ANSATZ LIBRARY FOR STRUCTURE SELECTION IN GLOBAL MODELING OF SCALAR EXPERIMENTAL TIME SERIES:

Application To Electrocardiogram Data

Erin Brown

Salk Institute Computational Neurobiology Laboratory

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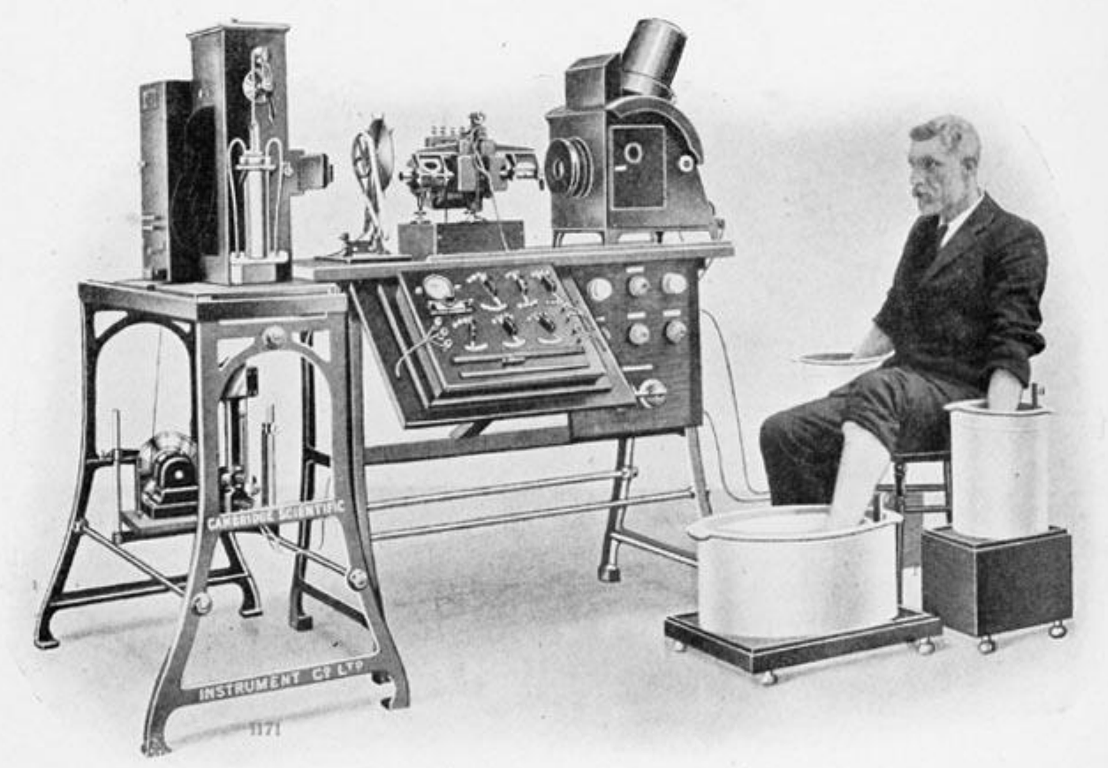
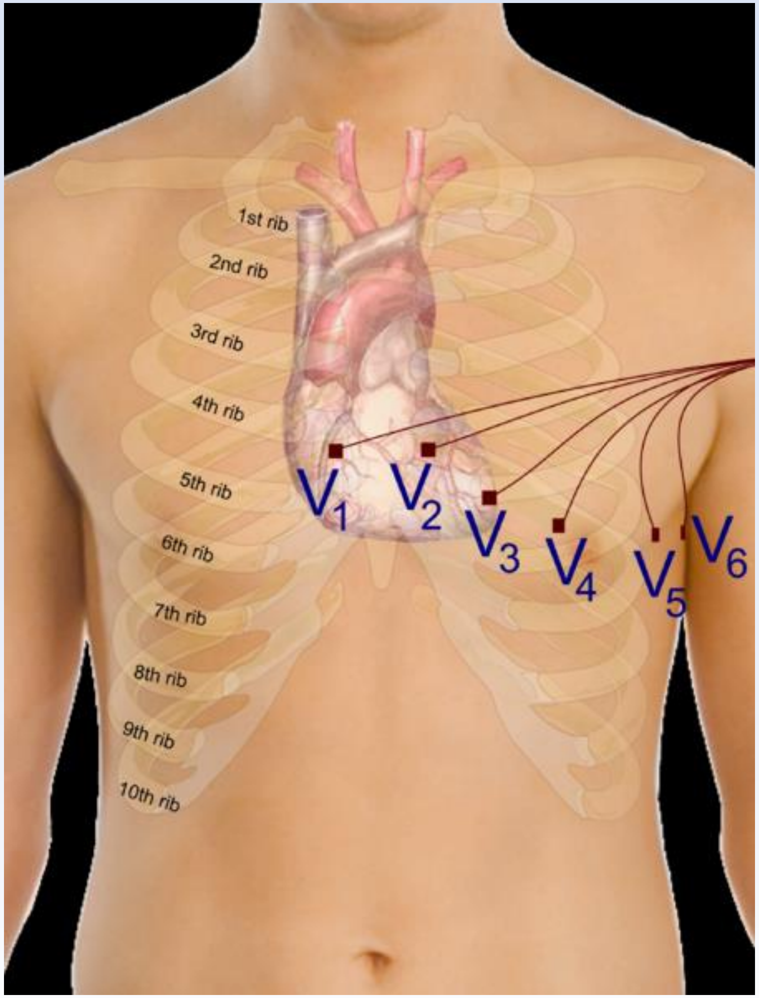
WILLEM EINTHOVEN



Photograph of a Complete Electrocardiograph, Showing the Manner in which the Electrodes are Attached to the Patient, In this Case the Hands and One Foot Being Immersed in Jars of Salt Solution

EKGs TODAY


1st rib
2nd rib
3rd rib
4th rib
5th rib
6th rib
7th rib
8th rib
9th rib
10th rib
V_1
V_2
V_3
V_4
V_5
V_6

PROJECT OVERVIEW

***Goal**:*

Reconstruct dynamical system from experimental scalar time series using *Ansatz Library*

