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Invasive Species Control Optimization as a Dynamic Spatial Process: An Application to Buffelgrass (*Pennisetum ciliare*) in Arizona

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Buffelgrass (*Pennisetum ciliare*) is a fire-prone, African bunchgrass spreading rapidly across the southern Arizona desert. This article introduces a model that simulates buffelgrass spread over a gridded landscape over time to evaluate strategies to control this invasive species. Weed-carrying capacity, treatment costs, and damages vary across grid cells. Damage from buffelgrass depends on its density and proximity to valued resources. Damages include negative effects on native species (through spatial competition) and increased fire risk to land and buildings. We evaluate recommended "rule of thumb" control strategies in terms of their ability to prevent weed establishment in newly infested areas and to reduce damage indices over time. Two such strategies—potential damage weighting and consecutive year treatment—used in combination, provided significant improvements in long-term control over no control and over a strategy of minimizing current damages in each year. Results suggest specific recommendations for deploying rapid-response teams to prevent establishment in new areas. The long-run population size and spatial distribution of buffelgrass is sensitive to the priority given to protecting different resources. Land managers with different priorities may pursue quite different control strategies, posing a challenge for coordinating control across jurisdictions.

Nomenclature: Buffelgrass, *Pennisetum ciliare* (L.) Link.

Key words: Biological invasion, buffelgrass, dynamic spatial processes, environmental studies, integer programming, invasive species, land management, optimal control.

This study examines the spread and management of invasive weeds as a spatial-dynamic problem (Smith et al. 2009; Wilen 2007), which Wilen (2007, p 1134) defines as "some (generally biophysical) process that generates potentially predictable patterns that evolve over space and time." Here, the underlying dynamics of biophysical (and economic) systems have important spatial dimensions. Although studies of invasive species account for spatial aspects of population growth, they usually do not consider other important aspects of spatial variations over landscapes. For example, they may treat control costs as independent of the terrain where invasive weeds are found, or they may model damage as a function of the total invasive weed population but not the location of that population. New work has begun to formally model critical spatial-dynamic relationships in the study of biological invasions. For example, Epanchin-Niell and Wilen (2012) consider how optimal control of invasive weeds is affected by landscape size and landscape shape and by where an initial invasion occurs.

Buffelgrass Invasion Risks in Southern Arizona. The spatial-dynamic framework is applied to answer questions about the management of buffelgrass [*Pennisetum ciliare* (L.) Link], an invasive, fire-prone, African bunchgrass that is spreading rapidly across the desert landscapes of southern Arizona. This region represents the northern stretches of the Sonoran Desert, home of unique species, such as saguaro [*Carnegiea gigantea* (Engelm.) Britton & Rose]. The Sonoran Desert ecosystem has sparse vegetation and is not adapted to fire (Burquez-Montijo et al. 2002; Rogstad et al. 2006; Stevens and Falk 2009). Buffelgrass forms dense stands, crowding out native species, reducing species diversity, and increasing wildfire risk (Bowers et al. 2006;

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> ### Management Implications
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> A key challenge facing land managers is how best to allocate limited resources to control invasive plant species across space and time. Optimization models are useful tools for exploring alternative strategies to optimally allocate scarce resources, such as treatment control teams and budgets, and to protect valued resources from invasion of nonnative species. In this article, we developed a mathematical model to provide guidance to land managers for addressing the following concerns: (1) the optimal size of treatment teams; (2) where, when, and what size of infestation those teams should target; and (3) the number of years for which follow-up treatments should continue. Because of the many variables interrelated across both space and time, solving such a completely forward-looking (i.e., takes full account of how all current decisions affect all future options and decisions) problem may prove intractable. Instead, we compare three "rules-of-thumb" strategies: (1) minimize current invasive species damage; (2) minimize current damage, given that any areas treated are treated in at least 3 consecutive yr; and (3) prioritize treatment based not only on current damages but also on the potential future damages of leaving an infested area untreated. The second and third strategies are also considered in combination. We evaluate those rules of thumb for their ability to prevent weed establishment in newly infested areas and to reduce damage indices over time. The rules have the advantage of telling land managers to "treat these lands now."
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> Another advantage of this approach is its applicability because Microsoft Excel spreadsheets—used broadly by land and resource managers in the area—are customized to (1) manage data layers, (2) use cell formulae to maintain relationships across space and time, and (3) use the chart function to produce maps of costs, damages, weed population, and treatment recommendations. The ILOG CPLEX software package (IBM), a powerful tool for solving linear integer (binary) programs, interfaces with Excel programs so that model solutions can be readily converted to treatment priority (and other) maps. We found that the long-run population size and spatial distribution of buffelgrass are sensitive to priority weights for protecting resources. Results also indicate that resources must be increased because they are currently insufficient to control the spread of buffelgrass.

Clarke et al. 2005; Jackson 2005; McDonald and McPherson 2011, 2013). The saguaro cactus, an iconic symbol in southern Arizona, is particularly vulnerable to fire (Esque et al. 2004). Betancourt (2007) has warned that buffelgrass and other invasive perennial grasses are "rapidly transforming fireproof desert into flammable grassland." Wildfire not only threatens native species but also poses risks to commercial and residential property in natural, urban, and suburban areas.

This work is an extension of Büyüktahtakin et al. (2011) that introduced a large-scale, nonlinear, zero-one integer programming model for the dynamic control of invasive weeds. That model used a rolling-horizon solution in which they solved single-period problems 1 yr at a time and fed the result into the next period's problem. The earlier study emphasized model construction, structure, and solution algorithm concepts, whereas this article focuses more heavily on the management implications of model results. This study considers explicitly how resource protection priorities can affect the long-term size and distribution of buffelgrass populations.

We begin by introducing a dynamic-spatial model of weed invasion with multiple sources of spatial heterogeneity developed in Büyüktahtakin et al. (2011). Buffelgrass spreads across a gridded landscape. Each cell in the grid represents 0.4 ha (1 ac) of land. The potential for an invasive weed to become established, the weed's carrying capacity (maximum achievable population density), the costs of its control, and the damage it causes can vary across the landscape. Previous work has focused on a subset of these features, usually treating damage as a function of total weed population. Here, we emphasize that damage caused by invasive species depends on their location relative to resources of value. Damage caused by buffelgrass in a given cell depends on the buffelgrass population density in that cell and whether valued, threatened resources are in or near that cell. A land manager's problem is to minimize damage over time, subject to budget and labor constraints. A damage index is specified as a weighted sum of damages to different resources, with weights reflecting management priorities. Buffelgrass can be treated in a cell, at most, once per period. Given constraints, the manager must choose which cells to treat during each period. In this article, an optimization model is developed and calibrated to replicate historical spread behavior.

An optimization model seeks the best (optimal) solution to a problem, often subject to some constraints. In our case, the problem is to minimize damage caused by buffelgrass invasions over time, sometimes given labor and budget constraints. A dynamic optimization model is forward looking; it fully accounts for how current decisions affect all future options and decisions. Choices to treat or not to treat certain locations of the grid today affect future treatment options and the costs and benefits of all future treatment choices. In a fully dynamic optimization model, the decision maker knows how each current decision affects future decisions. Thus, the decision maker accounts for how decisions taken in each period affect the best achievable long-run path of damage reduction. Although we introduce such a general dynamic optimization model as a guiding concept, our problem involves literally thousands of nonlinear, interrelated equations. A fully dynamic, optimal solution to the general model is not tractable. Therefore, we simplify the problems to address specific buffelgrass management questions. The initial approach is to simply allocate resources to minimize damage by buffelgrass in each current year. In each new year, past decisions and consequences are taken as givens. Then, we have the model follow "rules of thumb" that land managers actually implement or that have been recommended by the Buffelgrass Working Group (Rogstad

2008). As discussed below and in Büyüktahtakin et al. (2011), those rules of thumb mimic considerations of a full forward-looking decision maker solving a dynamic optimization problem.

First, we use the model to estimate labor requirements to prevent buffelgrass from becoming established in a recently invaded area. Costs of delay are evaluated in terms of growing labor requirements needed to eradicate new infestations. The National Invasive Species Council (NISC) was established by Executive Order 13112 in 1999 to improve coordination of invasive species control programs. The Council's management plan stresses the importance of rapid response to invasive species and calls for the use of "rapid-response teams" to control new invasions before they spread (NISC 2001). Model results have direct implications for the staffing and deployment of rapid response teams to prevent buffelgrass establishment. They suggest the following: (a) how large those teams should be, (b) what size of infestation those teams should target, and (c) how many years follow-up treatments should continue.

