of the seven regressors we initially proposed: *amount won*, the *prediction error* (*PE*), and the *absolute value* of the prediction error ( $|PE|$ ) (see also Section 3.3 for a re-analysis with more data). Crucially, *PE* and  $|PE|$  account for significant variance in emotion ratings after accounting for amount won. This suggests that affective cognition is remarkably consistent with economic and psychological models of subjective utility. In particular, emotion inferences exhibited *reference-dependence*—tracking prediction error in addition to amount won—and *loss aversion*—in that emotion inferences were more strongly predicted by negative, as opposed to positive prediction error. These features suggest that lay observers spontaneously use key features of prospect theory (Kahneman & Tversky, 1979, 1984) in reasoning about others’ emotions: a remarkable connection between formal and everyday theorizing. It is worth noting as well that the significant regressors for *surprise* followed a slightly different pattern from the rest of the other emotions, where the win probability, as well as regret and relief, seem just as important as the amount won, *PE*, and  $|PE|$ .

The aforementioned analysis suggests that amount won, *PE*, and  $|PE|$  are necessary to model emotion inferences in a gambling context and suggest a low dimensional structure for the situation features. Next, we explored the underlying dimensionality of participants’ inferences about agents’ emotions via an *a priori* planned Principal Component Analysis (PCA). Previous work on judgments of facial and verbal emotions (e.g., Russell, 1980; Schlosberg, 1954) and self-reported emotions (e.g., Kuppens, Tuerlinckx, Russell, & Barrett, 2012; Watson & Tellegen, 1985) have suggested a low-dimensional structure, and we planned this analysis to see if a similar low-dimensional structure might emerge in attributed emotions in our paradigm.

The first principal component (PC) accounted for 59% of the variance in participants’ ratings along all 8 emotions, while the second PC accounted for 16%; subsequent PCs individually explained less than 10% of the variance. The first PC accounted for most of the variance in the emotion ratings, although the second PC accounted for a far lower, but still noteworthy, amount of variance. Full details of the PCA procedure and loading results are given in Appendix A.

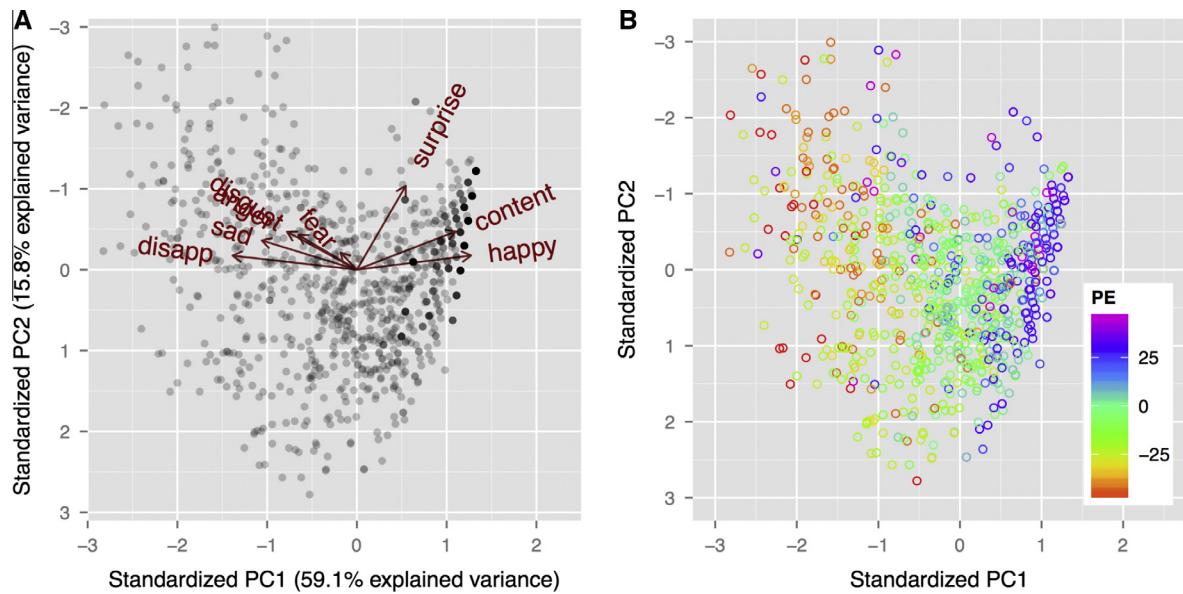
Post-hoc exploratory analysis of the first two PCs revealed that the first PC positively tracked happiness and contentment, while negatively tracking all negative emotions; by contrast, the second PC positively tracked the intensity of both positive and negative emotions (Fig. 4A). Interestingly, this connects with classic concepts of *valence* and *arousal*, respectively, which feature centrally in emotion science<sup>6</sup> (e.g., Kuppens et al., 2012; Russell, 1980; Schlosberg, 1954). In particular, some theorists view emotional valence as a crucial form of feedback to the agent: positively valenced emotions like happiness signal a positive prediction error—that the agent is doing better than expected—hence, positively reinforcing successful behavior. Conversely, negatively valenced emotions could signal to the agent to change some behavior to achieve a better outcome (e.g., Carver & Scheier, 2004; Ortony et al., 1988). In line with this, we find that the first PC (“valence”) of emotions attributed by the observer correlated strongly with the *PE* of the situation ( $r = 0.737$ , 95% C.I. =  $[0.707; 0.764]$ ). Additionally, we find that the second PC (“arousal”) correlated with  $|PE|$  ( $r = 0.280$   $[0.222, 0.364]$ ; Fig. 4B).

We started with a *a priori* predictions for a low-dimensional summary of outcome features, and followed up with a post hoc dimensionality reduction analysis of the emotion ratings. It is intriguing that the low-dimensional value computations (e.g. *PE*,  $|PE|$ ) are intimately tied with the principal components of the emotion ratings (“valence” and “arousal”). However, note that the second PC (“arousal”) accounts for much less variance than the first PC (“valence”). One possibility is that the paradigm we used is limited in the range of emotions that it elicits in an agent, which restricts the complexity of emotion inferences in this paradigm. A second possibility is that emotional valence is the central feature of affective cognition, and valence would carry most of the variance in emotion inferences across more complex scenarios. Although we are not able to address the second possibility in this paper, there is much theoretical evidence from affective science (e.g., Barrett & Russell, 1999; Russell, 1980; Schlosberg, 1954) in favor of the second possibility; future work is needed to explore this further.

Together, these findings provide two key insights into the structure of affective cognition: (i) lay theories of emotion are low dimensional, consistent with affective science concepts of valence and arousal, and (ii) these core dimensions of emotion inference also track aspects of the situation and outcome that reflect value computation parameters described by economic models such as prospect theory. Although the specific structure of affective

<sup>5</sup> We tried other non-linear transforms such as exponential and log transforms, which all performed comparably.

<sup>6</sup> We invite the reader to compare the striking similarity between our Fig. 4A with similar valence-arousal figures in the literature, such as Fig. 1 from Russell (1980) and Fig. 2A from Kuppens et al. (2012).



**Fig. 4.** (A) Participants' emotion ratings projected onto the dimensions of the first two principal components (PCs), along with the loadings of the PCs on each of the eight emotions. The loading of the PCs onto the eight emotions suggests a natural interpretation of the first two PCs as "valence" and "arousal" respectively. The labels for disgust and anger are overlapping. (B) Participant's emotion ratings projected onto the dimensions of the first two PCs, this time colored by the prediction error (PE = amount won – expected value of wheel). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

cognition likely varies depending on the complexity and details of a given context, we believe that observers' use of low-dimensional "active psychological ingredients" in drawing inferences constitutes a core feature of affective cognition.

## 2.2. Experiment 2: Outcomes from emotions

Experiment 1 established the features that allow observers to reason about agents' emotions given a gamble outcome: representing  $P(\mathbf{e}|\mathbf{o})$  in a simple linear model. In Experiment 2 (Fig. 5), we test the hypothesis that participants' lay theories are flexible enough to also allow "backward" inferences about the situation outcome based on emotions. These backward inferences are predicted from the forward regression model by Bayes' rule. To evaluate our model's predictions, we show participants the emotions of characters who have played the same game show—importantly, without showing the eventual outcome—and elicit participants' judgments about the likely outcome. We then compare the empirical judgments (in Experiment 2) to the posterior probabilities predicted by the model (based on data from Experiment 1).

### 2.2.1. Participants

We recruited one hundred twenty-five participants via Amazon Mechanical Turk. We excluded three participants because they reported technical difficulties with the animation, resulting in a final sample size of 122.

### 2.2.2. Stimuli

We generated graphical *emotion profiles* that ostensibly represented the emotions that the agent feels after each outcome (Fig. 5B). These emotion profiles were shown on continuous slider scales that corresponded to each individual emotion. For each of the outcomes, we used the average emotion ratings given by participants in Experiment 1 to generate the emotion profiles seen by participants in Experiment 2. Specifically, for each outcome, we drew eight emotion values from Gaussians with means and standard deviations equal to the means and standard deviations of the emotion ratings given by participants in response to that outcome in Experiment 1. This was meant to provide naturalistic emotion profiles for the participants in Experiment 2.

### 2.2.3. Procedures

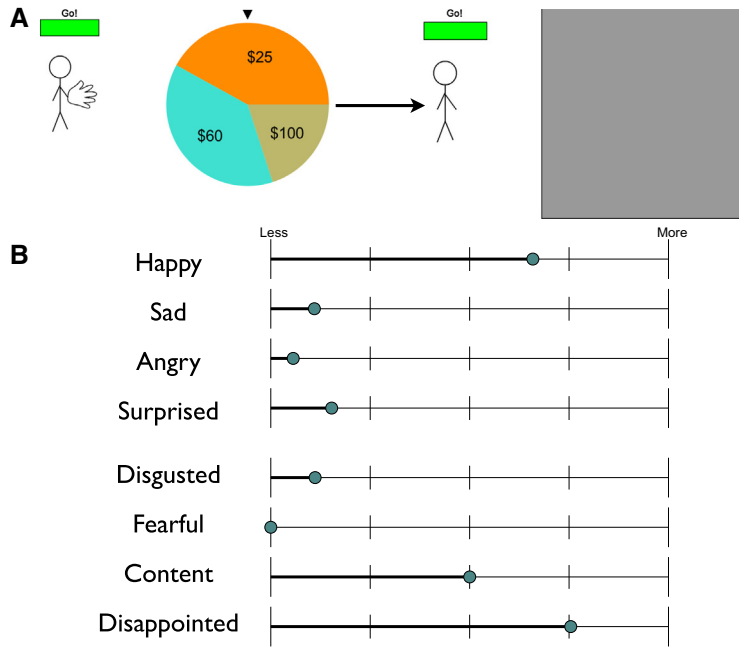
Each participant completed 10 trials, randomly drawn from the same 50 pre-generated scenarios used in Experiment 1. On each trial, participants were shown a stick figure and the game wheel, as before. This time, as the wheel spun, a gray square covered the wheel, occluding the outcome (Fig. 5A). Participants were then shown a graphical representation of the agent's emotions after seeing the outcome (Fig. 5B). For instance, on a particular trial, a participant might see three possible gamble outcomes of \$25, \$60, and \$100. Without being able to see where the wheel landed, they might then be told that the agent who spun the wheel, following its outcome, feels moderately happy (e.g., a slider at about a 6 on a 9 point scale), not at all sad (e.g., another slider that shows between a 1 and 2 on a 9 point scale), and so forth; see Fig. 5 for an illustration. Participants were then asked to infer how likely it was that each of the three possible outcomes had occurred. They gave probability judgments on 9 point Likert scales. Hence, on each trial, they gave three likelihood judgments, which corresponded to each of the three possible outcomes.

