Delay Differential Analysis of Time Series Data: Rössler Time Series and Mouse Schizophrenia Model EEG

Erin Brown

15 July 2014

Delay Differential Analysis (DDA)

Dynamical classification of time series

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  - Jonathan Weyhenmeyer
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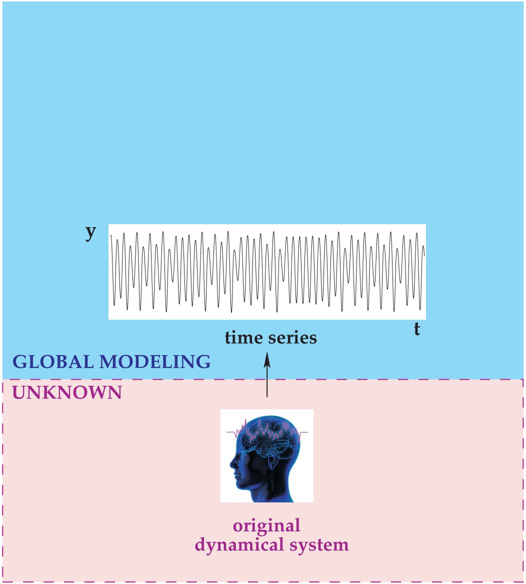
Dynamical System Analysis

Dynamical System Analysis

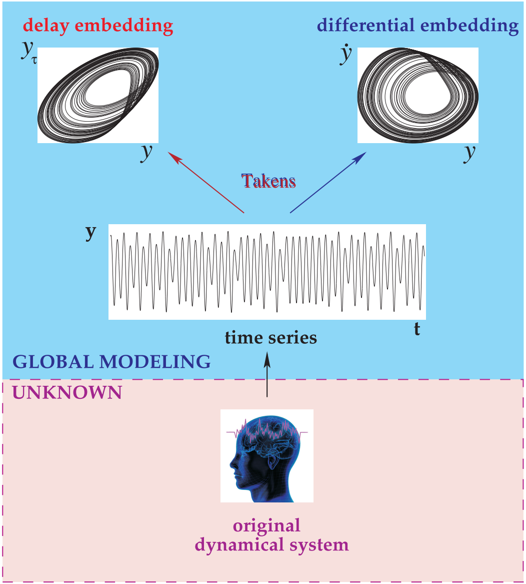
UNKNOWN

original dynamical system

Dynamical System Analysis



Dynamical System Analysis



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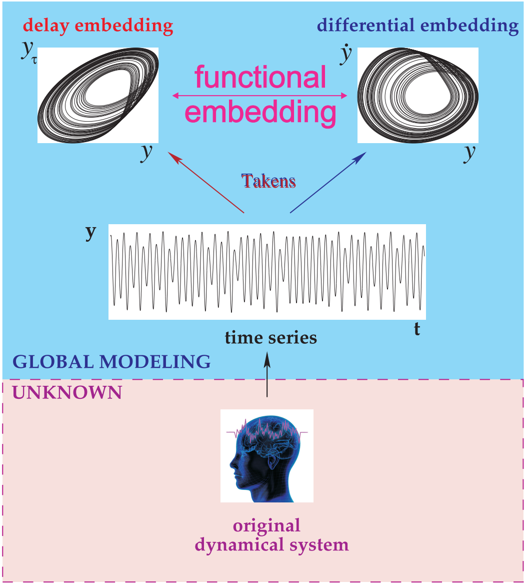
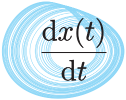
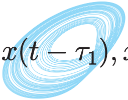


Delay differential analysis is done in the time domain,

not in the spectral domain!





