Answering questions with data

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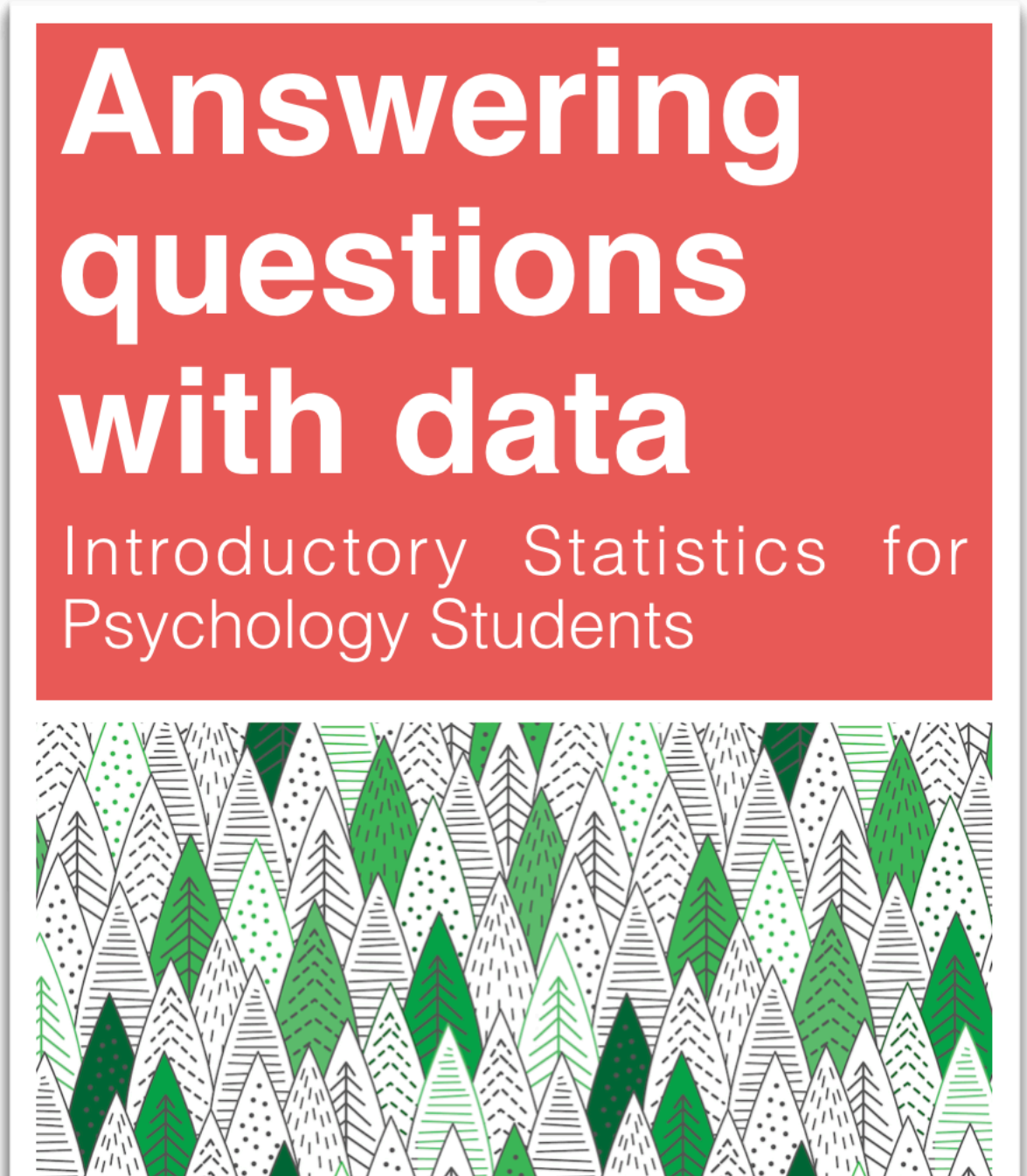
Preface


Answering questions with data
Introductory Statistics for Psychology Students

Last Compiled: 2025-10-03

This version of the textbook was first branched for PSYC 292 in Winter, 2021. The original textbook by Crump et al. can be found here: https://www.crumplab.com/statistics/

The main OSF project link associated with the original textbook is: https://osf.io/3s68c/.

The citation for the original textbook is: Crump, M. J. C., Navarro, D., & Suzuki, J. (2019, June 5). Answering Questions with Data (Textbook): Introductory Statistics for Psychology Students. https://doi.org/10.17605/OSF.IO/JZE52

0.1 Important notes

This is a free textbook teaching introductory statistics for undergraduates in Psychology. This textbook is part of a larger OER course package for teaching undergraduate statistics in Psychology, including this textbook, a lab manual, and a course website. All of the materials are free and copiable, with source code maintained in Github repositories. The links below connect to various components of the project.

0.1.1 Textbook

- website: https://www.erinmazerolle.com/statistics/
- Github: https://github.com/erinmaz/statistics

0.1.2 Lab Manual

- website: https://www.erinmazerolle.com/statisticsLab/
- Github: https://github.com/erinmaz/statisticsLab/

0.1.3 Contributors

Team members contributing new content include, Matthew Crump, Alla Chavarga, Anjali Krishnan, Jeffrey Suzuki, and Stephen Volz. This textbook was primarily written by Matthew J. C. Crump. Jeff contributed the YouTube videos peppered throughout the textbook. All of Jeff's statistics videos are available on his Youtube channel: Statistics Video playlist.

Alla, Anjali, and Stephen wrote the lab manual exercises for SPSS, JAMOVI, and Excel. Matt Crump wrote the lab manual exercises for R.

Matt Crump wrote a free and copiable course website, in R Markdown. The course website also contains slide decks for the lectures.

0.1.4 Attributions

Two of the chapters were adapted from Danielle Navarro's wonderful (and bigger) free textbook, also licensed under the same creative commons license. The citation for that textbook is: Navarro, D. (2018). Learning statistics with R: A tutorial for psychology students and other beginners (version 0.6). The website is https://compcogscisydney.org/learning-statistics-with-r/

Chapter notes within the book are provided to indicate sections where material from Navarro was included. A short summary is here

Chapter 1: Why statistics, Adapted nearly verbatim with some editorial changes from Chapters 1 and 2, Navarro, D.

Chapter 4: Probability, Sampling, and Estimation, Adapted and expanded from Chapters 9 and 10, Navarro D.

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0.2 Copying the textbook

This textbook was written in R-Studio, using R Markdown, and compiled into a web-book format using the bookdown package. In general, I thank the larger R community for all of the amazing tools they made, and for making those tools open, so that I could use them to make this thing.

All of the source code for compiling the book is available in the GitHub repository for this book:

https://github.com/erinmaz/statistics

In principle, anybody could fork or otherwise download this repository. Load the Rproj file in R-studio, and then compile the entire book. Then, the individual .rmd files for each chapter could be edited for content and style to better suit your needs.

If you want to contribute to this version of the textbook, you could make pull requests on GitHub, or discuss issues and request on the issues tab.

0.2.1 Acknowledgments

Thanks to the librarians at Brooklyn College of CUNY, especially Miriam Deutch, and Emily Fairey, for their support throughout the process. Thanks to CUNY for supporting OER development, and for the grant we received to develop this work. Thanks to Jenn Richler for letting me talk about statistics all summer long.

0.2.2 Why we did this

Why write another statistics textbook, aren't there already loads of those? Yes, there are. We had a couple reasons. First, we would like to make R more accessible for the undergraduate population, and we wrote this textbook around the capabilities of R. The textbook was written entirely in R-Studio, and most of the examples have associated R-code. R is not much of a focus in the textbook, but there is an introduction to using R to solve data-analysis problems in the lab manual. Many instructors still use SPSS, Excel, or newer free GUIs like JAMOVI, so we also made lab exercises for each of those as well.

This is a mildly opinionated, non-traditional introduction to statistics. It acknowledges some of the major ideas from traditional frequentist approaches, and some Bayesian approaches. Much of the conceptual foundation is rooted in simulations that can be conducted in R. We use some formulas, but mostly explain things without formulas. The textbook was written with math-phobia in mind, and attempts to reduce the phobia associated with arithmetic computations. There are many things missing that should probably be added. We will do our best to add necessary things as we update the textbook.

0.2.3 Hypothes.is

Hypothes.is is a web browser plug-in that lets you make comments on websites. You simply highlight text and then making comments. Feel free to use Hypothesis with this textbook. We will read your comments.

a. Go to Hypothes.is, and click "Get Started."

b. Install the the add-on for Chrome.

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Chapter 1

Why Statistics?

To call in statisticians after the experiment is done may be no more than asking them to perform a post-mortem examination: They may be able to say what the experiment died of. —Sir Ronald Fisher

1.1 On the psychology of statistics

Adapted nearly verbatim from Chapters 1 and 2 in Navarro, D. "Learning Statistics with R." https://compcogscisydney.org/learning-statistics-with-r/

To the surprise of many students, statistics is a fairly significant part of a psychological education. To the surprise of no-one, statistics is very rarely the *favorite* part of one's psychological education. After all, if you really loved the idea of doing statistics, you'd probably be enrolled in a statistics class right now, not a psychology class. So, not surprisingly, there's a pretty large proportion of the student base that isn't happy about the fact that psychology has so much statistics in it. In view of this, I thought that the right place to start might be to answer some of the more common questions that people have about stats…

A big part of this issue at hand relates to the very idea of statistics. What is it? What's it there for? And why are scientists so bloody obsessed with it? These are all good questions, when you think about it. So let's start with the last one. As a group, scientists seem to be bizarrely fixated on running statistical tests on everything. In fact, we use statistics so often that we sometimes forget to explain to people why we do. It's a kind of article of faith among scientists – and especially social scientists – that your findings can't be trusted until you've done some stats. Undergraduate students might be forgiven for thinking that we're all completely mad, because no-one takes the time to answer one very simple question:

> *Why do you do statistics? Why don't scientists just use common*

sense?

It's a naive question in some ways, but most good questions are. There's a lot of good answers to it, but for my money, the best answer is a really simple one: we don't trust ourselves enough. We worry that we're human, and susceptible to all of the biases, temptations and frailties that humans suffer from. Much of statistics is basically a safeguard. Using "common sense" to evaluate evidence means trusting gut instincts, relying on verbal arguments and on using the raw power of human reason to come up with the right answer. Most scientists don't think this approach is likely to work.

In fact, come to think of it, this sounds a lot like a psychological question to me, and since I do work in a psychology department, it seems like a good idea to dig a little deeper here. Is it really plausible to think that this "common sense" approach is very trustworthy? Verbal arguments have to be constructed in language, and all languages have biases – some things are harder to say than others, and not necessarily because they're false (e.g., quantum electrodynamics is a good theory, but hard to explain in words). The instincts of our "gut" aren't designed to solve scientific problems, they're designed to handle day to day inferences – and given that biological evolution is slower than cultural change, we should say that they're designed to solve the day to day problems for a *different world* than the one we live in. Most fundamentally, reasoning sensibly requires people to engage in "induction", making wise guesses and going beyond the immediate evidence of the senses to make generalisations about the world. If you think that you can do that without being influenced by various distractors, well, I have a bridge in Brooklyn I'd like to sell you. Heck, as the next section shows, we can't even solve "deductive" problems (ones where no guessing is required) without being influenced by our pre-existing biases.

1.1.1 The curse of belief bias

People are mostly pretty smart. We're certainly smarter than the other species that we share the planet with (though many people might disagree). Our minds are quite amazing things, and we seem to be capable of the most incredible feats of thought and reason. That doesn't make us perfect though. And among the many things that psychologists have shown over the years is that we really do find it hard to be neutral, to evaluate evidence impartially and without being swayed by pre-existing biases. A good example of this is the **belief bias effect** in logical reasoning: if you ask people to decide whether a particular argument is logically valid (i.e., conclusion would be true if the premises were true), we tend to be influenced by the believability of the conclusion, even when we shouldn't. For instance, here's a valid argument where the conclusion is believable:

- No cigarettes are inexpensive (Premise 1)
- Some addictive things are inexpensive (Premise 2)
- Therefore, some addictive things are not cigarettes (Conclusion)

And here's a valid argument where the conclusion is not believable:

- No addictive things are inexpensive (Premise 1)
- Some cigarettes are inexpensive (Premise 2)
- Therefore, some cigarettes are not addictive (Conclusion)

The logical *structure* of argument #2 is identical to the structure of argument #1, and they're both valid. However, in the second argument, there are good reasons to think that premise 1 is incorrect, and as a result it's probably the case that the conclusion is also incorrect. But that's entirely irrelevant to the topic at hand: an argument is deductively valid if the conclusion is a logical consequence of the premises. That is, a valid argument doesn't have to involve true statements.

