An Age Structured Model of the Impact of Buffelgrass on Saguaro Cacti and their Nurse Trees

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Institute for Strengthening the Understanding of Mathematics and Sciences

Importance of the Saguaro Cactus

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- The saguaro is also important for the **tourism industry** and has **cultural significance** to the Papago and Pima nations.

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www.wildsonora.com/image-content/saguaros-and-nurse-tree

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- A palo verde typically **cannot out-compete** a saguaro.

The Danger of Buffelgrass

www.desertmuseum.org/invaders/invaders_buffelgrass.php

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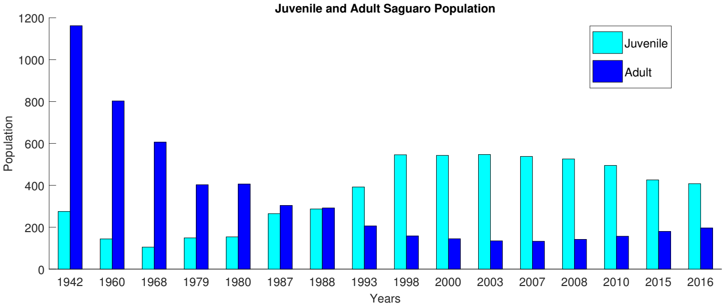
- Buffelgrass is an **invasive species** able to grow in close proximity to other plants.
- Buffelgrass,a **fire-adapted plant**, is able to survive fires and quickly regrow.
- Buffelgrass acts as a **fuel for fires**, quickly burning native plants like saguaros and nurse trees and replacing them within weeks.

The Saguaro Population has Begun to Decline



Data provided by Orum et.al. shows the changes in the distribution of adult and juvenile saguaro populations from 1942 to 2016 [1].

The Question

Under what conditions will buffelgrass propagated wildfires interrupt the natural life cycle between the saguaro cactus and its nurse trees?

www.pamperingcampers.wordpress.com/tag/saguaro

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www.desertmuseum.org/invaders/invaders_buffelgrass.php

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Assumptions of the Model

- Juvenile saguaros are in the age range of 1 to 35, while the natural life span of all adult saguaros is on average 175 years.

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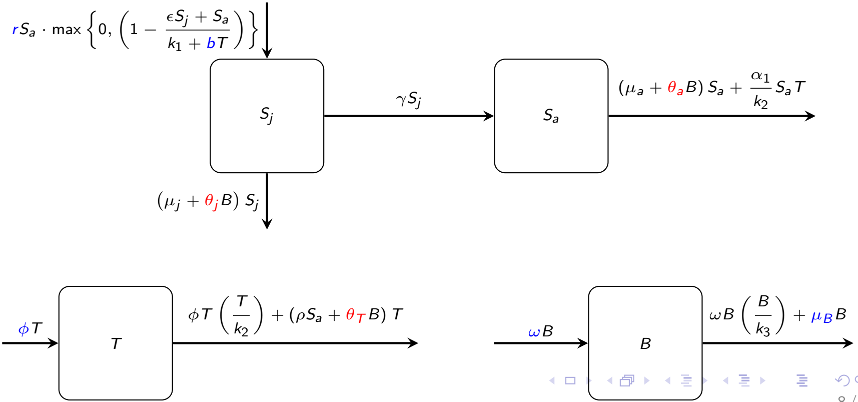
- Nurse trees do not out compete adult saguaros for resources. However, juvenile saguaros compete over space with adult saguaros.
- Nurse trees increase the available space for juvenile saguaros.

Flow Chart Diagram of the Model

Species interactions:

- Commensalism between juvenile saguaros and nurse trees
- Competition between adult saguaros and nurse trees
- Buffelgrass propagated wildfire.



Parameters of the Model

Parameter values were estimated through review of existing literature.