$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(a, x(t-\tau_1), x(t-\tau_2), \ldots)$$

linear terms: dominant time scales (frequencies)

non-linear terms: frequency/phase couplings, feedback

DDE has

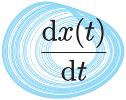
- n delays: $\tau_1, \tau_2, \ldots \tau_n$
- l terms with coefficients $a_1, a_2, \ldots, a_l$
- degree m of nonlinearity

$\Rightarrow$ coefficients a_k and model error ρ as features to identify **dynamical** differences in data

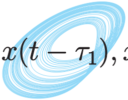
Why DDA?

- DDA is reliable even for short time series
- computations are fast
- DDA is noise insensitive
- no pre-processing of data
- small feature space compared to traditional techniques
- 3-4 features: no overfitting

differential embedding



delay embedding



$$\frac{dx(t)}{dt} = f(a, x(t-\tau_1), x(t-\tau_2), \dots)$$

$$\begin{aligned} \dot{x} \quad = \quad & a_1 x_{\tau_1} + a_2 x_{\tau_2} + a_3 x_{\tau_1}^2 + a_4 x_{\tau_1} x_{\tau_2} \\ & + a_5 x_{\tau_2}^2 + a_6 x_{\tau_1}^3 + a_7 x_{\tau_1}^2 x_{\tau_2} + a_8 x_{\tau_1} x_{\tau_2}^2 \\ & + a_9 x_{\tau_2}^3 + a_{10} x_{\tau_1}^4 + a_{11} x_{\tau_1}^3 x_{\tau_2} \\ & + a_{12} x_{\tau_1}^2 x_{\tau_2}^2 + a_{13} x_{\tau_1} x_{\tau_2}^3 + a_{14} x_{\tau_2}^4 \end{aligned}$$

$$x_{\tau_j} = x(t - \tau_j)$$

Structure (Model) Selection

Select **one** fixed model with a fixed set of delays
(e.g. $\dot{x} = a_1 x_{\tau_1} + a_2 x_{\tau_2}^2$)

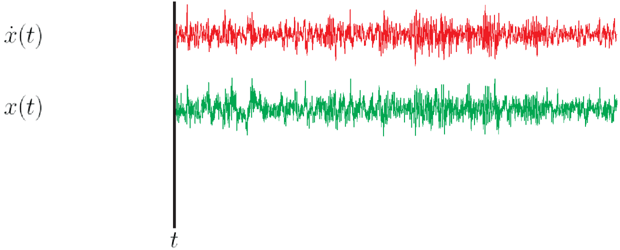
supervised or unsupervised

classification: best separation between classes of data

lowest error DDE model from data — dynamical deviation from that stage

Delay Differential Equation

$$\dot{x}(t) = a_1\, x(t - \tau_2) + a_2\, x(t - \tau_1)^2$$



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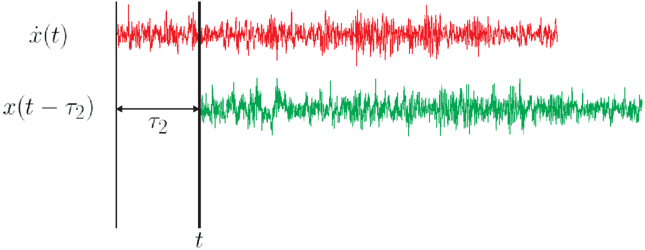

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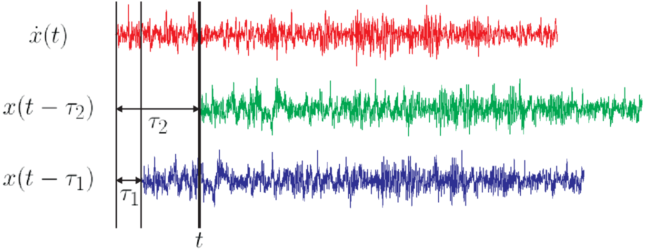