On the other hand, here's an invalid argument that has a believable conclusion:

- No addictive things are inexpensive (Premise 1)
- Some cigarettes are inexpensive (Premise 2)
- Therefore, some addictive things are not cigarettes (Conclusion)

And finally, an invalid argument with an unbelievable conclusion:

- No cigarettes are inexpensive (Premise 1)
- Some addictive things are inexpensive (Premise 2)
- Therefore, some cigarettes are not addictive (Conclusion)

Now, suppose that people really are perfectly able to set aside their pre-existing biases about what is true and what isn't, and purely evaluate an argument on its logical merits. We'd expect 100% of people to say that the valid arguments are valid, and 0% of people to say that the invalid arguments are valid. So if you ran an experiment looking at this, you'd expect to see data like this:

	conclusion feels true	conclusion feels false
argument is valid	100% say "valid"	100% say "valid"
argument is invalid	0% say "valid"	0% say "valid"

If the psychological data looked like this (or even a good approximation to this), we might feel safe in just trusting our gut instincts. That is, it'd be perfectly okay just to let scientists evaluate data based on their common sense, and not bother with all this murky statistics stuff. However, you guys have taken psych classes, and by now you probably know where this is going.

In a classic study, **?** ran an experiment looking at exactly this. What they found is that when pre-existing biases (i.e., beliefs) were in agreement with the structure of the data, everything went the way you'd hope:

	conclusion feels true	conclusion feels false
argument is valid	92% say "valid"	–
argument is invalid	–	8% say "valid"

Not perfect, but that's pretty good. But look what happens when our intuitive feelings about the truth of the conclusion run against the logical structure of the argument:

	conclusion feels true	conclusion feels false
argument is valid	92% say "valid"	**46% say "valid"**
argument is invalid	**92% say "valid"**	8% say "valid"

Oh dear, that's not as good. Apparently, when people are presented with a strong argument that contradicts our pre-existing beliefs, we find it pretty hard to even perceive it to be a strong argument (people only did so 46% of the time). Even worse, when people are presented with a weak argument that agrees with our pre-existing biases, almost no-one can see that the argument is weak (people got that one wrong 92% of the time!)

If you think about it, it's not as if these data are horribly damning. Overall, people did do better than chance at compensating for their prior biases, since about 60% of people's judgements were correct (you'd expect 50% by chance). Even so, if you were a professional "evaluator of evidence", and someone came along and offered you a magic tool that improves your chances of making the right decision from 60% to (say) 95%, you'd probably jump at it, right? Of course you would. Thankfully, we actually do have a tool that can do this. But it's not magic, it's statistics. So that's reason #1 why scientists love statistics. It's just *too easy* for us to "believe what we want to believe"; so if we want to "believe in the data" instead, we're going to need a bit of help to keep our personal biases under control. That's what statistics does: it helps keep us honest.

1.2 The cautionary tale of Simpson's paradox

The following is a true story (I think...). In 1973, the University of California, Berkeley had some worries about the admissions of students into their post-graduate courses. Specifically, the thing that caused the problem was that the gender breakdown of their admissions, which looked like this:

	Number of applicants	Percent admitted
Men	8442	44%
Women	4321	35%

and they were worried about being sued. Given that there were nearly 13,000 applicants, a difference of 9% in admission rates between men and women is just way too big to be a coincidence. Pretty compelling data, right? And if I were

to say to you that these data *actually* reflect a weak bias in favour of women (sort of!), you'd probably think that I was either crazy or sexist.

Earlier versions of these notes incorrectly suggested that they actually were sued – apparently that's not true. There's a nice commentary on this here: https://www.refsmmat.com/posts/2016-05-08-simpsons-paradox-berkeley.html. A big thank you to Wilfried Van Hirtum for pointing this out to me!

When people started looking more carefully at the admissions data (**?**) they told a rather different story. Specifically, when they looked at it on a department by department basis, it turned out that most of the departments actually had a slightly *higher* success rate for woman applicants than for man applicants. The table below shows the admission figures for the six largest departments (with the names of the departments removed for privacy reasons):

	Men		Women	
Department	Applicants	Percent admitted	Applicants	Percent admitted
A	825	62%	108	82%
B	560	63%	25	68%
C	325	37%	593	34%
D	417	33%	375	35%
E	191	28%	393	24%
F	373	6%	341	7%

Remarkably, most departments had a *higher* rate of admissions for women than for men! Yet the overall rate of admission across the university for women was *lower* than for men How can this be? How can both of these statements be true at the same time?

Here's what's going on. Firstly, notice that the departments are *not* equal to one another in terms of their admission percentages: some departments (e.g., engineering, chemistry) tended to admit a high percentage of the qualified applicants, whereas others (e.g., English) tended to reject most of the candidates, even if they were high quality. So, among the six departments shown above, notice that department A is the most generous, followed by B, C, D, E and F in that order. Next, notice that men and women tended to apply to different departments. If we rank the departments in terms of the total number of man applicants, we get **A**>**B**>D>C>F>E (the "easy" departments are in bold). On the whole, men tended to apply to the departments that had high admission rates. Now compare this to how the woman applicants distributed themselves. Ranking the departments in terms of the total number of woman applicants produces a quite different ordering C>E>D>F>**A**>**B**. In other words, what these data seem to be suggesting is that the woman applicants tended to apply to "harder" departments. And in fact, if we look at all Figure **??** we see that this trend is systematic, and quite striking. This effect is known as **Simpson's paradox**. It's not common, but it does happen in real life, and most people

are very surprised by it when they first encounter it, and many people refuse to even believe that it's real. It is very real. And while there are lots of very subtle statistical lessons buried in there, I want to use it to make a much more important point. Doing research is hard, and there are *lots* of subtle, counterintuitive traps lying in wait for the unwary. That's reason #2 why scientists love statistics, and why we teach research methods. Because science is hard, and the truth is sometimes cunningly hidden in the nooks and crannies of complicated data.

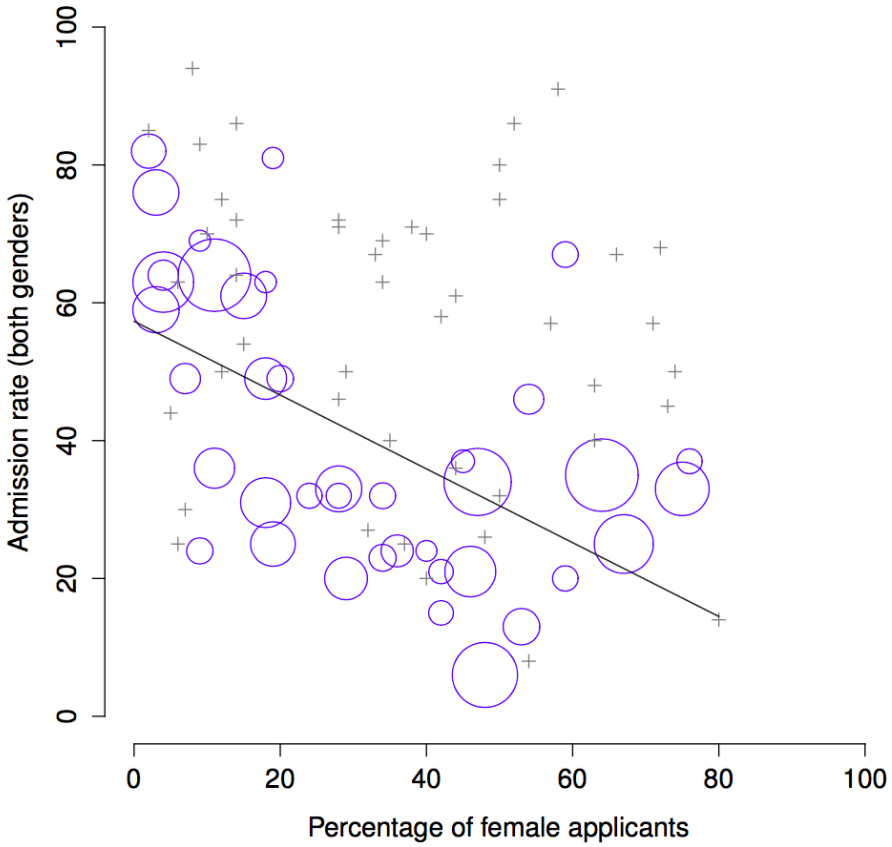


Figure 1.1: The Berkeley 1973 college admissions data. This figure plots the admission rate for the 85 departments that had at least one woman applicant, as a function of the percentage of applicants that were women. The plot is a redrawing of Figure 1 from Bickel et al. (1975). Circles plot departments with more than 40 applicants; the area of the circle is proportional to the total number of applicants. The crosses plot department with fewer than 40 applicants.

Before leaving this topic entirely, I want to point out something else really crit-

ical that is often overlooked in a research methods class. Statistics only solves *part* of the problem. Remember that we started all this with the concern that Berkeley's admissions processes might be unfairly biased against woman applicants. When we looked at the "aggregated" data, it did seem like the university was discriminating against women, but when we "disaggregate" and looked at the individual behaviour of all the departments, it turned out that the actual departments were, if anything, slightly biased in favour of women. The gender bias in total admissions was caused by the fact that women tended to self-select for harder departments. From a legal perspective, that would probably put the university in the clear. Postgraduate admissions are determined at the level of the individual department (and there are good reasons to do that), and at the level of individual departments, the decisions are more or less unbiased (the weak bias in favour of women at that level is small, and not consistent across departments). Since the university can't dictate which departments people choose to apply to, and the decision making takes place at the level of the department it can hardly be held accountable for any biases that those choices produce.

That was the basis for my somewhat glib remarks earlier, but that's not exactly the whole story, is it? After all, if we're interested in this from a more sociological and psychological perspective, we might want to ask *why* there are such strong gender differences in applications. Why do men tend to apply to engineering more often than women, and why is this reversed for the English department? And why is it the case that the departments that tend to have an application bias towards women tend to have lower overall admission rates than those departments that have an application bias towards men? Might this not still reflect a gender bias, even though every single department is itself unbiased? It might. Suppose, hypothetically, that men preferred to apply to "hard sciences" and women prefer "humanities". And suppose further that the reason for why the humanities departments have low admission rates is because the government doesn't want to fund the humanities (spots in Ph.D. programs, for instance, are often tied to government funded research projects). Does that constitute a gender bias? Or just an unenlightened view of the value of the humanities? What if someone at a high level in the government cut the humanities funds because they felt that the humanities are "useless chick stuff". That seems pretty *blatantly* gender biased. None of this falls within the purview of statistics, but it matters to the research project. If you're interested in the overall structural effects of subtle gender biases, then you probably want to look at *both* the aggregated and disaggregated data. If you're interested in the decision making process at Berkeley itself then you're probably only interested in the disaggregated data.

In short there are a lot of critical questions that you can't answer with statistics, but the answers to those questions will have a huge impact on how you analyse and interpret data. And this is the reason why you should always think of statistics as a *tool* to help you learn about your data, no more and no less. It's a powerful tool to that end, but there's no substitute for careful thought.

1.3 Statistics in psychology

I hope that the discussion above helped explain why science in general is so focused on statistics. But I'm guessing that you have a lot more questions about what role statistics plays in psychology, and specifically why psychology classes always devote so many lectures to stats. So here's my attempt to answer a few of them...

- **Why does psychology have so much statistics?**

To be perfectly honest, there's a few different reasons, some of which are better than others. The most important reason is that psychology is a statistical science. What I mean by that is that the "things" that we study are *people*. Real, complicated, gloriously messy, infuriatingly perverse people. The "things" of physics include objects like electrons, and while there are all sorts of complexities that arise in physics, electrons don't have minds of their own. They don't have opinions, they don't differ from each other in weird and arbitrary ways, they don't get bored in the middle of an experiment, and they don't get angry at the experimenter and then deliberately try to sabotage the data set. At a fundamental level psychology is harder than physics.