Parameter	Description	Parameter	Description
r	Germination rate	k_1	Adult saguaro carrying capacity
ϵ	Converts juveniles to adults	b	Average juveniles under nurse tree
γ	Maturation rate	μ_j	Juvenile death rate
α_1	Saguaro death rate by competition	k_2	Carrying capacity of adults and trees
μ_a	Adult death rate	ϕ	Growth tree population
ρ	Tree competition death rate	σ	Proportion carrying capacity
θ_j	Grass fire frequency effect-juvenile	θ_a	Grass fire frequency effect-adult
θ_t	Grass fire frequency effect-tree	ω	Grass growth rate
μ_B	grass harvesting	k_3	Carrying capacity of grass

The Model

$$\frac{dS_j}{dt} = rS_a \cdot \max\left\{0, \left(1 - \frac{\epsilon S_j + S_a}{k_1 + bT}\right)\right\} - \gamma S_j - \mu_j S_j - \theta_j B S_j,$$

$$\frac{dS_a}{dt} = \gamma S_j - \frac{\alpha_1}{k_2} S_a T - \mu_a S_a - \theta_a B S_a,$$

$$\frac{dT}{dt} = \phi T \left(1 - \frac{T}{k_2}\right) - \rho S_a T - \theta_T B T,$$

$$\frac{dB}{dt} = \omega B \left(1 - \frac{B}{k_3}\right) - \mu_B B$$

Buffelgrass grows independently of nurse trees and saguaros, so the model can be redimensionalized into a 3 compartment system by redefining 5 parameters, μ_j, μ_a, ϕ, k_2, and α_1, as $\tilde{\mu_j}$, $\tilde{\mu_a}$, $\tilde{\phi}$, k_4, and $\tilde{\alpha_1}$.

Buffelgrass Independence

$$\tilde{\mu}_a = \mu_a + B\theta_a$$

$$\tilde{\phi} = 1 - \frac{\theta_T B}{\phi}$$

$$\tilde{\alpha}_1 = \tilde{\phi}\alpha_1$$

$$\tilde{\mu}_j = \mu_j + B\theta_j$$

$$k_4 = k_2\tilde{\phi}$$

$$\frac{dS_j}{dt} = rS_a \cdot \max\left\{0, \left(1 - \frac{\epsilon S_j + S_a}{k_1 + bT}\right)\right\} - \gamma S_j - \tilde{\mu}_j S_j \qquad (1)$$

$$\frac{dS_a}{dt} = \gamma S_j - \frac{\tilde{\alpha}_1}{k_4} S_a T - \tilde{\mu}_a S_a \qquad (2)$$

$$\frac{dT}{dt} = \tilde{\phi} T \left(1 - \frac{T + \sigma S_a}{k_4}\right) \qquad (3)$$

Demographic Reproductive Numbers

$$R_{di} = \begin{pmatrix}\text{Proportion of juvenile} \\ \text{saguaros that become} \\ \text{adults}\end{pmatrix} \cdot \begin{pmatrix}\text{Average number of juvenile} \\ \text{saguaros produced in an} \\ \text{adult's life}\end{pmatrix}$$

$=$ The average number of adult saguaros produced by one adult saguaro during its lifetime.

	Without Trees	**With Trees**
Without Buffelgrass	$R_{d1} = \dfrac{\gamma}{\gamma + \mu_j} \cdot \dfrac{r}{\mu_a}$	$R_{d2} = \dfrac{\gamma}{\gamma + \mu_j} \cdot \dfrac{r}{\mu_a + \alpha_1}$
With Buffelgrass	$R_{d3} = \dfrac{\gamma}{\gamma + \tilde{\mu_j}} \cdot \dfrac{r}{\tilde{\mu_a}}$	$R_{d4} = \dfrac{\gamma}{\gamma + \tilde{\mu_j}} \cdot \dfrac{r}{\tilde{\mu_a} + \tilde{\alpha_1}}$