### 2.2.4. Bayesian model details

In order to generate model-based predictions about observers' "reverse inference" about outcomes given emotions, the posterior  $P(\mathbf{o}|\mathbf{e})$ , one needs three components:  $P(\mathbf{o})$ ,  $P(\mathbf{e})$ , and  $P(\mathbf{e}|\mathbf{o})$  (Eq. (1)). The prior probabilities of the outcomes  $P(\mathbf{o})$  here are given by the notational probability of each outcome on the wheel—i.e. the relative size of each outcome—and are transparent to an observer in our paradigm. Larger sectors have higher probabilities of that outcome occurring. One actually does not need to explicitly calculate the prior probability of the emotion  $P(\mathbf{e})$  as long as we properly normalize  $P(\mathbf{o}|\mathbf{e})$ .<sup>7</sup>

The crucial step lies in calculating the likelihood  $P(\mathbf{e}|\mathbf{o})$ . To calculate  $P(\mathbf{e}|\mathbf{o})$ , we drew on the "forward reasoning" data collected in

<sup>7</sup> Consider calculating the posteriors of the three possible outcomes given an observed value of happiness,  $\mathbf{h}$ :  $P(\mathbf{o}_1|\mathbf{h})$ ,  $P(\mathbf{o}_2|\mathbf{h})$ , and  $P(\mathbf{o}_3|\mathbf{h})$ , which are proportional to  $[P(\mathbf{h}|\mathbf{o}_1)P(\mathbf{o}_1)]$ ,  $[P(\mathbf{h}|\mathbf{o}_2)P(\mathbf{o}_2)]$ , and  $[P(\mathbf{h}|\mathbf{o}_3)P(\mathbf{o}_3)]$  respectively. The sum of the latter three quantities is simply the prior on emotion  $P(\mathbf{h})$ . Thus, as long as proper normalization of the probabilities is carried out (i.e. ensuring that the posteriors all sum to 1), we do not need to explicitly calculate  $P(\mathbf{h})$  in our calculation of  $P(\mathbf{o}|\mathbf{h})$ . This is true for the other emotions in the model.



**Fig. 5.** Screenshots from Experiment 2. (A) Participants were shown a character that spins a wheel, but a gray square then occludes the outcome on the wheel. (B) Participants then saw a graphical representation of the character's emotions, and inferred separate probabilities for each of the three possible outcomes having occurred.

Experiment 1. In particular, we leveraged Experiment 1's regression model to calculate the extent to which observers would be likely to infer different emotions of an agent based on each outcome. To do so, we applied the variables identified as most relevant to affective cognition in Experiment 1—*win*, *PE*, and  $|PE|$ —to predict the emotions observers would assign to agents given the novel outcomes of Experiment 2. For instance, in modeling happiness, this approach produces the following equation for each wheel outcome:

$$happy = c_{0,happy} + c_{1,happy}win + c_{2,happy}PE + c_{3,happy}|PE| + \varepsilon_{happy} \quad (2)$$

We employed similar regression equations to estimate  $P(e|o)$  for the other seven emotions, where  $\varepsilon_{emotion}$  (with zero mean and standard deviation  $\sigma_{emotion}$ ) represents the residual error terms of the regressions. The coefficients  $c_{i,emotion}$  are obtained numerically by fitting the data from Experiment 1; the linear model is fit across all participants and scenarios to obtain one set of coefficients per emotion. For a new scenario with  $\{win, PE, |PE|\}$ , the likelihood of observing a certain happy value  $h'$  is simply the probability that  $h'$  is drawn from the linear model. In other words, it is the probability that the error

$$h' - c_{0,happy} - c_{1,happy}win - c_{2,happy}PE - c_{3,happy}|PE| \quad (3)$$

is drawn from the residual error distribution for  $\varepsilon_{happy}$ .

The error distribution  $\varepsilon_{happy}$  resulting from the above regression captures intrinsic noise in the relation between outcomes and emotions of the agent—uncertainty in the participant's lay theory. However, in addition to this noise, there are several other sources of noise that may enter into participants' judgments. First, participants may not take the graphical sliders as an accurate representation of the agent's true emotions (and indeed we experimentally generated these displays with a small amount of noise, as described above). Secondly, participants might have some uncertainty around reading the values from the slider. Thirdly, participants may also have some noise in the prior estimates they use in each trial.

Instead of having multiple noise parameters to model these and other external sources of noise, we instead modified the intrinsic

noise in the regression model. We added a likelihood smoothing parameter  $\zeta$  (zeta), which amplifies the intrinsic noise in the regression model, such that the likelihood  $P(h'|o)$  is the probability that  $h' - c_{0,happy} - c_{1,happy}win - c_{2,happy}PE - c_{3,happy}|PE|$  is drawn from  $N(0, (\zeta\sigma_{happy})^2)$ , i.e. a normal distribution with mean 0 and standard deviation  $\zeta\sigma_{happy}$ .

Using Eq. (3) with the additional noise parameter, we can calculate the likelihood of observing a value  $h'$  as a result of an outcome  $o$ , i.e.  $P(h'|o)$ . We then calculate the joint likelihood of observing a certain combination of emotions  $e'$  for a particular outcome  $o$  as the product of the individual likelihoods,

$$P(e'|o) = P(happy'|o)P(sad'|o) \dots P(diapp'|o) \quad (4)$$

A note on Eq. (4): The only assumption we make is that the outcome  $o$  is the only cause of the emotions, i.e., there are no other hidden causes that might influence emotions. The individual emotions are *conditionally independent* given the outcome (common cause), and thus the joint likelihood is proportional to the product of the individual emotion likelihoods.

Next, to calculate the posterior as specified in Eq. (1), we multiply the joint likelihood  $P(e'|o)$  with the prior probability of the outcome  $o$  occurring,  $P(o)$ , which is simply the size of the sector. We performed this calculation for each individual outcome, before normalizing to ensure the posterior probabilities  $P(o|e')$  for a particular wheel sum to 1 (by dividing each probability by the sum of the posteriors). The normalization removes the need to calculate  $P(e')$  explicitly (see Footnote 3). The resulting model has only one free noise parameter, otherwise being fixed by hypothesis and the results of Experiment 1.

Up to this point, we had only used data from participants in Experiment 1 to build the model. To verify the model, we collapsed the empirical judgments that participants gave in Experiment 2 (an independent group compared to participants in Experiment 1) for each individual outcome. We then compared the model's predicted posterior probabilities for each outcome to the empirical judgments.

We optimized the one free noise parameter  $\zeta$  in the model to minimize the root-mean-squared-error (RMSE) of the model



residuals. We conducted a bootstrap with 5000 iterations to estimate the noise parameter, RMSE, and the model correlation, as well as their confidence interval.

### 2.2.5. Results

Model-based posterior probabilities tightly tracked the observer judgments in Experiment 2 (Fig. 6B). The optimal noise parameter was 3.2 [2.9, 3.6], which resulted in a model RMSE of 0.116 [0.112, 0.120]. The model's predictions explained much of the variance in participants' judgments, achieving a high correlation of 0.806 [0.792, 0.819]. For comparison, the bootstrapped split-half correlation of the posterior probability estimates in Experiment 2 is 0.895 [0.866, 0.920]. The split-half correlation for the emotion attributions in Experiment 1, the data that this model is fit to, is 0.938 [0.925, 0.950]. Together these two split-half reliabilities give an upper-bound for model performance, and our model performs very comparably to these upper limits.

This results suggest that a Bayesian framework can accurately describe how observers make reverse inferences,  $P(\mathbf{o}|\mathbf{e})$ , given how they make forward inferences,  $P(\mathbf{e}|\mathbf{o})$ . At a broader level, the results imply that the causal knowledge in an observer's lay theory of emotion is abstract enough to use for multiple patterns of reasoning. In the next section, we extend this work further by considering inferences about emotion from *multiple* sources of information.

## 3. Emotional cue integration

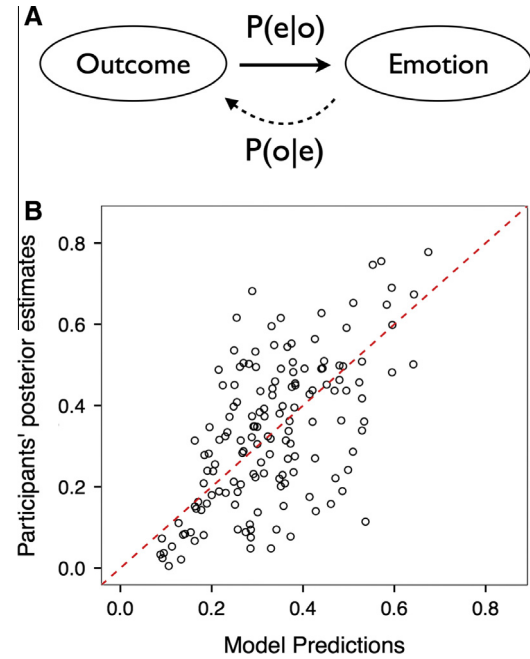
Thus far, we have examined how observers reason from outcomes to emotions, and likewise use emotions to draw inferences about the outcomes that caused those emotions. However, unlike the observers in Experiment 2 who were given a graphical read out of agents' emotional states, real-world observers rarely have direct access to others' emotions. Instead observers are forced to draw inferences about invisible but crucial emotional states based on agents' outward, observable cues. In fact, observers often are tasked with putting together multiple, sometimes competing emotional cues, such as facial expression, body language, situation outcome, and verbal utterances. How does the observer integrate this information—performing *emotional cue integration*—to infer the agent's underlying emotional state? We propose that as with other forms of cognition, observers' performance might be similar to an ideal observer that rationally integrates cues to an agent's emotion using Bayesian inference.

The model we introduced earlier (Fig. 2) can be extended to integrate multiple cues to emotion. Assume that the observer has access to both the outcome  $\mathbf{o}$  and the facial expression  $\mathbf{f}$ . The posterior probability of an underlying emotion  $\mathbf{e}$  given *both*  $\mathbf{o}$  and  $\mathbf{f}$  is given by the following equation:

$$P(\mathbf{e}|\mathbf{o}, \mathbf{f}) \propto \frac{P(\mathbf{e}|\mathbf{f})P(\mathbf{e}|\mathbf{o})}{P(\mathbf{e})} \quad (5)$$

A complete derivation of this equation is given in Appendix B. The joint-cue posterior—that is,  $P(\mathbf{e}|\mathbf{o}, \mathbf{f})$ —is proportional to the product of the individual cue likelihoods  $P(\mathbf{e}|\mathbf{f})$  and  $P(\mathbf{e}|\mathbf{o})$ , normalized by the prior probability of the emotion occurring  $P(\mathbf{e})$ . This mathematical expression captures the intuition that judgments using two cues should be related to the judgments using each individual cue.

Not all cues provide equal information about the latent emotion—some cues are more *reliable* than others. Consider  $P(\mathbf{e}|\mathbf{f})$  or the probabilities of observing different intensities of emotion  $\mathbf{e}$ , given a face  $\mathbf{f}$ . A big frown, for instance, often signals low happiness; thus, if we operationalize *happiness* as a variable ranging from 1 to 9, we can represent  $P(\text{happy}|\text{frown})$  as a curve that is



**Fig. 6.** (A) Simple causal model. The solid arrow represents a causal relationship (outcomes “cause” emotion;  $P(\mathbf{e}|\mathbf{o})$  from Experiment 1), while the dashed arrow represents an inference that can be made ( $P(\mathbf{o}|\mathbf{e})$  from Experiment 2). (B) Comparison of participants' estimates of the posterior probability from Experiment 2 with the predictions of the model built in Experiment 1. There is a strong correlation of 0.806 [0.792, 0.819]. The dashed red line has intercept 0 and slope 1 and is added for reference. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

sharply peaked at a low value of *happy*, like 3 (see the red curve in the top panel in Fig. 7.). In this case, when the observer can be fairly confident of the inference of emotion given the face, we say that this face is a *reliable* cue.

