A Stage Structured Model of the Impact of Buffelgrass on Saguaro Cacti and their Nurse Trees

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1. Abstract

The saguaro cactus (Carnegiea gigantea), a keystone species in the Sonoran Desert, has faced population decline in recent years. The immediate threat to the saguaro cactus is the increase in wildfires fueled by the invasive species buffelgrass (Pennisetum ciliare). The increasing rate of wildfires could result in the collapse of the Sonoran Desert ecosystem. A stage structured model is used to capture interactions between saguaro cacti (juvenile and adults), their nurse trees, and buffelgrass. The introduction of buffelgrass to the model demonstrates its influence on the natural life cycles of the saguaro and nurse tree populations. This model consists of a system of non–linear ordinary differential equations which considers commensalism between juvenile saguaros, their nurse trees, and the eventual competition between adults and nurse trees. Both qualitative analysis of the equilibria of the system as well as a numerical analysis of the sensitivity of key parameters are done. In the interest of preserving the saguaro cactus population, the buffelgrass population must be limited through controlled herbicide and harvesting. **Under what conditions will buffelgrass, and buffelgrass propagated wildfires, interrupt the natural life cycle between the saguaro cactus and its nurse trees?**

2. Introduction

Changes in the saguaro cactus population indicates the health of the Saguaro National Park ecosystem. This cactus is important for the tourism industry and has cultural significance to the Papago and Pima nations. In 1989, buffelgrass, an invasive species with the ability to spread wildfires, was introduced to Saguaro National Park. As a fire-adapted plant, buffelgrass is able to survive fires and quickly regrow. Therefore, buffelgrass fuels fires, burning native plants like saguaros and nurse trees and replacing them within weeks. Figure 1 shows the changes in population among adult and juvenile saguaro cacti from 1942-2015 at Saguaro National Park.

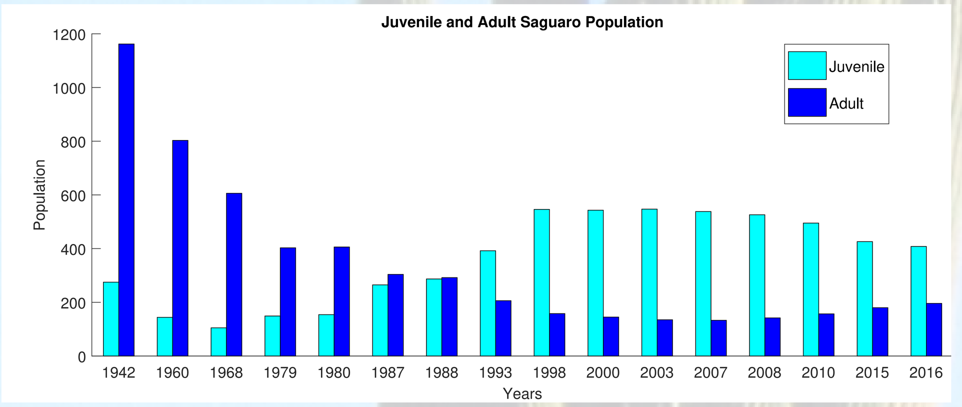


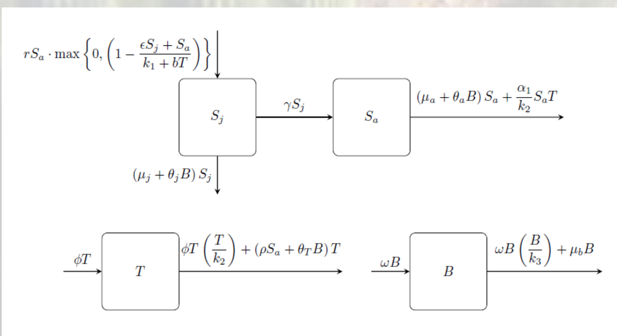
Figure 1: *Population data collected for the saguaro population from 1942 to 2016 [1]*

3. The Model Assumptions

- Juvenile saguaros are assumed to be cacti in the age range of 1 to 35.
- The natural life span of all adult saguaros is 175 years, which is the average lifespan.
- Nurse trees are not able to out compete adult saguaros for resources
- Juvenile saguaros compete over space with adult saguaros.
- Nurse trees increase the available space for juvenile saguaros
- Competition or commensalism between buffelgrass and other species is minimal and is neglected.
- Buffelgrass harms saguaros and nurse trees through the propagation of wildfire.
- Since buffelgrass is a fire-adapted species, it is not harmed by wildfire and regrows quickly after fires.

