

# Knowing how to act in an uncertain world: a model of task-set inference

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## Abstract

To cope with the difficulty of making decisions in a complicated world, people often employ task-sets that define action policies over a range of stimuli. By retrieving task-sets in novel situations, people can directly benefit from past experience, potentially sidestepping the slow process of reinforcement learning. Whether a potential task-set is deemed applicable and thus useful for influencing decisions directly depends on the person's assessment of the current world structure. An open question is how environmental factors influence this assessment and the resulting selection of task-sets. Here, we use a novel task in which probabilistic cues predict one of two task-sets on a trial-by-trial basis. We explore a number of Bayesian models that explain subjects' task-set choices as a function of their underlying confidence in the two potential task-sets. Of primary interest is the weighting of different information sources that should, in principle, contribute to task-set selection. We find that people are generally biased towards contextual cues over task history when selecting task-sets, although there is pronounced individual variation in the specific weighting.

## Introduction

Humans face the constant challenge of making decisions in an uncertain and open-ended world. In such a world, learning a repertoire of independent stimulus-response mappings is impractically slow and fails to capitalize on the natural structure inherent in tasks. Task-sets that define action policies across a range of stimuli can potentially ease the learning problem by providing flexible representations that can be leveraged in multiple contexts.

Extensive work on task-sets has emphasized task-switching, or the process by which an individual shifts from one task-set to another (e.g. Herd, 2014; Monsell, 2003; Ardid & Wang, 2013). Even when people have full information about which task to perform, task-

set switching is inherently costly; responses are both slower and more error-prone (Monsell, 2003). In addition to switch-costs, a "mixing cost" leads to a slower response rate when more than one task is required in a given experiment block. While the costs of task-switching are clear, less work has explored the circumstances under which people proactively choose to switch task-sets.

The bulk of current research in this domain pertains to model-based decision-making (Churchland & Ditterich, 2012; Collette, 2012; Karayanidis et al, 2010). In that framework, a person learns a goal-sensitive action policy based on an assumed model of the world. This concept has been integral in outlining how task-sets may be learned given a model of the world, but it ignores how the models underlying decisions are created and selected when multiple models may apply in a particular environment (so-called 'structure learning').

Some research has directly addressed the problem of structure learning, proposing that animals simultaneously attempt to infer the latent causal structure of the world while identifying the appropriate task-set given that inferred structure (Gershman & Niv, 2010; Redish et al, 2007). From this perspective, structure learning is intimately tied with stimulus-response learning. This view admits the compelling prediction that people will reuse task-sets whenever they infer that the latent structure of the world conforms to the structure in which the task-set was first learned.

This process is closely related to categorization, a procedure in which people construct a useful organizational framework to group instances that are functionally or perceptually equivalent on some level of abstraction. Similarly, many environments may differ in superficial perceptual details while sharing the same latent structure; in such cases, task-sets potentially reduce the decision process from generating and testing a large set of actions to making a single inference over latent

world states.

This idea has some empirical and computational support. Work by Collins and colleagues have shown that people reuse task-sets in an approximately optimal way based on contextual support (Collins & Koechlin, 2012; Collins & Frank, 2013). However, the tasks studied in those experiments employed deterministic cues that directly indicated the appropriate task-set, greatly simplifying the learning problem and potentially obscuring the factors underlying task-set inference in general.

In this project, we aimed to resolve these problems and clarify the process underlying task-set inference. We introduced a novel task-switching paradigm that required subjects to reason over probabilistic environmental cues to select the appropriate task-set on a trial-by-trial basis. With this paradigm, we anticipated that subjects would use multiple sources of information such that their decisions related both to contextual cues and task-set transition probabilities. As these different sources contribute to behavior in subtle ways, we developed explicit quantitative models to assess the information subjects access to infer task-sets. These models differ based on the environment statistics to which they have access as well as the weighting of current contextual information versus task history.

As a first step, we explore how an optimal observer would navigate the task environment. We then introduce a parameter controlling the relative weighting of contextual evidence over task-set history; higher values create a model that undervalues priors or partially neglects the ‘base rate.’ We then fit this parameter to individual subjects’ behavioral data.

## **Methods**

### ***Task Description & Behavior***

24 subjects completed a task-switching experiment composed of two phases: training (832 trials) and testing (800 trials). In the training phase, subjects learned to select one of four keys in response to two-dimensional stimuli varying in color (red and blue) and shape (circle and square). Each key corresponded to one of those features (e.g. a

‘red’ key). On each trial, one dimension was relevant for responses, defining the appropriate task-set for that trial (referred to as “CTS” or “STS” for color or shape task-sets, respectively). The operating task-set changed probabilistically from trial to trial such that the probability of repeating the same task-set on  $\text{trial}_{t+1} = 0.9$ . The vertical position of the stimulus also varied across trials, drawn from a Gaussian distribution parameterized by the operating task-set. These distributions differed in their mean height such that each task-set was primarily associated with either the top or bottom of the screen, randomized across subjects.