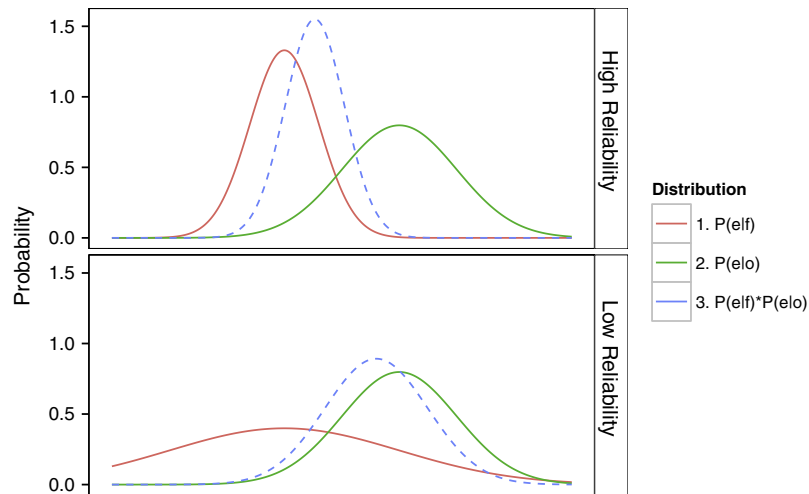
By contrast, if a second face has an ambiguous expression, this face might not give the observer much information about the unobserved emotion. In this case, we represent  $P(\text{happy}|\text{ambiguous expression})$  as a curve with a larger variance. If asked to make an inference about an agent's emotion based on this expression, observers make an inference on a value of happiness, but they may not be confident in the accuracy of that estimate; this face is an *unreliable* cue (see the red curve in the bottom panel in Fig. 7).

Consider the two faces described above, now each paired with a medium-reliability context. Imagine an observer who sees an agent unwrap a parcel from home, a context that usually elicits some joy. If the observer sees the context paired with an ambiguous facial expression (a relatively low-reliability cue), the observer's judgments would tend to rely more on the context, because the facial expression contributes little information (bottom panel, Fig. 7). In contrast, if the observer sees instead a big frown (a relatively high-reliability cue) with the context, the observer's judgments would tend to favor the facial expression over the context (top panel, Fig. 7). Importantly, reliability gives a quantifiable way of measuring how much each cue should be weighted with respect to each other.

We operationalize the reliability of a cue by calculating the information theoretic entropy (Shannon & Weaver, 1949),  $H$ , of the emotion-given-cue (e.g.  $\mathbf{e}|\mathbf{f}$ ) distribution:

$$H[P(\mathbf{e}|\mathbf{f})] = - \sum_{\mathbf{f} \in \mathcal{F}} P(\mathbf{f}) \sum_{\mathbf{e}} P(\mathbf{e}|\mathbf{f}) \log P(\mathbf{e}|\mathbf{f}) \quad (6)$$

where the sum over  $\mathbf{e}$  is taken over the different values of the emotion (in our model, emotions take discrete values from 1 to



**Fig. 7.** Illustration of the effect of the reliability of a cue in cue combination. The same contextual outcome cue distribution (green solid line) is given in both panels. Top: the emotion distribution for a reliable face (red solid line). The distribution is relatively sharply peaked. Bottom: the emotion distribution for a less reliable face with the same mean (red solid line), which results in a larger uncertainty in the distribution as compared to the top panel. The blue dashed line in both panels shows the product of the two distributions (note that this is *only the numerator* in the cue integration equation, Eq. (5), and does not include the normalizing prior term in the denominator). One can see that the low reliability cue has a relatively smaller effect on the distribution of the product: the distribution  $P(e|f) * P(e|o)$  in the bottom panel is very similar to the distribution  $P(e|o)$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

9 inclusive).<sup>8</sup> The entropy of a distribution captures the amount of information that can be obtained from the distribution, with a more informative (“sharply-peaked”) distribution having lower entropy. Thus, the higher the reliability of the cue, the lower the entropy of the emotion-given-cue probability distribution.

The resulting model predictions are sensitive to the reliability of a given cue. To illustrate, consider two faces,  $f_1$  and  $f_2$ , whose distributions  $P(e|f)$  have the same mean. Let  $f_1$  have a much smaller variance so that  $P(e|f_1)$  is very sharply peaked (red curve, top panel, Fig. 7), and let  $f_2$  have a much larger variance so that  $P(e|f_2)$  is almost uniform (red curve, bottom panel, Fig. 7) i.e.,  $f_1$  is a more reliable cue and has less entropy than  $f_2$ . In a single cue emotion-perception scenario, where an observer has to make an inference about the emotion  $e$  given only either  $f_1$  or  $f_2$ , the distribution of the observer's inferences would follow  $P(e|f)$ ; in both cases, the observer would give the same mean, but perhaps with more variance in the case of  $f_2$ . Consider next a multi-cue integration scenario, where the observer is given two cues, the outcome context  $o$  (green curve, Fig. 7), and either  $f_1$  or  $f_2$ . Because  $f_2$  is less reliable than  $f_1$ ,  $f_2$  will be weighted less with respect to  $o$ , than  $f_1$  will be weighted relative to  $o$ . Mathematically, this follows from Eq. (5): multiplying the distribution  $P(e|o)/P(e)$  by  $P(e|f_2)$  will have little effect because  $P(e|f_2)$  has very similar values across a large domain of emotion values, and so would modify each potential emotion by a similar amount. Conversely, multiplying by  $P(e|f_1)$  will have a larger effect (see Fig. 7 for an illustration).

Finally, we consider two approximations—or simplifications—that observers could use during affective cognition, as alternatives to our fully Bayesian cue integration model. Under a *face-cue-only* model, an observer presented with a face  $f$  and an outcome  $o$  would rely *only* on the face, and would thus use  $P(e|f)$  to approximate  $P(e|o, f)$ . Under a second *outcome-cue-only* model, the observer instead relies exclusively on the outcome and uses  $P(e|o)$  to approximate  $P(e|o, f)$ . These approximations are meant to model the predictions made by “face dominance” and “context dominance” accounts (see above).

In Experiments 3 and 4, we calculate the performance of these two approximate models in addition to the full Bayesian model. In Experiment 3, we examined cue integration with outcomes and faces, and in Experiment 4, we examined integration of evidence from outcomes and verbal utterances, showing the generalizability of the model to other types of emotional cues.

### 3.1. Experiment 3: Cue Integration from outcomes and faces

In Experiment 3, we tested our cue integration model by examining its correspondence with human judgments of combinations of facial expressions and situation outcomes. On one third of the trials, participants saw the outcome of a gamble that a character plays, as in Experiment 1. On another third of the trials, participants saw only the facial expression following the outcome, but not the outcome itself. On the final third of “joint-cue” trials, participants saw both the facial expression and the outcome. On all trials, participants attributed emotions to the character, as in Experiment 1.

#### 3.1.1. Participants

We recruited four hundred sixty-five participants through Amazon's Mechanical Turk. We planned for a larger number than were included in Studies 1 and 2 because of the large number of face stimuli tested.

#### 3.1.2. Stimuli

The gambles were identical to Experiments 1 and 2, except that we used only 10 possible scenarios for this experiment. We generated eighteen facial expressions, shown in Fig. 8, using the software *FaceGen*. The first 12 faces varied in emotional valence and arousal. Here, we operationalized emotional valence by parametrically varying the curvature of the lips—for positive valence: an upward turn in the lips for a smile; and for negative valence: a downward turn in the lips for a frown or scowl—and the shape of the eyes and eyebrows, with larger eyes and relaxed eyebrows signaling positive valence, and smaller eyes and furrowed eyebrows signaling negative valence. We operationalized emotional arousal by varying the gap between the lips, with low arousal faces having no gap between the lips and high arousal faces showing a

<sup>8</sup> In our experiments, faces are chosen at random, hence we use a uniform distribution for the prior on faces,  $P(f)$ .

wide gap (a wide mouthed smile or scowl with teeth). We designed the final 6 faces to be “ambiguous”, i.e. a mix of different emotions like sad and angry, or sad and happy. We made these using a combination of FaceGen’s preset discrete emotion settings. Exact combinations are given in Fig. 8 caption.

### 3.1.3. Procedures

Participants completed ten trials. On each trial, participants watched a character play a gamble in the form of a wheel (Fig. 9A). Participants saw the character spin the wheel, and were then shown one of three possibilities. On *Outcome-Only* trials, participants saw only the outcome on the wheel (similar to Experiment 1). On *Face-Only* trials, a gray square would occlude the outcome on the wheel, and participants saw only the facial expression of the character after the character sees the outcome on the wheel (we told participants that the character still could see the outcome). Finally, on *Joint-Cue* trials, participants saw both the outcome and the facial expression. See Fig. 9B for an illustration. These three types of trials (two types of single cue trials and one joint-cue trial) occurred with equal probability, so on average each participant encountered about three of each type of trial, out of ten total trials. On the joint-cue trials where both facial and outcome cues are shown, we randomly matched the outcome and face, so there was no correlation between the emotions typically conveyed by the two cues. This ensured a random mix of congruent (e.g., a large outcome on the wheel accompanied by a positive face) and incongruent combinations (e.g., a large outcome on the wheel accompanied by a negative face). On all trials, participants rated the extent to which the character felt each of eight discrete emotions—*happy*, *sad*, *angry*, *surprised*, *fearful*, *disgusted*, *content*, and *disappointed*—using 9-point Likert scales.

### 3.1.4. Model details

We used participants’ responses to the single-cue *Outcome-Only* and *Face-Only* trials to construct empirical distributions for  $P(\mathbf{e}|\mathbf{o})$  and  $P(\mathbf{e}|\mathbf{f})$  respectively (Fig. 9C). For the outcome model,  $P(\mathbf{e}|\mathbf{o})$ , we used the set of features isolated in Experiment 1 (*win*, *PE*, *|PE|*). The ratings for the outcome-only trials were used to model  $P(\mathbf{e}|\mathbf{o})$ , in a similar fashion to Experiment 1. The set of features isolated in Experiment 1 enables a reduction in dimensionality of the situation features, which allows the estimation of a less noisy statistical model. For faces, it is less clear what a corresponding set of low dimensional features would be, and so we estimated  $P(\mathbf{e}|\mathbf{f})$  from the raw density-smoothed<sup>9</sup> empirical ratings to the face-only trials. Using the face-only model  $P(\mathbf{e}|\mathbf{f})$  and the wheel-only model  $P(\mathbf{e}|\mathbf{o})$ , we can construct the full Bayesian cue integration model (Eq. (5)), as well as the approximate face-only and wheel-only models.

We compared these three models against participants’ responses to the *Joint-Cue* trials. First, we used participants’ responses to the *Joint-Cue* trials to generate an empirical (density-smoothed) probability distribution of emotion given a particular face and outcome combination  $P(\mathbf{e}|\mathbf{o}, \mathbf{f})$ . From this, we calculated the expected emotion rating (i.e., a real value from 1 to 9) for each emotion for each combination. Finally, we compared the empirical expectations with the expectations of the different models. To clarify, though this was a within-subject paradigm, we did not have enough statistical power to build a cue integration model for each individual participant. Instead, we constructed these models collapsed across participants.

To compare the performance of the models, we calculated two quantities, the root-mean-squared-error (RMSE) and the correlation, both with respect to the empirical data. We bootstrapped

the two values, along with their 95% confidence intervals, from bootstrap calculations with 5000 iterations.