Buffelgrass Extinction Equilibria and Stability

For all stability of equilibria without buffelgrass, $\omega < \mu_B$

	Equilibrium	Existence	Stability
$B_1^* = 0$	$E_1 = (0, 0, 0, 0)$	always	unstable
	$E_2 = (0, 0, T_2^*, 0)$	always	$R_{d2} < 1$
	$E_3 = (S_{j3}^*, S_{a3}^*, 0, 0)$	$R_{d1} > 1$	$\rho S_a^* > \phi$
	$E_4 = (S_{j4}^*, S_{a4}^*, T_4^*, 0)$	$\rho S_a^* < \phi$ and $R_{d2} > 1$	Stable(numerical)

Equilibria and Stability with Buffelgrass

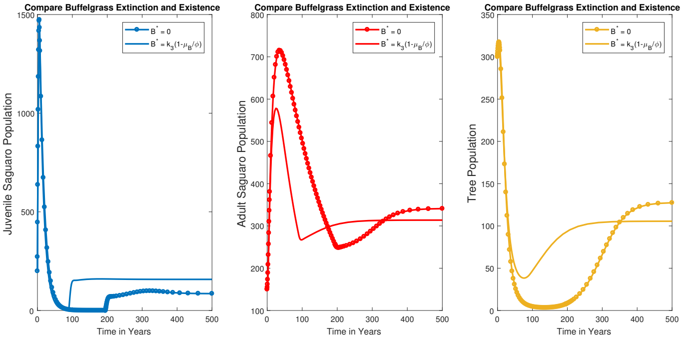
For all stability of equilibria with buffelgrass, $\omega > \mu_B$

	Equilibrium	Existence	Stability
$B_2^* = k_3\left(1 - \frac{\mu_B}{\omega}\right)$	$E_5 = (0, 0, 0, B_2^*)$	always	unstable
	$E_6 = (0, 0, T_2^*, B_2^*)$	always	$R_{d4} < 1$
	$E_7 = (S_{j7}^*, S_{a7}^*, 0, B_2^*)$	$R_{d3} > 1$	$\rho S_a^* > \tilde{\phi}$
	$E_8 = (S_{j8}^*, S_{a8}^*, T_8^*, B_2^*)$	$\rho S_a^* < \tilde{\phi}$ and $R_{d4} > 1$	Stable (numerical)

When is Coexistence Possible?

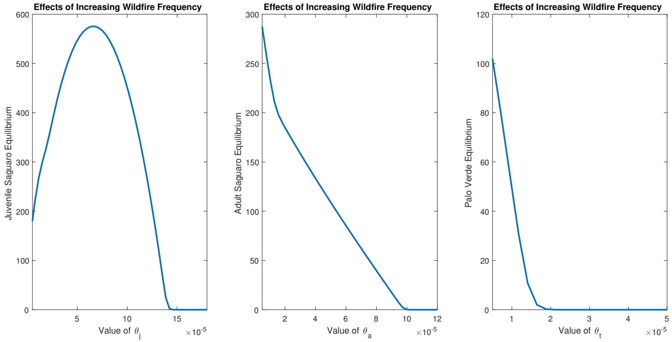
Theorem: If $\dfrac{\tilde{\phi}}{\rho} > \dfrac{k_1}{1+\tilde{E}}\left(1 - \dfrac{1}{R_{d3}}\right)$ and $R_{d4} > 1$ then there exists at least one coexistence equilibrium.

Simulations with Baseline Parameters



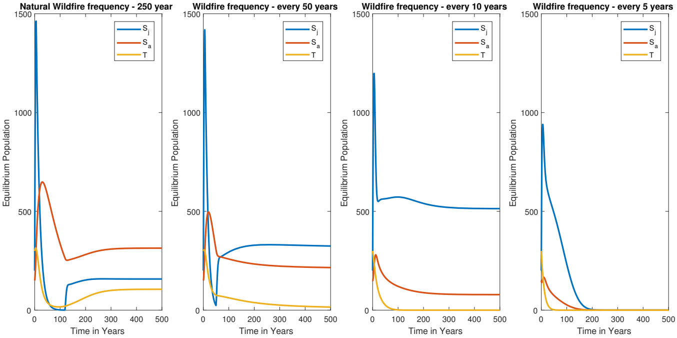
Comparing Dynamics at $B^* = 0$ and $B^* = k_3\left(1 - \dfrac{\mu_B}{\omega}\right)$ equilibria, it can be seen that at $B^* = 0$ equilibrium population values of adult saguaros and trees are increased.