Second, we conduct positive analysis of treatment recommendations from the Southern Arizona Buffelgrass Strategic Plan (Rogstad 2008). That plan recommended using potential-damage weighting and consecutive-year treatment rules to prioritize which areas to treat. Those recommendations are specified as heuristic treatment rules and are applied as integer programming problems in the spatial-dynamic framework. Those heuristic rules do not represent fully dynamic optimization; however, they do optimize objective functions that account for certain dynamic relationships. The heuristic rules are evaluated in terms of their effectiveness at (1) preventing buffelgrass from becoming established in a newly invaded area, and (2) reducing damage over time. They are also evaluated in terms of damage reduction compared with no treatment and with treatments minimizing current damages in each year.

The approach here is in the tradition of research comparing specific strategies for invasive species management. For example, Moody and Mack (1988) and Martin et al. (2007) compare the efficiency of targeting new, small, invasive weed populations over larger, established populations. They found that, for the landscapes with lower levels of infestation, detecting and eliminating small, new infestations was more effective than controlling large, known infestations, although with higher levels of infestation, containing the edges of large infestations and simultaneously detecting and eliminating new foci is an ideal strategy. Using a state-and-transition model, Frid and Wilmhurst (2009) and Frid et al. (2013) formally evaluated similar strategies for invasive species control and obtained the same results regarding different level of infestations. Wadsworth et al. (2000) compared random treatment with alternative strategies based on proximity to human settlements and weed population size, age, and spatial distribution. Jetter et al. (2003) estimated the benefits and costs of biological control programs and subsidies for private rangeland restoration to control yellow starthistle (*Centaurea solstitialis* L.). Cacho et al. (2004) compared the net benefits of immediate eradication vs. containment and no-control strategies, examining under what conditions each of the three alternatives dominate.

A limitation to this approach is that we identify superior strategies among selected strategies, but do not know if there are other, even better strategies. However, many optimal control or dynamic programming models of invasive species management often fail to provide specific, useful recommendations. As Wilen (2007) points out, "the more important questions seem to be where to spray, when, and at what intensity in a landscape setting (p 1139)." The rules introduced here have the advantage of telling land managers, "treat these locations now."

Materials and Methods

Ecological and Economic Components of the Problem. In this article, a gridded landscape model is considered. Treatment decisions are taken as a function of the possible damages and costs estimated across time and space from an invasion emanating from already-invaded areas. We model growth using a logistic growth function that is spatially and population dependent, incorporating the dispersal that a cell receives from neighboring cells. In our case, we use a negative exponential-kernel function to formulate the dispersal in the grid model.

We model treatment costs as an area-specific linear function of population, and damages as area-specific functions of resources threatened by the species' presence. A numerical, biological spread model is calibrated using historical data (aerial photography and population monitoring data) from the University of Arizona Desert Laboratory and environs laid out on a 40 by 50 cell grid. The Desert Laboratory on Tumamoc Hill is a 370 ha reserve west of downtown Tucson, AZ, where ecological research has been conducted for more than 100 yr. The 809-ha study area includes Desert Laboratory lands; Sentinel Peak (the A Mountain), a city-managed park; other open space; and some homes. More homes, commercial real estate, and schools surround the area. Buffelgrass populations have been monitored regularly around the Desert Laboratory since 1983 (Bowers et al. 2006). Figure 1 shows the study area in relationship to the city of Tucson, AZ, and Pima County, AZ. Parameters of the numerical buffelgrass-spread model were calibrated to replicate actual, historic spread behavior.

Buffelgrass Population Spread and Growth Equations. Let $t \in \{0, \ldots, T\}$ be any year of the entire time horizon T. Define an index on the x-axis $i \in X = \{1, \ldots, I\}$ and an index on the y-axis $j \in Y = \{1, \ldots, J\}$ giving the coordinates

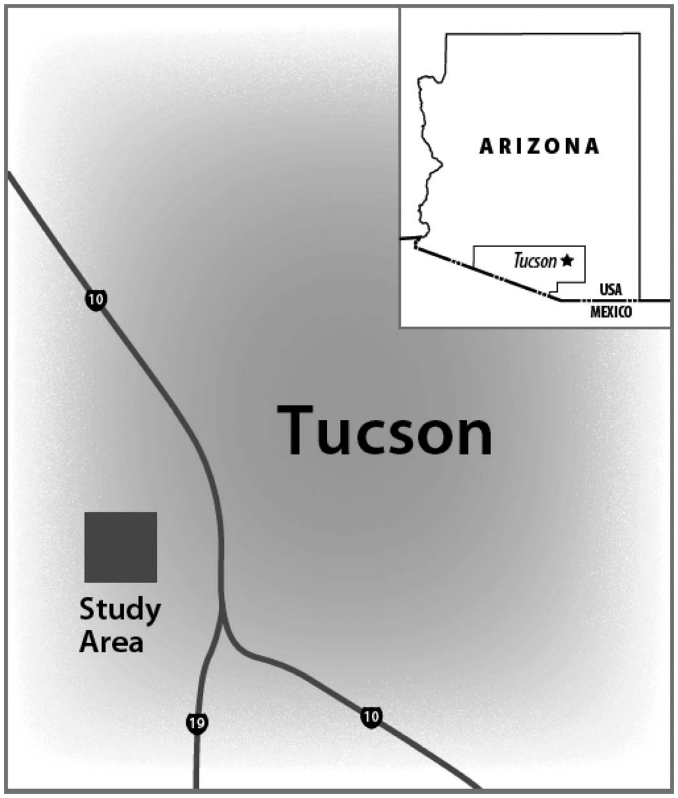

Figure 1. The study area: Tumamoc Hill and environs, west of Tucson, AZ, in Pima County.

of cell (i,j). At any time t, the pretreatment population density of buffelgrass in a cell depends on the population density in that cell and in surrounding cells in the previous year:

$$N_{i,j,t} = g\left[N_{i,j,t-1},\ \boldsymbol{N}_{\sim i,\sim j,t-1},\ K(\mathbf{s})_{i,j}\right] \qquad [1]$$

where $N_{i,j,t}$ is the pretreatment buffelgrass population density in cell (i,j) in year t; $\boldsymbol{N}_{\sim i,\sim j,t}$ is the pretreatment buffelgrass population density in eight cells surrounding cell (i,j) in year t; $K(\boldsymbol{s})_{i,j}$ is the carrying capacity (maximum buffelgrass population density) possible in cell (i,j); and $\boldsymbol{s}$ is the vector of attributes affecting carrying capacity, such as soils, altitude, climate, slope, aspect, and past land disturbance.

The function $g(.)$ has a logistic growth form, where population grows at an increasing rate at first, then at a decreasing rate as the population approaches the cell's carrying capacity. Growth slows as the cell becomes saturated with buffelgrass. The logistic growth function is shown in Equation 2:

$$g\left[N_{i,j,t},\ \boldsymbol{N}_{\sim i,\sim j,t},\ K(\mathrm{s})_{i,j}\right] = \left[\exp(r)K(\mathrm{s})_{i,j}d\left(N_{i,j,t},\ \boldsymbol{N}_{\sim i,\sim j,t}\right)\right] \Big/ \left[K(\mathrm{s})_{i,j} + d\left(N_{i,j,t},\ \boldsymbol{N}_{\sim i,\sim j,t}\right)(\exp(r)-1)\right] \qquad [2]$$

where function $d(.)$ represents the population in cell (i,j) in year t after accounting for the dispersal from neighboring cells, and r is the intrinsic growth rate. Using the published assessments of individual plant growth rates (Halvorson and Guertin 2003) and following discussion with local buffelgrass experts, the term r is estimated to be 1. The term $K(\boldsymbol{s})_{i,j}$, carrying capacity, is computed based on the predicted suitability of the landscape, considering attributes such as soils, altitude, climate, slope, aspect, and past land disturbance (Frid et al. 2013). Using a logistic regression model, $K(\boldsymbol{s})_{i,j}$, is estimated to vary between 0 and 6 plants m^{-2} (between 0 and 5 plants yd^{-2}).