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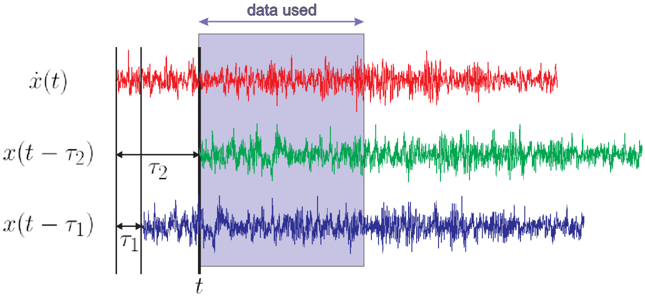

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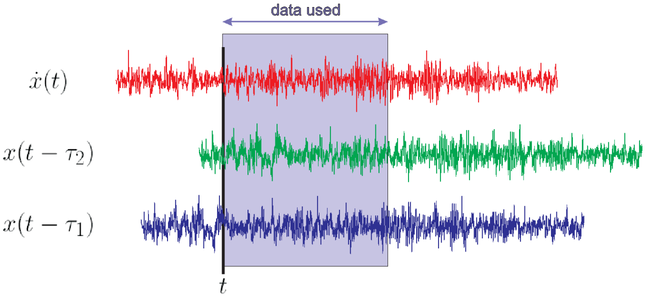

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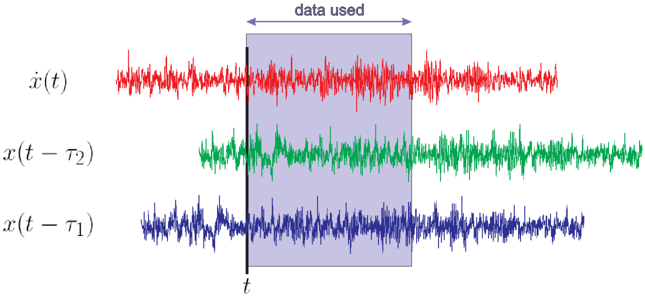

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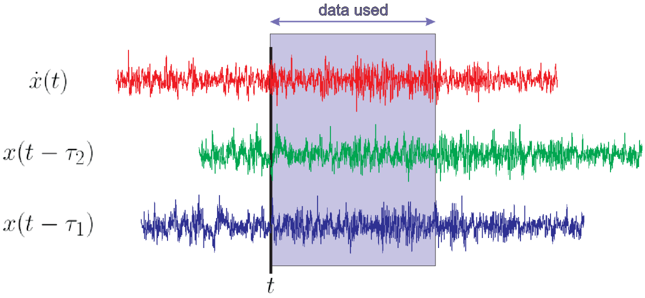

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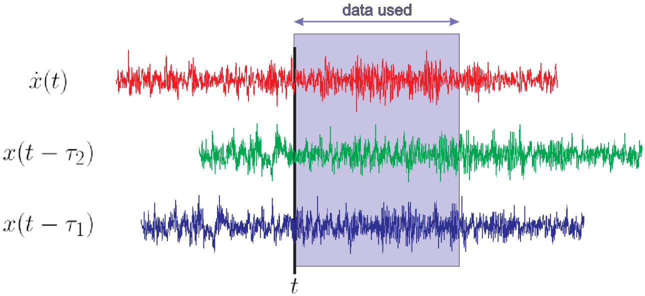
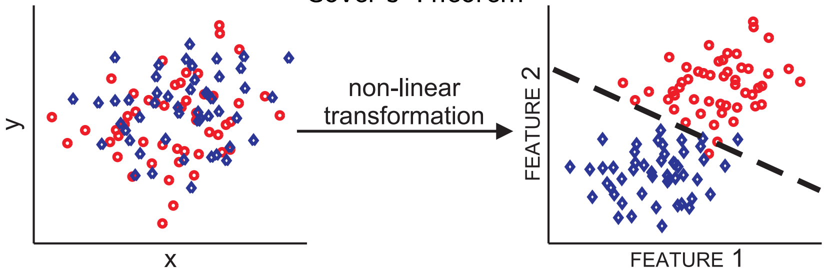
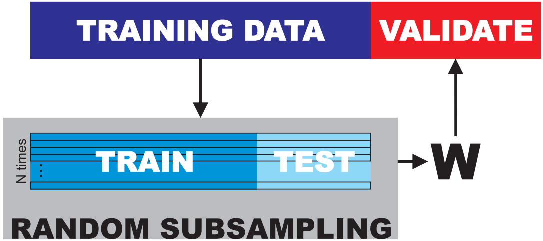