### 3.1.5. Results

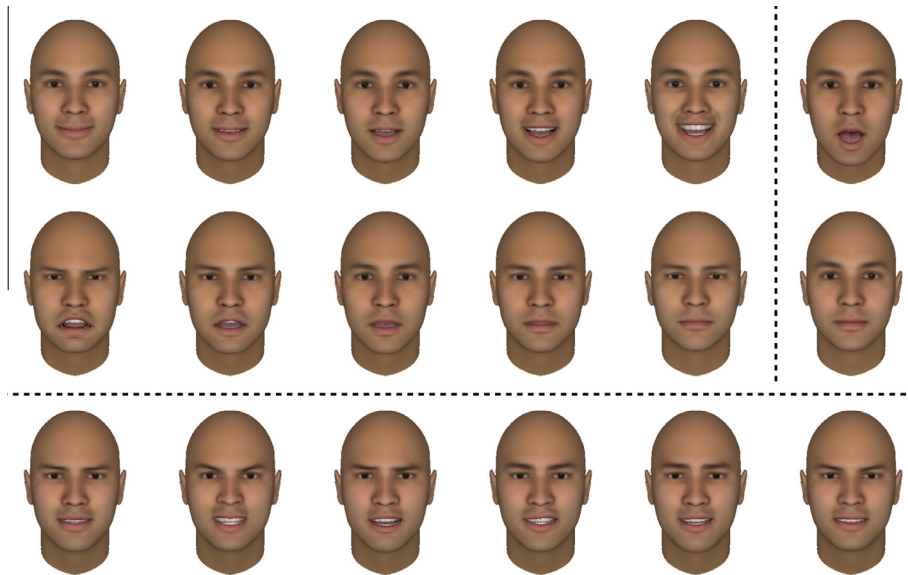
We present a summary of the results in Fig. 10. Comparing the RMSE on all the trials (Fig. 10A), the Bayesian model performed the best with the lowest RMSE of 1.218 [1.149, 1.287], outperforming<sup>10</sup> both the Face-only model with 1.386 [1.314, 1.464] ( $t(49) = -3.30$ ,  $p = .002$ ) and the Outcome-only model with 1.524 [1.462, 1.589] ( $t(49) = -6.46$ ,  $p < .001$ ). Correspondingly, the Bayesian model also had the highest correlation with the empirical data ( $r = 0.812$  [0.790, 0.833]), performing significantly better than the Face-only ( $r = 0.733$  [0.706, 0.759];  $t(49) = 4.55$ ,  $p < .001$ ) and the Outcome-only ( $r = 0.692$  [0.664, 0.717];  $t(49) = 7.06$ ,  $p < .001$ ) models (Fig. 10B).

Next, we analyzed two subsets of the data that contained incongruent cue combinations: *Joint-Cue* trials that had a negatively-valenced face presented with a “positive” wheel (defined as winning the largest amount on the wheel), and *Joint-Cue* trials that had a positively-valenced face presented with a “negative” wheel (defined as winning the smallest amount on the wheel). Although the pattern of RMSEs did not differ from the set of all trials to the incongruent combinations (Fig. 10A), we found an interesting result when we examined the correlations of the single-cue-only models in the incongruent cue-combination trials (Fig. 10B). When a negative face is presented with a positive wheel, the Face-only model produced a lower correlation of  $r = 0.343$  [0.213, 0.468] than the Outcome-only model ( $r = 0.465$  [0.332, 0.576];  $t(49) = -1.31$ ,  $p = .20$ , not significant) and the full Bayesian model ( $r = 0.571$  [0.454, 0.668];  $t(49) = -2.61$ ,  $p = .012$ , significant with some overlap of the 95% CIs). Conversely, when a positive face is presented with a negative wheel, the Outcome-only model did significantly worse with a correlation of 0.297 [0.165, 0.422], as compared to the Face-only model ( $r = 0.678$  [0.551, 0.778];  $t(49) = -4.19$ ,  $p < .001$ ) and the full Bayesian model ( $r = 0.667$  [0.568, 0.748];  $t(49) = -4.55$ ,  $p < .001$ ). In other words, if we compare the single-cue-only models, when a negative cue is presented with a positive cue, the model that *only* considers the positive cue better approximated observer judgments than the model that only considers the negative cue.

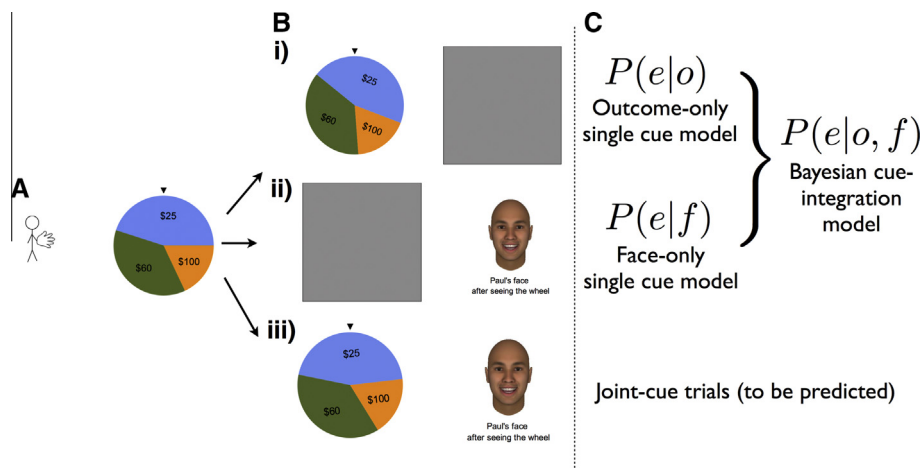
This “valence-dominance” result is surprising, and in fact, is not predicted by the literature, which, as laid out earlier, predicts specific-cue-dominance. The Bayesian model, however, seems to account for this valence effect extremely well. In particular, the Bayesian model automatically weights the positive cues to a greater extent than the negative cues. Because the Bayesian model weights cues according to their reliability (see discussion above, especially Fig. 7), this suggests that the positive cues have a higher reliability than the negative cues. If this is true, then the positive cues distributions (both  $P(\mathbf{e}|\mathbf{o})$  and  $P(\mathbf{e}|\mathbf{f})$ ) should have a lower entropy than the negative cue distributions. Post-hoc analyses confirmed this predicted difference in distribution entropies. The negative faces have a significantly higher entropy (mean entropy of the emotion given negative face distributions = 2.66 bits,  $SD = 0.43$  bits) as compared to the positive faces (mean entropy = 1.60 bits,  $SD = 0.97$  bits;  $t(7) = 2.70$ ,  $p = .03$ ). Similarly, the negative wheels have a significantly higher entropy (mean entropy = 2.39 bits,  $SD = 0.37$  bits) than the positive wheels (mean entropy = 1.99 bits,  $SD = 0.43$  bits;  $t(7) = 4.10$ ,  $p = .005$ ). This highlights an interesting result: the reason why there seems to be some

<sup>9</sup> We used R’s *density* function with its default settings to perform density smoothing using a Gaussian kernel.

<sup>10</sup> In this section we bootstrapped parameter estimates and 95% confidence intervals. This allows judgment statistical significance of differences by CI non-overlap, a non-parametric approach. For ease of comparison, we additionally estimate and report *t*-statistics, bearing in mind the hidden parametric assumptions that may not be satisfied.



**Fig. 8.** Face stimuli used in Experiment 1, created using FaceGen. The 12 faces in the top and middle rows vary in both valence and arousal. Top row: positively valenced faces, increasing in valence and arousal from left to right. Middle row: negatively valenced faces, decreasing in valence and arousal from left to right. The top and middle right-most faces are neutral valence high arousal and neutral valence low arousal, respectively. Bottom row: set of “ambiguous” faces made using combinations of FaceGen’s pre-defined discrete emotions. From left to right: (Sad, Surprised, and Happy), (Angry, Happy, and Surprised), (Fear, Happy, and Disgust), (Disgust, Surprised, and Happy), (Sad, Happy, Disgust and Fear), and (Sad, Happy, and Angry).



**Fig. 9.** (A) Screenshot from a trial from Experiment 3. Participants saw a character about to spin a wheel with three possible outcomes. (B) Each trial resulted in one of three possibilities: the participant is shown (i) only the outcome, (ii) only the character’s facial expression, or (iii) both the outcome and the facial expression. Following this, the participant is asked to judge the character’s emotions. (C) The single cue trials are used to model  $P(e|o)$  and  $P(e|f)$  respectively, which serve as single-cue only models. The single-cue models are used to calculate the Bayesian cue-integration model. These three models are evaluated using empirical judgments made by participants in the joint-cue trials.

evidence for a “positive-valence-dominance” is because positive cues tend to have higher reliability. This result implies that—at least in the context of our task—participants tend to be more certain when making emotion attributions to agents given positive, as compared to negative, cues.

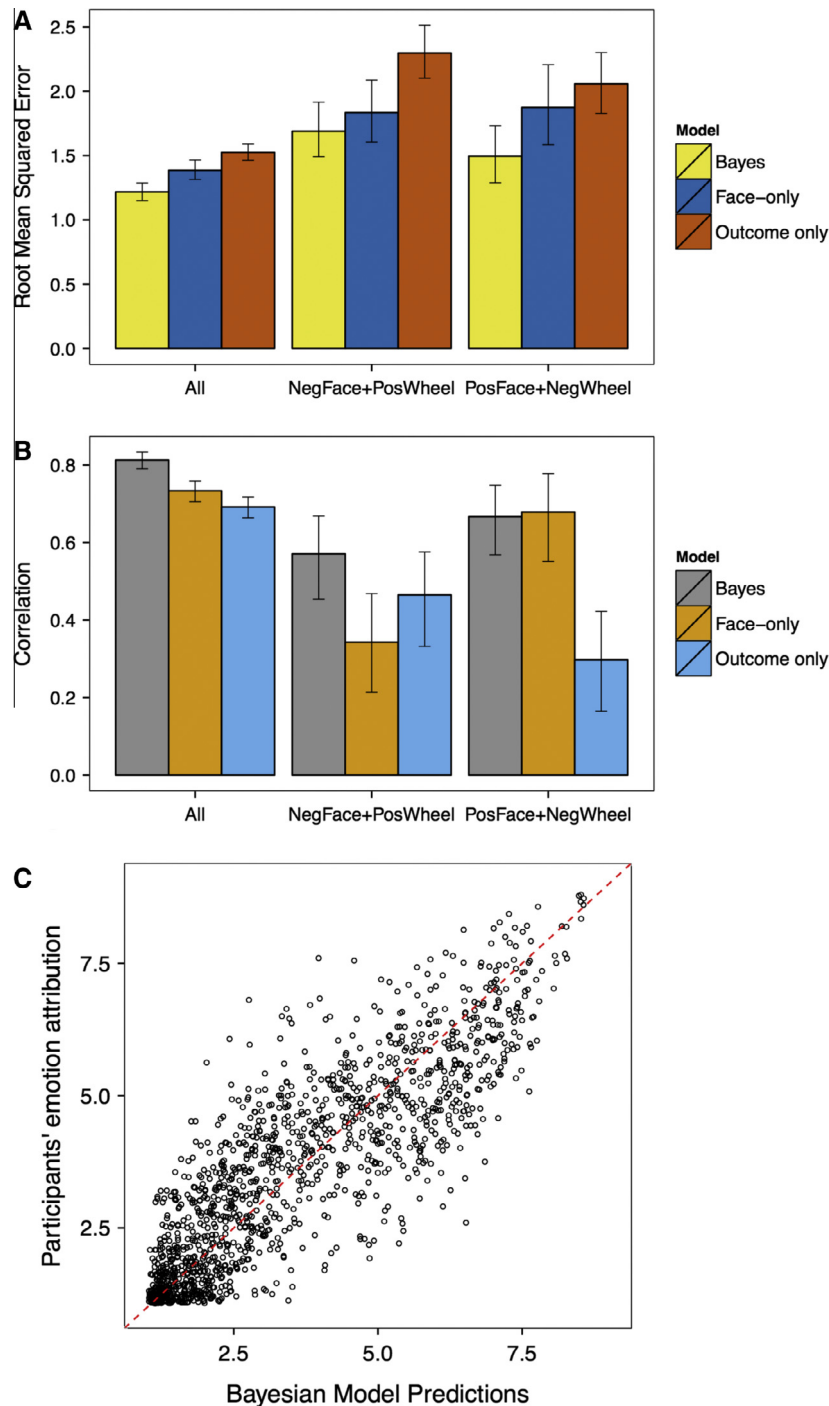
In sum, the results from Experiment 3 showed that the Bayesian model best predicts participants’ judgments of emotions in multiple-cue scenarios. In addition, this quantitative paradigm allowed us to examine participants’ emotion attributions in incongruent cue combinations, and uncovered evidence for a different type of dominance: in our paradigm, positively-valenced cues have greater reliability and tend to dominate negatively-valenced cues. However, we do not want the take home message to be that “positive-valence-dominance” is a better rule than face or context dominance to resolve conflicts; in fact, this is antithetical to the spirit of