A cell receives propagules from plants within the cell and from the eight neighbor cells surrounding it. The rate at which a cell receives propagules from neighboring cells is obtained using an exponential decay function, which is a simple, but widespread, dispersal-kernel model (Levin et al. 2003). Therefore $d(.)$ represents a linear combination of $N_{i,j,t}$, and $N_{\sim i,\sim j,t}$, where $N_{\sim i,\sim j,t}$ is weighted using the negative exponential dispersal kernel:

$$y = \lambda \exp(-\lambda z) \qquad [3]$$

where z is the distance from the cell center (i,j) to the neighboring cell center, and λ is a parameter. Using a nonlinear least-squares regression based on historical reconstructions of the spread in the nearby Catalina Mountains (Olsson et al. 2012), the term λ is estimated to be 1.

Buffelgrass Treatment Equations. The most effective means of controlling buffelgrass is treatment with the herbicide glyphosate. Buffelgrass can be manually removed using pry bars, but that method is labor intensive. Moreover, many sites in Arizona (including the Desert Laboratory) have Native American cultural resources lying below ground, limiting the extent to which removal via digging is permitted.

The decision about whether to treat a cell is a discrete choice, such that a cell is either treated (sprayed) or not. In each year t, the decision variables represent the treatment choice defined as

$$x_{i,j,t} = \begin{cases} 1 & \text{if cell } (i,j) \text{ is treated in year } t \\ 0 & \text{otherwise} \end{cases} \qquad [4]$$

for all i, j, and t.

The posttreatment buffelgrass population density in a cell, $n_{i,j,t}$, is

$$n_{i,j,t} = N_{i,j,t}\left(1 - kx_{i,j,t}\right) \qquad [5]$$

where k is the kill rate of the herbicide treatment, where $k = 0.9$ if $N_{i,j,t} > \underline{N}_{i,j,t}$ and $k = 1.0$ if $N_{i,j,t} \leq \underline{N}_{i,j,t}$; and $\underline{N}_{i,j,t}$ is the critical population, below which, it is possible to remove buffelgrass completely from a cell.

Treatment reduces the buffelgrass population by 90% in each year of treatment. Because herbicide treatment is only effective for a short time following (rare) rainfall events, we assume that cells are treated once a year, at most. Successive treatments reduce the population by 90%, based on recent data for treatment effectiveness in the Saguaro National Park and the Organ Pipe National Monument (Hunter 2011). If the population falls below the minimum threshold $\underline{N}_{i,j,t}$, however, we allow for the possibility that an additional treatment can drive the population to zero in a cell. In our computations, we set $\underline{N}_{i,j,t}$ to 0.00001 plants 0.4 ha^{-1}.

Treatment Cost Equations. The cost of treating a cell (*i,j*), $C_{i,j}$ increases with pretreatment buffelgrass population, average cell slope, and distance of the cell from the closest road:

$$C_{i,j} = c_1 + c_2 N_{i,j,t} + c_3 slope_{i,j} + c_4 distance_{i,j} \qquad [6]$$

where the coefficients c_1, c_2, c_3, and c_4 are estimated by the least-squares method based on recent treatment records in and around the Desert Laboratory (Bowers et al. 2006) and were set to U.S. \$0.31, \$2.91, \$0.19, and \$0.04, respectively. The cost model was developed based on daily progress maps of treatment made at the Desert Laboratory in the summer of 2006. The boundary of each day's work was digitized, and the area was calculated. The mean buffelgrass density (based on mapping performed in 2005), the mean distance from the nearest road (which served as staging areas for herbicide treatments), and mean slope (based on U.S. Geological Survey [USGS] National Elevation Data Set 30-m digital elevation models) were calculated and treated as data samples. The cost for each sample was registered as the cost of herbicide and of crew hours for each day. Treatment costs can vary for each cell, but the cost of treating an *individual* cell in a *given* year is constant. The cost of treating an individual cell can change because pretreatment of the buffelgrass population, $N_{i,j,t}$, changes. Without treatments to reduce the buffelgrass population, the cost of treating a landscape will increase over time. Treatment costs increase until they reach a maximum, where the buffelgrass population is at its carrying capacity in each cell.

Resource Constraints. The land manager faces a budget constraint in treating buffelgrass as follows:

$$\sum\nolimits_{i \in X} \sum\nolimits_{j \in Y} C_{i,j,t} x_{i,j,t} \leq B_t \qquad [7]$$

where B_t is the annual control budget in time *t*. In reality, land managers are likely to face both a monetary budget constraint and a labor availability constraint. Volunteer labor provides a significant amount of buffelgrass treatment. Moreover, chemical treatment is only effective at certain times of the year (not too long after rainfall), so time constraints can be as important as monetary ones.

Buffelgrass Damage Function. Posttreatment damage caused by buffelgrass in a cell (*i,j*) depends on its density in the cell, whether there are resources that it threatens in that cell, and whether there are resources in neighboring cells that are threatened:

$$D_{i,j,t} = D_{i,j,t}\left(n_{i,j,t},\ R_{i,j,t},\ \boldsymbol{R}_{\sim i,\sim j,t}\right) \qquad [8]$$

where $D_{i,j,t}$ is the damage caused by buffelgrass in cell (*i,j*); $n_{i,j,t}$ is the posttreatment buffelgrass population; $R_{i,j,t}$ is the proportion of resource at risk in cell (*i,j*); $\boldsymbol{R}_{\sim i,\sim j,t}$ is the proportion of resource at risk in cells surrounding cell (*i,j*).

The term $R_{i,j,t} \in [0,\ 1]$ is obtained by a distribution map of different resources in the landscape using the exotic plant surveys performed in 1983 and 2005 at Tumamoc Hill (Bowers et al. 2006). We identified the following values at risk at Tumamoc Hill: saguaros, riparian areas, historic buildings, critical infrastructure, viewsheds, and residences. For each value at risk, a new *R* was calculated. Different values of *R* can be combined linearly to represent different value viewpoints because different stakeholders would view resource values differently.

Damage from buffelgrass follows an exponential decay pattern. Buffelgrass in any cell contributes to damage by threatening resources. As a resource at risk is further away from the buffelgrass, the buffelgrass causes less damage. Distance is measured from a centroid of a cell to the centroid of another cell. We assume damage depends on resources in a cell (*i,j*) and the eight cells adjacent to it. The relevant risk factors for cell (*i,j*) are shown in Table 1.

Damage from buffelgrass depends not only on the total buffelgrass population, but also on its location relative to resources of value throughout the landscape. There can be more than one resource at risk, so there is a different damage function for each resource.

In this article, we focus on risks to buildings, to saguaro cactus, and to (ephemeral) riparian vegetation. Saguaros and vegetation may be threatened by crowding out from dense buffelgrass stands. Buffelgrass can increase the frequency and intensity of wild fires. This could create a positive feedback loop where fire-prone grassland replaces fire-vulnerable native desert vegetation (MacDonald and McPherson 2011, 2013; Stevens and Falk 2009). Buildings, saguaros, and vegetation may all be at increased risk from wildfires. The Southwest's iconic saguaro cactus can suffer 68 to 85% mortality after fires. (MacDonald and

Table 1. Relevant risk factors for cell (*i,j*).

$R_{i-1,j-1}$	$R_{i-1,j}$	$R_{i-1,j+1}$
$R_{i,j-1}$	$R_{i,j}$	$R_{i,j+1}$
$R_{i+1,j-1}$	$R_{i+1,j}$	$R_{i+1,j+1}$

McPherson 2013). According to the Southern Arizona Buffelgrass Strategic Plan (Rogstad 2008, p 6), "Areas most at-risk to buffelgrass invasion are also highly prized real estate where multi-million dollar homes are nestled among the saguaros and paloverde trees of the mountain foothills." Concerns over buffelgrass-induced fire risk to homes have also been raised in a Congressional field hearing (Subcommittee on National Parks, Forests and Public Lands, 2010).