Non-Linear Separation

Cover's Theorem



Repeated Random Subsampling



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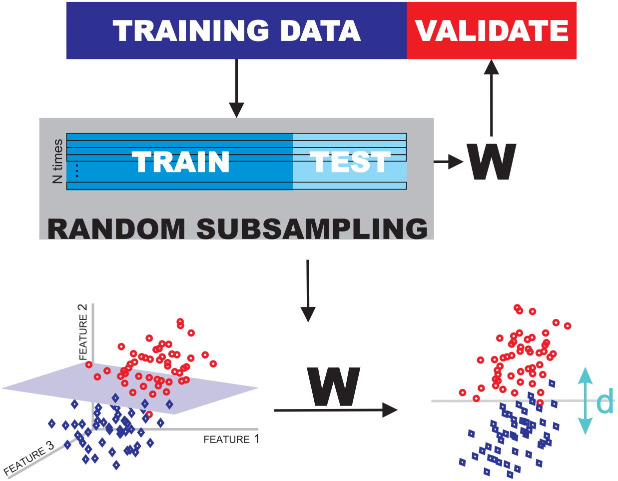

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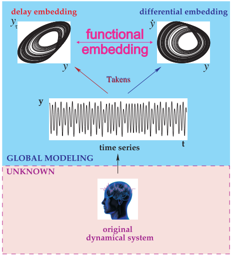
Rössler System

$$
\begin{aligned}
\dot{x} &= -y - z \\
\dot{y} &= x + a\,y \\
\dot{z} &= \mathbf{b} - c\,z + x\,z
\end{aligned}
$$