- Develop set of ordinary differential equations that describe the underlying dynamics of electrocardiogram data

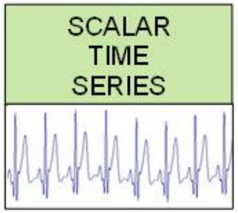
Implications:

Medical

- Improve understanding of underlying dynamics of heart
- Faster, more accurate diagnostic method for heart conditions

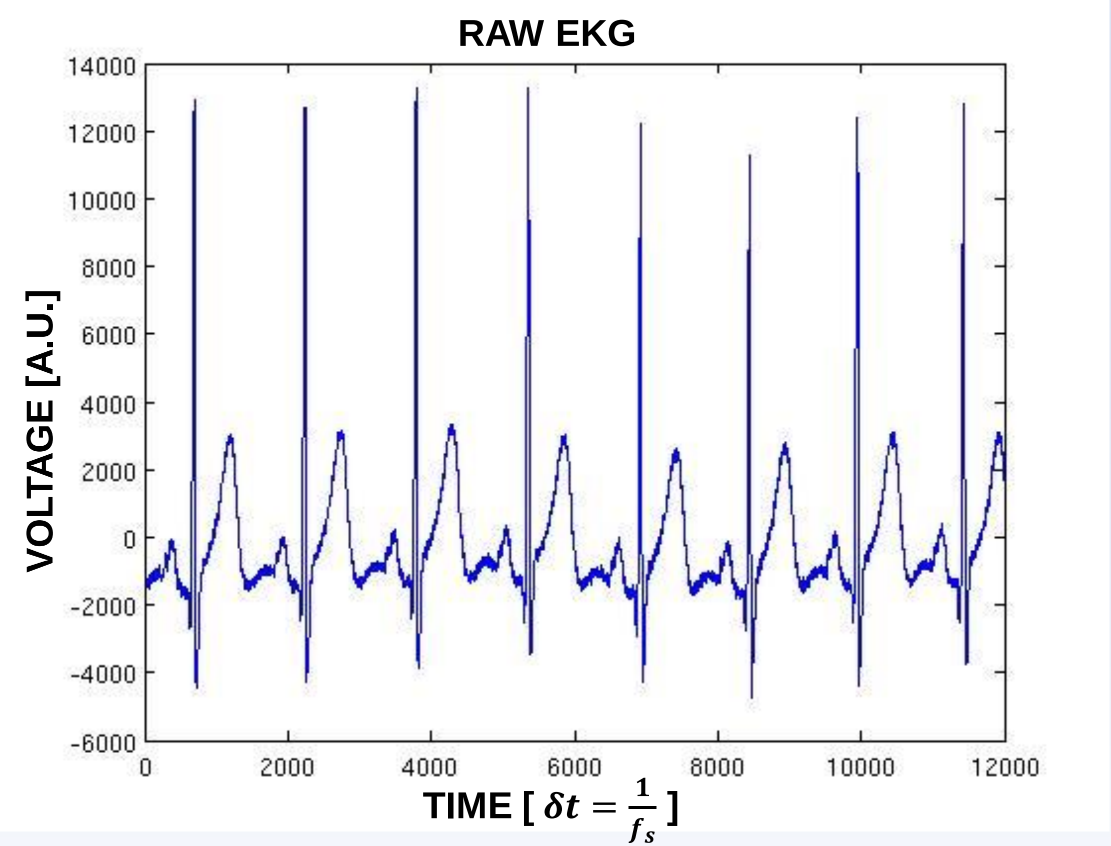
TIME DOMAIN TIME SERIES ANALYSIS

- Start with scalar time series of observed variable



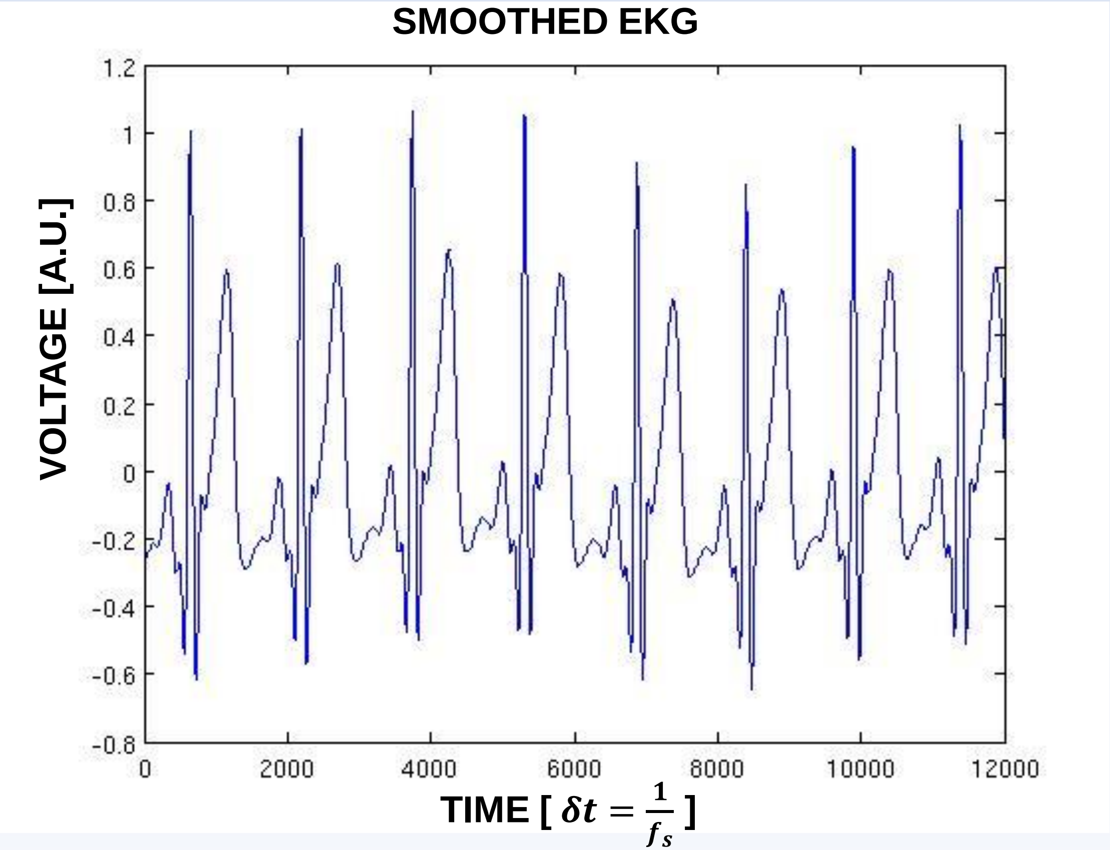