the model. The Bayesian model makes one simple assumption: that observers weigh cues according to the cues’ reliability. In this gambling paradigm, positive cues have higher reliability, but we do not want to generalize that positive cues in other contexts are more reliable as well. The Bayesian model accounted for this valence effect even without an explicit assumption, further suggesting that a rational approach to emotional cue integration is well able to capture these intricacies in affective cognition.

### 3.2. Experiment 4: Cue integration from outcomes and utterances

Experiment 3 examined combinations of facial expressions and situation outcomes. In Experiment 4, we show that our model generalizes to other cues by examining combinations of verbal utterances and situation outcomes.





**Fig. 10.** Results of Experiment 3. Error bars indicate bootstrapped 95% confidence intervals. (A) Root-mean-squared-error for the different models. From left to right, performance on all data; performance on the negative face, positive wheel trials; and performance on the positive face, negative wheel trials. (B) Correlations of the different models with empirical judgments. (C) Performance of the Bayesian model on all trials. On the vertical axis are participants' judgments of all eight emotions in the joint-cue trials, and the horizontal axis indicates the models' predicted emotions. The red-dashed line has an intercept of 0 and a slope of 1, and is added for reference. The model's correlation with the empirical joint-cue predictions was 0.812 [0.790, 0.833].

### 3.2.1. Participants

We recruited one hundred fifty participants through Amazon's Mechanical Turk.

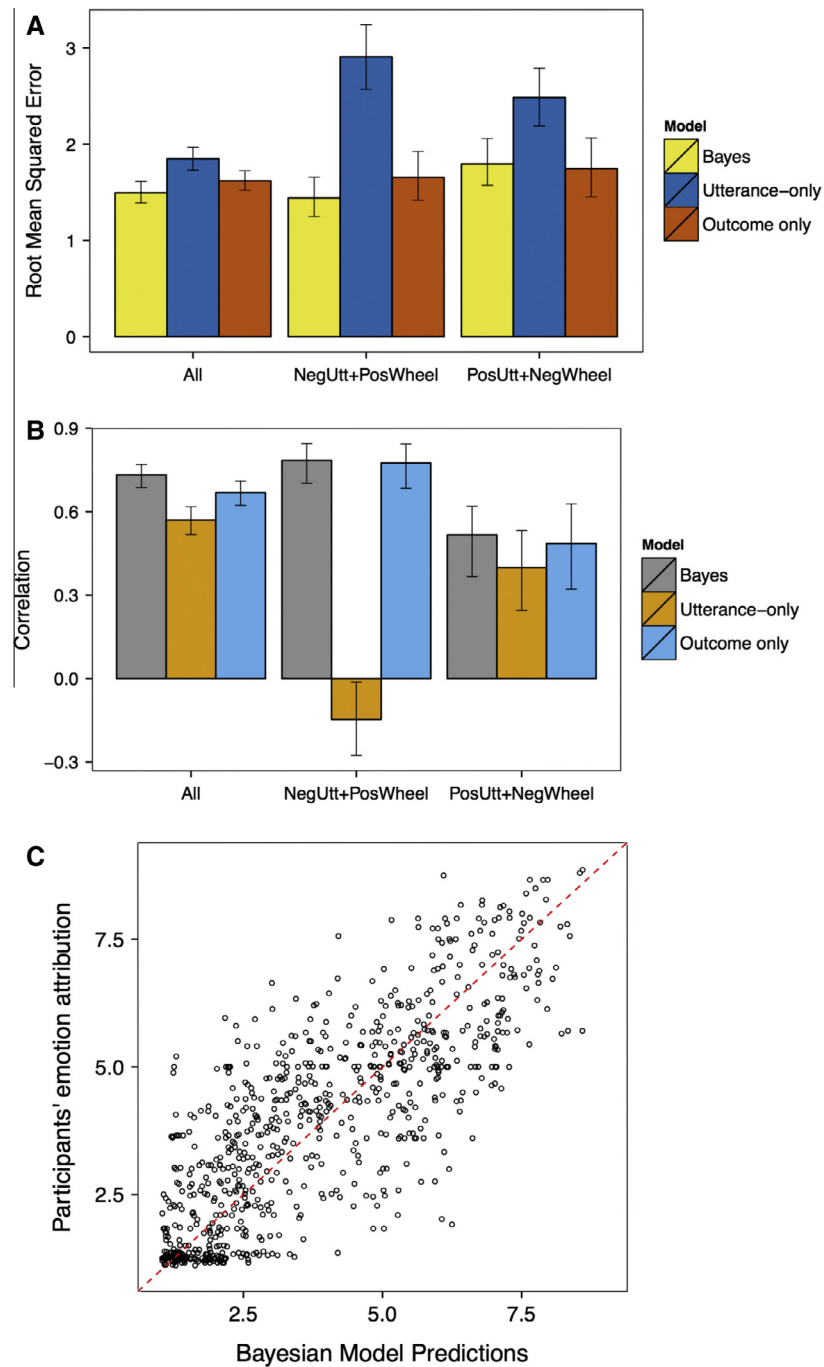
### 3.2.2. Stimuli

We used identical gambles to Experiment 3. We replaced the facial expression with an utterance, ostensibly made by the character after seeing the outcome of the wheel. The list of the 10 utterances ("cool", "awesome", "yay", "wow", "man", "oh", "damn",

"dang", "meh" and "yikes") included a mix of clearly valenced utterances and ambiguous utterances.

### 3.2.3. Procedures

We used identical procedures to Experiment 3, except that instead of face stimuli, participants saw an utterance that ostensibly was made by the character after seeing the outcome of the wheel. Each participant completed 10 trials. On each trial, participants saw either (i) an *Outcome-Only* trial, (ii) an *Utterance-Only*



**Fig. 11.** Results of Experiment 4. (A) Root-mean-squared-error for the different models. From left to right, performance on all data; performance on the negative utterance and positive wheel trials; and performance on the positive utterance and negative wheel trials. (B) Correlations of the different models. (C) Performance of the Bayesian model on all trials, showing a correlation with the empirical joint-cue predictions of 0.733 [0.687, 0.770].

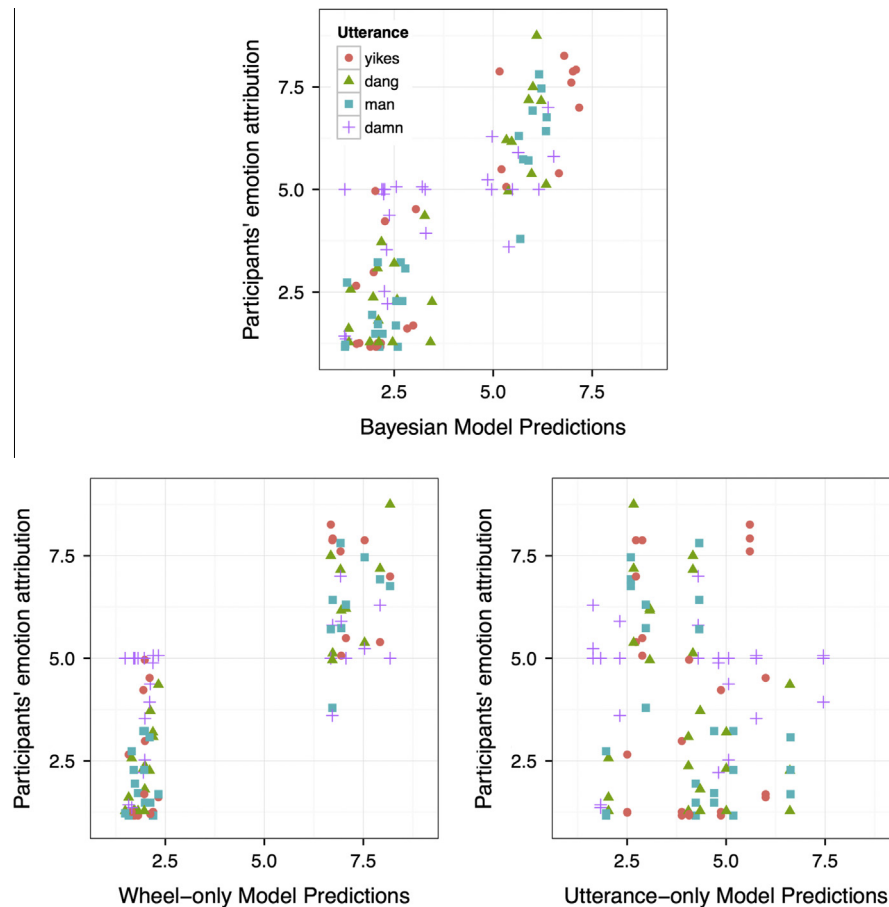
trial, or (iii) a *Joint-Cue* trial with both outcomes and utterances, randomly paired. Participants then attributed emotions to the character as in Experiment 3.

### 3.2.4. Results

We repeated the same analysis procedure as in Experiment 3. We used participants' responses to the *Utterance-Only* and *Outcome-Only* trials to construct the empirical distributions for  $P(e|u)$  and  $P(e|o)$  respectively. Next, we used these two models to construct the full Bayesian model  $P(e|o, u)$ . Replicating the results of Experiment 3, on all trials, the Bayesian model performs the best with the lowest RMSE of 1.494 [1.391, 1.612], performing

significantly better as compared to the Utterance-only model at 1.847 [1.731, 1.969] ( $t(49) = -4.18$ ,  $p < .001$ ), and better, but not significantly, as compared to the Outcome-only model at 1.619 [1.512, 1.724] ( $t(49) = -1.54$ ,  $p = .13$ ). When we examined the correlation with the empirical judgments, the Bayesian model achieved a correlation of  $r = 0.733$  [0.687, 0.770], again, significantly better compared with the Utterance-only model at  $r = 0.570$  [0.518, 0.618] ( $t(49) = 4.81$ ,  $p < .001$ ), and the Outcome-only model at  $r = 0.668$  [0.623, 0.710] ( $t(49) = 2.05$ ,  $p = .046$ , significant with some overlap of the 95% CIs) (Fig. 11).