Spatial Dynamic Model of Buffelgrass Control. The land manager's problem of controlling buffelgrass is to minimize long-term damage by choosing which cells to treat. Cells are either treated or not. The land manager's objective function is a damage index (*DI*), which is the sum of damages caused by buffelgrass in each cell over the time horizon (*T*). Formally, the land manager's objective is as follows:

$$\text{minimize } DI = \sum_{i \in X} \sum_{j \in Y} \sum_{t \in T} D_{i,j,t} \qquad [9]$$

subject to constraints in Equations 4 through 8.

For completeness, optimization is also subject to initial conditions at $t = 0$. The problem can be generalized further to account for multiple types of damage:

$$\min DI = \sum_{i \in X} \sum_{j \in Y} \sum_{t \in T} \sum_{r \in R} \rho_r D_{i,j,t} \qquad [10]$$

subject to constraints in Equation 4 through 8, where r denotes different resources the manager wants to protect, and ρ_r represents the relative-importance weight placed on protecting resource r in the overall objective function. The term ρ_r measures how much priority a manager places on protecting the given resource r. Different managers may have different priorities. For example, a natural resource specialist may be interested in protecting saguaros and riparian areas from the harmful effects of buffelgrass invasion as well as preserving biodiversity, whereas a county manager may be more interested in lowering the fire risk around critical infrastructure and buildings. For simulations, we examine extreme cases where all weight is given to protecting a single resource to highlight the sensitivity of results with different priorities. Other weighting schemes are possible (e.g., equal weighting). Economic estimates of the costs of different damages if they were available could also be used to assign weights to different types of damage.

For a 40 by 50 cell grid, the problem involves 2,000 nonlinear, interrelated state equations. Full, dynamic optimization of this problem is not tractable. Instead, we consider the resource requirements necessary to prevent buffelgrass from becoming established in an area. Critical issues here are the costs of delay in response to new invasions and the implications for the design of invasive-species rapid-response teams. Next, we consider alternative rules of thumb to minimize buffelgrass damage under resource constraints. The rules are evaluated in terms of their ability to reduce the path of damage over time.

Model Implementation. Our first simulations consider how much labor is required to prevent buffelgrass from becoming established after it first appears in an area. A related question is how much any delays in initiating a treatment regime would increase those labor requirements. We focus on labor requirements because land managers in Arizona frequently face binding labor constraints for buffelgrass control.

It is assumed that buffelgrass is initially discovered on 48 cells of the 40 by 50-cell grid (about 2.4% of cells). The 48-cell invasion represents the population at the time that a systematic control regime was initiated in 2005. Actual control practices from this initial period to the present were used to estimate control costs and labor requirements in the model simulations. In the initial year (year 0), median, mean, and maximum buffelgrass densities on infested cells are 0.2, 0.5, and 2.6 plants m^{-2}. The maximum density possible is about 6 plants m^{-2}. Next, we consider a program of most rapid local eradication (MRLE). Under MRLE, each infested cell is treated each year until the population across the entire area is driven to zero, thus preventing buffelgrass establishment. Labor requirements can be measured in hours or in terms of 400-hr team-wk. Each team-wk represents a 40-hr work week of a 10-person team. Currently in the Tucson, AZ, metro area, various volunteer groups supply roughly 6,000 labor h yr^{-1} toward buffelgrass control (or the equivalent of about 15 team-wk). This is in addition to any professional control labor applied by federal, state, or local entities.

We consider labor required for MRLE given different start years for the local eradication program: years 1, 3, 5, 9, and 13 (Figure 2). If the local eradication program is initiated in year 1 or 3, labor requirements are modest. Fewer than 3 team-weeks would be required in any single year. It takes at least 6 years, however, to drive the population to zero. If treatment is delayed until year 5, then 5 team-weeks are needed in year 5, with declining labor requirements in subsequent years. If treatment is delayed to year 9, however, 15 team-weeks are needed initially. By year 13, requirements exceed 27 team-weeks in the initial year of treatment.

Treatment on a scale of 15 team-wk or more is likely infeasible for two reasons. First, land management agencies face budget and labor time constraints. Second, backpack spraying with glyphosate is only effective when the plants have turned green after sufficient rainfall. In Arizona's arid climate, there may simply be too few weeks in a year when glyphosate treatment is viable. The short time window when laborers can be deployed is thus an additional constraint.

One may also consider how delaying treatment increases the costs of preventing buffelgrass establishment. The

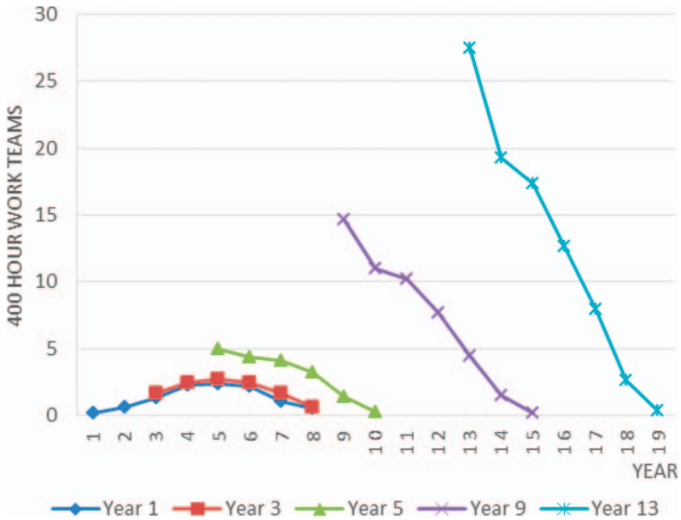

Figure 2. Labor required per year to prevent buffelgrass establishment in a newly infested area, varied by the starting year of the control efforts. Labor is measured for 400-h work teams. (Color for this figure is available in the online version of this paper.)

Buffelgrass Strategic Action Plan prices labor currently at \$18.50 h^{-1}, based on trained applicator costs (Rogstad 2008). If one assumes costs rise at the rate of inflation, and using a real discount rate of 4%, the cumulative labor costs of MRLE can be calculated. If treatment begins by year 1, total discounted costs are \$64,000. If treatment begins in year 3, costs rise to \$70,000. Delays beyond year 5 begin to increase costs more sharply. Costs are \$105,000 when starting in year 5; \$367,000 for year 13; and \$422,000 for year 17. All values are in 2012 constant (inflation-adjusted) dollars.

Our results have direct implications for the staffing and deployment of rapid response teams to prevent buffelgrass establishment. They suggest the following: (1) how large those teams should be, (2) what size of infestation those teams should target, and (3) how many years the follow-up treatments should continue. Our results suggest that three work-teams operating 400 h yr^{-1} each would be sufficient to prevent buffelgrass from becoming established in a newly infested area if the following apply: (1) they began treatment within 3 yr of initial infestation, and (2) they continue with follow-up treatments over 6 to 8 yr. In most years, just two work-teams would be sufficient. Delaying treatment from 1 to 3 yr has little effect on overall labor requirements. Beyond year 3, however, preventing buffelgrass establishment in the area begins to have large labor requirement. Beyond year 3, land managers would need to consider shifting strategies from local eradication in an area to longer-term management and damage containment.

Heuristic Decision Rules. We now consider the effectiveness of rules of thumb to reduce different types of damage. In southern Arizona, a Buffelgrass Working Group was established through a Memorandum of Understanding between federal, state, and county agencies along with private organizations. In 2008, that Working Group published a Strategic Plan, which included recommendations for coordinating and implementing buffelgrass control across jurisdictions (Rogstad 2008). One Working Group recommendation was to "Set and implement control priorities based on actual and *potential* impacts (p vii) (emphasis added)." Another recommendation was for land managers to "institute a minimum three-year treatment and management program" (Rogstad 2008, p 16, 32) to control buffelgrass.

In this section, we specify how these heuristic rules are incorporated as decision rules in our dynamic spatial model. Although fully dynamic optimization is not tractable, we can obtain solutions following those rules of thumb. In subsequent sections, we examine how those rules perform in terms of their ability to prevent buffelgrass establishment and in terms of reducing the long-run path of damage indices.

Rule 1—Static Optimization. We first establish static optimization, which involves solving single-period optimization problems using a rolling-horizon method as a baseline rule (Büyüktahtakin et al. 2011). Subsequent rules may be evaluated both for their performance relative to no treatment and relative to this static rule. The static optimization decision rule is as follows:

1. Reduce current damage as much as possible, subject to a labor constraint.
2. If all cells generating positive, current damage are treated, and labor remains, then treat cells to minimize buffelgrass population, subject to remaining labor availability.