ORIGINAL TIME SERIES

- One healthy subject
- 20 minutes
- Sampled at 50 kHz
- One electrode



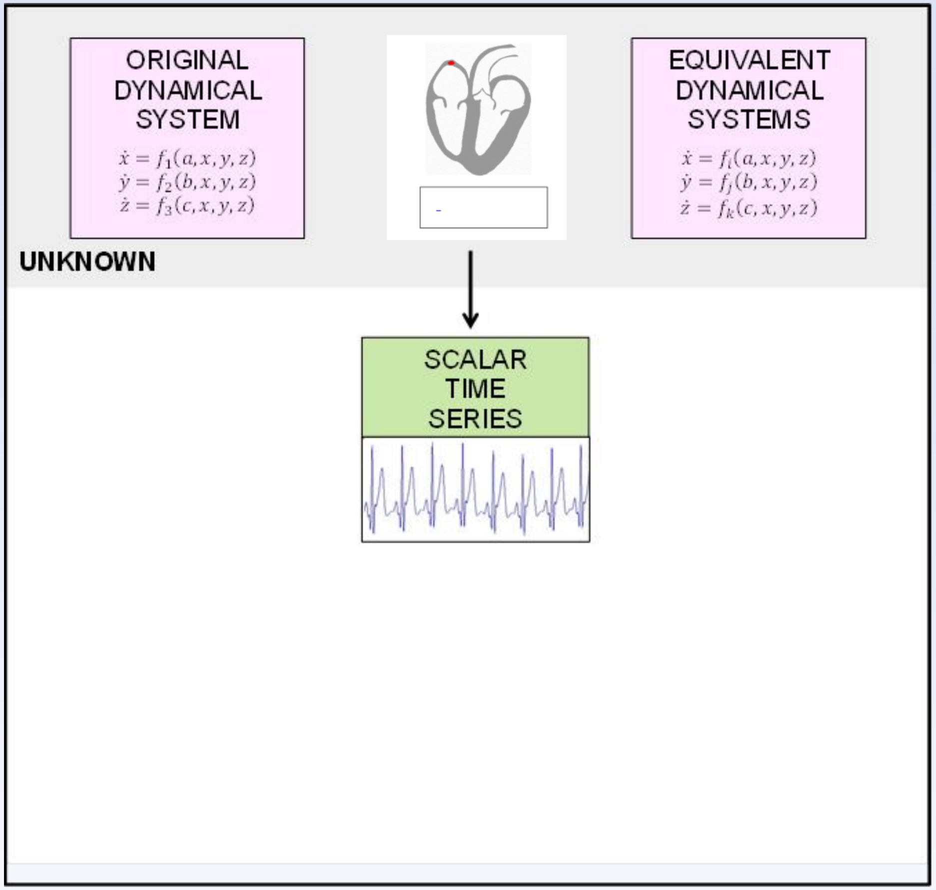
FILTERING AND NORMALIZATION

- Downsampled to 2.5 kHz
- 4^{th} order Butterworth filter; normalized cutoff frequency of 0.016
- Normalized



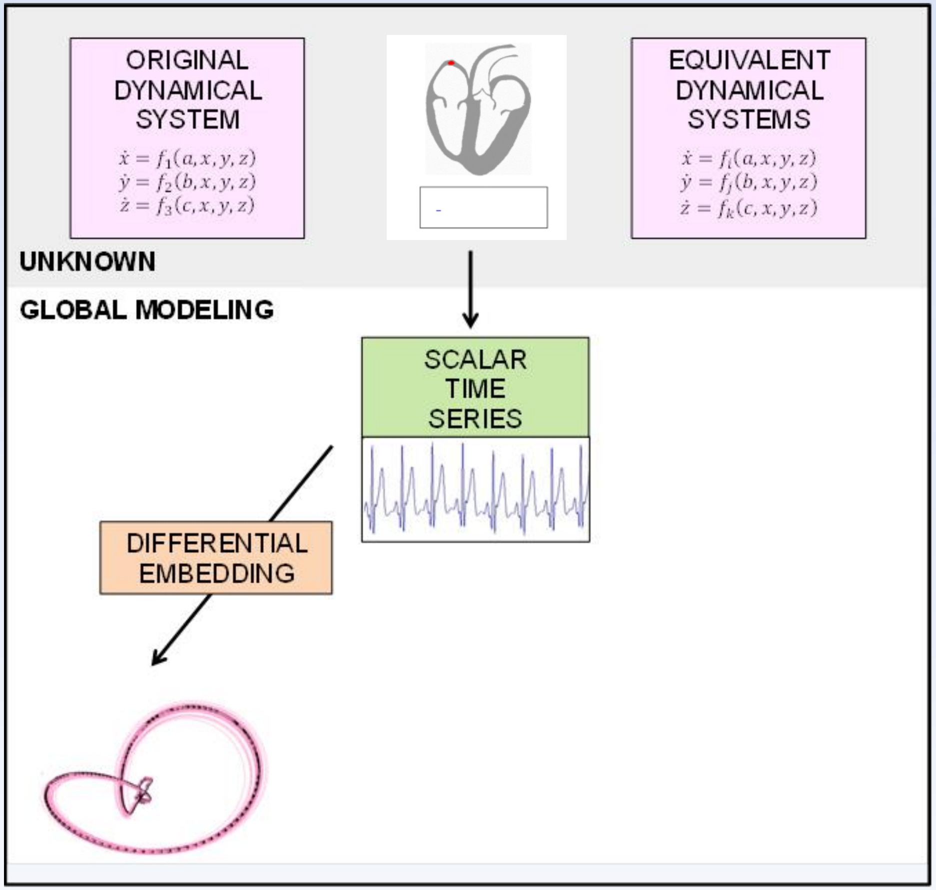
TIME DOMAIN TIME SERIES ANALYSIS

- Start with scalar time series of observed variable
- What can we say about underlying dynamics?