Basically, we teach statistics to you as psychologists because you need to be better at stats than physicists. There's actually a saying used sometimes in physics, to the effect that "if your experiment needs statistics, you should have done a better experiment". They have the luxury of being able to say that because their objects of study are pathetically simple in comparison to the vast mess that confronts social scientists. It's not just psychology, really: most social sciences are desperately reliant on statistics. Not because we're bad experimenters, but because we've picked a harder problem to solve. We teach you stats because you really, really need it.

- **Can't someone else do the statistics?**

To some extent, but not completely. It's true that you don't need to become a fully trained statistician just to do psychology, but you do need to reach a certain level of statistical competence. In my view, there's three reasons that every psychological researcher ought to be able to do basic statistics:

1. There's the fundamental reason: statistics is deeply intertwined with research design. If you want to be good at designing psychological studies, you need to at least understand the basics of stats.
2. If you want to be good at the psychological side of the research, then you need to be able to understand the psychological literature, right? But almost every paper in the psychological literature reports the results of statistical analyses. So if you really want to understand the psychology, you need to be able to understand what other people did with their data. And that means understanding a certain amount of statistics.
3. There's a big practical problem with being dependent on other people to

do all your statistics: statistical analysis is *expensive*. In almost any real life situation where you want to do psychological research, the cruel facts will be that you don't have enough money to afford a statistician. So the economics of the situation mean that you have to be pretty self-sufficient.

Note that a lot of these reasons generalize beyond researchers. If you want to be a practicing psychologist and stay on top of the field, it helps to be able to read the scientific literature, which relies pretty heavily on statistics.

- **I don't care about jobs, research, or clinical work. Do I need statistics?**

Okay, now you're just messing with me. Still, I think it should matter to you too. Statistics should matter to you in the same way that statistics should matter to *everyone*: we live in the 21st century, and data are *everywhere*. Frankly, given the world in which we live these days, a basic knowledge of statistics is pretty damn close to a survival tool! Which is the topic of the next section…

1.4 Statistics in everyday life

"We are drowning in information,
but we are starved for knowledge"

– Various authors, original probably John Naisbitt

When I started writing up my lecture notes I took the 20 most recent news articles posted to the ABC news website. Of those 20 articles, it turned out that 8 of them involved a discussion of something that I would call a statistical topic; 6 of those made a mistake. The most common error, if you're curious, was failing to report baseline data (e.g., the article mentions that 5% of people in situation X have some characteristic Y, but doesn't say how common the characteristic is for everyone else!) The point I'm trying to make here isn't that journalists are bad at statistics (though they almost always are), it's that a basic knowledge of statistics is very helpful for trying to figure out when someone else is either making a mistake or even lying to you. Perhaps, one of the biggest things that a knowledge of statistics does to you is cause you to get angry at the newspaper or the internet on a far more frequent basis :).

1.5 There's more to research methods than statistics

So far, most of what I've talked about is statistics, and so you'd be forgiven for thinking that statistics is all I care about in life. To be fair, you wouldn't be far wrong, but research methodology is a broader concept than statistics. So most research methods courses will cover a lot of topics that relate much

more to the pragmatics of research design, and in particular the issues that you encounter when trying to do research with humans. However, about 99% of student *fears* relate to the statistics part of the course, so I've focused on the stats in this discussion, and hopefully I've convinced you that statistics matters, and more importantly, that it's not to be feared. That being said, it's pretty typical for introductory research methods classes to be very stats-heavy. This is not (usually) because the lecturers are evil people. Quite the contrary, in fact. Introductory classes focus a lot on the statistics because you almost always find yourself needing statistics before you need the other research methods training. Why? Because almost all of your assignments in other classes will rely on statistical training, to a much greater extent than they rely on other methodological tools. It's not common for undergraduate assignments to require you to design your own study from the ground up (in which case you would need to know a lot about research design), but it *is* common for assignments to ask you to analyse and interpret data that were collected in a study that someone else designed (in which case you need statistics). In that sense, from the perspective of allowing you to do well in all your other classes, the statistics is more urgent.

But note that "urgent" is different from "important" – they both matter. I really do want to stress that research design is just as important as data analysis, and this book does spend a fair amount of time on it. However, while statistics has a kind of universality, and provides a set of core tools that are useful for most types of psychological research, the research methods side isn't quite so universal. There are some general principles that everyone should think about, but a lot of research design is very idiosyncratic, and is specific to the area of research that you want to engage in. To the extent that it's the details that matter, those details don't usually show up in an introductory stats and research methods class.

1.6 A brief introduction to research design

In this chapter, we're going to start thinking about the basic ideas that go into designing a study, collecting data, checking whether your data collection works, and so on. It won't give you enough information to allow you to design studies of your own, but it will give you a lot of the basic tools that you need to assess the studies done by other people. However, since the focus of this book is much more on data analysis than on data collection, I'm only giving a very brief overview. Note that this chapter is "special" in two ways. Firstly, it's much more psychology-specific than the later chapters. Secondly, it focuses much more heavily on the scientific problem of research methodology, and much less on the statistical problem of data analysis. Nevertheless, the two problems are related to one another, so it's traditional for stats textbooks to discuss the problem in a little detail. This chapter relies heavily on **?** for the discussion of study design, and **?** for the discussion of scales of measurement. Later versions

will attempt to be more precise in the citations.

1.7 Introduction to psychological measurement

The first thing to understand is data collection can be thought of as a kind of **measurement**. That is, what we're trying to do here is measure something about human behaviour or the human mind. What do I mean by "measurement"?

1.7.1 Some thoughts about psychological measurement

Measurement itself is a subtle concept, but basically it comes down to finding some way of assigning numbers, or labels, or some other kind of well-defined descriptions to "stuff". So, any of the following would count as a psychological measurement:

- My **age** is *33 years*.
- I *do not* **like anchovies**.
- My **chromosomal sex** is *male*.
- My **self-identified gender** is a *man*.

In the short list above, the **bolded part** is "the thing to be measured", and the *italicized part* is "the measurement itself". In fact, we can expand on this a little bit, by thinking about the set of possible measurements that could have arisen in each case:

- My **age** (in years) could have been *0, 1, 2, 3 ...*, etc. The upper bound on what my age could possibly be is a bit fuzzy, but in practice you'd be safe in saying that the largest possible age is *150*, since no human has ever lived that long.
- When asked if I **like anchovies**, I might have said that *I do*, or *I do not*, or *I have no opinion*, or *I sometimes do*.
- My **chromosomal sex** is almost certainly going to be *male (XY)* or *female (XX)*, but there are a few other possibilities. I could also have *Klinfelter's syndrome (XXY)*, which is more similar to male than to female. And I imagine there are other possibilities too.
- My **self-identified gender** is also very likely to be a *man* or *woman*, but it doesn't have to agree with my chromosomal sex. I may also identify with *neither*, or *transgender*, for example.

As you can see, for some things (like age) it seems fairly obvious what the set of possible measurements should be, whereas for other things it gets a bit tricky. But I want to point out that even in the case of someone's age, it's much more subtle than this. For instance, in the example above, I assumed that it was

okay to measure age in years. But if you're a developmental psychologist, that's way too crude, and so you often measure age in *years and months* (if a child is 2 years and 11 months, this is usually written as "2;11"). If you're interested in newborns, you might want to measure age in *days since birth*, maybe even *hours since birth*. In other words, the way in which you specify the allowable measurement values is important.

Looking at this a bit more closely, you might also realise that the concept of "age" isn't actually all that precise. In general, when we say "age" we implicitly mean "the length of time since birth". But that's not always the right way to do it. Suppose you're interested in how newborn babies control their eye movements. If you're interested in kids that young, you might also start to worry that "birth" is not the only meaningful point in time to care about. If Baby Alice is born 3 weeks premature and Baby Bianca is born 1 week late, would it really make sense to say that they are the "same age" if we encountered them "2 hours after birth"? In one sense, yes: by social convention, we use birth as our reference point for talking about age in everyday life, since it defines the amount of time the person has been operating as an independent entity in the world, but from a scientific perspective that's not the only thing we care about. When we think about the biology of human beings, it's often useful to think of ourselves as organisms that have been growing and maturing since conception, and from that perspective Alice and Bianca aren't the same age at all. So you might want to define the concept of "age" in two different ways: the length of time since conception, and the length of time since birth. When dealing with adults, it won't make much difference, but when dealing with newborns it might.

Moving beyond these issues, there's the question of methodology. What specific "measurement method" are you going to use to find out someone's age? As before, there are lots of different possibilities:

- You could just ask people "how old are you?" The method of self-report is fast, cheap and easy, but it only works with people old enough to understand the question, and some people lie about their age.

- You could ask an authority (e.g., a parent) "how old is your child?" This method is fast, and when dealing with kids it's not all that hard since the parent is almost always around. It doesn't work as well if you want to know "age since conception", since a lot of parents can't say for sure when conception took place. For that, you might need a different authority (e.g., an obstetrician).

- You could look up official records, like birth certificates. This is time consuming and annoying, but it has its uses (e.g., if the person is now dead).

1.7.2 Operationalization: defining your measurement

All of the ideas discussed in the previous section all relate to the concept of **operationalization**. To be a bit more precise about the idea, operationalization is the process by which we take a meaningful but somewhat vague concept, and turn it into a precise measurement. The process of operationalization can involve several different things:

- Being precise about what you are trying to measure: For instance, does "age" mean "time since birth" or "time since conception" in the context of your research?
- Determining what method you will use to measure it: Will you use self-report to measure age, ask a parent, or look up an official record? If you're using self-report, how will you phrase the question?
- Defining the set of the allowable values that the measurement can take: Note that these values don't always have to be numerical, though they often are. When measuring age, the values are numerical, but we still need to think carefully about what numbers are allowed. Do we want age in years, years and months, days, hours? Etc. For other types of measurements (e.g., gender), the values aren't numerical. But, just as before, we need to think about what values are allowed. If we're asking people to self-report their gender, what options to we allow them to choose between? Is it enough to allow only "man" or "woman"? Do you need an "other" option? Or should we not give people any specific options, and let them answer in their own words? And if you open up the set of possible values to include all verbal response, how will you interpret their answers?

Operationalization is a tricky business, and there's no "one, true way" to do it. The way in which you choose to operationalize the informal concept of "age" or "gender" into a formal measurement depends on what you need to use the measurement for. Often you'll find that the community of scientists who work in your area have some fairly well-established ideas for how to go about it. In other words, operationalization needs to be thought through on a case by case basis. Nevertheless, while there a lot of issues that are specific to each individual research project, there are some aspects to it that are pretty general.

Before moving on, I want to take a moment to clear up our terminology, and in the process introduce one more term. Here are four different things that are closely related to each other:

- **A theoretical construct**. This is the thing that you're trying to take a measurement of, like "age", "gender" or an "opinion". A theoretical construct can't be directly observed, and often they're actually a bit vague.
- **A measure**. The measure refers to the method or the tool that you use to make your observations. A question in a survey, a behavioural observation or a brain scan could all count as a measure.

- **An operationalization**. The term "operationalization" refers to the logical connection between the measure and the theoretical construct, or to the process by which we try to derive a measure from a theoretical construct.
- **A variable**. Finally, a new term. A variable is what we end up with when we apply our measure to something in the world. That is, variables are the actual "data" that we end up with in our data sets.

In practice, even scientists tend to blur the distinction between these things, but it's very helpful to try to understand the differences.

1.8 Scales of measurement

As the previous section indicates, the outcome of a psychological measurement is called a variable. But not all variables are of the same qualitative type, and it's very useful to understand what types there are. A very useful concept for distinguishing between different types of variables is what's known as **scales of measurement**.

1.8.1 Nominal scale

A **nominal scale** variable (also referred to as a **categorical** variable) is one in which there is no particular relationship between the different possibilities: for these kinds of variables it doesn't make any sense to say that one of them is "bigger' or "better" than any other one, and it absolutely doesn't make any sense to average them. The classic example for this is "eye colour". Eyes can be blue, green and brown, among other possibilities, but none of them is any "better" than any other one. As a result, it would feel really weird to talk about an "average eye colour". Similarly, gender is nominal too: a man isn't better or worse than a woman, neither does it make sense to try to talk about an "average gender". In short, nominal scale variables are those for which the only thing you can say about the different possibilities is that they are different. That's it.