Before the training phase, subjects were explicitly told about the two task-sets and informed that the vertical position of the stimulus partially determined the operating task-set. They were instructed to use feedback in the training phase to learn how the task-sets switched and what the correct responses to each feature were (i.e. key 1 for red shapes); however, they were told not to rely on this feedback, as the test phase did not include it.

During the training phase, subjects received deterministic feedback indicating whether they selected the correct response on that trial. Because feedback was omitted during the subsequent test phase, subjects had to respond based on their understanding of the task-sets’ relationship with the probabilistic cues in the environment (stimulus position and the previous operating task-set). Subjects’ performance was assessed using an ideal observer with access to the same information as the subject.

Of the 24, four subjects failed to learn the correspondence between features and keys and so were excluded. Six additional subjects performed at chance on the test phase indicating that if they internalized a rule it was orthogonal to the true dimensions of the task. Although we acknowledge that future analysis of such ‘non-learners’ may be interesting, we exclude them in the subsequent analysis.

### ***Computational Modeling***

Optimal task-set inference can be formalized as Bayesian inference over contextual cues and task history. The task is

structured such that:

$$P(TS_t = TS^* | TS_{t+1} = TS^*) = 0.9$$

Thus the prior over task-sets on trial<sub>t</sub> is proportional to the posterior over task-sets after trial<sub>t-1</sub>. This prior information is then combined with the stimulus position's likelihood under each task-set's positional distribution to arrive at a posterior over task-sets. When needed, task-set choice is then achieved through MAP estimation.

To probe which components of this process reflect human judgments, we used the Church programming language (Goodman et al, 2012). With this language, we were able to quantitatively assess how task-set choice depends on different aspects of the environment. For example, if people only used positional information to infer task-sets (ignoring the prior over task-sets), their behavior should closely match a model that optimally reasons over only this information. Critically, we assumed that subjects have gained an approximately accurate model of the task during training such that deviations from optimal inference could arise because of decision-related processes, rather than a failure to successfully encode the parameters of the environment. Within this framework we are unable to distinguish between these two alternatives.

We defined two models to assess human performance: an optimal observer model and a biased observer model. The optimal observer encodes a generative model of the environment and inverts it to arrive at trial-by-trial task-sets. The biased observer operates in the same manner but has an additional parameter representing the belief about how probable task-sets are to repeat from trial to trial. This parameter can vary from 0 to 1, with 0, 0.5, and 1 corresponding to the beliefs that task-sets deterministically alternate, randomly switch, or deterministically persevere, respectively. Consequently, a value of 0.5 is equivalent to the base-rate neglect model above, while a value of 0.9 (the true task probability) is equivalent to the optimal model. Values between these two can be interpreted as an overreliance on stimulus position. Parameter estimation was

accomplished using Church's enumeration query, which computes the analytic solution to all possible parameter values.

The output of the model is a vector of task-set posteriors over trials, which can be used to compute the overall likelihood of a particular subject's choices given different parameter values, resulting in a distribution over parameter values. To calculate a point estimate of the parameter we integrated over this distribution, using a weighted average in proportion to each value's relative likelihood.

Model selection was accomplished using Bayesian information criterion (BIC: Schwarz, 1978), and was performed on all trials across the group and for each participant.

We were also interested in whether more difficult decisions were related to RT, as predicted by a number of studies relating choice confidence and RT (e.g. Henmon, 1911; Smith, 1968; Roitman & Shadlen, 2002). We defined a trial-by-trial estimate of model choice confidence based on the average distance from 0.5 across the task-set posteriors, ranging from 0 (indifference) to 1 (certainty). Because there were only two possible task-sets, this is equivalent to calculating the distance from the choice boundary between the two task-sets. We assessed this relationship with a mixed-effects linear regression with participant as a random effect, using the lme4 package (Pinheiro & Bates, 2010) in the R programming language.

## Results

Model comparison across the population (11,171 trials) showed that the biased observer model was a significantly better fit than the optimal model (biased BIC = 7611, Optimal = 8535). Moreover, individual analysis showed that the biased observer model fit better than the optimal models for all but one participant. Converting the biased observer posteriors into task-set choices via MAP estimation, we found that model choices matched individual subject choices on 87.8% of trials. Individual subjects fits were qualitatively the same ( $\mu=87.8\%$ ,  $\sigma=6.8$ ). One participant was not fit well by the model, matching model choices only 67% of the time. Removing this one subject revealed that the ability for the model to predict participant

choices was highly consistent ( $\mu=89.4\%$ ,  $\sigma=3.0$ ).