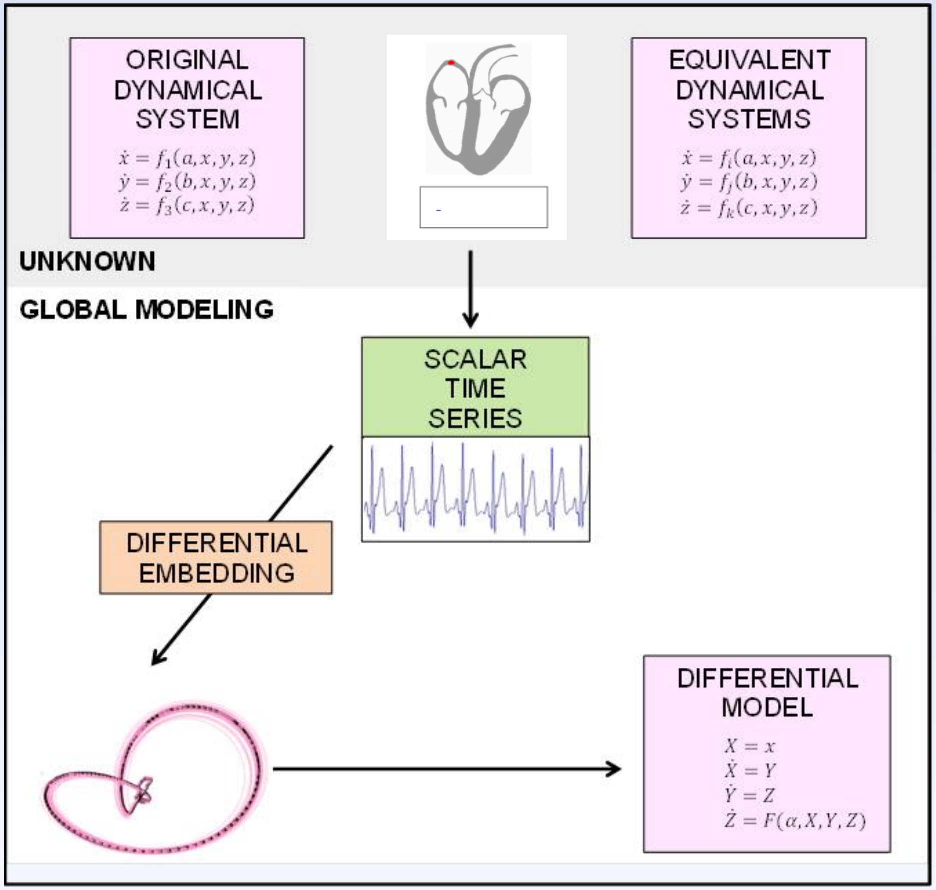
TIME DOMAIN TIME SERIES ANALYSIS

- Start with scalar time series of observed variable
- What can we say about underlying dynamics?
- Differential embedding



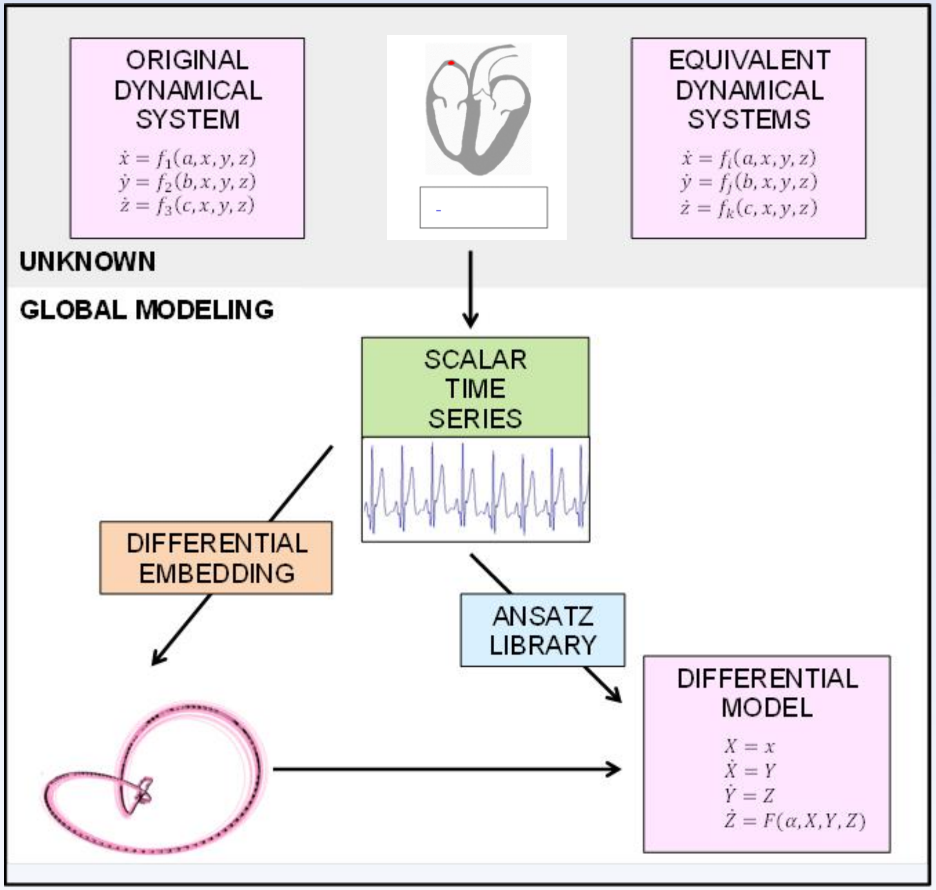
TIME DOMAIN TIME SERIES ANALYSIS

- Start with scalar time series of observed variable
- What can we say about underlying dynamics?
- Differential embedding
- Functional form of differential embedding



TIME DOMAIN TIME SERIES ANALYSIS

- Start with scalar time series of observed variable
- What can we say about underlying dynamics?
- Differential embedding
- Functional form of differential embedding
- Ansatz library for structure selection of differential model