Effects of Increased Wildfire Frequency on Equilibrium Populations



All equilibrium populations will eventually approach extinction as the frequency of wildfires is increased.

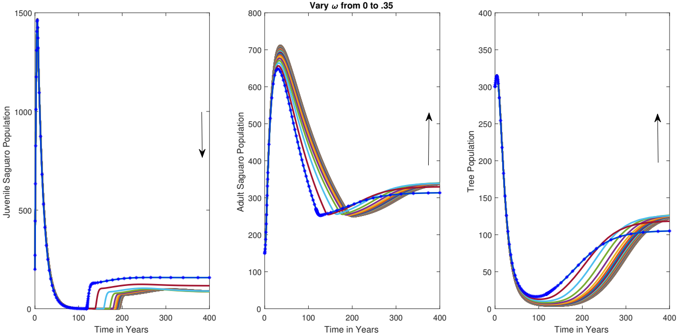
Effects of Increased Wildfire Frequency over Time



$\frac{1}{250}$ - Baseline $\frac{1}{50}$ - Real-World Dynamics $\frac{1}{10}$ - Tree Extinction $\frac{1}{5}$ - Saguaro Extinction

An increase in wildfire frequency can cause the extinction of the saguaros and nurse trees.

Effects of Reducing Buffelgrass by Chemical Spraying



The use of herbicide on buffelgrass will increase the adult saguaro and nurse trees population, but it will decrease the juvenile population.

Local Sensitivity Analysis Without Buffelgrass

Sensitivity Analysis	r	b	ϕ
S_j	0.0013	0.1006	0.6534
S_a	0.0013	0.1006	0.6534
T	-0.0084	-0.6534	2.2520

Table: Sensitivity index of the population values of S_j, S_a, and T at equilibrium when changing the parameter values by 1% from their baseline values.

Local Sensitivity Analysis with Buffelgrass

Sensitivity Analysis	θ_j	θ_a	θ_T	μ_B	ω	ϕ
S_j	-3.92×10^{-4}	0.4916	-0.0753	-8.3189	8.3232	0.7213
S_a	-3.92×10^{-4}	-0.0111	-0.0753	1.7351	-1.7354	0.7213
T	0.0028	0.0803	-0.2980	4.2973	-4.2973	2.8558
B	N/A	N/A	N/A	-20	20	N/A

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Conclusions

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  1. Since the sensitivities of the equilibrium populations to these parameters are about the same, which is the more efficient strategy depends on cost.

Future Work

Future work for this model should be the inclusion of:

- An optimal control analysis for the most efficient way of reducing the buffelgrass population.
- Adding stochastic wildfires instead of averaging death due to wildfire over time.
- Add biological relationships of buffelgrass with native plants

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Questions?

Selected Sources

M. J. Orum TV, Ferguson N, “”Data from: Saguaro mortality and population regeneration in the cactus forest of Saguaro National Park: seventy-five years and counting”,” 2016.

- D. Swann. Personal communication. July 5th 2017.

More Information on Sensitivity Analysis

A local sensitivity analysis was performed by finding the percent change in the quantity on interest due to a one percent change in the parameter of interest.

$$S_p^q := \frac{\hat{p}}{\hat{q}} \times \frac{\partial q}{\partial p}\bigg|_{p=\hat{p}} = \frac{\theta_q}{\theta_p}$$

The terms $\hat{p}$ and $\hat{q}$ represent the baseline values of the parameter of interest and quantity of interest, respectively. The θ's give the changes in the parameter values and resulting changes in quantity values.