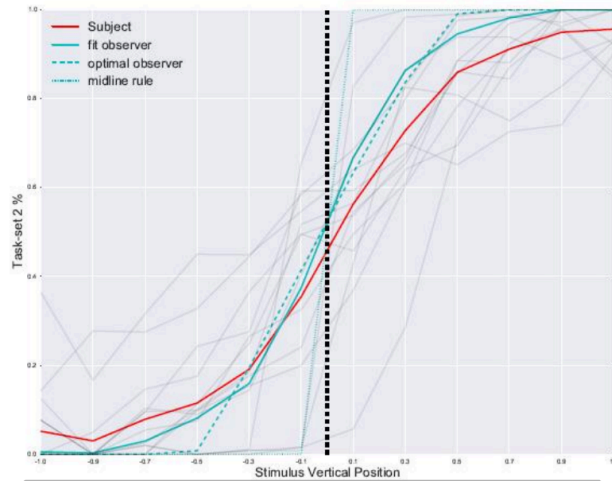


Figure 1. Percentage of trials for each of the 12 vertical position bins where responses reflected STS selection. The stimulus was never shown exactly at the midline. The red line shows the average percentage chosen across all participants. Individual curves are shown in light gray.

Parameter estimates showed that people were highly variable in their estimations of the transition probabilities (recursion probability:  $\mu=0.86$ ,  $\sigma=0.13$ ), but were consistent with the true statistics at a population level (true recursion probability = 0.9). Participant-reported estimates during the post-task questionnaire were less accurate and less consistent ( $\mu=0.71$ ,  $\sigma=0.19$ ) and were not significantly related to the biased observer parameter estimates ( $r = 0.15(14)$ ,  $p = 0.6$ ).

To visualize the model fits, we calculated the proportion of times participants selected the STS at each vertical position (Figure 1). All models predict that the STS should be chosen more frequently for higher contextual values. We repeated this calculation for optimal model and biased observer model. Individual participant results were similar. We also determined how often participants switched from one task-set to the other based on the vertical position (Figure 2). If participants were factoring in the prior task-set probabilities, they should switch less often than the midline rule at intermediate position values (near the center of the screen).

Finally, decision confidence as estimated by the biased observer model was

inversely related to RT (with  $\beta = -0.34(.03)$ ,  $t = -10.4$ ). The regression predicts that when choice confidence equaled 1, participants responded 252 ms faster than when it equaled 0. Random effects analysis showed that this trend was true for all participants except the one subject who was poorly fit by the biased observer model.

To visualize this effect, the regression was repeated independently for each subject (Figure 3), demonstrating the inverse relationship between confidence and RT.

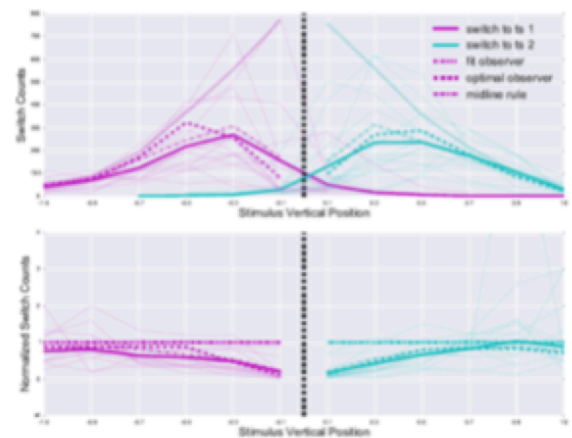
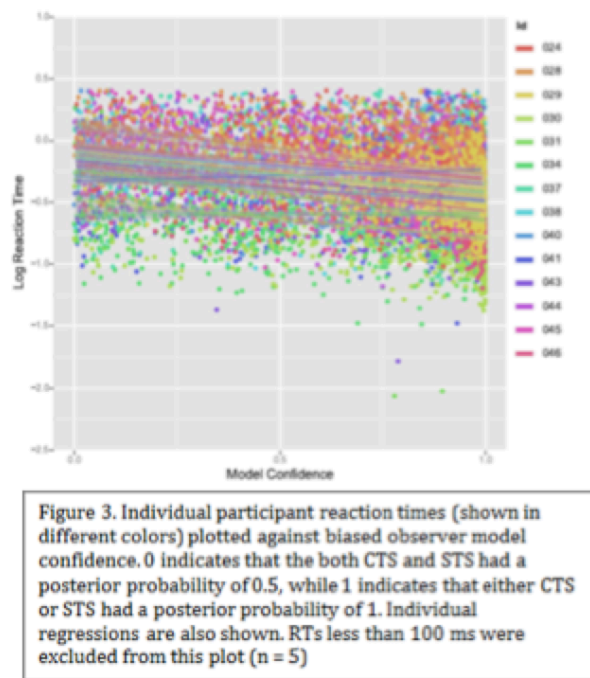


Figure 2. **top:** Group trial counts at each vertical position when a task-set switch occurred. The solid lines reflect participant counts: lighter colors are individual participant curves, scaled for plotting purposes. Participant curves largely overlap the biased observer curves. Because the distribution of vertical positions is a Gaussian around the midline, counts are not directly comparable across positions. To account for this, the **bottom** graph shows switch counts as a proportion of the midline rule counts. Note one participant switched to the STS 8 times as often as the midline rule when the stimulus position equaled 0.9, which is not shown.