ANSATZ LIBRARY

Set of all analytically derivable **maps** φ_i between sets of **ordinary differential equations** of polynomial form and **differential models** expressed in terms of Lie derivatives

Set of Polynomial ODEs

$$\dot{x} = f_1(a, x, y, z)$$
$$\dot{y} = f_2(b, x, y, z)$$
$$\dot{z} = f_3(c, x, y, z)$$

$$\xleftrightarrow{\varphi_i}$$

Differential Models

$$X = x$$
$$\dot{X} = Y$$
$$\dot{Y} = Z$$
$$\dot{Z} = F(\alpha, X, Y, Z)$$

POSSIBLE DIFFERENTIAL MODEL TERMS

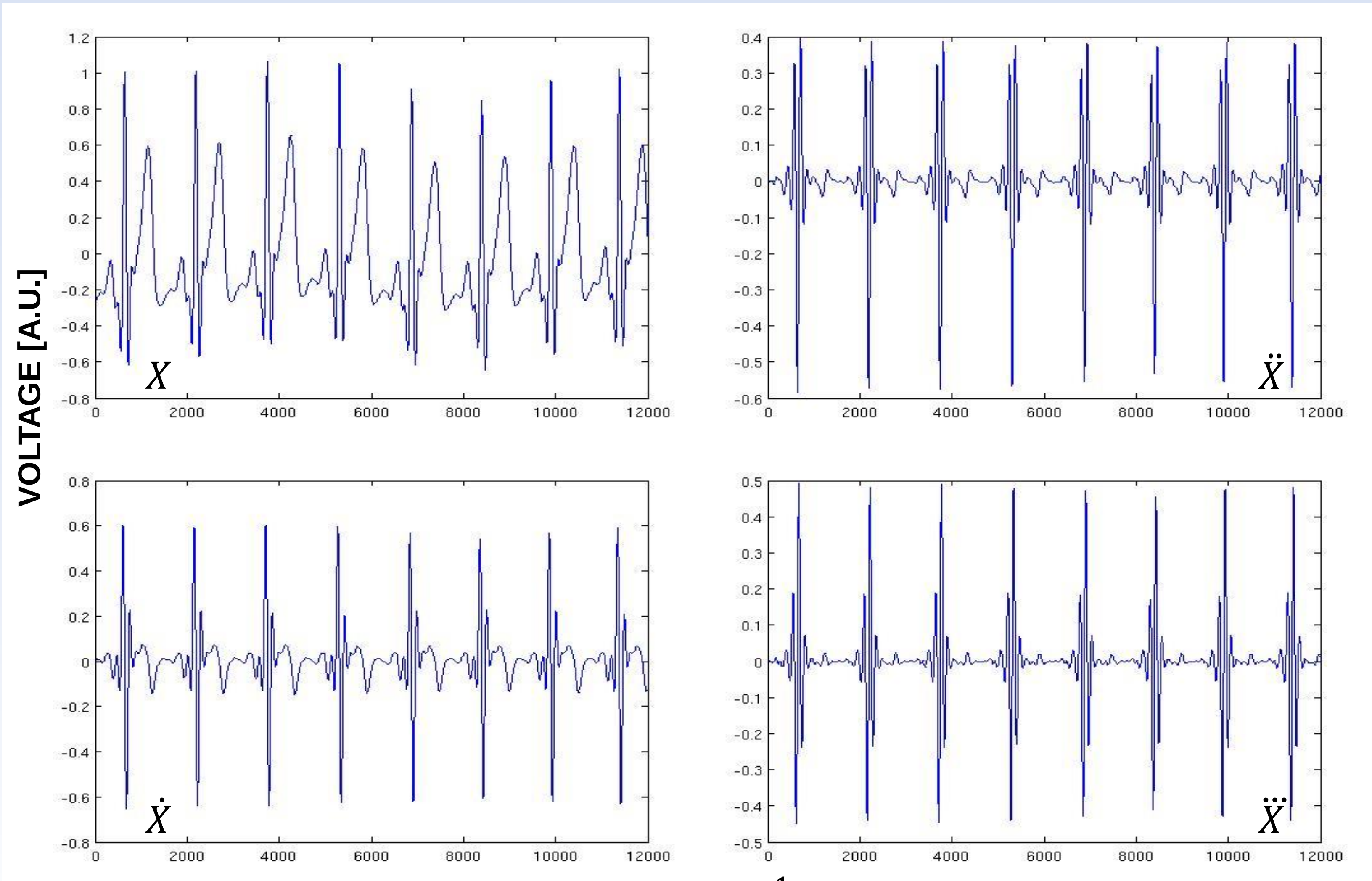
$$
\begin{aligned}
\dot{X} &= Y \\
\dot{Y} &= Z \\
\dot{Z} &= \alpha_1 + \alpha_2 \frac{1}{X^4} + \alpha_3 \frac{1}{X^3} + \alpha_4 \frac{1}{X^2} + \alpha_5 \frac{1}{X} + \alpha_6 X + \alpha_7 X^2 + \\
& \alpha_8 X^3 + \alpha_9 X^4 + \alpha_{10} X^5 + \alpha_{11} X^6 + \alpha_{12} X^7 + \alpha_{13} X^8 + \alpha_{14} \frac{1}{Y} + \\
& \alpha_{15} \frac{X}{Y} + \alpha_{16} \frac{X^2}{Y} + \alpha_{17} \frac{X^3}{Y} + \alpha_{18} \frac{X^4}{Y} + \alpha_{19} \frac{X^5}{Y} + \alpha_{20} \frac{X^6}{Y} + \alpha_{21} Y + \\
& \alpha_{22} \frac{Y}{X^4} + \alpha_{23} \frac{Y}{X^3} + \alpha_{24} \frac{Y}{X^2} + \alpha_{25} \frac{Y}{X} + \alpha_{26} X Y + \alpha_{27} X^2 Y + \alpha_{28} X^3 Y + \\
& \alpha_{29} X^4 Y + \alpha_{30} X^5 Y + \alpha_{31} X^6 Y + \alpha_{32} Y^2 + \alpha_{33} \frac{Y^2}{X^4} + \alpha_{34} \frac{Y^2}{X^3} + \\
& \alpha_{35} \frac{Y^2}{X^2} + \alpha_{36} \frac{Y^2}{X} + \alpha_{37} X Y^2 + \alpha_{38} X^2 Y^2 + \alpha_{39} X^3 Y^2 + \alpha_{40} X^4 Y^2 + \alpha_{41} Y^3 + \\
& \alpha_{42} \frac{Y^3}{X^4} + \alpha_{43} \frac{Y^3}{X^3} + \alpha_{44} \frac{Y^3}{X^2} + \alpha_{45} \frac{Y^3}{X} + \alpha_{46} X Y^3 + \alpha_{47} X^2 Y^3 + \alpha_{48} Y^4 + \\
& \alpha_{49} \frac{Y^4}{X^4} + \alpha_{50} \frac{Y^4}{X^3} + \alpha_{51} \frac{Y^4}{X} + \alpha_{52} Z + \alpha_{53} \frac{Z}{X^3} + \alpha_{54} \frac{Z}{X^2} + \alpha_{55} \frac{Z}{X} + \\
& \alpha_{56} X Z + \alpha_{57} X^2 Z + \alpha_{58} X^3 Z + \alpha_{59} X^4 Z + \alpha_{60} \frac{Z}{Y} + \alpha_{61} \frac{X Z}{Y} + \alpha_{62} \frac{X^2 Z}{Y} + \\
& \alpha_{63} \frac{X^3 Z}{Y} + \alpha_{64} Y Z + \alpha_{65} \frac{Y Z}{X^3} + \alpha_{66} \frac{Y Z}{X^2} + \alpha_{67} \frac{Y Z}{X} + \alpha_{68} X Y Z + \alpha_{69} X^2 Y Z + \\
& \alpha_{70} Y^2 Z + \alpha_{71} \frac{Y^2 Z}{X^3} + \alpha_{72} \frac{Y^2 Z}{X^2} + \alpha_{73} \frac{Y^2 Z}{X} + \alpha_{74} Z^2 + \\
& \alpha_{75} \frac{Z^2}{X^2} + \alpha_{76} \frac{Z^2}{X} + \alpha_{77} \frac{Z^2}{Y}
\end{aligned}
$$