When we repeated the analysis for the incongruent combination subsets, we find that the Bayesian model again tightly predicts



**Fig. 12.** Scatterplot results of Experiment 4, only on the trials with the incongruent combination of Negative Utterance and Positive Outcomes. All emotions are plotted. Participants' judgments are plotted on the y-axis on all three graphs. The Utterance-only model, in the bottom right, has a negative correlation with the data, due to the larger spread, possibly due to potential pragmatic interpretations that the utterance-only model cannot account for.

observers' judgments, with a high correlation with the empirical judgments (Negative Utterance and Positive Outcome,  $r = 0.784$  [0.702, 0.845]; Positive Utterance and Negative Outcome,  $r = 0.517$  [0.367, 0.620]), although it did not significantly outperform the Outcome-only model (Negative Utterance and Positive Outcome,  $r = 0.775$  [0.684, 0.844];  $t(49) = 0.16$ ,  $p = .87$ ; Positive Utterance and Negative Outcome,  $r = 0.486$  [0.321, 0.628];  $t(49) = 0.29$ ,  $p = .77$ ) in this experiment (see Fig. 11).

Unexpectedly, the Utterance-only model showed a negative correlation of  $r = -0.147$  [−0.276, −0.012] with the empirical judgments in trials in which a negative utterance was presented with a positive outcome. We suspect that this is because utterances could be interpreted *pragmatically*, i.e., in a non-literal way. Consider an utterance like “dang”: this utterance carries a negative literal connotation (e.g. expected happiness given “dang” alone was 2.66 on a 1–9 scale). However, when paired with a positive outcome—an example outcome which, rated alone, produced an expected happiness rating of 8.18—the combination of “dang” and the outcome could be interpreted as being *even more* positive (expected happiness of the combination of “dang” and the outcome is 8.75). Thus, although observers might interpret an individual verbal cue to be negative, they might interpret the same cue, presented with a positive outcome, to be positive, perhaps as a form of sarcasm or hyperbole. Note that there is much heterogeneity in the way that participants interpret these combinations: not everyone treats them pragmatically, and not every such combination is treated pragmatically. This heterogeneity results in a larger spread in

participant responses to the joint combinations relative to the utterance-only results (see Fig. 12). This effect likely drives the negative correlation of the (literal) Utterance-only model with the judgments of the combinations given by participants. It is worth noting that there has been recent success in applying Bayesian models to study interpretation of non-literal language (Frank & Goodman, 2012; Kao, Wu, Bergen, & Goodman, 2014). Future work could extend our model of affective cognition to include these pragmatic effects.

Finally, as in Experiment 3, we find an effect of valence on cue reliability: positive cues have higher reliability and lower entropy than negative cues. For outcomes alone, the negative wheels show a significantly higher entropy (mean entropy = 2.38 bits, SD = 0.37 bits) than the positive wheels (mean entropy = 1.98 bits, SD = 0.40 bits;  $t(7) = 5.01$ ,  $p = .002$ ). This was the case for utterances as well; entropy for negative utterances (mean entropy = 2.69 bits, SD = 0.45 bits) was greater than for the positive utterances (mean entropy = 1.77 bits, SD = 0.80 bits;  $t(7) = 2.83$ ,  $p = .03$ ). This reinforces the finding from Experiment 3 that participants seem to be more certain in their judgments given positive cues than negative cues, at least in this gambling context.

The results of this experiment show that the emotional cue integration model we propose, derived from a Bayesian treatment of lay theories, generalizes beyond facial expressions to other cues. Experiment 4 also replicates many of the critical results from Experiment 3, including the consistent performance of the Bayesian model.

### 3.3. Revisiting emotions from outcomes: a combined analysis

Experiments 3 and 4 were designed to test our cue integration predictions, using analyses that built upon the results of Experiment 1, which explored the important outcome features for predicting participants' affective judgments. However, Experiments 3 and 4 also produced additional data with which we can re-examine Experiment 1's results, that is, the statistical causal model of outcomes to emotions. This combined analysis would allow us to test the reliability of the results of the statistical model that we built in Experiment 1 (and used in the cue integration analyses above). In order to do this, we proceeded to isolate the wheel-only trials from Experiments 3 and 4, together with all the trials from Experiment 1, creating a dataset of 3048 observations from 690 participants.

With this larger dataset, we repeated the model selection, with the full set of regressors (as in [Appendix A](#)), to determine which outcome features significantly predict attributed emotions. As expected, the amount won, *PE*, and  $|PE|$  again came out as significant predictors for the various emotions (details in [Appendix A](#)), with several additional patterns of interest: e.g., *fear* is only predicted by the amount won. *Surprise* now strongly depends on  $|PE|$ , regret, and the winning probability, which suggests a different structure for *surprise* than the rest of the emotions (additional evidence for this is also given by the different loadings in the PCA analysis).

One additional regressor of note became significant with this larger dataset: the near-miss term predicted happiness ( $b = -3.32e-05 [-5.51e-05 - 1.12e-05]$ ,  $t(682) = -2.963$ ,  $p = 0.003$ ). We tested for, but did not find a significant difference between positive and negative near-misses ( $\chi^2(1) = 0.089$ ,  $p = 0.77$ ), i.e., whether the next-nearer outcome was a larger or smaller payoff, although this could just be from a lack of power. To better understand the magnitude of the near-miss term, let us consider a concrete near-miss example using the slopes on win ( $b = 0.0405 [0.029, 0.052]$ ,  $t(682) = 7.08$ ,  $p < .001$ ), *PE* ( $b = 0.036 [0.024, 0.048]$ ,  $t(682) = 5.95$ ,  $p < .001$ ), and  $|PE|$  ( $b = -0.015 [-0.025, -0.005]$ ,  $t(682) = -2.84$ ,  $p = .006$ ), and the \$25/\$60/\$100 wheel in [Fig. 2](#). Not considering the near-miss term, and all else being equal, if the result had changed from \$60 to \$100, there would be an increase in happiness of  $40 * (.0405 + 0.036 - 0.015) = 2.46$  points on a 9 point Likert scale. By contrast, if the outcome result (the exact point the black pointer indicated) moved from the center of the \$60 sector to a near-miss distance of 1% of the sector size away from the \$60/\$100 boundary, there would be a decrease in happiness of  $40 * (1/0.5 - 1/0.01) * (-3.32 * 10^{-5}) = 0.130$  points on a 9 point scale. Thus, in this gambling scenario, the effect of a near-miss on subjective happiness attributed is on the order of 5% of the relative happiness of winning the next higher amount. Getting a near-miss on the \$60 wheel in [Fig. 2](#) and narrowly missing the \$100 sector (narrowly missing winning \$40 more) has a subjective cost equivalent to losing about \$2, compared with a far-miss (landing in the center of the sector). This is a small effect relative to actually winning—and indeed, we could not detect it with Experiment 1 data alone—yet it is a large and not insignificant effect considering that it does not depend on changing actual payoffs, but merely relative closeness. This result builds upon the results of Experiment 1 by investigating what additional situation features might factor (perhaps more weakly) into affective cognition in this paradigm.

## 4. Discussion

We have proposed a framework for studying affective cognition as domain-general reasoning applied to a domain-specific lay theory of emotions; the lay theory is described as a statistical causal knowledge of others' emotions, and reasoning as Bayesian

inference based on this knowledge. Observers' lay theories consist of a consistent structure that captures causal relationships between situation outcomes (or emotion-eliciting events), the agent's emotions, and the agent's observable behaviors. Each of these causal relationships contains complex knowledge: for example, an observer incorporates appraisal processes when reasoning about how an agent feels after the outcome of a situation. This framework makes detailed quantitative predictions that were borne out in a series of experiments. We demonstrated that observers are able to consistently reason both forward, along the causal direction, and “backwards”, about causes based on emotional reaction. The forward causal model relied on a small set of situation features (Experiment 1), and backward reasoning was well-described as Bayesian inference given this forward model (Experiment 2). This approach provides further traction in understanding how observers infer unobservable emotional states from diverse observable cues. In particular, we have shown that integrating multiple sources of information as prescribed by Bayesian statistics explains human judgments of emotions from facial expressions and outcomes (Experiment 3), and from utterances and outcomes (Experiment 4). Our results showed an interesting “valence dominance” effect, whereby positively-valenced emotional cues tended to have higher reliabilities and were weighted more so than negatively-valenced cues. Our model was able to account for this valence effect without *a priori* specification, attesting to the robustness of a probabilistic model of affective cognition. Our studies contrast with previous studies that have found face or context dominance (e.g., [Aviezer et al., 2008](#); [Nakamura et al., 1990](#); [Russell et al., 2003](#)) in that our paradigm involves a restricted gambling context; it is possible that our valence result may not generalize to other contexts, though we believe that the underlying model of cue integration will still be applicable.

Emotions are numerous and complex, and we have only examined emotional reasoning in very constrained gambling scenarios. These scenarios constitute a reasonable starting point for studying affective cognition, because they afford consistent assumptions and quantitative manipulations. For instance, an agent playing a gamble almost certainly wants to win more money, giving observers a clear cornerstone for understanding the affective meaning of different gamble outcomes. Even in these simple scenarios, with ostensibly simple economic motivations, participants were sensitive to subtle “irrationalities” that they expected the agent to evince. For instance, by pooling data from three of our experiments, we found that the nearness of alternative, better outcomes (or worse outcomes) factored into our participants' inferences about agents' emotion. Further work will likely uncover additional features that factor into affective cognition.

Not all emotions are relevant in our scenarios—*fear*, for example, may be irrelevant, as may a more complex emotion like *pride*—and furthermore, we might reason about emotions like happiness differently in a gambling context than in other contexts. One avenue of further research includes extending our model to more complex situations. For instance, people do not live in a social vacuum, and emotions often depend on interactions with others: a comprehensive model would include how observers reason about emotions in social interactions, leading to emotions like *pride* and *jealousy*.

The work presented here was concerned with high-level reasoning, and abstracted out, for example, the process by which we perceive facial expressions (How does an observer look at a face and decide that the agent is happy? Which facial features are important?), or how we interpret the linguistic content of emotional utterances (How do we interpret “dang” as negative?). In this work, we measured the connection between these percepts and emotional states, rather than explaining them, then investigated how observers utilized them to make higher-level



inferences. The details of these processes are important in a full description of affective cognition, and each of these will require further, domain-specific study into the lay theory of emotion. That said, we believe that the framework considered here—treating affective cognition using domain-general reasoning—is generalizable and will extend to more detailed knowledge about emotions.

#### 4.1. Relation to modeling of social cognition

In Fig. 2 and throughout this paper, we alluded to other mental states, such as the agent's goals, that are important in a lay theory of emotion but that we did not consider in detail. This tight link between emotions and other mental states translates into a strong parallel of our work with other models of social cognition (e.g., Goodman, Baker, & Tenenbaum, 2009). For example, Baker et al. (2009) proposed a computational model of Theory of Mind that incorporates an agent's beliefs and desires into an observer's lay theory of behavior. This was a formalization of earlier work in belief-desire psychology, describing a lay theory of how an agent rationally chooses actions given his beliefs and desires (Dennet, 1989; Gopnik & Meltzoff, 1997). In a similar fashion, the recently proposed Rational Speech Act (RSA) model (e.g., Frank & Goodman, 2012; Goodman & Stuhlmüller, 2013; Kao et al., 2014) treats language understanding between agents as rational social cognition. Again, this work has its roots in earlier work on communication between rational agents (e.g., Clark, 1996). The RSA model assumes that the speaker rationally chooses his utterance to convey his desired inference to the listener, and the listener recursively reasons about the speaker's mental state to infer the goal of the speaker. These and other models of social cognition are helping to shed light on how naïve observers reason about the cognitive states of those around them.