4. The Model

The model is formulated using the dynamics between the adult and juvenile saguaro cacti, their nurse trees, and buffelgrass. The interactions considered are given by the following nonlinear system of ordinary differential equations, where S_j denotes juvenile saguaro populations, S_a denotes adult saguaro populations, T denotes nurse tree populations, and B denotes buffelgrass populations. The non-linear system of differential equations is given by:



$$\frac{dS_j}{dt} = rS_a \cdot \max\left\{0, \left(1 - \frac{\epsilon S_j + S_a}{k_1 + bT}\right)\right\} - \gamma S_j - \mu_j S_j - \theta_j B S_j,$$
$$\frac{dS_a}{dt} = \gamma S_j - \frac{\alpha_1}{k_2} S_a T - \mu_a S_a - \theta_a B S_a,$$
$$\frac{dT}{dt} = \phi T\left(1 - \frac{T + \sigma S_a}{k_2}\right) - \theta_T B T \text{ and,}$$
$$\frac{dB}{dt} = \omega B\left(1 - \frac{B}{k_3}\right) - \mu_B B.$$

5. Parameters

Parameter	Description	Value	Parameter	Description	Value
r	Germination rate	$\frac{4.725 \cdot juveniles}{adult \cdot year}$	k_1	Adult saguaro carrying capacity	$\frac{250}{hectare}$
ϵ	Converts juveniles to adults	.113	b	Average juveniles under nurse tree	.8 Indv
γ	Maturation rate	.0285 $year^{-}1$	μ_j	Juvenile death rate	.042 $year^{-}1$
α_1	Saguaro death rate by competition	0.000001	k_2	Carrying capacity of adults and trees	975.5 Indv
μ_a	Adult death rate	$\frac{1}{140}$ $year^{-}1$	ϕ	Growth tree population	.07 $year^{-}1$
ρ	Tree competition death rate	$\frac{.8}{4500}$	σ	Proportion carrying capacity	2.432
θ_j	Grass fire frequency effect-juvenile	3.61×10^{-6} $(year \cdot grass)$	θ_a	Grass fire frequency effect-adult	2.71×10^{-6} $(year \cdot grass)$
θ_t	Grass fire frequency effect-tree	2.27×10^{-6} $(year \cdot grass)^{-}1$	ω	Grass growth rate	.35 $year^{-}1$
μ_B	grass harvesting	$\frac{1}{3}$ $year^{-}1$	k_3	Carrying capacity of grass	60000 Indv

Table 1: *This table gives the meaning of the parameters used in the model and their baseline values*

6. Results of the Equilibria Analysis

In order to maintain a stable population equilibrium, several requirements were found. In the absence of buffelgrass, the expected number of adult saguaro cacti produced by one adult saguaro must be greater than one for population maintenance among the cacti. Without nurse trees, this condition is given by,

$$R_{d1} = \frac{\gamma}{\gamma + \mu_j} \cdot \frac{r}{\mu_a} > 1$$

In the presence of nurse trees, this condition is given by,

$$R_{d2} = \frac{r}{\mu_a + \alpha} \cdot \frac{\gamma}{\gamma + \mu_j} > 1.$$

Through rescaling certain parameters directly affected by the buffelgrass induced wildfires, R_{d1} and R_{d2} can be redefined as R_{d3} and R_{d4}.

Equilibrium	Existence	Stability
$E_1 = (0,0,0,0)$	always	unstable
$E_2 = (0,0,T_2^*,0)$	always	$R_{d1} < 1$ and $\omega < \mu_B$
$E_3 = (S_{j3}^*, S_{a3}^*, 0, 0)$	$R_{d1} > 1$	$\frac{\phi}{\rho} < \frac{k_1}{1+E}\left(1 - \frac{1}{R_{d1}}\right)$ and $\omega < \mu_B$
$E_4 = (S_{j4}^*, S_{a4}^*, T_4^*, 0)$	If, with $S_a^* < \frac{\phi}{\rho}, R_{d1} > 1$	Stable(numerical) and $\omega < \mu_B$
$E_5 = (0,0,0,B^*)$	always	unstable
$E_6 = (0,0,T_6^*,B^*)$	always	$R_{d4} < 1$ and $\omega > \mu_B$
$E_7 = (S_{j7}^*, S_{a7}^*, 0, B^*)$	$R_{d3} > 1$	$\frac{\tilde{\phi}}{\rho} < \frac{k_1}{1+\tilde{E}}\left(1 - \frac{1}{R_{d3}}\right)$
$E_8 = (S_{j8}^*, S_{a8}^*, T_8^*, B^*)$	If, with $S_a^* < \frac{\phi}{\rho}$, $R_{d4} > 1$ and $\frac{\tilde{\phi}}{\rho} > \frac{k_1}{1+\tilde{E}}\left(1 - \frac{1}{R_{d3}}\right)$ then there is at least one coexistence equilibrium.	Stable (numerical)