We define the damage function such that buffelgrass causes damage only if a resource of value is either in that same cell or in an adjacent cell. This leaves open the possibility that buffelgrass would not be treated if it first appeared in a cell distant from resources of value, even though it could contribute considerably to future damages. Hence, the second rule prevents lands from going untreated when the labor constraint is not binding. (An alternative approach is to reduce the decay rate of the damage function so that buffelgrass damage depends on more-distant cells; that, however, increases the computational complexity of the model.) Rule 1 does not consider how current treatment affects future damages or subsequent treatment costs.

Minimizing current damage is equivalent to maximizing the reduction in current damage. The reduction in damage from treating a cell is

$$DR_{i,j,t}x_{i,j,t} = \Big[D_{i,j,t}\Big(N_{i,j,t},\, R_{i,j,t},\, \boldsymbol{R}_{\sim i,\sim j,t}\Big) - D_{i,j,t}\Big(n_{i,j,t},\, R_{i,j,t},\, \boldsymbol{R}_{\sim i,\sim j,t}\Big)\Big]x_{i,j,t} \qquad [11]$$

where $x_{i,j,t}$ denotes the binary decision of whether to treat the cell, with the first term in brackets being the damage due to the pretreatment population, and the second term being the posttreatment damage.

The first part of rule 1 is treated as a static integer linear programming (ILP) problem. The first objective is

$$\max DR_1 = \sum_{i \in X} \sum_{j \in Y} DR_{i,j,t} \qquad [12]$$

subject to the constraints in Equations 4 through 8 from the dynamic spatial model, with an additional labor constraint as follows:

$$\sum_{i \in X} \sum_{j \in Y} L_{i,j,t} x_{i,j,t} \leq \underline{L}_t \qquad [13]$$

where $\underline{L}_t$ is a labor-availability constraint. Labor requirements are assumed to be linearly increasing in the pretreatment buffelgrass population, cell average slope, and cell distance from the nearest road. We assume the labor constraint becomes binding before the monetary budget constraint does, rendering the latter redundant; therefore, throughout the rest of the article, we focus on labor constraints. This model includes the well-known zero-one knapsack problem formulation as a subproblem (Wolsey 1998).

The second part of rule 1 takes effect if the damage reduction function (Equation 12) is maximized and the labor constraint is not binding. In this case, *current* damage is reduced to zero. However, buffelgrass may remain in the landscape that is *currently* distant from resources of value. It may not contribute to the current damage index but can increase potential future damage. Let L_t^* represent the optimal amount of labor used to maximize Equation 12. If $L_t^* < \underline{L}_t$, then the second part of rule 1 implies that the land manager faces the following problem:

$$\min \sum_{i \in X} \sum_{j \in Y} n_{i,j,t} \qquad [14]$$

subject to the constraints in Equations 4 through 8, similar to before, with a labor constraint:

$$\sum_{i \in X} \sum_{j \in Y} l_{i,j,t} x_{i,j,t} \leq \underline{L}_t - L_t^* \qquad [15]$$

where $n_{i,j,t}$ is the total posttreatment buffelgrass population, $\underline{L}_t - L_t^*$ is left over (if any) labor after current damage is reduced to zero, and $l_{i,j,t}$ is the application of remaining labor to treatment. Rule 1 might be summarized as follows: first, minimize current buffelgrass damage; second, if damage is reduced to zero, use any remaining labor to minimize the current buffelgrass population.

Rule 2—Potential Damage Weighting. Under rule 1, cells are prioritized for treatment based on their contribution to *current* damage. Rule 2 simulates the recommendation of the Buffelgrass Working Group to prioritize areas to treat "based on actual and potential impacts." Rule 2 employs potential damage weighting as a way of simulating the recommendation of the Buffelgrass Working Group. Cells are prioritized for treatment based not only on their contribution to the current damage but also on their potential contribution to future damages. The maximum potential damage $D^+_{i,j,t}$ that buffelgrass can cause in cell (i,j) depends on resources of value in proximity to that cell and the buffelgrass carrying capacity $K(\boldsymbol{s})_{i,j}$ of the cell (i,j) if the cell is left untreated from the current period to the end of the planning horizon:

$$D^+_{i,j,t} = D^+_{i,j,t}\left[K(\mathbf{s})_{i,j}, R_{i,j,t}, \boldsymbol{R}_{\sim i,\sim j,t}\right] \qquad [16]$$

Rule 2 is, therefore,

$$\max DR_2 = \sum_{i \in X} \sum_{j \in Y} \left[w DR_{i,j} + (1-w) D^+_{i,j,t} \right] x_{i,j,t} \qquad [17]$$

subject to labor and other constraints (as under rule 1). The first objective is to maximize DR_2, whereas the second objective is to minimize buffelgrass population with any remaining labor after maximizing DR_2. Rule 1 is simply a special case of rule 2, where $w = 1$. In subsequent discussion, we focus on an equal weighting scheme where $w = 0.5$.

Rule 2 prioritizes cell treatment considering the following: (1) how much current damage buffelgrass causes, and (2) how much potential damage could be caused if the population were allowed to reach its carrying capacity. Cells with higher carrying capacity will receive higher priority for treatment. This rule accounts for factors that affect the suitability of an area to foster buffelgrass establishment and growth, such as soils, aspect, elevation, or climate. Low populations in suitable areas may cause more *future* damage than higher populations in less-suitable areas. Although rule 2 is not a dynamic optimization, it is forward looking in one sense. It considers potential future damage of leaving a cell untreated.

Rule 3—Treat Three Times. This rule simulates the recommendation of the Buffelgrass Working Group to treat areas in at least 3 consecutive yr. Because of the logistic growth of buffelgrass populations, treating a cell with a population near its carrying capacity will push the population back to the fast part of its growth path. Thus, if a high-population cell is treated only once, the population will rebound quickly the following year. Repeated treatments can push populations down to the slow portions of their growth paths and may even reduce cell populations to zero. Specifically, rule 3 is as follows:

1. First, treat any cell that was treated in the previous year and that has not already been treated for 3 consecutive yr.
2. Next, use any remaining resources to follow rule 1 above.

In the initial year, the treatment strategies under rules 1 and 3 are identical. After that, priorities shift to emphasize repeated treatments of cells.

Rule 4—Treat Three Times with Potential Damage Weighting. Rule 4 combines the heuristics of the previous rules (treat three times consecutively, then follow rule 2, potential damage weighting):

1. Treat any cell that was treated in the previous year that has not already been treated in 3 consecutive yr.
2. With remaining resources, follow rule 2 above, assuming $w = 0.5$.

Solution Algorithm, Software, and Data Management. A linear programming–based tree-search algorithm, called *branch and bound methods* (Nemhauser and Wolsey 1988), was used to derive model solutions. Büyüktahtakin et al. (2011) provide more details on the solution methods. We used ILOG CPLEX (Version 10.0, IBM ILOG CPLEX), which has a straightforward interface with Microsoft Excel spreadsheets. Data inputs and outputs can be managed and represented in Excel, whereas computations can be carried out efficiently using ILOG CPLEX. Data layers for buffelgrass population, treatment costs, resources at risk, and damages are maintained as Excel worksheets. Each cell in the worksheets corresponds to a specific 0.4-ha area of land. Three resources-at-risk layers are measured in terms of saguaro density, presence or absence of buildings/ structures, and presence or absence of ephemeral riparian vegetation. In principle, money metrics for these risk layers could be developed and applied.

The interface with Excel also makes it possible to generate simple, gridded maps. Thus, land managers following heuristic decision rules could print out maps indicating which lands to treat. Our study area is a 40 by 50 gridded rectangle west of downtown Tucson, AZ (Figure 1). Figure 3 is a portion of that rectangle from a simple Excel worksheet with binary code transformed to symbols (0 is no treatment to a blank cell; 1 is treatment to a marked cell). Each cell of Figure 3 represents a specific 0.4-ha area of land on Tumamoc Hill and its environs (Figure 1). Marked cells indicate the lands that the model recommends be treated, given resource-protection priorities and labor constraints. These priorities and constraints are inputs entered by the model user.