Integrating the lay theories of cognitive states (beliefs and desires) and lay theories of emotions will have explanatory benefits for both lines of work. For instance, in this paper we implicitly assumed the goals of the agent, and showed how observers reason about an agent's emotions with respect to the prediction error of the situation, or how well the situation compared to the agent's expectations. In our paradigm it was a safe assumption that agents wished to win larger amounts of money; in more general situations the observer would have to make an inference about the agent's goals in order to evaluate the “prediction error”. Thus, one would have to incorporate inferences over goals into a comprehensive theory of affective cognition. In the other direction—adding emotions into other models of social cognition—attributed emotions become a potential cause for otherwise “irrational” actions that are observed. This is an important extension to current models of social cognition, which must explain actions as the result of rational, purposive behavior or of simple noise.

#### 4.2. Relation to scientific theories of emotion

Here we have focused on lay theories that observers apply to understand the emotions of those around them, but people likely apply an analogous reasoning process to reason about their own emotions as well. This is consistent with evidence that people use similar strategies when reasoning about their own traits and preferences (Bem, 1972; Gopnik, 1993; Nisbett & Wilson, 1977). Contrary to many people's intuition, it appears that individuals do not have privileged access to their “true” mental states; rather, they use observations of their own behaviors and situations to reason about themselves using lay theories—similar to how they might reason about other people.

A similar paradigm shift in affective science increasingly suggests that people do not have “privileged access” to their emotional experience, but instead reason about their experiences in a

contextualized manner. For instance, under Feldman Barrett et al.'s Conceptual Act Theory (Barrett, 2006; Lindquist & Barrett, 2008), a typical emotion experience starts with the agent experiencing a stimulus. The agent collects information about the context as well as visceral signals of valence (e.g. pleasurable feelings) and arousal (e.g. heart rate, clammy skin)—what Barrett calls “core affect” (Russell & Barrett, 1999). The agent then uses these pieces of information as inputs into a *conceptualization* process wherein the agent labels their emotion using emotion concepts—the agent's own lay theory of emotion.

Our approach to understanding third-person affective cognition mirrors the emerging prominence of concepts and lay theories in first-person emotional experience. There is much room for overlap and progress down these parallel and complementary paths. On the one hand, studying first-person emotional experience might give insight into what factors go into a third-person lay theory. In our work, for example, we build off prior work in appraisal theory (e.g., Ortony et al., 1988) and behavioral economics (e.g. Kahneman & Tversky, 1979) to inform our *a priori* assumptions of PE and loss aversion, which we can then test in a third-person context. Conversely, studying third-person lay theories might allow testing of predictions that might be hard to manipulate in a first-person context. For example, in a first-person context, it is difficult to experimentally separate the affective component of a response from the cognitive processes of reasoning and categorization. If a researcher finds a phenomenon in a first-person context and cannot distinguish whether it might be due to a cognitive process, then translating the paradigm into a third-person context might allow the researcher to isolate the cognitive components of that reasoning. Thus our work may prove useful in understanding emotional *experience* in addition to emotional reasoning.

#### 4.3. Applications

An affective cognition approach also holds a great deal of applied potential. One natural application is to artificial agents capable of interacting with emotional users (e.g., Breazeal, 2004; Gratch & Marsella, 2004; Hudlicka, 2003; Picard, 1997; Wehrle & Scherer, 2001). There are many specific applications of emotionally aware agents. Researchers have started using “virtual humans” in training physicians in patient interaction (Raij et al., 2007; see also Medical Cyberworlds, Inc.), and using realistic avatars to populate immersive virtual reality environments in order to improve users' social cognition (Bailenson, Yee, Merget, & Schroeder, 2006; Swartout et al., 2006). Other researchers have worked on building robots that can provide companionship—often termed Social Robotics, for example, Kismet, from the MIT Media Lab (Brooks, Breazeal, Marjanović, Scassellati, & Williamson, 1999). Building on our framework here, we can imagine endowing artificial observers with a human-like theory of emotions. If one uses a psychologically validated computational model to allow artificial observers to reason about emotions, this could result in near-human-level ability to attribute emotions, but importantly, do so in a way that a human partner would expect. One could imagine incorporating a lay theory of emotion into personal digital assistants (like Apple's Siri and Google's Google Now) which, when combined with the enormous amount of information they “observe” about their users, would allow digital assistants to reason about and react to their users' emotions. This would have far reaching implications for the efficiency of these products (digital assistants pre-empting users' emotions and choices) and also improve users' likeability of the product. We might also start seeing these digital assistants doubling up as conversation partners, and perhaps providing basic advice and counseling services (e.g., modern successors to the “computer therapist” ELIZA).

This segues into another potential area in which a theory affective cognition could prove useful: the diagnosis and treatment of psychopathology. The idea that disorders might arise from a bias in affective cognition dovetails with popular approaches in clinical psychology, such as Cognitive Behaviorism, which conceptualizes affective disorders as stemming from maladaptive cognitive biases (Beck, 1979). Our work might serve as an additional tool that the cognitive behavior therapist may use to help identify patients' biases in reasoning about others', and possibly the patients' own, emotions. One noteworthy and emerging field is that of Computational Psychiatry (Huys et al., 2012; Montague, Dolan, Friston, & Dayan, 2012; Sharp, Monterosso, & Montague, 2012), which assumes that social and emotional functioning can be characterized by computational "phenotypes", which comprise genetic, neural, and behavioral characteristics described in a computational model of cognitive functioning. Similarly, our work provides a way to model behavioral attributions of emotions that could form part of these models of psychiatric disorders. One could imagine using a model of affective cognition to help identify characteristic biases in certain psychiatric populations: for example, perhaps patients who suffer from anxiety might also attribute increased emotional reactivity to agents (greater reliance on *PE*, *|PE|*) as compared to a typical observer. Finally, combining the applications to technology and psychopathology, our work could enable novel technologies for scalable diagnosis and treatment. Our work could inform automated cognitive monitoring applications and automated dialog agents that can measure early warning diagnostic signs or serve as artificial therapists (e.g., Barak & Grohol, 2011; Helgadottir, Menzies, Onslow, Packman, & O'Brian, 2009; Kenny, Parsons, Gratch, & Rizzo, 2008). In sum, building a computational model of affective cognition has many potential applications, and here we have listed only a few that we feel are most promising.

## 5. Conclusion

Humans are lay psychologists that use theories to understand the people around them, and often how, when, and why these people experience emotions. In this paper, we propose a framework for studying affective cognition—reasoning about emotion—using a computational, lay theory approach. This approach concentrates

on how observers reason about emotions, and our results show a surprising rationality and flexibility in the way human observers perform affective cognition. Specifically, the way that observers reason between pairs of variables (emotions and the outcomes that cause them), and the way that observers combine information from multiple sources to infer emotion (emotional cue integration), can both be described using domain-general reasoning similar to other forms of cognition. Emotions are immensely complicated, and have complex interactions with other psychological states—these relationships are captured in a domain-specific lay theory of emotions. Yet, the way that an observer reasons about these complex emotions can be described and understood using the same inferential processes that underlie reasoning in other psychological domains. It is our hope that the study of affective cognition—treated as another form of cognition—will drive forward theory and empirical studies in affective science, social cognition, and the crucial bridge between them.

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## Appendix A. Model selection and Principal Component Analysis

### A.1. Experiment 1 model selection details

For the analysis in Experiment 1 (Section 2.1.3), we performed model selection from the full set of *a priori* specified features (*win*, *PE*, *|PE|*, *Regret*, *Relief*, *logWinProb* and *nearMiss*). In the table below, we report coefficients for the full model (with all 7 predictors). We can see from the full model that *win*, *PE* and *|PE|* predicts the majority of all emotions. In particular, *PE* and *|PE|* predict all except *fear*, and *win* predicts all except *fear* and *anger*. *Fear* is a strange emotion in this paradigm, and so not much inference should be drawn from the results for *fear*. The other notable emotion to note is *surprise*, which has significant loadings on some of the other predictors. Note that we can also see interesting

**Table A1**

Table of coefficients (with *p*-values in parentheses) for the models with all seven regressors. Total significant refers to the total count of significant predictors from the full models (i.e. from the table). Total after model selection indicates number of regressors remaining after model selection.

	Win	PE	PE	Regret	Relief	logWinProb	nearMiss
Happy	0.036 (<0.001 <sup>***</sup> )	0.027 (<0.001 <sup>***</sup> )	−0.020 (<0.001 <sup>***</sup> )	0.0055 (0.36)	0.0056 (0.24)	−0.31 (0.032 <sup>*</sup> )	−1.5e−05 (0.23)
Sad	−0.024 (<0.001 <sup>***</sup> )	−0.018 (0.022 <sup>**</sup> )	0.028 (<0.001 <sup>***</sup> )	−0.0018 (0.80)	−0.0026 (0.64)	0.10 (0.54)	−2.6e−06 (0.86)
Anger	−0.0041 (0.31)	−0.025 (<0.001 <sup>***</sup> )	0.027 (<0.001 <sup>***</sup> )	0.0013 (0.83)	−0.0011 (0.81)	−0.10 (0.48)	−2.1e−06 (0.87)
Surprise	0.019 (0.029 <sup>*</sup> )	−0.019 (0.035 <sup>*</sup> )	0.027 (<0.001 <sup>***</sup> )	0.024 (0.003 <sup>**</sup> )	0.015 (0.016 <sup>*</sup> )	−1.94 (<0.001 <sup>***</sup> )	2.2e−05 (0.22)
Disgust	−0.0084 (0.022 <sup>*</sup> )	−0.026 (<0.001 <sup>***</sup> )	0.022 (<0.001 <sup>***</sup> )	0.0075 (0.16)	0.00063 (0.88)	−0.029 (0.82)	3.3e−06 (0.78)
Fear	−0.0021 (0.36)	−0.0039 (0.31)	0.0034 (0.063.)	−0.00031 (0.93)	0.0012 (0.64)	0.013 (0.87)	−1.3e−05 (0.071.)
Content	0.034 (<0.001 <sup>***</sup> )	0.024 (0.0135 <sup>*</sup> )	−0.014 (0.0025 <sup>**</sup> )	−0.0032 (0.71)	−0.0001 (0.98)	−0.26 (0.20)	−1.3e−05 (0.48)
Disappointment	−0.025 (<0.001 <sup>***</sup> )	−0.035 (<0.001 <sup>***</sup> )	0.023 (<0.001 <sup>***</sup> )	−0.0097 (0.19)	−0.0036 (0.52)	0.17 (0.32)	−1.6e−05 (0.33)
Total significant	6	7	7	1	1	2	0
Total after model selection	5 (ex. anger, disgust, fear)	8	8	1 surprise	1 surprise	2 surprise, happy	0

<sup>\*</sup> *p* < .05.

<sup>\*\*</sup> *p* < .01.

<sup>\*\*\*</sup> *p* < .001.

**Table A2**

Table of coefficients (with *p*-values in parentheses) for the final model used in the text, with only win, PE, |PE|.

	Intercept	Win	PE	PE
Happy	4.515 ( $<0.001^{***}$ )	0.0386 ( $<0.001^{***}$ )	0.0350 ( $<0.001^{***}$ )	−0.0177 ( $<0.001^{***}$ )
Sad	3.380 ( $<0.001^{***}$ )	−0.0245 ( $<0.001^{***}$ )	−0.0217 ( $<0.001^{***}$ )	0.0268 ( $<0.001^{***}$ )
Anger	1.721 ( $<0.001^{***}$ )	−0.00375 (0.251)	−0.0251 ( $<0.001^{***}$ )	0.0267 ( $<0.001^{***}$ )
Surprise	3.251 ( $<0.001^{***}$ )	0.0222 ( $<0.001^{***}$ )	0.012 (0.013 <sup>*</sup> )	0.0373 ( $<0.001^{***}$ )
Disgust	1.764 ( $<0.001^{***}$ )	−0.00536 (0.066.)	−0.0211 ( $<0.001^{***}$ )	0.0217 ( $<0.001^{***}$ )
Fear	1.381 ( $<0.001^{***}$ )	−0.00225 (0.23)	−0.00325 (0.09.)	0.00342 (0.037 <sup>*</sup> )
Content	3.709 ( $<0.001^{***}$ )	0.0321 ( $<0.001^{***}$ )	0.0221 ( $<0.001^{***}$ )	−0.0124 (0.0025 <sup>**</sup> )
Disappointment	4.724 ( $<0.001^{***}$ )	−0.0292 ( $<0.001^{***}$ )	−0.0441 ( $<0.001^{***}$ )	0.0219 ( $<0.001^{***}$ )

<sup>\*</sup> *p* < .05.