Table 2 Provides the equilibria obtained as well as their conditions for existence and stability.

7. Results of the Numerical Simulation

Numerical simulations and a local sensitivity analysis were performed. From the numerical simulations, it can be seen that with the current wildfire frequency, the saguaro and nurse tree populations are able to coexist with buffelgrass.

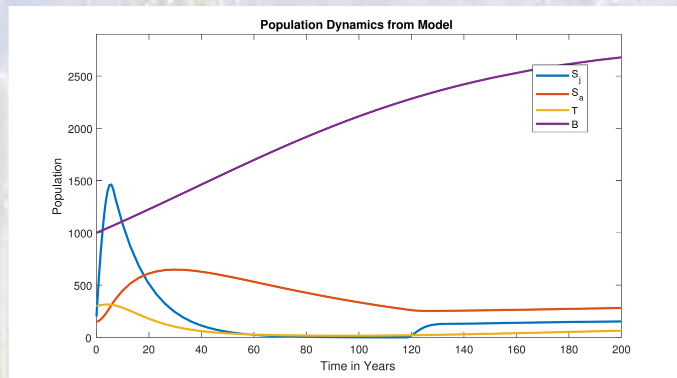


Figure 3: *The population dynamics for saguaros, nurse trees, and buffelgrass given in the figure use the baseline parameter values and show that no population approaches extinction. Although the tree population appears to die out, this does not occur at any point in the time span.*

However, if the wildfire frequency increases, saguaros and nurse trees would be put at risk.

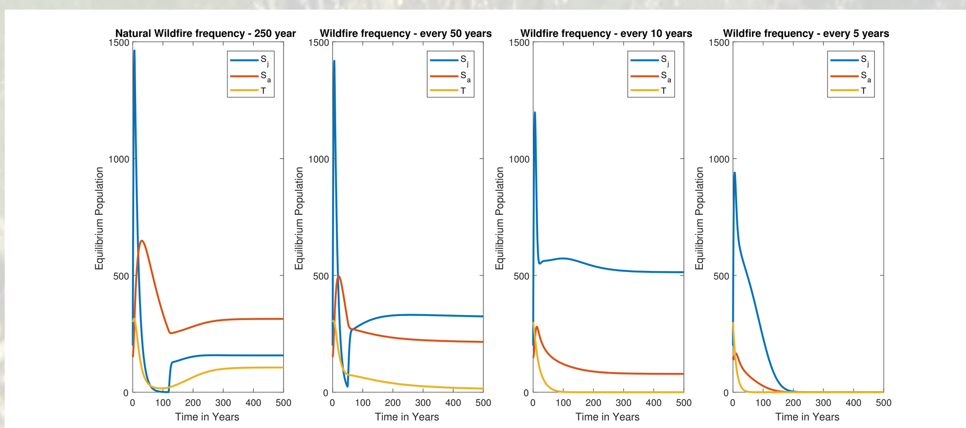


Figure 4: *This figure show the population dynamic of saguaros and nurse trees changing in response to increased wildfire frequency.*

Since the frequency of wildfires is currently increasing, this means saguaros and nurse trees are at a greater risk. The best way to reduce this risk according to both the simulation and sensitivity analysis is to reduce the buffelgrass population by decreasing ω, the growth rate of the buffelgrass population, and increasing μ_B, the harvesting rate affecting the buffelgrass population.