C. Lainscsek, C. Letellier, I. Gorodnitsky, Phys. Lett. A 314 (2003) 409.

DIFFERENTIAL MODEL SELECTION

1. Compute derivatives using Taylor expansion
2. Determine value of α coefficients for 77 term model from data over many windows and look for coefficients that are stable across windows
3. Keep terms with highly significant coefficients
4. Look for lowest term differential model in Ansatz library containing chosen coefficients
5. Calculate value of α coefficients for specific model chosen

TIME SERIES DERIVATIVES



TIME [$\delta t = \frac{1}{f_s}$]

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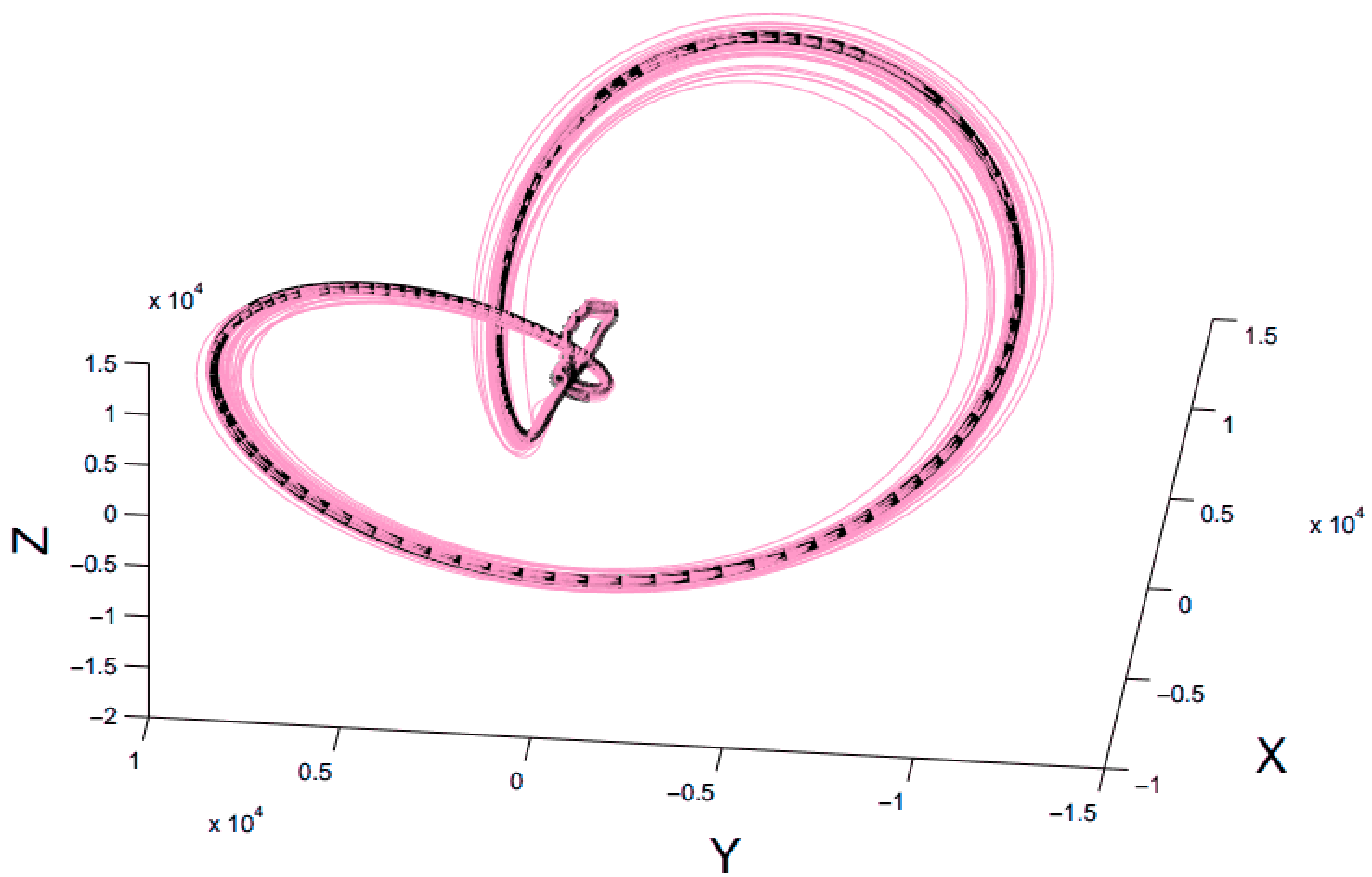
STABILITY ACROSS TIME