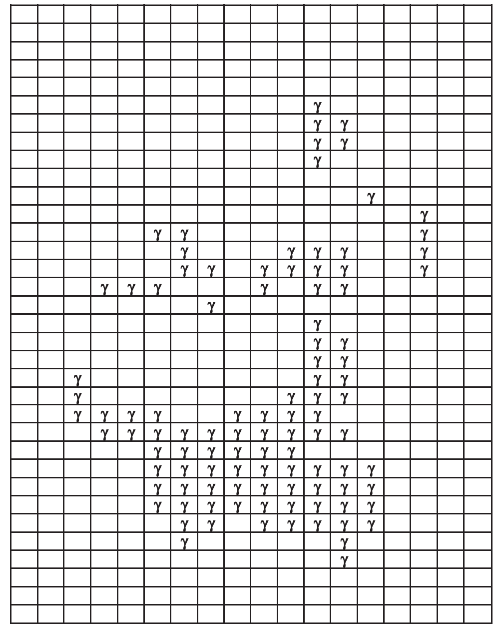

Figure 3. Buffelgrass treatment recommendations. Each cell represents a specific 0.4-ha area of land on Tumamoc Hill. Marked cells indicate the lands the model recommends be treated, given resource-protection priorities and labor constraints entered by the model user.

Results and Discussion

Simulation Results and Sensitivity Analysis. We now compare the performance of the four decision rules for their scope in preventing buffelgrass establishment (achieving local eradication) under binding labor constraints. Our previous analysis of MRLE assumed that labor supplies were unconstrained. Using our 40 by 50–cell grid and initial infestation assumption as in the MRLE problem, we consider three different damage indices: risk to buildings and structures, risk to saguaro cacti, and risk to (ephemeral) riparian vegetation. The four decision rules are applied to maximize damage reduction separately to the three different resources at risk. Given binding labor constraints, we are interested in whether these rules can achieve local eradication.

MRLE of buffelgrass was possible using no more than 1,200 h of labor in any single year with treatment initiated by year 3 in the model (Figure 2). If treatment did not begin until year 5, then nearly 2,000 h were needed in the first year, more than 1,600 h were needed in years 6 and 7, and more than 1,200 h were needed in year 8.

Local eradication is also possible using less labor than under the MRLE rule, although it takes more years to accomplish (Table 2). Under rules 3 and 4, which call for treating infested cells a minimum of 3 consecutive yr, local eradication is possible using no more than 800 labor h yr^{-1} if treatment is initiated in year 1. Under rules 1 and 2, however, buffelgrass is eradicated when the objective is to minimize risk to saguaros but not when it is attempting to reduce the other risk factors. Ironically, because minimizing saguaro risk leads to local eradication, it performs better at reducing risk to buildings or risk to vegetation than rules directly targeting those risks. This is a peculiarity (and problem) of relying on rules of thumb, instead of true, constrained, dynamic optimization.

Table 2. Occurrence of eradication depending on labor constraints, treatment starting year, and decision rule followed.

	Rule 1	Rule 2	Rule 3	Rule 4
	Static optimization	Potential damage weighting	Treat 3×	Treat 3× + potential damage weighting
Labor, 800 h; start, year 1				
Vegetation risk	No	No	Yes	Yes
Building risk	No	No	Yes	Yes
Saguaro risk	Yes	Yes	Yes	Yes
Labor, 1,200 h; start, year 5				
Vegetation risk	No	No	No	No
Building risk	No	No	No	No
Saguaro risk	No	No	No	Yes
Labor, 1,600; start, year 5				
Vegetation risk	No	No	Yes	Yes
Building risk	No	No	Yes	Yes
Saguaro risk	Yes	Yes	Yes	Yes

If treatment is delayed until year 5, and labor is constrained to 1,200 hours, local eradication is possible only by following rule 4 to minimize saguaro risk. If the labor constraint is relaxed to 1,600 hours, then rules 3 and 4 (requiring three treatments) lead to eradication, whereas the other two, again, lead to eradication only when targeting saguaro risk.

Long-Term Damage Reduction. The decision rules may also be compared for their effects on long-run damage paths. Damage indices $DI_{r,t}$ for each resource r (property, saguaros, riparian vegetation) are as follows:

$$DI_{r,t} = \sum_{i \in X} \sum_{j \in Y} D_{r,i,j,t} \qquad [18]$$

which are just the single-year values of the objective function from the full dynamic programming problem, as shown previously in Equation 9. The indices for each resource are scaled so that, absent any treatment, each index approaches 1,000 after 30 yr. We then evaluate the four decision rules for how well they reduce the path of each damage index over time, given varying labor constraints.

We can examine how well the four decision rules reduce damage indices for vegetation (Figure 4) and for saguaros (Figure 5) when treatment begins in year 9 and labor is constrained at 400 h. In both cases, rule 4 (combining the treatment three times with potential damage weighting) reduces the path of the damage index the most. Without treatment, the damage indices approach 1,000 by year 29. Simple, static optimization (rule 1) consistently performs the worst. Even under static optimization, however, the terminal value of the vegetation damage index is 20% lower than under no treatment (Figure 4). Following rule 4, however, the terminal value of the vegetation damage index falls about by 33%. Under rule 1, the terminal value of the saguaro damage index is reduced by 22%, whereas under Rule 4, the index's terminal value falls 40%

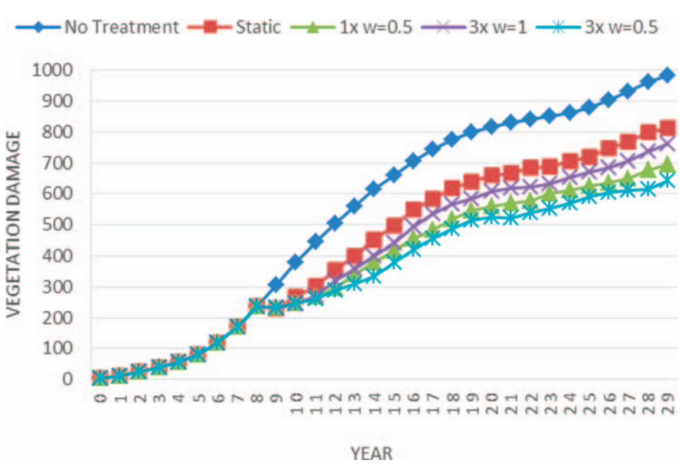

Figure 4. Long-term path of the vegetation damage index (from buffelgrass invasion) without treatment and with alternative treatment rules and a constraint of 400 labor h yr^{-1}. Rule 1—minimizing current damage in each year. Rule 2—weight priority based on current damages and maximum, potential, future damages from infestation of 4,047 m^2 of land; assign 50% weight to current damage reduction and 50% to potential damage reduction: 1 × w = 0.5. Rule 3—minimizing current damage in each year, given that all treated cells must be treated for 3 consecutive yr; giving 100% weight to minimizing current damage: 3 × w = 1. Rule 4—combine rules 2 and 3: minimizing the average of current and potential damage in each year, given that all treated cells must be treated for 3 consecutive yr. All treatments begin in year 9. (Color for this figure is available in the online version of this paper.)

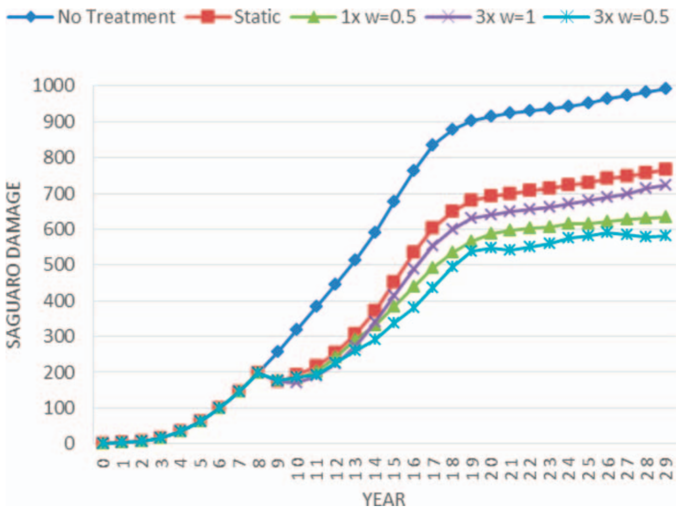

Figure 5. Long-term path of the saguaro damage index (from buffelgrass invasion) without treatment and with alternative treatment rules and a constraint of 400 labor h yr^{-1}. Rule 1—static optimization. Rule 2—uses potential damage weighting to prioritize treatments; $1 \times w = 0.5$. Rule 3—requires treated cells to be treated in 3 consecutive yr; $3 \times w = 1$. Rule 4—combines rules 2 and 3; $3 \times w = 0.5$. (Color for this figure is available in the online version of this paper.)