Let's take a slightly closer look at this. Suppose I was doing research on how people commute to and from work. One variable I would have to measure would be what kind of transportation people use to get to work. This "transport type" variable could have quite a few possible values, including: "train", "bus", "car", "bicycle", etc. For now, let's suppose that these four are the only possibilities, and suppose that when I ask 100 people how they got to work today, and I get this:

Transportation	Number of people
(1) Train	12
(2) Bus	30
(3) Car	48

Transportation	Number of people
(4) Bicycle	10

So, what's the average transportation type? Obviously, the answer here is that there isn't one. It's a silly question to ask. You can say that travel by car is the most popular method, and travel by train is the least popular method, but that's about all. Similarly, notice that the order in which I list the options isn't very interesting. I could have chosen to display the data like this and nothing really changes.

Transportation	Number of people
(3) Car	48
(1) Train	12
(4) Bicycle	10
(2) Bus	30

1.8.2 Ordinal scale

Ordinal scale variables have a bit more structure than nominal scale variables, but not by a lot. An ordinal scale variable is one in which there is a natural, meaningful way to order the different possibilities, but you can't do anything else. The usual example given of an ordinal variable is "finishing position in a race". You *can* say that the person who finished first was faster than the person who finished second, but you *don't* know how much faster. As a consequence we know that 1st $>$ 2nd, and we know that 2nd $>$ 3rd, but the difference between 1st and 2nd might be much larger than the difference between 2nd and 3rd.

Here's an more psychologically interesting example. Suppose I'm interested in people's attitudes to climate change, and I ask them to pick one of these four statements that most closely matches their beliefs:

(1) Temperatures are rising, because of human activity

(2) Temperatures are rising, but we don't know why

(3) Temperatures are rising, but not because of humans

(4) Temperatures are not rising

Notice that these four statements actually do have a natural ordering, in terms of "the extent to which they agree with the current science". Statement 1 is a close match, statement 2 is a reasonable match, statement 3 isn't a very good match, and statement 4 is in strong opposition to the science. So, in terms of the thing I'm interested in (the extent to which people endorse the science), I

can order the items as $1 > 2 > 3 > 4$. Since this ordering exists, it would be very weird to list the options like this…

(3) Temperatures are rising, but not because of humans

(4) Temperatures are rising, because of human activity

(5) Temperatures are not rising

(6) Temperatures are rising, but we don't know why

…because it seems to violate the natural "structure" to the question.

So, let's suppose I asked 100 people these questions, and got the following answers:

	Number
(1) Temperatures are rising, because of human activity	51
(2) Temperatures are rising, but we don't know why	20
(3) Temperatures are rising, but not because of humans	10
(4) Temperatures are not rising	19

When analysing these data, it seems quite reasonable to try to group (1), (2) and (3) together, and say that 81 of 100 people were willing to *at least partially* endorse the science. And it's *also* quite reasonable to group (2), (3) and (4) together and say that 49 of 100 people registered *at least some disagreement* with the dominant scientific view. However, it would be entirely bizarre to try to group (1), (2) and (4) together and say that 90 of 100 people said…what? There's nothing sensible that allows you to group those responses together at all.

That said, notice that while we *can* use the natural ordering of these items to construct sensible groupings, what we *can't* do is average them. For instance, in my simple example here, the "average" response to the question is 1.97. If you can tell me what that means, I'd love to know. Because that sounds like gibberish to me!

1.8.3 Interval scale

In contrast to nominal and ordinal scale variables, **interval scale** and ratio scale variables are variables for which the numerical value is genuinely meaningful. In the case of interval scale variables, the *differences* between the numbers are interpretable, but the variable doesn't have a "natural" zero value. A good example of an interval scale variable is measuring temperature in degrees celsius. For instance, if it was 15° yesterday and 18° today, then the 3° difference between the two is genuinely meaningful. Moreover, that 3° difference is *exactly the same*

as the 3° difference between 7° and 10°. In short, addition and subtraction are meaningful for interval scale variables.

However, notice that the 0° does not mean "no temperature at all": it actually means "the temperature at which water freezes", which is pretty arbitrary. As a consequence, it becomes pointless to try to multiply and divide temperatures. It is wrong to say that 20° is *twice as hot* as 10°, just as it is weird and meaningless to try to claim that 20° is negative two times as hot as −10°.

Again, lets look at a more psychological example. Suppose I'm interested in looking at how the attitudes of first-year university students have changed over time. Obviously, I'm going to want to record the year in which each student started. This is an interval scale variable. A student who started in 2003 did arrive 5 years before a student who started in 2008. However, it would be completely insane for me to divide 2008 by 2003 and say that the second student started "1.0024 times later" than the first one. That doesn't make any sense at all.

1.8.4 Ratio scale

The fourth and final type of variable to consider is a **ratio scale** variable, in which zero really means zero, and it's okay to multiply and divide. A good psychological example of a ratio scale variable is response time (RT). In a lot of tasks it's very common to record the amount of time somebody takes to solve a problem or answer a question, because it's an indicator of how difficult the task is. Suppose that Alan takes 2.3 seconds to respond to a question, whereas Ben takes 3.1 seconds. As with an interval scale variable, addition and subtraction are both meaningful here. Ben really did take $3.1 - 2.3 = 0.8$ seconds longer than Alan did. However, notice that multiplication and division also make sense here too: Ben took $3.1/2.3 = 1.35$ times as long as Alan did to answer the question. And the reason why you can do this is that, for a ratio scale variable such as RT, "zero seconds" really does mean "no time at all".

1.8.5 Continuous versus discrete variables

There's a second kind of distinction that you need to be aware of, regarding what types of variables you can run into. This is the distinction between continuous variables and discrete variables. The difference between these is as follows:

- A **continuous variable** is one in which, for any two values that you can think of, it's always logically possible to have another value in between.
- A **discrete variable** is, in effect, a variable that isn't continuous. For a discrete variable, it's sometimes the case that there's nothing in the middle.

These definitions probably seem a bit abstract, but they're pretty simple once you see some examples. For instance, response time is continuous. If Alan takes

3.1 seconds and Ben takes 2.3 seconds to respond to a question, then it's possible for Cameron's response time to lie in between, by taking 3.0 seconds. And of course it would also be possible for David to take 3.031 seconds to respond, meaning that his RT would lie in between Cameron's and Alan's. And while in practice it might be impossible to measure RT that precisely, it's certainly possible in principle. Because we can always find a new value for RT in between any two other ones, we say that RT is continuous.

Discrete variables occur when this rule is violated. For example, nominal scale variables are always discrete: there isn't a type of transportation that falls "in between" trains and bicycles, not in the strict mathematical way that 2.3 falls in between 2 and 3. So transportation type is discrete. Similarly, ordinal scale variables are always discrete: although "2nd place" does fall between "1st place" and "3rd place", there's nothing that can logically fall in between "1st place" and "2nd place". Interval scale and ratio scale variables can go either way. As we saw above, response time (a ratio scale variable) is continuous. Temperature in degrees celsius (an interval scale variable) is also continuous. However, the year you went to school (an interval scale variable) is discrete. There's no year in between 2002 and 2003. The number of questions you get right on a true-or-false test (a ratio scale variable) is also discrete: since a true-or-false question doesn't allow you to be "partially correct", there's nothing in between 5/10 and 6/10. The table summarizes the relationship between the scales of measurement and the discrete/continuity distinction. Cells with a tick mark correspond to things that are possible. I'm trying to hammer this point home, because (a) some textbooks get this wrong, and (b) people very often say things like "discrete variable" when they mean "nominal scale variable". It's very unfortunate.

Table 1.9: The relationship between the scales of measurement and the discrete/continuity distinction. Cells with an x correspond to things that are possible.

	continuous	discrete
nominal		x
ordinal		x
interval	x	x
ratio	x	x

1.8.6 Some complexities

Okay, I know you're going to be shocked to hear this, but ...the real world is much messier than this little classification scheme suggests. Very few variables in real life actually fall into these nice neat categories, so you need to be kind of careful not to treat the scales of measurement as if they were hard and fast rules. It doesn't work like that: they're guidelines, intended to help you think about the situations in which you should treat different variables differently. Nothing

more.

So let's take a classic example, maybe *the* classic example, of a psychological measurement tool: the **Likert scale**. The humble Likert scale is the bread and butter tool of all survey design. You yourself have filled out hundreds, maybe thousands of them, and odds are you've even used one yourself. Suppose we have a survey question that looks like this:

> Which of the following best describes your opinion of the statement that "all pirates are freaking awesome" ...

and then the options presented to the participant are these:

> (1) Strongly disagree
> (2) Disagree
> (3) Neither agree nor disagree
> (4) Agree
> (5) Strongly agree

This set of items is an example of a 5-point Likert scale: people are asked to choose among one of several (in this case 5) clearly ordered possibilities, generally with a verbal descriptor given in each case. However, it's not necessary that all items be explicitly described. This is a perfectly good example of a 5-point Likert scale too:

> (1) Strongly disagree
> (2)
> (3)
> (4)
> (5) Strongly agree

Likert scales are very handy, if somewhat limited, tools. The question is, what kind of variable are they? They're obviously discrete, since you can't give a response of 2.5. They're obviously not nominal scale, since the items are ordered; and they're not ratio scale either, since there's no natural zero.

But are they ordinal scale or interval scale? One argument says that we can't really prove that the difference between "strongly agree" and "agree" is of the same size as the difference between "agree" and "neither agree nor disagree". In fact, in everyday life it's pretty obvious that they're not the same at all. So this suggests that we ought to treat Likert scales as ordinal variables. On the other hand, in practice most participants do seem to take the whole "on a scale from 1 to 5" part fairly seriously, and they tend to act as if the differences between the five response options were fairly similar to one another. As a consequence, a lot of researchers treat Likert scale data as if it were interval scale. It's not interval scale, but in practice it's close enough that we usually think of it as being **quasi-interval scale**.

1.9 Assessing the reliability of a measurement

At this point we've thought a little bit about how to operationalize a theoretical construct and thereby create a psychological measure; and we've seen that by applying psychological measures we end up with variables, which can come in many different types. At this point, we should start discussing the obvious question: is the measurement any good? We'll do this in terms of two related ideas: *reliability* and *validity*. Put simply, the **reliability** of a measure tells you how *precisely* you are measuring something, whereas the validity of a measure tells you how *accurate* the measure is.

Reliability is actually a very simple concept: it refers to the repeatability or consistency of your measurement. The measurement of my weight by means of a "bathroom scale" is very reliable: if I step on and off the scales over and over again, it'll keep giving me the same answer. Measuring my intelligence by means of "asking my mom" is very unreliable: some days she tells me I'm a bit thick, and other days she tells me I'm a complete moron. Notice that this concept of reliability is different to the question of whether the measurements are correct (the correctness of a measurement relates to it's validity). If I'm holding a sack of potatos when I step on and off of the bathroom scales, the measurement will still be reliable: it will always give me the same answer. However, this highly reliable answer doesn't match up to my true weight at all, therefore it's wrong. In technical terms, this is a *reliable but invalid* measurement. Similarly, while my mom's estimate of my intelligence is a bit unreliable, she might be right. Maybe I'm just not too bright, and so while her estimate of my intelligence fluctuates pretty wildly from day to day, it's basically right. So that would be an *unreliable but valid* measure. Of course, to some extent, notice that if my mum's estimates are too unreliable, it's going to be very hard to figure out which one of her many claims about my intelligence is actually the right one. To some extent, then, a very unreliable measure tends to end up being invalid for practical purposes; so much so that many people would say that reliability is necessary (but not sufficient) to ensure validity.

Okay, now that we're clear on the distinction between reliability and validity, let's have a think about the different ways in which we might measure reliability:

- **Test-retest reliability**. This relates to consistency over time: if we repeat the measurement at a later date, do we get a the same answer?
- **Inter-rater reliability**. This relates to consistency across people: if someone else repeats the measurement (e.g., someone else rates my intelligence) will they produce the same answer?
- **Parallel forms reliability**. This relates to consistency across theoretically-equivalent measurements: if I use a different set of bathroom scales to measure my weight, does it give the same answer?
- **Internal consistency reliability**. If a measurement is constructed from lots of different parts that perform similar functions (e.g., a personality

questionnaire result is added up across several questions) do the individual parts tend to give similar answers.