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EKG SHORTEST DIFFERENTIAL MODEL

- $X = x$
- $\dot{X} = Y$
- $\dot{Y} = Z$
- $\dot{Z} = \alpha_1 X^2 + \alpha_2 X^4 + \alpha_3 X^5 + \alpha_4 X^6 + \alpha_5 X^7 + \alpha_6 X^8 + \alpha_7 Y + \alpha_8 XY + \alpha_9 X^2 Y + \alpha_{10} X^3 Y + \alpha_{11} X^4 Y + \alpha_{12} X^5 Y + \alpha_{13} X^6 Y + \alpha_{14} Y^2 + \alpha_{15} XY^2 + \alpha_{16} X^2 Y^2 + \alpha_{17} X^3 Y^2 + \alpha_{18} X^4 Y^2 + \alpha_{19} Y^3 + \alpha_{20} XY^3 + \alpha_{21} X^2 Y^3 + \alpha_{22} Y^4 + \alpha_{23} Z + \alpha_{24} XZ + \alpha_{25} X^2 Z + \alpha_{26} X^3 Z + \alpha_{27} X^4 Z + \alpha_{28} YZ + \alpha_{29} XYZ + \alpha_{30} X^2 YZ + \alpha_{31} Y^2 Z + \alpha_{32} Z^2$

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Differential Model in Embedding Space ✓

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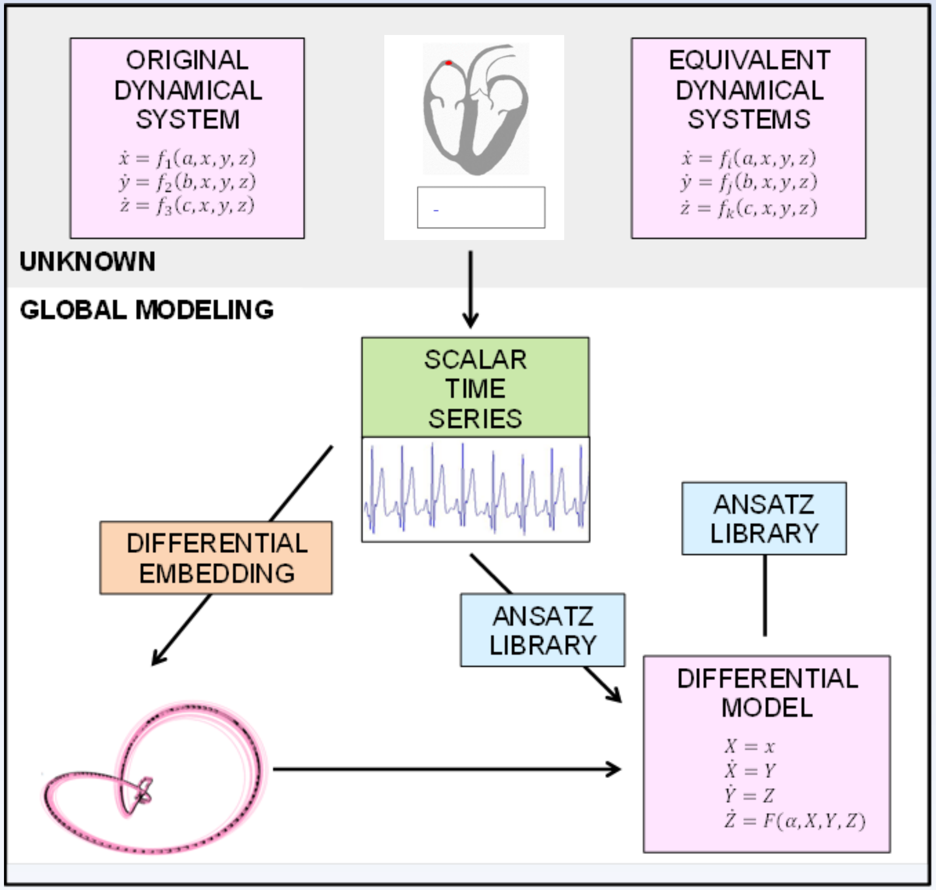
- What can we say about underlying dynamics?
- Differential embedding
- Functional form of differential embedding
- Ansatz library for structure selection of differential model
- Ansatz library for map inversion



SELECTED MINIMUM MODELS

Differential Model:

- $X = x$
- $\dot{X} = Y$
- $\dot{Y} = Z$
- $\dot{Z} = \alpha_1 X^2 + \alpha_2 X^4 + \alpha_3 X^5 + \alpha_4 X^6 + \alpha_5 X^7 + \alpha_6 X^8 + \alpha_7 Y + \alpha_8 XY + \alpha_9 X^2 Y + \alpha_{10} X^3 Y + \alpha_{11} X^4 Y + \alpha_{12} X^5 Y + \alpha_{13} X^6 Y + \alpha_{14} Y^2 + \alpha_{15} XY^2 + \alpha_{16} X^2 Y^2 + \alpha_{17} X^3 Y^2 + \alpha_{18} X^4 Y^2 + \alpha_{19} Y^3 + \alpha_{20} XY^3 + \alpha_{21} X^2 Y^3 + \alpha_{22} Y^4 + \alpha_{23} Z + \alpha_{24} XZ + \alpha_{25} X^2 Z + \alpha_{26} X^3 Z + \alpha_{27} X^4 Z + \alpha_{28} YZ + \alpha_{29} XYZ + \alpha_{30} X^2 YZ + \alpha_{31} Y^2 Z + \alpha_{32} Z^2$

Dynamical Model:

- $\dot{x} = a_2 y + a_4 x^2$
- $\dot{y} = b_2 y + b_3 z + b_5 xy + b_7 y^2$
- $\dot{z} = c_2 y + c_9 z^2$

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- Start with scalar time series of observed variable
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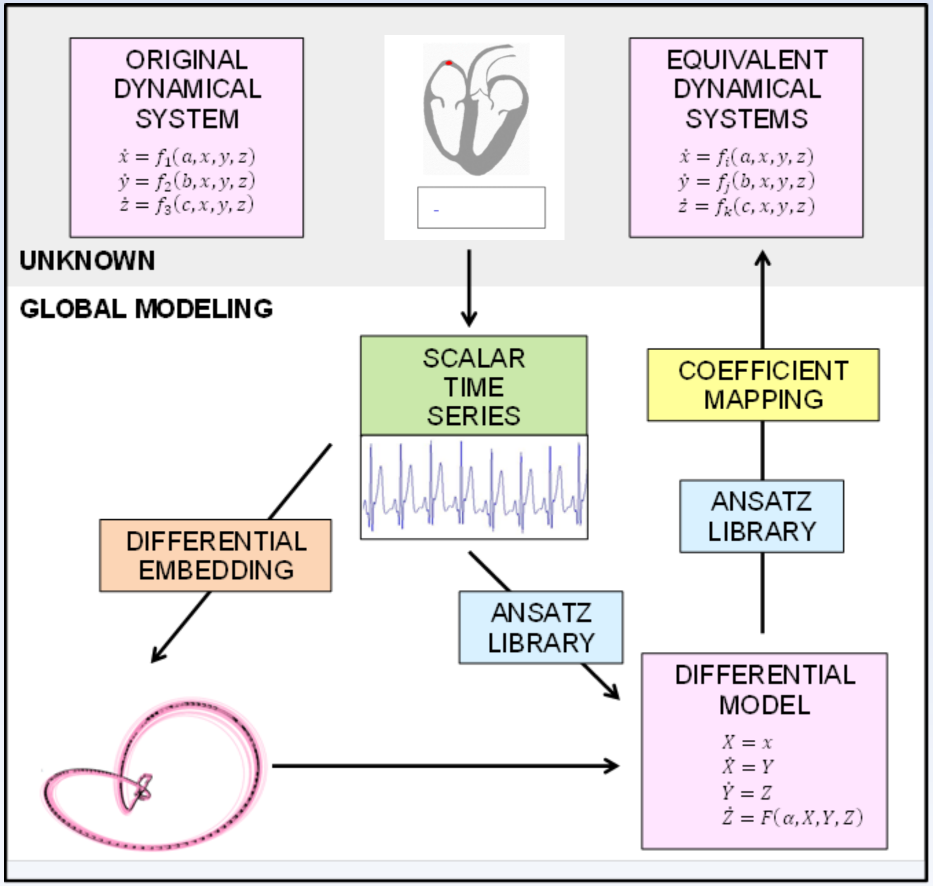
- Genetic algorithm for coefficient mapping



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- Start with scalar time series of observed variable
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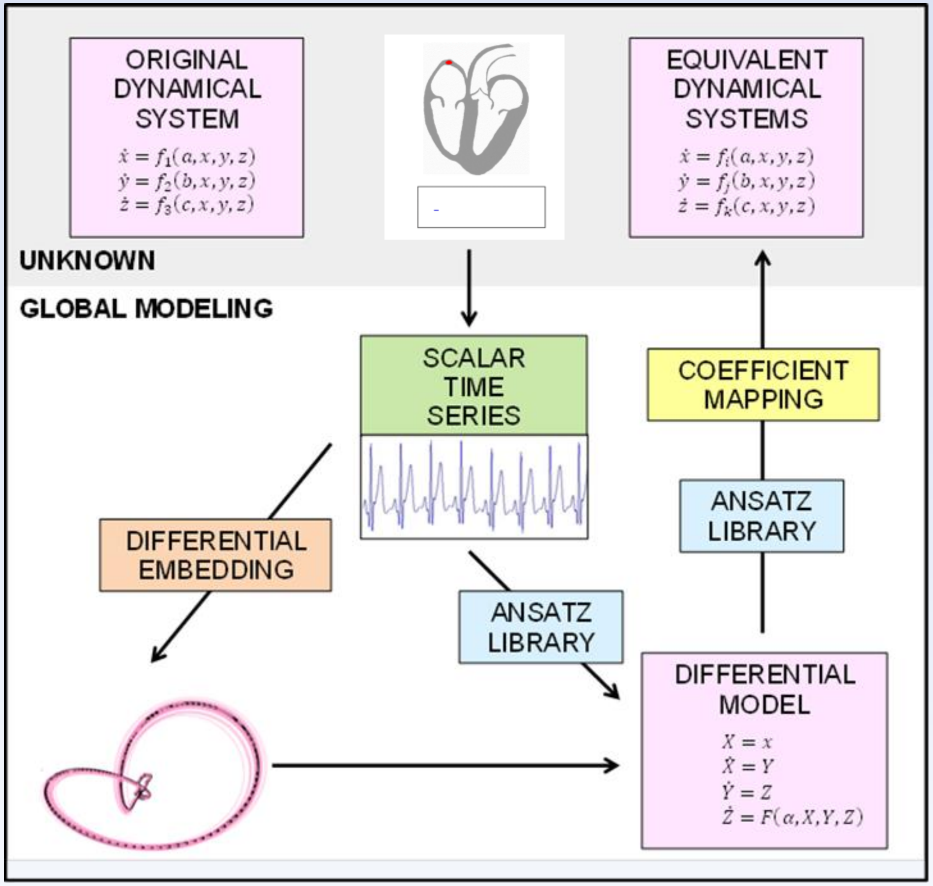
- Genetic algorithm for coefficient mapping
- Integrate ODE model to reconstruct time series



DISCUSSION

Still searching for model that is stable when integrated

<u>Future Directions:</u>

To obtain model –

- Better filtering
- Process automation to check more possibilities
- Incorporation of higher order nonlinearities

After model is obtained –

- More subjects
- Diverse heart conditions
- Diagnostic tool – differences in coefficients, model structure, time scaling, etc. between conditions

ACKNOWLEDGEMENTS

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