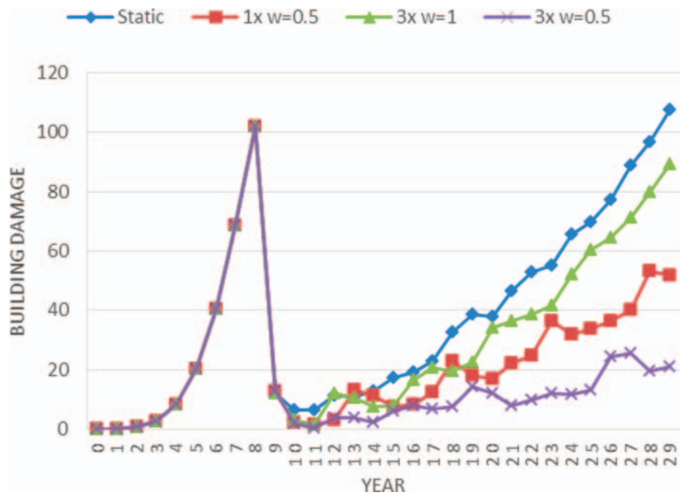

Figure 6. Long-term path of the damage index for building risk from wildfires without treatment and with alternative treatment rules and a constraint of 800 labor h yr^{-1}. Rule 1—static optimization. Rule 2—uses potential damage weighting to prioritize treatments; $1 \times w = 0.5$. Rule 3—requires treated cells to be treated in 3 consecutive yr; $3 \times w = 1$. Rule 4—combines rules 2 and 3; $3 \times w = 0.5$. (Color for this figure is available in the online version of this paper.)

(Figure 5). Treating at least three times (rule 3) or applying potential damage weighting (rule 2) are modest improvements over static optimization. Combining both approaches (rule 4) provides the greatest damage reduction.

Reducing fire risk to buildings is relatively easy because structures are primarily on the periphery of the grid, whereas initial infestations are not close to that periphery (Figure 6). Protecting buildings, then, involves maintaining a "defensible space" in front of properties. Starting treatment in year 9 with 800 h of labor yr^{-1}, each rule reduces the terminal value of damage below 120 on a 1,000-point scale (Figure 6). Again, rule 4 outperforms the others. Under rule 1, however, the buffelgrass populations exceed pretreatment levels by year 27, even with constant use of 800 labor h yr^{-1}.

The ordering of how well each rule performed was consistent across the three resources at risk and at different labor levels (see Büyüktahtakin et al. 2011 for more scenarios varying labor availability). Rule 1 always resulted in the highest damage trajectory, whereas rule 4 always resulted in the lowest.

Figures 7 and 8 illustrate how labor constraints affect damage index trajectories for buildings and saguaros. Trajectories are shown when rule 4 is applied, labor is constrained at constant annual levels, and treatment commences in year 13. For saguaros, treatment stabilizes damages at decreasing levels as more annual labor is applied. The damage trajectories have relatively small slopes after year 20. With 400 h of labor annually, the damage index stabilizes at about 600 (compared with the no-treatment baseline). With 800 labor h, the index stabilizes around 400; with 1,200 h, around 225. It requires about 2,000 h yr^{-1} to drive the damage index to zero and keep it there. For building damage, the damage index is driven close to zero if 800 h or more of labor are applied annually. Although buffelgrass populations near structures are kept at low levels, they are continually

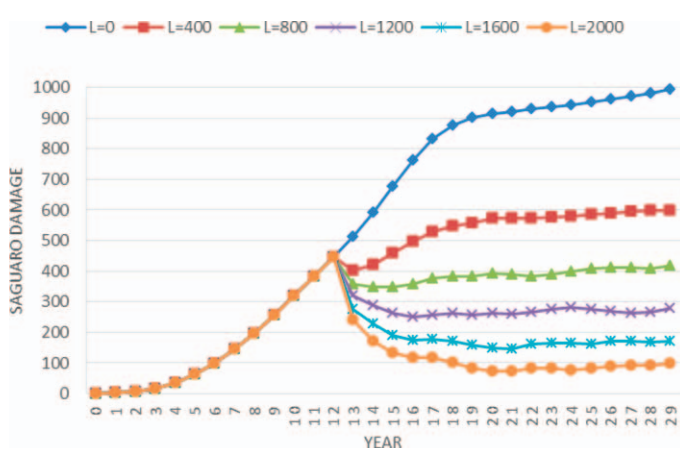

Figure 7. Long-run effect of increasing annual labor hours devoted to buffelgrass control on the saguaro damage index. All treatments begin in year 13. Abbreviation: L, annual labor hours. (Color for this figure is available in the online version of this paper.)

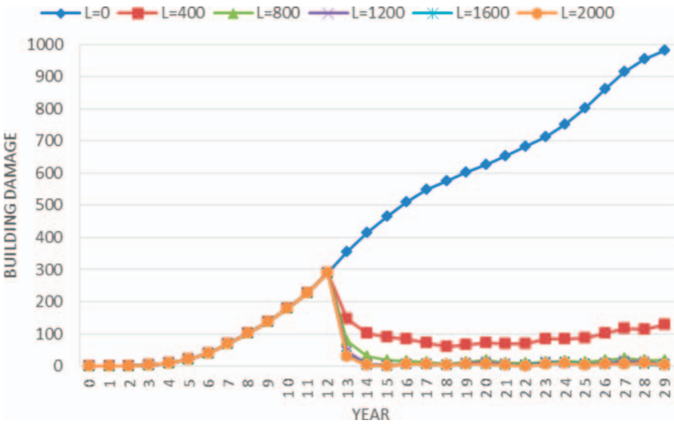

Figure 8. Long-run effect of increasing annual labor hours devoted to buffelgrass control on the damage index for building risk from wildfire. All treatments begin in year 13. Abbreviation: L, annual labor hours. (Color for this figure is available in the online version of this paper.)

reinfested from untreated cells that are farther away. Thus, creating a defensive space requires modest but continuous applications of labor. There is virtually no improvement in building damage reduction beyond 800 labor h (Figure 8).

Resource Protection Priorities and Long-Run Invasive Species Populations. The resources that a land manager chooses to protect can have important effects on the total number and spatial distribution of the invasive species. Reducing the damage indices is not the same as reducing the buffelgrass population. Figure 9 shows buffelgrass population densities of buffelgrass in year 29, assuming treatment commences in year 9, rule 4 is used to control buffelgrass, and 2,000 h yr^{-1} of labor are applied. The objectives are to minimize damage to saguaros (Figure 9a), buildings (Figure 9b), and riparian vegetation (Figure 9c). Recall that buildings border the northern, eastern, and southern edges of the grid. When the objective is to minimize risk to buildings, the terminal population of buffelgrass is cleared from these boundary regions. In contrast, when the objective is to protect saguaros, buffelgrass is allowed to grow along the upper edge of the grid. However, terminal buffelgrass populations are cleared from a patch in the central part of the grid where there is a large stand of saguaros.