Not all measurements need to possess all forms of reliability. For instance, educational assessment can be thought of as a form of measurement. One of the subjects that I teach, *Computational Cognitive Science*, has an assessment structure that has a research component and an exam component (plus other things). The exam component is *intended* to measure something different from the research component, so the assessment as a whole has low internal consistency. However, within the exam there are several questions that are intended to (approximately) measure the same things, and those tend to produce similar outcomes; so the exam on its own has a fairly high internal consistency. Which is as it should be. You should only demand reliability in those situations where you want to be measure the same thing!

1.10 The role of variables: predictors and outcomes

Okay, I've got one last piece of terminology that I need to explain to you before moving away from variables. Normally, when we do some research we end up with lots of different variables. Then, when we analyse our data we usually try to explain some of the variables in terms of some of the other variables. It's important to keep the two roles "thing doing the explaining" and "thing being explained" distinct. So let's be clear about this now. Firstly, we might as well get used to the idea of using mathematical symbols to describe variables, since it's going to happen over and over again. Let's denote the "to be explained" variable Y, and denote the variables "doing the explaining" as X_1, X_2, etc.

Now, when we're doing an analysis, we have different names for X and Y, since they play different roles in the analysis. The classical names for these roles are **independent variable** (IV) and **dependent variable** (DV). The IV is the variable that you use to do the explaining (i.e., X) and the DV is the variable being explained (i.e., Y). The logic behind these names goes like this: if there really is a relationship between X and Y then we can say that Y depends on X, and if we have designed our study "properly" then X isn't dependent on anything else. However, I personally find those names horrible: they're hard to remember and they're highly misleading, because (a) the IV is never actually "independent of everything else" and (b) if there's no relationship, then the DV doesn't actually depend on the IV. And in fact, because I'm not the only person who thinks that IV and DV are just awful names, there are a number of alternatives that I find more appealing.

For example, in an experiment the IV refers to the **manipulation**, and the DV refers to the **measurement**. So, we could use **manipulated variable** (independent variable) and **measured variable** (dependent variable).

Table 1.10: The terminology used to distinguish between different roles that a variable can play when analysing a data set.

role of the variable	classical name	modern name
"to be explained"	dependent variable (DV)	Measurement
"to do the explaining"	independent variable (IV)	Manipulation

We could also use **predictors** and **outcomes**. The idea here is that what you're trying to do is use X (the predictors) to make guesses about Y (the outcomes). This is summarized in the table:

Table 1.11: The terminology used to distinguish between different roles that a variable can play when analysing a data set.

role of the variable	classical name	modern name
"to be explained"	dependent variable (DV)	outcome
"to do the explaining"	independent variable (IV)	predictor

1.11 Experimental and non-experimental research

One of the big distinctions that you should be aware of is the distinction between "experimental research" and "non-experimental research". When we make this distinction, what we're really talking about is the degree of control that the researcher exercises over the people and events in the study.

1.11.1 Experimental research

The key features of **experimental research** is that the researcher controls all aspects of the study, especially what participants experience during the study. In particular, the researcher manipulates or varies something (IVs), and then allows the outcome variable (DV) to vary naturally. The idea here is to deliberately vary the something in the world (IVs) to see if it has any causal effects on the outcomes. Moreover, in order to ensure that there's no chance that something other than the manipulated variable is causing the outcomes, everything else is kept constant or is in some other way "balanced" to ensure that they have no effect on the results. In practice, it's almost impossible to *think* of everything else that might have an influence on the outcome of an experiment, much less keep it constant. The standard solution to this is **randomization**: that is, we randomly assign people to different groups, and then give each group a different treatment (i.e., assign them different values of the predictor variables). We'll talk more about randomization later in this course, but for now, it's enough to

say that what randomization does is minimize (but not eliminate) the chances that there are any systematic difference between groups.

Let's consider a very simple, completely unrealistic and grossly unethical example. Suppose you wanted to find out if smoking causes lung cancer. One way to do this would be to find people who smoke and people who don't smoke, and look to see if smokers have a higher rate of lung cancer. This is *not* a proper experiment, since the researcher doesn't have a lot of control over who is and isn't a smoker. And this really matters: for instance, it might be that people who choose to smoke cigarettes also tend to have poor diets, or maybe they tend to work in asbestos mines, or whatever. The point here is that the groups (smokers and non-smokers) actually differ on lots of things, not *just* smoking. So it might be that the higher incidence of lung cancer among smokers is caused by something else, not by smoking per se. In technical terms, these other things (e.g. diet) are called "confounds", and we'll talk about those in just a moment.

In the meantime, let's now consider what a proper experiment might look like. Recall that our concern was that smokers and non-smokers might differ in lots of ways. The solution, as long as you have no ethics, is to *control* who smokes and who doesn't. Specifically, if we randomly divide participants into two groups, and force half of them to become smokers, then it's very unlikely that the groups will differ in any respect other than the fact that half of them smoke. That way, if our smoking group gets cancer at a higher rate than the non-smoking group, then we can feel pretty confident that (a) smoking does cause cancer and (b) we're murderers.

1.11.2 Non-experimental research

Non-experimental research is a broad term that covers "any study in which the researcher doesn't have quite as much control as they do in an experiment". Obviously, control is something that scientists like to have, but as the previous example illustrates, there are lots of situations in which you can't or shouldn't try to obtain that control. Since it's grossly unethical (and almost certainly criminal) to force people to smoke in order to find out if they get cancer, this is a good example of a situation in which you really shouldn't try to obtain experimental control. But there are other reasons too. Even leaving aside the ethical issues, our "smoking experiment" does have a few other issues. For instance, when I suggested that we "force" half of the people to become smokers, I must have been talking about *starting* with a sample of non-smokers, and then forcing them to become smokers. While this sounds like the kind of solid, evil experimental design that a mad scientist would love, it might not be a very sound way of investigating the effect in the real world. For instance, suppose that smoking only causes lung cancer when people have poor diets, and suppose also that people who normally smoke do tend to have poor diets. However, since the "smokers" in our experiment aren't "natural" smokers (i.e., we forced non-smokers to become smokers; they didn't take on all of the other normal, real life characteristics that smokers might tend to possess) they probably have

better diets. As such, in this silly example they wouldn't get lung cancer, and our experiment will fail, because it violates the structure of the "natural" world (the technical name for this is an "artifactual" result; see later).

One distinction worth making between two types of non-experimental research is the difference between **quasi-experimental research** and **case studies**. The example I discussed earlier – in which we wanted to examine incidence of lung cancer among smokers and non-smokers, without trying to control who smokes and who doesn't – is a quasi-experimental design. That is, it's the same as an experiment, but we don't control the predictors (IVs). We can still use statistics to analyse the results, it's just that we have to be a lot more careful.

The alternative approach, case studies, aims to provide a very detailed description of one or a few instances. In general, you can't use statistics to analyse the results of case studies, and it's usually very hard to draw any general conclusions about "people in general" from a few isolated examples. However, case studies are very useful in some situations. Firstly, there are situations where you don't have any alternative: neuropsychology has this issue a lot. Sometimes, you just can't find a lot of people with brain damage in a specific area, so the only thing you can do is describe those cases that you do have in as much detail and with as much care as you can. However, there's also some genuine advantages to case studies: because you don't have as many people to study, you have the ability to invest lots of time and effort trying to understand the specific factors at play in each case. This is a very valuable thing to do. As a consequence, case studies can complement the more statistically-oriented approaches that you see in experimental and quasi-experimental designs. We won't talk much about case studies in these lectures, but they are nevertheless very valuable tools!

1.12 Assessing the validity of a study

More than any other thing, a scientist wants their research to be "valid". The conceptual idea behind **validity** is very simple: can you trust the results of your study? If not, the study is invalid. However, while it's easy to state, in practice it's much harder to check validity than it is to check reliability. And in all honesty, there's no precise, clearly agreed upon notion of what validity actually is. In fact, there's lots of different kinds of validity, each of which raises it's own issues, and not all forms of validity are relevant to all studies. I'm going to talk about five different types:

- Internal validity
- External validity
- Construct validity
- Face validity
- Ecological validity

To give you a quick guide as to what matters here...(1) Internal and external validity are the most important, since they tie directly to the fundamental question of whether your study really works. (2) Construct validity asks whether you're measuring what you think you are. (3) Face validity isn't terribly important except insofar as you care about "appearances". (4) Ecological validity is a special case of face validity that corresponds to a kind of appearance that you might care about a lot.

1.12.1 Internal validity

Internal validity refers to the extent to which you are able draw the correct conclusions about the causal relationships between variables. It's called "internal" because it refers to the relationships between things "inside" the study. Let's illustrate the concept with a simple example. Suppose you're interested in finding out whether a university education makes you write better. To do so, you get a group of first year students, ask them to write a 1000 word essay, and count the number of spelling and grammatical errors they make. Then you find some third-year students, who obviously have had more of a university education than the first-years, and repeat the exercise. And let's suppose it turns out that the third-year students produce fewer errors. And so you conclude that a university education improves writing skills. Right? Except... the big problem that you have with this experiment is that the third-year students are older, and they've had more experience with writing things. So it's hard to know for sure what the causal relationship is: Do older people write better? Or people who have had more writing experience? Or people who have had more education? Which of the above is the true *cause* of the superior performance of the third-years? Age? Experience? Education? You can't tell. This is an example of a failure of internal validity, because your study doesn't properly tease apart the *causal* relationships between the different variables.

1.12.2 External validity

External validity relates to the **generalizability** of your findings. That is, to what extent do you expect to see the same pattern of results in "real life" as you saw in your study. To put it a bit more precisely, any study that you do in psychology will involve a fairly specific set of questions or tasks, will occur in a specific environment, and will involve participants that are drawn from a particular subgroup. So, if it turns out that the results don't actually generalize to people and situations beyond the ones that you studied, then what you've got is a lack of external validity.

The classic example of this issue is the fact that a very large proportion of studies in psychology will use undergraduate psychology students as the participants. Obviously, however, the researchers don't care *only* about psychology students; they care about people in general. Given that, a study that uses only psych students as participants always carries a risk of lacking external validity. That

is, if there's something "special" about psychology students that makes them different to the general populace in some *relevant* respect, then we may start worrying about a lack of external validity.

That said, it is absolutely critical to realize that a study that uses only psychology students does not necessarily have a problem with external validity. I'll talk about this again later, but it's such a common mistake that I'm going to mention it here. The external validity is threatened by the choice of population if (a) the population from which you sample your participants is very narrow (e.g., psych students), and (b) the narrow population that you sampled from is systematically different from the general population, *in some respect that is relevant to the psychological phenomenon that you intend to study*. The italicized part is the bit that lots of people forget: it is true that psychology undergraduates differ from the general population in lots of ways, and so a study that uses only psych students *may* have problems with external validity. However, if those differences aren't very relevant to the phenomenon that you're studying, then there's nothing to worry about. To make this a bit more concrete, here's two extreme examples:

- You want to measure "attitudes of the general public towards psychotherapy", but all of your participants are psychology students. This study would almost certainly have a problem with external validity.
- You want to measure the effectiveness of a visual illusion, and your participants are all psychology students. This study is very unlikely to have a problem with external validity

Having just spent the last couple of paragraphs focusing on the choice of participants (since that's the big issue that everyone tends to worry most about), it's worth remembering that external validity is a broader concept. The following are also examples of things that might pose a threat to external validity, depending on what kind of study you're doing:

- People might answer a "psychology questionnaire" in a manner that doesn't reflect what they would do in real life.
- Your lab experiment on (say) "human learning" has a different structure to the learning problems people face in real life.

1.12.3 Construct validity

Construct validity is basically a question of whether you're measuring what you want to be measuring. A measurement has good construct validity if it is actually measuring the correct theoretical construct, and bad construct validity if it doesn't. To give very simple (if ridiculous) example, suppose I'm trying to investigate the rates with which university students cheat on their exams. And the way I attempt to measure it is by asking the cheating students to stand up in the lecture theatre so that I can count them. When I do this with a class of 300 students, 0 people claim to be cheaters. So I therefore conclude that the

proportion of cheaters in my class is 0%. Clearly this is a bit ridiculous. But the point here is not that this is a very deep methodological example, but rather to explain what construct validity is. The problem with my measure is that while I'm *trying* to measure "the proportion of people who cheat" what I'm actually measuring is "the proportion of people stupid enough to own up to cheating, or bloody minded enough to pretend that they do". Obviously, these aren't the same thing! So my study has gone wrong, because my measurement has very poor construct validity.