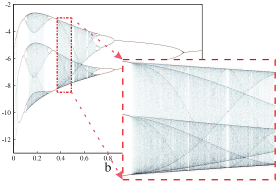
bifurcation parameter **b**

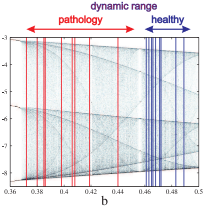
$\downarrow$

dynamical parameter

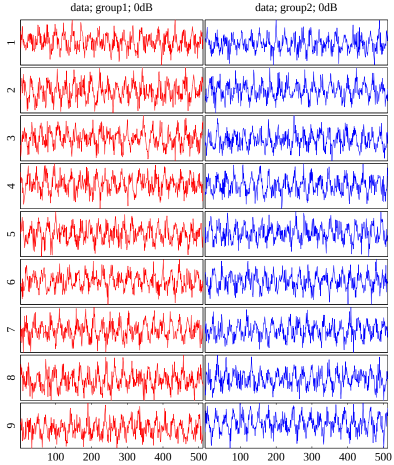


Delay Differential Analysis of Rössler System

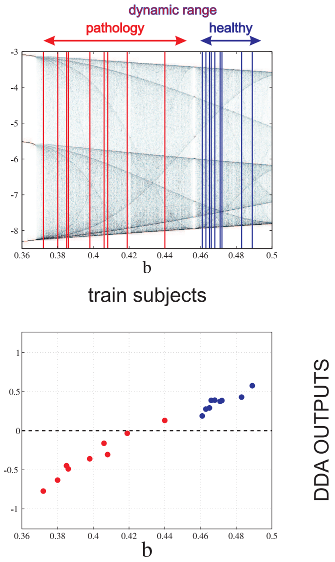


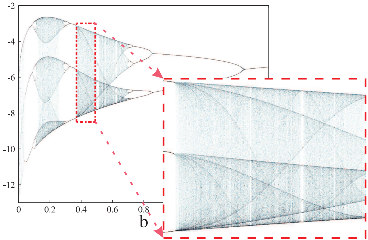


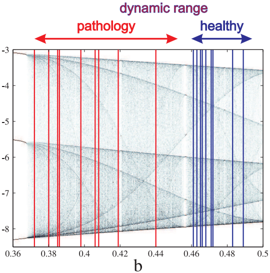
train subjects



Rössler results







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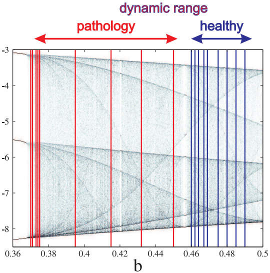
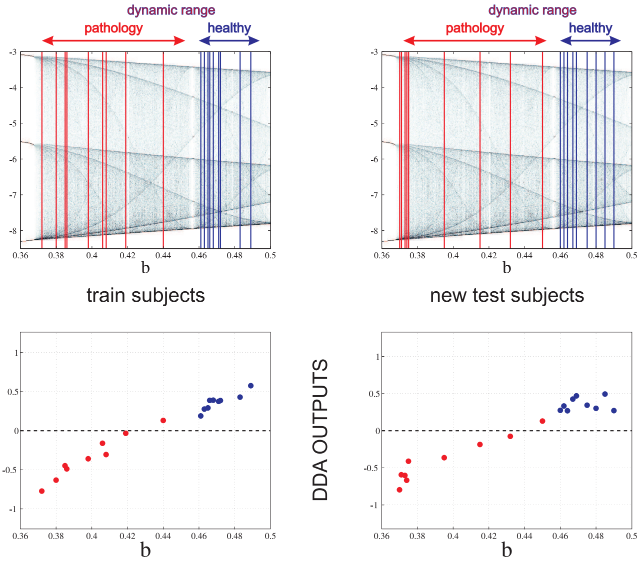


new test subjects

Rössler results - additional subjects



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Mouse Schizophrenia EEG Data

Two classes - FF and KO

FF: controls

KO: genetically modified to display schizophrenia type behaviors

Question: Are there dynamical differences between the two classes?

Mouse EEG Data

- Sampling rate: 1017 Hz
- Time series length: 1s
- Number of trials: 400
- Channels: 5 used

Subjects

Channel	FF Subjects	KO Subjects
1	32	33
2	38	33
3	33	28
4	–	–
5	30	31
6	34	32

Methods

1. Get features–3 coefficients and the error–for each channel, subject, model, delay pair combination
   - Use 3 term models with up to quadratic nonlinearity and two delays
2. Rewrite outputs into matrices containing features for all subjects
3. Perform cross validation
4. Select best performing model and delay pair combination for each channel
   - Use area under the ROC curve to quantify performance
5. Test classification using associated weight matrix
6. Combine results for each channel for optimal classification performance

Results

CH	Model	Tau 1	Tau 2	A' Ind. Trials	A' Meaned Trials
1	123	20	7	0.6687	0.7860
2	123	9	17	0.6467	0.7753
3	123	1	22	0.6575	0.7853
5	123	7	19	0.6947	0.8067
6	123	9	16	0.6567	0.8066
ALL	123	–	–	–	0.9149
ALL	ALL	–	–	–	**0.9226**

Conclusion

- Dynamical classification of time series
  - Toy data - Rössler system
    - Good performance for high noise (0 dB)
    - Linear dependence of outputs with dynamical bifurcation parameter
    - DDA is a successful dynamical classifier
  - Mouse Schizophrenia EEG data
    - Good classification between FF and KO subjects
    - Dynamical difference between FF and KO classes