Figure 9 also illustrates that the long-run populations of invasive species can be quite sensitive to the choice of weights in a multiattribute damage index. For policy makers, this means that the choice of weights is not an innocuous assumption. The approach described here can be used to develop maps illustrating the consequences of different weighting schemes. The figures also illustrate what could happen if different agents have different priorities in

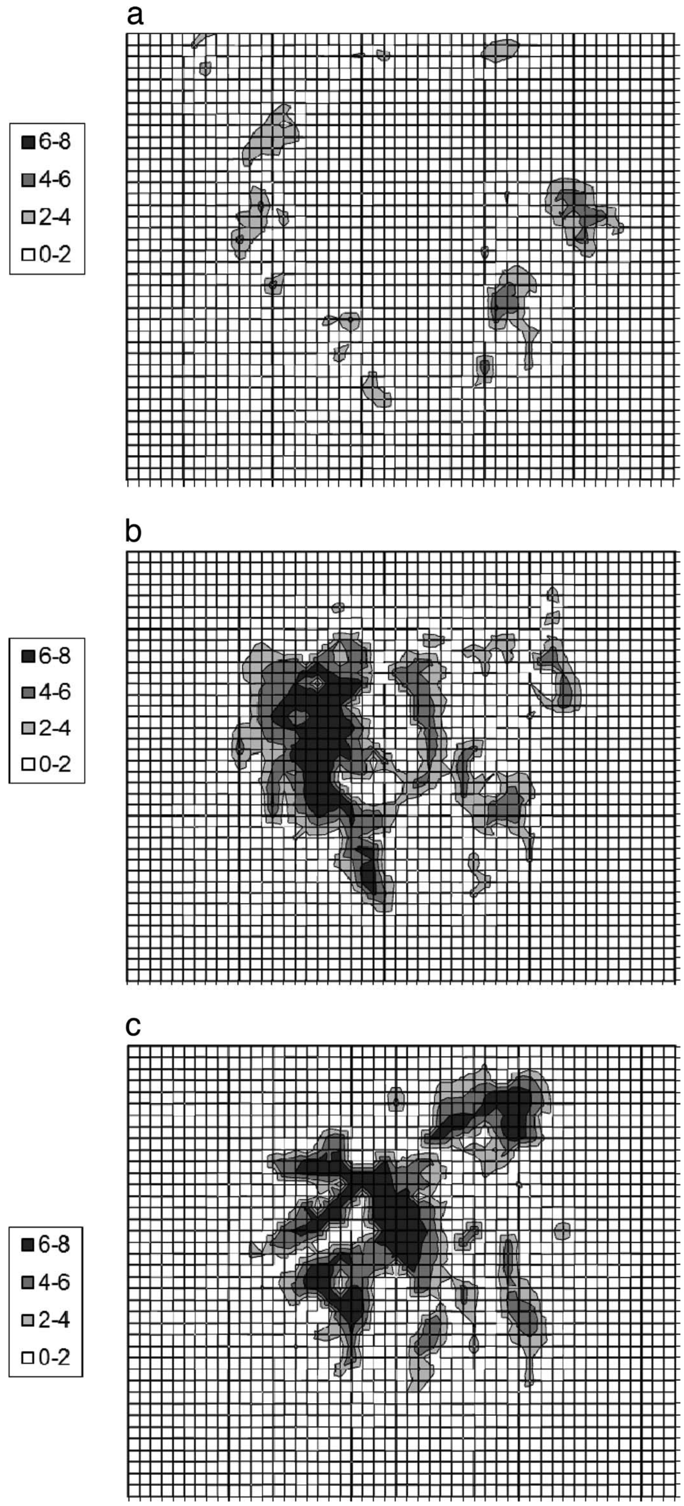

Figure 9. (a) Buffelgrass population density (plants m^{-2}) in year 29; 100% priority weight given to protecting saguaros. (b) Buffelgrass population density (plants m^{-2}) in year 29; 100% priority weight given to reducing wildfire risk to buildings. (c) Buffelgrass population density (plants m^{-2}) in year 29; 100% priority weight given to protecting riparian vegetation.

damage reduction. For example, private homeowners or the city government may care about protecting buildings and structures, whereas federal land agencies may have mandates to protect endangered species. Different land managers may treat lands quite differently. This may pose challenges for coordinating control across jurisdictions.

The model presented here is neither the only nor the first model developed to address buffelgrass control in southern Arizona. Frid et al. (2013) presented a spatial state-and-transition simulation-modeling framework, the Tool for Exploratory Landscape Scenario Analyses (TELSA) to inform allocation of resources between treating buffelgrass and gathering information about new infestations. They integrated data management systems, vulnerability, and risk assessments into their decision-analysis framework, which accounted for habitat suitability, invasion rates, dispersal dynamics, and treatment costs and effectiveness.

The modeling approach presented by Frid et al. (2013) and the one presented here differ in some key respects, emphasizing different aspects of invasive species management. Although differing, the approaches can be seen as complementing each other. For example, the Frid et al. (2013) approach emphasizes trade-offs between managing known populations and searching for newly emerging ones. This trade-off is of keen interest to the Southern Arizona Buffelgrass Coordinating Committee that must struggle with decisions over allocating scarce resources across the two activities. Our model does not address the search for new populations. However, it (1) identifies the resources needed to prevent nascent populations from becoming established, and (2) defines how soon treatment needs to begin once buffelgrass is introduced into a new area to prevent establishment (3 yr or less). The value of finding new outbreaks depends on whether there are sufficient resources to act on that information. Our model incorporates varying damages and control costs over a landscape. Frid et al. (2013) did not consider spatial differences in control costs. Those differences, however, can have important implications for strategy. For example, the finding that treating nascent foci is superior to treating established populations is sensitive to assumptions about the costs of reaching and treating nascent foci in remote areas. Our own findings corroborate those of Frid et al. (2013, p 44), which point out that "a large upfront investment can reduce total management cost substantially over the long term." In our "Results" section, we provide dollar-value estimates of the costs of delaying treatments to prevent buffelgrass establishment in new areas.

Managerial Implications. This article developed a general spatial-dynamic model of invasive weed spread and management and applied it to address questions about management of buffelgrass in southern Arizona. A numerical simulation model was developed and calibrated to match historic buffelgrass spread, treatment effectiveness, and treatment cost data. Although full dynamic optimization of the model proved intractable, we were nevertheless able to solve simplified problems to address relevant policy questions.

First, the Management Plan of the National Invasive Species Council (NISC 2001) calls for "rapid-response teams" to control new invasions before they spread. Our first simulations quantified labor requirements needed for such teams to prevent new buffelgrass establishment. Those simulations also illustrated how requirements increase with delay of program initiation. Results quantified the following: (1) how large the response teams need to be (2) what size of infestation the teams should target, and (3) how many years the follow-up treatments should continue for team efforts to be effective. The approach developed here is readily applicable to rapid response to other invasive species. Furthermore, results show that current resources must be increased because they are currently not sufficient to control the spread of buffelgrass.

Next, we evaluated two control recommendations—potential damage weighting and consecutive-year treatment rules—from the Southern Arizona Buffelgrass Strategic Plan (Rogstad 2008). These recommendations were modeled as heuristic treatment rules and solved as special-case integer programming problems in a spatial-dynamic framework. Applying those rules together increased the scope for preventing buffelgrass establishment under resource constraints. They also reduced buffelgrass damage trajectories substantially, both relative to the no-treatment option and relative to static optimization.

Third, the long-run population size and spatial distribution of buffelgrass are sensitive to priority weights for protecting different resources. Land managers with different priorities may pursue quite different control strategies, which may pose a challenge for coordinating control across jurisdictions. Extensions of this model could consider coordination problems between land managers with different priorities for buffelgrass control. Büyüktahtakin et al. (2013) considered how land managers with different resource-protection priorities might develop cooperative strategies for buffelgrass control. Work by Grimsrud et al. (2008) suggested that such multiagent problems could provide important insights concerning invasive species control.

Optimization Implications. Although the simulation results showed that heuristic rules could be significant improvements over static optimization, static optimization is a lower bound of performance. The key question is this: "How far are these heuristic rules from full, dynamic optimization?" Our ongoing research seeks to answer that important question. A weakness of many invasive species optimal-control models is their failure to provide specific, useful recommendations. If those heuristic rules are good approximations of the dynamic optimum, it means that easy-to-determine treatment strategies can be effective. If, however, those rules are not good approximations of the optimal solution, then that information is also important. The rules could be investigated to determine conditions where they were (or were not) reasonable approximations. This could lead to other rules of thumb that are easy to implement, but closer to optimal.

Conclusions: From Theory to Practice. As illustrated in Figure 3, the model presented here uses an interface with Excel to print out worksheet maps that recommend to land managers which lands should be treated in the current year given inputted information about the labor constraints and resource protection priorities. Neither our approach, however, nor the TELSA-based approach of Frid et al. (2013) is yet user friendly enough to work as a "turnkey" technology, where the researchers simply hand the models off to managers. That could, however, be achieved through an extension activity in which the university keeps engaged with the community.

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