1.12.4 Face validity

Face validity simply refers to whether or not a measure "looks like" it's doing what it's supposed to, nothing more. If I design a test of intelligence, and people look at it and they say "no, that test doesn't measure intelligence", then the measure lacks face validity. It's as simple as that. Obviously, face validity isn't very important from a pure scientific perspective. After all, what we care about is whether or not the measure *actually* does what it's supposed to do, not whether it *looks like* it does what it's supposed to do. As a consequence, we generally don't care very much about face validity. That said, the concept of face validity serves three useful pragmatic purposes:

- Sometimes, an experienced scientist will have a "hunch" that a particular measure won't work. While these sorts of hunches have no strict evidentiary value, it's often worth paying attention to them. Because often times people have knowledge that they can't quite verbalize, so there might be something to worry about even if you can't quite say why. In other words, when someone you trust criticizes the face validity of your study, it's worth taking the time to think more carefully about your design to see if you can think of reasons why it might go awry. Mind you, if you don't find any reason for concern, then you should probably not worry: after all, face validity really doesn't matter much.

- Often (very often), completely uninformed people will also have a "hunch" that your research is crap. And they'll criticize it on the internet or something. On close inspection, you'll often notice that these criticisms are actually focused entirely on how the study "looks", but not on anything deeper. The concept of face validity is useful for gently explaining to people that they need to substantiate their arguments further.

- Expanding on the last point, if the beliefs of untrained people are critical (e.g., this is often the case for applied research where you actually want to convince policy makers of something or other) then you *have* to care about face validity. Simply because – whether you like it or not – a lot of people will use face validity as a proxy for real validity. If you want the government to change a law on scientific, psychological grounds, then it won't matter how good your studies "really" are. If they lack face validity, you'll find that politicians ignore you. Of course, it's somewhat unfair

that policy often depends more on appearance than fact, but that's how things go.

1.12.5 Ecological validity

Ecological validity is a different notion of validity, which is similar to external validity, but less important. The idea is that, in order to be ecologically valid, the entire set up of the study should closely approximate the real world scenario that is being investigated. In a sense, ecological validity is a kind of face validity – it relates mostly to whether the study "looks" right, but with a bit more rigour to it. To be ecologically valid, the study has to look right in a fairly specific way. The idea behind it is the intuition that a study that is ecologically valid is more likely to be externally valid. It's no guarantee, of course. But the nice thing about ecological validity is that it's much easier to check whether a study is ecologically valid than it is to check whether a study is externally valid. An simple example would be eyewitness identification studies. Most of these studies tend to be done in a university setting, often with fairly simple array of faces to look at rather than a line up. The length of time between seeing the "criminal" and being asked to identify the suspect in the "line up" is usually shorter. The "crime" isn't real, so there's no chance that the witness being scared, and there's no police officers present, so there's not as much chance of feeling pressured. These things all mean that the study *definitely* lacks ecological validity. They might (but might not) mean that it also lacks external validity.

1.13 Confounds, artifacts and other threats to validity

If we look at the issue of validity in the most general fashion, the two biggest worries that we have are *confounds* and *artifact*. These two terms are defined in the following way:

- **Confound**: A confound is an additional, often unmeasured variable that turns out to be related to both the predictors and the outcomes. The existence of confounds threatens the internal validity of the study because you can't tell whether the predictor causes the outcome, or if the confounding variable causes it, etc.
- **Artifact**: A result is said to be "artifactual" if it only holds in the special situation that you happened to test in your study. The possibility that your result is an artifact describes a threat to your external validity, because it raises the possibility that you can't generalize your results to the actual population that you care about.

What is the difference between a confound and an artifact? If you realize you have a confound, that means you don't know whether it was the confound or

the predictor that caused your result. If you realize you have an artifact, that means you know the artifact caused your result.

As a general rule confounds are a bigger concern for non-experimental studies, precisely because they're not proper experiments: by definition, you're leaving lots of things uncontrolled, so there's a lot of scope for confounds working their way into your study. Experimental research tends to be much less vulnerable to confounds: the more control you have over what happens during the study, the more you can prevent confounds from appearing.

However, there's always swings and roundabouts, and when we start thinking about artifacts rather than confounds, the shoe is very firmly on the other foot. For the most part, artifactual results tend to be a concern for experimental studies than for non-experimental studies. To see this, it helps to realize that the reason that a lot of studies are non-experimental is precisely because what the researcher is trying to do is examine human behaviour in a more naturalistic context. By working in a more real-world context, you lose experimental control (making yourself vulnerable to confounds) but because you tend to be studying human psychology "in the wild" you reduce the chances of getting an artifactual result. Or, to put it another way, when you take psychology out of the wild and bring it into the lab (which we usually have to do to gain our experimental control), you always run the risk of accidentally studying something different than you wanted to study: which is more or less the definition of an artifact.

Be warned though: the above is a rough guide only. It's absolutely possible to have confounds in an experiment, and to get artifactual results with non-experimental studies. This can happen for all sorts of reasons, not least of which is researcher error. In practice, it's really hard to think everything through ahead of time, and even very good researchers make mistakes. But other times it's unavoidable, simply because the researcher has ethics (e.g., see "differential attrition").

Okay. There's a sense in which almost any threat to validity can be characterized as a confound or an artifact: they're pretty vague concepts. So let's have a look at some of the most common examples…

1.13.1 History effects

History effects refer to the possibility that specific events may occur during the study itself that might influence the outcomes. For instance, something might happen in between a pre-test and a post-test. Or, in between testing participant 23 and participant 24. Alternatively, it might be that you're looking at an older study, which was perfectly valid for its time, but the world has changed enough since then that the conclusions are no longer trustworthy. Examples of things that would count as history effects:

- You're interested in how people think about risk and uncertainty. You started your data collection in December 2010. But finding participants

and collecting data takes time, so you're still finding new people in February 2011. Unfortunately for you (and even more unfortunately for others), the Queensland floods occurred in January 2011, causing billions of dollars of damage and killing many people. Not surprisingly, the people tested in February 2011 express quite different beliefs about handling risk than the people tested in December 2010. Which (if any) of these reflects the "true" beliefs of participants? I think the answer is probably both: the Queensland floods genuinely changed the beliefs of the Australian public, though possibly only temporarily. The key thing here is that the "history" of the people tested in February is quite different to people tested in December.

- You're testing the psychological effects of a new anti-anxiety drug. So what you do is measure anxiety before administering the drug (e.g., by self-report, and taking physiological measures, let's say), then you administer the drug, and then you take the same measures afterwards. In the middle, however, because your labs are in Los Angeles, there's an earthquake, which increases the anxiety of the participants.

1.13.2 Maturation effects

As with history effects, **maturational effects** are fundamentally about change over time. However, maturation effects aren't in response to specific events. Rather, they relate to how people change on their own over time: we get older, we get tired, we get bored, etc. Some examples of maturation effects:

- When doing developmental psychology research, you need to be aware that children grow up quite rapidly. So, suppose that you want to find out whether some educational trick helps with vocabulary size among 3 year olds. One thing that you need to be aware of is that the vocabulary size of children that age is growing at an incredible rate (multiple words per day), all on its own. If you design your study without taking this maturational effect into account, then you won't be able to tell if your educational trick works.
- When running a very long experiment in the lab (say, something that goes for 3 hours), it's very likely that people will begin to get bored and tired, and that this maturational effect will cause performance to decline, regardless of anything else going on in the experiment

1.13.3 Repeated testing effects

An important type of history effect is the effect of **repeated testing**. Suppose I want to take two measurements of some psychological construct (e.g., anxiety). One thing I might be worried about is if the first measurement has an effect on the second measurement. In other words, this is a history effect in which the "event" that influences the second measurement is the first measurement itself! This is not at all uncommon. Examples of this include:

- *Learning and practice*: e.g., "intelligence" at time 2 might appear to go up relative to time 1 because participants learned the general rules of how to solve "intelligence-test-style" questions during the first testing session.
- *Familiarity with the testing situation*: e.g., if people are nervous at time 1, this might make performance go down; after sitting through the first testing situation, they might calm down a lot precisely because they've seen what the testing looks like.
- *Auxiliary changes caused by testing*: e.g., if a questionnaire assessing mood is boring, then mood at measurement at time 2 is more likely to become "bored", precisely because of the boring measurement made at time 1.

1.13.4 Selection bias

Selection bias is a pretty broad term. Suppose that you're running an experiment with two groups of participants, where each group gets a different "treatment", and you want to see if the different treatments lead to different outcomes. However, suppose that, despite your best efforts, you've ended up with a gender imbalance across groups (say, group A has 80% women and group B has 50% women). It might sound like this could never happen, but trust me, it can. This is an example of a selection bias, in which the people "selected into" the two groups have different characteristics. If any of those characteristics turns out to be relevant (say, your treatment works better on women than men), then you're in a lot of trouble. For example, if Group A was our treatment group and Group B was our control group, we might overestimate how well our treatment works on average.

One thing to note is that this could even happen if a researcher did random assignment to the groups. It's very unlikely that such a larger gender bias would result from random assignment, but not impossible. Even if that case, the impact of the gender discrepancy between groups is important to keep in mind when interpreting the results.

1.13.5 Differential attrition

One quite subtle danger to be aware of is called **differential attrition**, which is a kind of selection bias that is caused by the study itself. Suppose that, for the first time ever in the history of psychology, I manage to find the perfectly balanced and representative sample of people. I start running "Dan's incredibly long and tedious experiment" on my perfect sample, but then, because my study is incredibly long and tedious, lots of people start dropping out. I can't stop this: as we'll discuss later in the chapter on research ethics, participants absolutely have the right to stop doing any experiment, any time, for whatever reason they feel like, and as researchers we are morally (and professionally) obliged to remind people that they do have this right. So, suppose that "Dan's incredibly long and tedious experiment" has a very high drop out rate. What do you suppose

the odds are that this drop out is random? Answer: zero. Almost certainly, the people who remain are more conscientious, more tolerant of boredom etc than those that leave. To the extent that (say) conscientiousness is relevant to the psychological phenomenon that I care about, this attrition can decrease the validity of my results.

When thinking about the effects of differential attrition, it is sometimes helpful to distinguish between two different types. The first is **homogeneous attrition**, in which the attrition effect is the same for all groups, treatments or conditions. In the example I gave above, the differential attrition would be homogeneous if (and only if) the easily bored participants are dropping out of all of the conditions in my experiment at about the same rate. In general, the main effect of homogeneous attrition is likely to be that it makes your sample unrepresentative. As such, the biggest worry that you'll have is that the generalisability of the results decreases: in other words, you lose external validity.

The second type of differential attrition is **heterogeneous attrition**, in which the attrition effect is different for different groups. This is a much bigger problem: not only do you have to worry about your external validity, you also have to worry about your internal validity too. To see why this is the case, let's consider a very dumb study in which I want to see if insulting people makes them act in a more obedient way. Why anyone would actually want to study that I don't know, but let's suppose I really, deeply cared about this. So, I design my experiment with two conditions. In the "treatment" condition, the experimenter insults the participant and then gives them a questionnaire designed to measure obedience. In the "control" condition, the experimenter engages in a bit of pointless chitchat and then gives them the questionnaire. Leaving aside the questionable scientific merits and dubious ethics of such a study, let's have a think about what might go wrong here. As a general rule, when someone insults me to my face, I tend to get much less co-operative. So, there's a pretty good chance that a lot more people are going to drop out of the treatment condition than the control condition. And this drop out isn't going to be random. The people most likely to drop out would probably be the people who don't care all that much about the importance of obediently sitting through the experiment. Since the most bloody minded and disobedient people all left the treatment group but not the control group, we've introduced a confound: the people who actually took the questionnaire in the treatment group were *already* more likely to be dutiful and obedient than the people in the control group. In short, in this study insulting people doesn't make them more obedient: it makes the more disobedient people leave the experiment! The internal validity of this experiment is completely shot.