## Discussion

Using a novel task-switching task, we investigated how people integrate probabilistic evidence in the service of task selection. For the most part, people based their decisions on both the probabilistic cues and transition probabilities, consistent with their internalizing the latent structure of the environment. On the population level, participants appeared to correctly identify the true statistics of the environment, giving the impression that they behaved in accordance with optimal Bayesian inference. However, individual participants differed in their weighting of the two sources of

information, such that some overvalued the probabilistic cue when making their choice. The importance of this distinction is particularly clear when predicting RT from model estimates of trial-by-trial choice confidence. As choice confidence is a continuous metric, it is particularly sensitive to specific trial sequences, as well as parameter estimates. During test, each trial's task-set is estimated based on the encoded transition probabilities, the posteriors over task-sets on the previous trial, and the probabilistic cue. Thus it is imperative to have an individual-specific estimate of the encoded transition probabilities to analyze trial-by-trial performance.



In this task, the estimate of model confidence was inversely related to RT. This parameter was defined by the absolute distance from the choice boundary between the two task-sets, suggesting that this distance may relate to the speed of evidence accumulation in a way analogous to perceptual decision tasks (Roitman & Shadlen, 2012). Evidence accumulation in higher-level decision-making has been suggested before by Shadlen & Kiani (2013); they advance the idea that accumulators may serve as a general algorithmic description of many cognitive computations. The relationship between RT and choice confidence would support this description.

A related idea is that RT relates to decision conflict. Difficult decisions are, by definition, closer to the choice boundary indicating that the evidence does not clearly favor a particular action. On a neural level, this conflict may stem from the concurrent representation of multiple task-sets which must compete in a winner-take-all fashion before gating lower-level actions (Collins & Frank, 2013). If this competition is probabilistically resolved in proportion to each task-set's representational strength, this idea is just a restatement of evidence accumulation for mutually exclusive alternatives.

The biased observer model had one free parameter, which we interpret as reflecting a bias such that prior information is undervalued at the time of decision-making. However, any encoding errors such that subjects inappropriately estimate the task statistics would lead to an identical model. For instance, if participants encoded the true transition probabilities but only attended to the stimulus position when making a choice, the model would estimate an “encoded” recursion probability of 0.5. While it is impossible to completely disentangle these two alternatives, the lack of correspondence between the parameter estimates and the participant estimates on the post-task questionnaire suggests a decision bias, rather than an encoding bias. However, due to the possibility that encoded task statistics are not directly available to semantic retrieval during the questionnaire, we cannot rule out either possibility.

Regardless of whether variability is linked to encoding or the decision process, an obvious question emerges: What underlies these individual differences? Participants undoubtedly arrived at the experiment with different prior expectation for the kinds of rules that may be operating within a psychology experiment. While we attempted to normalize their expectations by orienting them to the task-sets of interest (shape or color), the prior expectations for higher-order rules may have prevented some people from appropriately integrating certain information. This may partially explain why some people were unable



to learn any rule – their prior beliefs constrained the search space, preventing the encoding of the relevant variables.

Similarly, early identification of a particular pattern may have stifled later learning – a type of confirmation bias that may have attentional roots. Participants who identified the relationship between task-set and vertical position may have been less motivated to search for more complicated relationships. While we expect that the relationship between transition probabilities and task-set selection relates more to unconscious statistical reasoning than explicit rules, it may be that explicit adherence to a particular rule may have overwhelmed other potential factors.

### Future Directions

The biased observer model is a simple extension of the normative optimal observer model, which inevitably falls short in capturing the range of strategies a person could employ on this task. Excluded subjects who performed at chance probably create some (inappropriate) rule governing their behavior on the test phase. Future work could extend the range of strategies under consideration.

In a similar vein, both models operate by inverting a relatively accurate generative model of the task. This is clearly overly optimistic in regards to subject's encoding, as subjects may fail to encode any aspect of the environment (e.g. the Gaussian form of the position distributions, the parameters of these Gaussians). It may be possible to build a more flexible model which infers subject's task understanding from their performance. In addition, simple biases have not been included which could help improve individual fits. Some subjects may have a tendency to select STS over CTS, even without evidence.

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