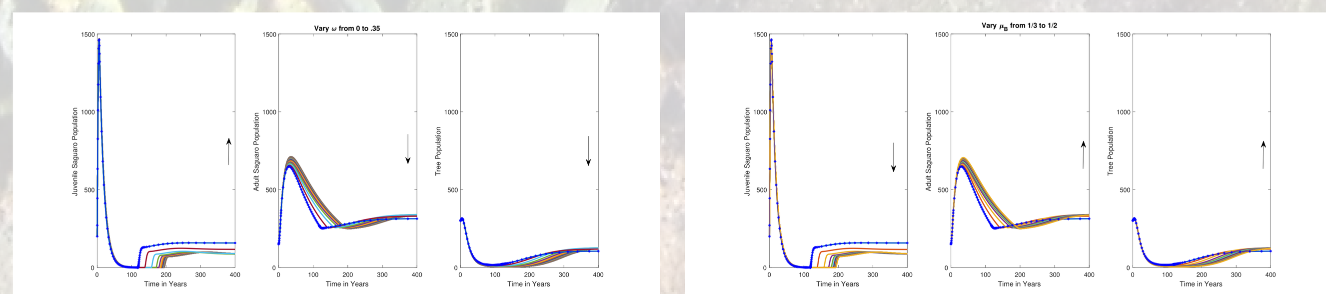


Figure 5: *ω, the buffelgrass growth rate, and increasing μ_B, the harvesting rate of buffelgrass, is shown to increase adult saguaro and nurse tree populations at equilibrium.*

8. Local Sensitivity Analysis

Local sensitivity analysis quantifies the effects of slightly perturbing the value of a critical parameter on a quantity on interest. In this case the value of the critical parameter is changed by one percent change and the quantities of interest are the equilibrium populations. The sign of each sensitivity index gives the direction of the change in the quantity of interest, and the value gives the magnitude of the change in percentage. Sensitivity analyzed without the inclusion of buffelgrass, that is at $B^* = 0$ equilibrium, gives the growth rate of trees, ϕ as the most sensitive parameter. This means planting more trees will increase both the nurse tree and saguaro populations.

Sensitivity Analysis	r	b	ϕ
S_j	0.0013	0.1006	0.6534
S_a	0.0013	0.1006	0.6534
T	-0.0084	-0.6534	2.2520

Sensitivity analysis after the inclusion of buffelgrass shows that the equilibrium populations are most sensitive to the growth rate, ω, and harvesting rate of buffelgrass, μ_B.

Sensitivity Analysis	θ_j	θ_a	θ_T	μ_b	ω
S_j	-3.92×10^{-4}	0.4916	-0.0753	-8.3189	8.3232
S_a	-3.92×10^{-4}	-0.0111	-0.0753	1.7351	-1.7354
T	0.0028	0.0803	-0.2980	4.2973	-4.2973
B	N/A	N/A	N/A	-20	20

9. Conclusions

A model was built that implements numerical simulations to determine if the saguaro and nurse tree populations are able to coexist with buffelgrass. In the absence of buffelgrass, the most efficient way to influence the production of more adult saguaros is to increase the growth rate of trees. With the natural wildfire frequency, the saguaro and nurse tree populations are able to coexist with buffelgrass. If the wildfire frequency increases, saguaros and nurse trees would be put at risk of extinction. The best way to reduce this risk was found to be through the reduction of the buffelgrass population by decreasing the growth rate of the buffelgrass population through herbicide spraying, and increasing the harvesting rate of buffelgrass population through manual pulling. Future work of this study would be an optimal control analysis; the results would shed some insight to which strategies are better than others to keep the cost low and make an impact on reducing buffelgrass.

10. Selected References

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11. Acknowledgements:

We would like to thank the Mathematical and Theoretical Biology Institute (MTBI) co-Directors Dr. Carlos Castillo-Chavez, and Dr. Anuj Mubayi for giving us the opportunity to participate in this research program. We would also like to thank associate director Sherry Woodley, coordinator Ciera Duran and, management intern, Sabrina Avila for their efforts in providing logistics for activities during MTBI. We also want to give special thanks to our co-authors, Dr. Karen Ríos-Soto, Dr. Christopher Kribs, as well as mentors, Fan Yu, Juan Meléndez-Alvarez, Dr. Leon M Arriola, and Dr. Anuj Mubayi for their contributions throughout this project. The research has been carried at the MTBI which is a Research Experience for Undergraduate (REU) summer program at the Simon A. Levin Mathematical, Computational and Modeling Sciences Center (SAL MCMSC) at Arizona State University (ASU). This project has been partially supported by grants from the National Science Foundation (DMS1263374), the National Security Agency (H98230-15-1-0021), the Office of the President of ASU, and the Office of the Provost at ASU.