1.13.6 Non-response bias

Non-response bias is closely related to selection bias, and to differential attrition. The simplest version of the problem goes like this. You mail out a survey to 1000 people, and only 300 of them reply. The 300 people who replied are

almost certainly not a random subsample. People who respond to surveys are systematically different to people who don't. This introduces a problem when trying to generalize from those 300 people who replied, to the population at large; since you now have a very non-random sample. The issue of non-response bias is more general than this, though. Among the (say) 300 people that did respond to the survey, you might find that not everyone answers every question. If (say) 80 people chose not to answer one of your questions, does this introduce problems? As always, the answer is maybe. If the question that wasn't answered was on the last page of the questionnaire, and those 80 surveys were returned with the last page missing, there's a good chance that the missing data isn't a big deal: probably the pages just fell off. However, if the question that 80 people didn't answer was the most confrontational or invasive personal question in the questionnaire, then almost certainly you've got a problem. In essence, what you're dealing with here is what's called the problem of **missing data**. If the data that is missing was "lost" randomly, then it's not a big problem. If it's missing systematically, then it can be a big problem.

1.13.7 Regression to the mean

Regression to the mean is a curious variation on selection bias. It refers to any situation where you select data based on an extreme value on some measure. Because the measure has natural variation, it almost certainly means that when you take a subsequent measurement, that later measurement will be less extreme than the first one, purely by chance.

Here's an example. Suppose I'm interested in whether a psychology education has an adverse effect on very smart kids. To do this, I find the 20 psych I students with the best high school grades and look at how well they're doing at university. It turns out that they're doing a lot better than average, but they're not topping the class at university, even though they did top their classes at high school. What's going on? The natural first thought is that this must mean that the psychology classes must be having an adverse effect on those students. However, while that might very well be the explanation, it's more likely that what you're seeing is an example of "regression to the mean". To see how it works, let's take a moment to think about what is required to get the best mark in a class, regardless of whether that class be at high school or at university. When you've got a big class, there are going to be *lots* of very smart people enrolled. To get the best mark you have to be very smart, work very hard, and be a bit lucky. The exam has to ask just the right questions for your idiosyncratic skills, and you have to not make any dumb mistakes (we all do that sometimes) when answering them. And that's the thing: intelligence and hard work are transferrable from one class to the next. Luck isn't. The people who got lucky in high school won't be the same as the people who get lucky at university. That's the very definition of "luck". The consequence of this is that, when you select people at the very extreme values of one measurement (the top 20 students), you're selecting for hard work, skill and luck. But because

the luck doesn't transfer to the second measurement (only the skill and work), these people will all be expected to drop a little bit when you measure them a second time (at university). So their scores fall back a little bit, back towards everyone else. This is regression to the mean.

Regression to the mean is surprisingly common. For instance, if two very tall people have kids, their children will tend to be taller than average, but not as tall as the parents. The reverse happens with very short parents: two very short parents will tend to have short children, but nevertheless those kids will tend to be taller than the parents. It can also be extremely subtle. For instance, there have been studies done that suggested that people learn better from negative feedback than from positive feedback. However, the way that people tried to show this was to give people positive reinforcement whenever they did good, and negative reinforcement when they did bad. And what you see is that after the positive reinforcement, people tended to do worse; but after the negative reinforcement they tended to do better. But! Notice that there's a selection bias here: when people do very well, you're selecting for "high" values, and so you should *expect* (because of regression to the mean) that performance on the next trial should be worse, regardless of whether reinforcement is given. Similarly, after a bad trial, people will tend to improve all on their own. The apparent superiority of negative feedback is an artifact caused by regression to the mean (**?**)

1.13.8 Experimenter bias

Experimenter bias can come in multiple forms. The basic idea is that the experimenter, despite the best of intentions, can accidentally end up influencing the results of the experiment by subtly communicating the "right answer" or the "desired behaviour" to the participants. Typically, this occurs because the experimenter has special knowledge that the participant does not – either the right answer to the questions being asked, or knowledge of the expected pattern of performance for the condition that the participant is in, and so on. The classic example of this happening is the case study of "Clever Hans", which dates back to 1907, **?** (**?**; **?**). Clever Hans was a horse that apparently was able to read and count, and perform other human like feats of intelligence. After Clever Hans became famous, psychologists started examining his behaviour more closely. It turned out that – not surprisingly – Hans didn't know how to do maths. Rather, Hans was responding to the human observers around him. Because they did know how to count, and the horse had learned to change its behaviour when people changed theirs.

The general solution to the problem of experimenter bias is to engage in double blind studies, where neither the experimenter nor the participant knows which condition the participant is in, or knows what the desired behaviour is. This provides a very good solution to the problem, but it's important to recognize that it's not quite ideal, and hard to pull off perfectly. For instance, the obvious way that I could try to construct a double blind study is to have one of my

Ph.D. students (one who doesn't know anything about the experiment) run the study. That feels like it should be enough. The only person (me) who knows all the details (e.g., correct answers to the questions, assignments of participants to conditions) has no interaction with the participants, and the person who does all the talking to people (the Ph.D. student) doesn't know anything. Except, that last part is very unlikely to be true. In order for the Ph.D. student to run the study effectively, they need to have been briefed by me, the researcher. And, as it happens, the Ph.D. student also knows me, and knows a bit about my general beliefs about people and psychology (e.g., I tend to think humans are much smarter than psychologists give them credit for). As a result of all this, it's almost impossible for the experimenter to avoid knowing a little bit about what expectations I have. And even a little bit of knowledge can have an effect: suppose the experimenter accidentally conveys the fact that the participants are expected to do well in this task. Well, there's a thing called the "Pygmalion effect": if you expect great things of people, they'll rise to the occasion; but if you expect them to fail, they'll do that too. In other words, the expectations become a self-fulfilling prophesy.

1.13.9 Demand effects and reactivity

When talking about experimenter bias, the worry is that the experimenter's knowledge or desires for the experiment are communicated to the participants, and that these effect people's behaviour **?**. However, even if you manage to stop this from happening, it's almost impossible to stop people from knowing that they're part of a psychological study. And the mere fact of knowing that someone is watching/studying you can have a pretty big effect on behaviour. This is generally referred to as **reactivity** or **demand effects**. The basic idea is captured by the Hawthorne effect: people alter their performance because of the attention that the study focuses on them. The effect takes its name from a the "Hawthorne Works" factory outside of Chicago (**?**). A study done in the 1920s looking at the effects of lighting on worker productivity at the factory turned out to be an effect of the fact that the workers knew they were being studied, rather than the lighting.

To get a bit more specific about some of the ways in which the mere fact of being in a study can change how people behave, it helps to think like a social psychologist and look at some of the *roles* that people might adopt during an experiment, but might not adopt if the corresponding events were occurring in the real world:

- The *good participant* tries to be too helpful to the researcher: he or she seeks to figure out the experimenter's hypotheses and confirm them.
- The *negative participant* does the exact opposite of the good participant: he or she seeks to break or destroy the study or the hypothesis in some way.
- The *faithful participant* is unnaturally obedient: he or she seeks to follow

instructions perfectly, regardless of what might have happened in a more realistic setting.

- The *apprehensive participant* gets nervous about being tested or studied, so much so that his or her behaviour becomes highly unnatural, or overly socially desirable.

1.13.10 Placebo effects

The **placebo effect** is a specific type of demand effect that we worry a lot about. It refers to the situation where the mere fact of being treated causes an improvement in outcomes. The classic example comes from clinical trials: if you give people a completely chemically inert drug and tell them that it's a cure for a disease, they will tend to get better faster than people who aren't treated at all. In other words, it is people's belief that they are being treated that causes the improved outcomes, not the drug.

1.13.11 Situation, measurement and subpopulation effects

In some respects, these terms are a catch-all term for "all other threats to external validity". They refer to the fact that the choice of subpopulation from which you draw your participants, the location, timing and manner in which you run your study (including who collects the data) and the tools that you use to make your measurements might all be influencing the results. Specifically, the worry is that these things might be influencing the results in such a way that the results won't generalize to a wider array of people, places and measures.

1.13.12 Fraud, deception and self-deception

> *It is difficult to get a man to understand something, when his salary depends on his not understanding it.*
>
> – Upton Sinclair

One final thing that I feel like I should mention. While reading what the textbooks often have to say about assessing the validity of the study, I couldn't help but notice that they seem to make the assumption that the researcher is honest. I find this hilarious. While the vast majority of scientists are honest, in my experience at least, some are not. Not only that, as I mentioned earlier, scientists are not immune to belief bias – it's easy for a researcher to end up deceiving themselves into believing the wrong thing, and this can lead them to conduct subtly flawed research, and then hide those flaws when they write it up. So you need to consider not only the (probably unlikely) possibility of outright fraud, but also the (probably quite common) possibility that the research is unintentionally "slanted". I opened a few standard textbooks and didn't find much of a discussion of this problem, so here's my own attempt to list a few ways in which these issues can arise are:

- **Data fabrication**. Sometimes, people just make up the data. This is occasionally done with "good" intentions. For instance, the researcher believes that the fabricated data do reflect the truth, and may actually reflect "slightly cleaned up" versions of actual data. On other occasions, the fraud is deliberate and malicious. Some high-profile examples where data fabrication has been alleged or shown include Cyril Burt (a psychologist who is thought to have fabricated some of his data), Andrew Wakefield (who has been accused of fabricating his data connecting the MMR vaccine to autism) and Hwang Woo-suk (who falsified a lot of his data on stem cell research).

- **Hoaxes**. Hoaxes share a lot of similarities with data fabrication, but they differ in the intended purpose. A hoax is often a joke, and many of them are intended to be (eventually) discovered. Often, the point of a hoax is to discredit someone or some field. There's quite a few well known scientific hoaxes that have occurred over the years (e.g., Piltdown man) some of were deliberate attempts to discredit particular fields of research (e.g., the Sokal affair).

- **Data misrepresentation**. While fraud gets most of the headlines, it's much more common in my experience to see data being misrepresented. When I say this, I'm not referring to newspapers getting it wrong (which they do, almost always). I'm referring to the fact that often, the data don't actually say what the researchers think they say. My guess is that, almost always, this isn't the result of deliberate dishonesty, it's due to a lack of sophistication in the data analyses. For instance, think back to the example of Simpson's paradox that I discussed in the beginning of these notes. It's very common to see people present "aggregated" data of some kind; and sometimes, when you dig deeper and find the raw data yourself, you find that the aggregated data tell a different story to the disaggregated data. Alternatively, you might find that some aspect of the data is being hidden, because it tells an inconvenient story (e.g., the researcher might choose not to refer to a particular variable). There's a lot of variants on this; many of which are very hard to detect.

- **Study "misdesign"**. Okay, this one is subtle. Basically, the issue here is that a researcher designs a study that has built-in flaws, and those flaws are never reported in the paper. The data that are reported are completely real, and are correctly analysed, but they are produced by a study that is actually quite wrongly put together. The researcher really wants to find a particular effect, and so the study is set up in such a way as to make it "easy" to (artifactually) observe that effect. One sneaky way to do this – in case you're feeling like dabbling in a bit of fraud yourself – is to design an experiment in which it's obvious to the participants what they're "supposed" to be doing, and then let reactivity work its magic for you. If you want, you can add all the trappings of double blind experimentation etc. It won't make a difference, since the study materials themselves are

subtly telling people what you want them to do. When you write up the results, the fraud won't be obvious to the reader: what's obvious to the participant when they're in the experimental context isn't always obvious to the person reading the paper. Of course, the way I've described this makes it sound like it's always fraud: probably there are cases where this is done deliberately, but in my experience the bigger concern has been with unintentional misdesign. The researcher *believes* …and so the study just happens to end up with a built in flaw, and that flaw then magically erases itself when the study is written up for publication.

- **Data mining & post hoc hypothesising**. Another way in which the authors of a study can more or less lie about what they found is by engaging in what's referred to as "data mining". As we'll discuss later in the class, if you keep trying to analyse your data in lots of different ways, you'll eventually find something that "looks" like a real effect but isn't. This is referred to as "data mining". It used to be quite rare because data analysis used to take weeks, but now that everyone has very powerful statistical software on their computers, it's becoming very common. Data mining per se isn't "wrong", but the more that you do it, the bigger the risk you're taking. The thing that is wrong, and I suspect is very common, is *unacknowledged* data mining. That is, the researcher run every possible analysis known to humanity, finds the one that works, and then pretends that this was the only analysis that they ever conducted. Worse yet, they often "invent" a hypothesis after looking at the data, to cover up the data mining. To be clear: it's not wrong to change your beliefs after looking at the data, and to reanalyse your data using your new "post hoc" hypotheses. What is wrong (and, I suspect, common) is failing to acknowledge that you've done so. If you acknowledge that you did it, then other researchers are able to take your behaviour into account. If you don't, then they can't. And that makes your behaviour deceptive. Bad!