<sup>\*\*</sup> *p* < .01.

<sup>\*\*\*</sup> *p* < .001.

differences between *surprise* and the other emotions, from the Principal Component Analysis (see below in [Appendix A.3](#))

Next, we performed stepwise backward elimination using the *step* function in the *lmerTest* package in R. Starting from the full model, the variables are considered one at a time for elimination, and the function calculates an anova to compare the model with and without the variable. According to the final model selection, the prediction error (PE) and its absolute value (|PE|) predicted was significant in predicting all of the emotions (last row of [Table A1](#)). For the amount won, it was significant after selection for five emotions, and trending for one, so we included it in the final set of regressors. The last four regressors did not have much explanatory power. Finally, in [Table A2](#), we report the coefficients in the reduced model that we used in Experiments 1–2. (Note also that although we use the same regressors (*win*, *PE*, |*PE*|) in Experiments 3 and 4, the coefficients were fitted to the data in Experiments 3 and 4 respectively.)

**Table A3**

Similar to [Table A1](#), now with combined data from Experiments 1, 3 and 4. Table of coefficients (with *p*-values in parentheses) for the models with all seven regressors. Total significant refers to the total count of significant predictors from the full models (i.e. from the table). Total after model selection indicates number of regressors remaining after model selection.

	Win	PE	PE	Regret	Relief	logWinProb	nearMiss
Happy	0.036 <sup>†</sup> ( $<0.001^{***}$ )	0.021 <sup>†</sup> (0.094.)	−0.017 <sup>†</sup> (0.0057 <sup>**</sup> )	0.013 (0.23)	0.0067 (0.43)	−0.32 (0.21)	−3.3e−05 <sup>†</sup> (0.003 <sup>**</sup> )
Sad	−0.027 <sup>†</sup> (0.012 <sup>**</sup> )	−0.014 <sup>†</sup> (0.32)	0.028 <sup>†</sup> ( $<0.001^{***}$ )	−0.0038 (0.75)	−0.0039 (0.67)	0.19 (0.52)	4.4e−06 (0.73)
Anger	−0.0015 <sup>†</sup> (0.038 <sup>*</sup> )	−0.017 <sup>†</sup> (0.171)	0.028 <sup>†</sup> ( $<0.001^{***}$ )	0.0014 (0.89)	−0.0030 (0.72)	0.022 (0.93)	4.7e−06 (0.69)
Surprise	0.0042 (0.532)	−0.019 (0.107)	0.031 <sup>†</sup> ( $<0.001^{***}$ )	0.033 <sup>†</sup> (0.002 <sup>**</sup> )	0.016 (0.052.)	−1.66 <sup>†</sup> ( $<0.001^{***}$ )	3.9e−05 (0.80)
Disgust	−0.015 <sup>†</sup> (0.026 <sup>*</sup> )	−0.020 <sup>†</sup> (0.093.)	0.023 <sup>†</sup> ( $<0.001^{***}$ )	0.0051 (0.59)	−0.0014 (0.85)	0.016 (0.95)	−5.9e−07 (0.96)
Fear	−0.0083 <sup>†</sup> ( $<0.001^{***}$ )	−0.0020 (0.58)	0.0027 (0.13)	0.0031 (0.26)	−0.0001 (0.95)	0.056 (0.46)	5.2e−06 (0.47)
Content	0.028 <sup>†</sup> ( $<0.001^{***}$ )	0.011 (0.34)	−0.015 <sup>†</sup> (0.011 <sup>**</sup> )	0.010 (0.28)	0.0089 <sup>†</sup> (0.25)	−0.25 (0.31)	−2.0e−05 (0.21)
Disappointment	−0.027 <sup>†</sup> ( $<0.001^{***}$ )	−0.030 <sup>†</sup> (0.020 <sup>*</sup> )	0.023 <sup>†</sup> ( $<0.001^{***}$ )	−0.013 (0.21)	−0.0065 (0.44)	0.14 (0.58)	−2.1e−06 (0.88)
Total significant	7	1	7	1	0	1	1
Total after model selection	7 (ex. surprise)	5 (ex. fear, content, surprise)	7 (ex. fear)	2 surprise content	1 content	1 surprise	1 happy

<sup>\*</sup> *p* < .05.

<sup>\*\*</sup> *p* < .01.

<sup>\*\*\*</sup> *p* < .001.

<sup>†</sup> Indicates the regressor was significant after model selection.

## A.2. Combined analysis model selection details

We repeated this model selection analysis with the data from Experiments 1, 3 and 4 (discussed in [Section 3.3](#)). The table for the combined data is given in [Table A3](#), with the coefficients for the full model as well as the total after model selection. The results are qualitatively similar to those reported in [Table A1](#) above. In the full model, the amount won and |*PE*| predict 7 emotions, while *PE* strangely predicts only 1 emotion, but after model selection, amount won and |*PE*| predict 7, and *PE* predicts 5. For illustration, we have indicated in [Table A3](#) the regressors that survived model selection. Similar to above, we noticed that the pattern of coefficient loadings on *surprise* and *fear* formed a different profile as compared to the other emotions.

The model selection results for Contentment was initially puzzling to us: if we include just win, *PE* and |*PE*| in the regression model (i.e., the formula we used in Experiments 3 and 4), all three are significant predictors, as we expect from [Table A1](#) (model log-likelihood = −6439.2, AIC = 12,894, BIC = 12,943). However, starting from the full set of regressors and performing backward step selection (which maximizes log-likelihood) yields win, |*PE*|, Regret and Relief as significant predictors (model log-likelihood = −6438.7, AIC = 12,895, BIC = 12,950). It seems that in this particular case for contentment, the two models are very similar in performance (with the log-likelihood of the latter model barely outperforming the former by 0.5, but the AIC and BIC of the former model also barely outperforms the latter).

## A.3. Principal Component Analysis details

We performed a Principal Component Analysis (PCA) of participants' emotion ratings using R's *princomp* function (discussed in [Section 2.1.4](#)). The loadings of each of the PCs, as well as their standard deviations and proportion of total variance explained, are given in [Table A4](#). In the main text, we limit the discussion to the first two PCs (see [Fig. 4](#)). The first PC, with 59.1% of the variance, loads positively with positively-valenced emotions, and negatively with negatively-valenced emotions, resulting in our interpretation of PC1 representing “emotional valence”. Although

**Table A4**

Results of the Principal Component Analysis. The top half shows the loadings of the various Principal Components (PCs) on the emotions. The values shown indicate the loading of a particular PC on a particular emotion. Missing values were non-significant ( $p > .05$ ) loadings. The bottom half of the table shows the standard deviation and the proportion of total variance explained by each PC.

Loadings	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
Happy	0.473	−0.126	0.121	0.250	0.260	0.775	0.112	
Sad	−0.394	−0.260	−0.127	0.181	−0.654	0.309	0.445	
Anger	−0.289	−0.337		0.451		0.115	−0.730	0.215
Surprise	0.202	−0.745	0.520	−0.290		−0.199		
Disgust	−0.243	−0.308	−0.127	0.320	0.507	−0.226	0.219	−0.609
Fear		−0.151	−0.103	0.259	0.316	−0.174	0.451	0.750
Content	0.409	−0.340	−0.808	−0.216				
Disappointment	−0.515	−0.125	−0.113	−0.633	0.360	0.406		
Standard deviation	4.68	2.42	1.91	1.39	1.15	1.06	0.84	0.77
Proportion of variance explained	0.591	0.158	0.098	0.052	0.035	0.030	0.019	0.016

the second PC explains much less (15.8%) of the variance, analyzing this does provide some insight. The second PC loads in the same direction with all the emotions; hence we interpret it to be tracking the magnitude of all the emotions. Note that in Table A4, the loadings of the second PC are all negative; however, the sign of a PC is arbitrary—what is relevant are the *relative* signs of the different loadings. Thus, we might interpret PC2 as being proportional to “emotional arousal” (or PC2 as “negative emotional arousal”). Subsequent PCs explain less than 10% of the variance.

## Appendix B. Derivation of the multi-cue combination equation

Recall that in our model (Fig. 2), we assume that outcome **o** causes emotion **e**, and emotion causes facial expressions **f**. If we treat this model as a Bayesian network, we can factor the network to get the probability of observing a certain combination of variables (**o**, **e**, **f**) into:

$$P(\mathbf{o}, \mathbf{e}, \mathbf{f}) = P(\mathbf{f}|\mathbf{e})P(\mathbf{e}|\mathbf{o})P(\mathbf{o}).$$

We are interested in inferring **e** given **o** and **f**,  $P(\mathbf{e}|\mathbf{o}, \mathbf{f})$ . Using the identity that:

$$P(A|B) = \frac{P(A, B)}{P(B)},$$

we can write the desired quantity as:

$$P(\mathbf{e}|\mathbf{o}, \mathbf{f}) = \frac{P(\mathbf{o}, \mathbf{e}, \mathbf{f})}{P(\mathbf{o}, \mathbf{f})} = \frac{P(\mathbf{f}|\mathbf{e})P(\mathbf{e}|\mathbf{o})P(\mathbf{o})}{P(\mathbf{o}, \mathbf{f})}.$$

Using Bayes' rule, we can rewrite  $P(\mathbf{f}|\mathbf{e})$  in terms of  $P(\mathbf{e}|\mathbf{f})$ :

$$P(\mathbf{e}|\mathbf{f}) = \frac{P(\mathbf{f}|\mathbf{e})P(\mathbf{e})}{P(\mathbf{f})}.$$

Making this substitution for  $P(\mathbf{f}|\mathbf{e})$  and rearranging, we arrive at:

$$P(\mathbf{e}|\mathbf{o}, \mathbf{f}) = \left( \frac{P(\mathbf{e}|\mathbf{f})P(\mathbf{f})}{P(\mathbf{e})} \right) \frac{P(\mathbf{e}|\mathbf{o})P(\mathbf{o})}{P(\mathbf{o}, \mathbf{f})} = \frac{P(\mathbf{e}|\mathbf{f})P(\mathbf{e}|\mathbf{o})}{P(\mathbf{e})} \left( \frac{P(\mathbf{f})P(\mathbf{o})}{P(\mathbf{o}, \mathbf{f})} \right)$$

The terms in the right-most parenthesis do not depend on **e**, and is a constant for a fixed (**o**, **f**) combination. Thus, we arrive at the crucial cue-integration equation:

$$P(\mathbf{e}|\mathbf{o}, \mathbf{f}) \propto \frac{P(\mathbf{e}|\mathbf{f})P(\mathbf{e}|\mathbf{o})}{P(\mathbf{e})},$$

where the joint-cue posterior  $P(\mathbf{e}|\mathbf{o}, \mathbf{f})$  is proportional to the individual (emotion|cue) probabilities  $P(\mathbf{e}|\mathbf{f})$  and  $P(\mathbf{e}|\mathbf{o})$ , and normalized by the prior probability of the emotion  $P(\mathbf{e})$ .

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