- **Publication bias & self-censoring**. Finally, a pervasive bias is "non-reporting" of negative results. This is almost impossible to prevent. Journals don't publish every article that is submitted to them: they prefer to publish articles that find "something". So, if 20 people run an experiment looking at whether reading *Finnegans Wake* causes insanity in humans, and 19 of them find that it doesn't, which one do you think is going to get published? Obviously, it's the one study that did find that *Finnegans Wake* causes insanity. This is an example of a *publication bias*: since no-one ever published the 19 studies that didn't find an effect, a naive reader would never know that they existed. Worse yet, most researchers "internalize" this bias, and end up *self-censoring* their research. Knowing that negative results aren't going to be accepted for publication, they never even try to report them. As a friend of mine says "for every experiment that you get published, you also have 10 failures". And she's right. The catch is, while some (maybe most) of those studies are failures for boring reasons (e.g. you stuffed something up) others might be genuine "null"

results that you ought to acknowledge when you write up the "good" experiment. And telling which is which is often hard to do. A good place to start is a paper by **?** with the depressing title "Why most published research findings are false". I'd also suggest taking a look at work by **?** presenting statistical evidence that this actually happens in psychology.

There's probably a lot more issues like this to think about, but that'll do to start with. What I really want to point out is the blindingly obvious truth that real world science is conducted by actual humans, and only the most gullible of people automatically assumes that everyone else is honest and impartial. Actual scientists aren't usually *that* naive, but for some reason the world likes to pretend that we are, and the textbooks we usually write seem to reinforce that stereotype.

1.14 Summary

This chapter isn't really meant to provide a comprehensive discussion of psychological research methods: it would require another volume just as long as this one to do justice to the topic. However, in real life statistics and study design are tightly intertwined, so it's very handy to discuss some of the key topics. In this chapter, I've briefly discussed the following topics:

- **Introduction to psychological measurement**: What does it mean to operationalize a theoretical construct? What does it mean to have variables and take measurements?
- **Scales of measurement and types of variables**: Remember that there are *two* different distinctions here: there's the difference between discrete and continuous data, and there's the difference between the four different scale types (nominal, ordinal, interval and ratio).
- **Reliability of a measurement**: If I measure the "same" " thing twice, should I expect to see the same result? Only if my measure is reliable. But what does it mean to talk about doing the "same" thing? Well, that's why we have different types of reliability. Make sure you remember what they are.
- **Terminology: predictors and outcomes**: What roles do variables play in an analysis? Can you remember the difference between predictors and outcomes? Dependent and independent variables? Etc.
- **Experimental and non-experimental research designs**: What makes an experiment an experiment? Is it a nice white lab coat, or does it have something to do with researcher control over variables?
- **Validity and its threats**: Does your study measure what you want it to? How might things go wrong? And is it my imagination, or was that a very long list of possible ways in which things can go wrong?

All this should make clear to you that study design is a critical part of research methodology. I built this chapter from the classic little book by **?**, but there are of course a large number of textbooks out there on research design. Spend a few minutes with your favourite search engine and you'll find dozens.

1.15 Videos

1.15.1 Terms of Statistics

Chapter 2

Describing Data

Far better an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise.
—John W. Tukey

Chapter by Matthew Crump

This chapter is about **descriptive statistics**. These are tools for describing data. Some things to keep in mind as we go along are:

1. There are lots of different ways to describe data
2. There is more than one "correct" way, and you get to choose the most "useful" way for the data that you are describing
3. It is possible to invent new ways of describing data, all of the ways we discuss were previously invented by other people, and they are commonly used because they are useful.
4. Describing data is necessary because there is usually too much of it, so it doesn't make any sense by itself.

2.1 This is what too many numbers looks like

Let's say you wanted to know how happy people are. So, you ask thousands of people on the street how happy they are. You let them pick any number they want from negative infinity to positive infinity. Then you record all the numbers. Now what?

Well, how about you look at the numbers and see if that helps you determine anything about how happy people are. What could the numbers look like. Perhaps something like this:

-193	481	399	-142	-23	469	557	-210	-291	-147
274	398	236	139	-247	-755	-587	-155	76	-305
243	182	401	-227	-193	386	427	-765	-568	87
158	-346	521	679	-840	245	-213	773	643	59
152	-410	-378	537	-491	-144	-605	421	12	62
53	-72	706	-594	620	639	298	-503	-111	-1065
676	-377	-67	498	722	403	-41	-143	32	-602
515	-615	-1053	-111	-102	-156	540	304	787	828
25	-92	86	975	170	6	-729	522	-342	921
-742	-732	-92	114	175	-121	116	178	285	1092
157	408	-535	201	0	74	-339	81	441	359
-184	-707	264	28	235	472	-460	1011	73	-279
146	352	44	17	975	-178	549	373	103	-243
75	562	485	691	2	-525	637	600	-110	888
-282	3	300	-193	490	-386	-226	1298	-374	-277
172	-184	-627	-584	-704	154	-387	-318	271	-167
714	-820	474	-48	-377	89	152	-233	-191	26
168	353	188	-938	432	363	-452	-397	-437	333
554	-396	276	-66	-173	-353	-620	32	-398	261
354	793	1220	767	408	-50	-598	-183	541	184
-4	571	-527	93	-845	678	-599	587	-239	149
336	610	880	1524	73	345	560	400	273	-418
275	292	115	-111	108	349	-681	-867	104	-862
-1016	241	413	-521	120	139	-121	-167	1157	825
-158	-566	-947	-206	444	-140	335	98	508	212
20	-50	331	515	-425	706	491	241	-764	-320
389	487	-311	78	391	733	994	630	178	379
711	227	258	396	-498	421	572	183	-1071	261
-188	251	264	-904	-30	-159	360	184	212	21
308	776	212	446	-266	954	281	541	178	-125
-975	-416	-891	61	-345	-267	-144	385	142	-620
379	-423	559	363	-189	-208	-133	-115	-892	-241
-823	-189	227	-361	-40	-547	-198	423	-228	87
643	262	-426	80	199	437	-346	369	797	328
95	248	513	-463	1051	252	156	398	81	146
582	752	-985	441	235	1089	232	206	229	-847
-357	233	457	578	-124	-16	-123	-190	-288	-189
-325	-1268	86	249	319	-199	394	921	-39	-90
426	922	293	-706	241	184	329	-8	-436	694
-676	-222	-356	-655	1169	209	4	-34	219	-272
-788	789	444	968	745	-190	-835	-106	-111	746
229	389	-1215	792	274	223	1124	-552	86	554
-155	-367	-122	-46	-22	-77	627	771	268	-389
325	208	583	-300	51	-925	156	576	-1219	142
471	489	-267	957	432	-197	-125	788	81	-48
-10	-618	-357	531	437	-384	-427	-800	312	-566
-73	392	-167	-100	-235	277	-229	397	164	-869
-502	-301	-429	-23	154	-515	-450	93	128	191
149	101	630	459	-584	-287	295	964	-45	384
22	-53	558	10	-115	-455	588	-179	-751	856

Now, what are you going to with that big pile of numbers? Look at it all day long? When you deal with data, it will deal so many numbers to you that you will be overwhelmed by them. That is why we need ways to describe the data in a more manageable fashion.

The complete description of the data is always the data itself. **Descriptive statistics** and other tools for describing data go one step further to summarize aspects of the data. Summaries are a way to compress the important bits of a thing down to a useful and manageable tidbit. It's like telling your friends why they should watch a movie: you don't replay the entire movie for them, instead you hit the highlights. Summarizing the data is just like a movie preview, only for data.

2.2 Look at the data

We already tried one way of looking at the numbers, and it wasn't useful. Let's look at some other ways of looking at the numbers, using graphs.

2.2.1 Plot the data

Let's turn all of the numbers into dots, then show them in a graph. Note, when we do this, we have not yet summarized anything about the data. Instead, we just look at all of the data in a visual format, rather than looking at the numbers.

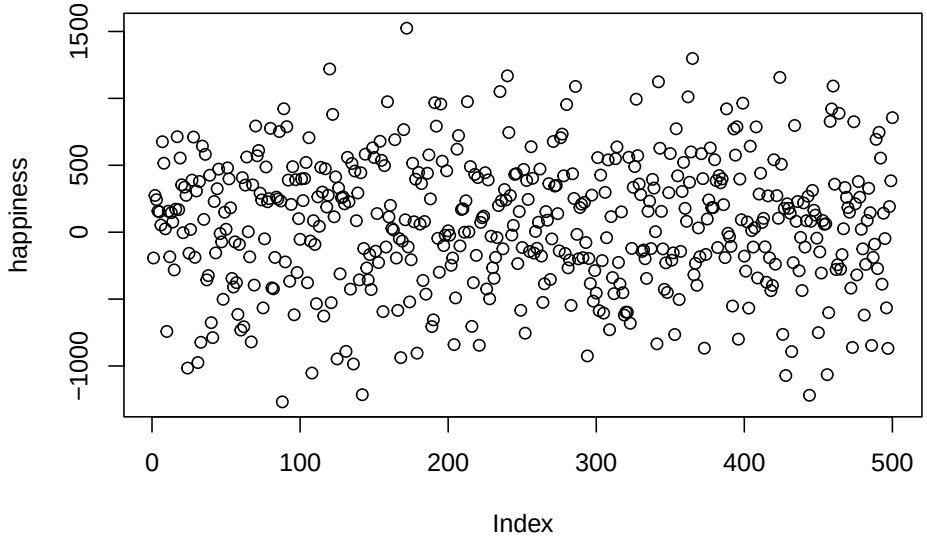


Figure 2.1: Pretend happiness ratings from 500 people

Figure **??** shows 500 measurements of happiness. The graph has two axes. The

horizontal **x-axis**, going from left to right is labeled "Index". The vertical **y-axis**, going up and down, is labelled "happiness". Each dot represents one measurement of every person's happiness from our pretend study. Before we talk about what we can and cannot see about the data, it is worth mentioning that the way you plot the data will make some things easier to see and some things harder to see. So, what can we now see about the data?

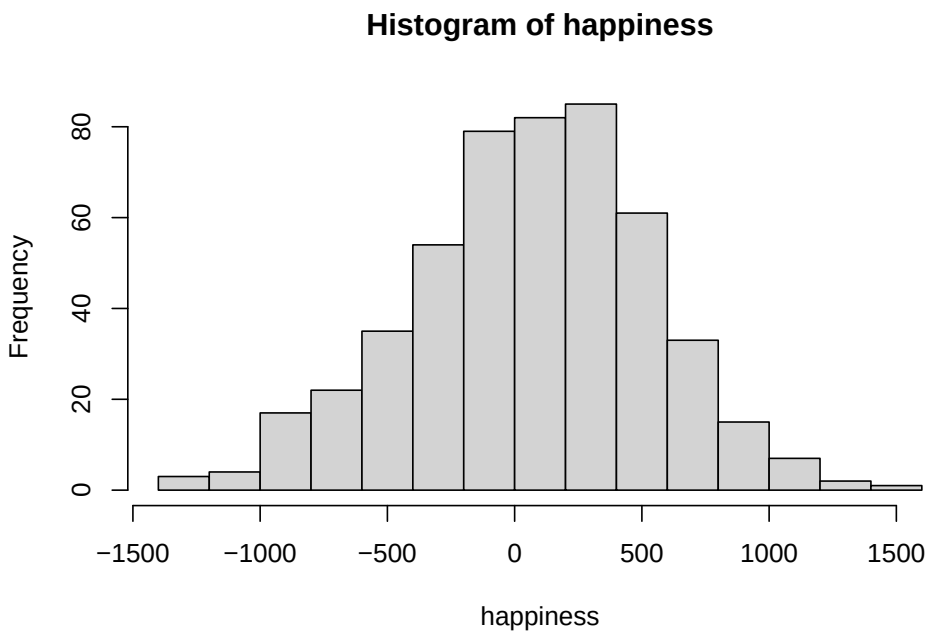
There are lots of dots everywhere. It looks like there are 500 of them because the index goes to 500. It looks like some dots go as high as 1000-1500 and as low as -1500. It looks like there are more dots in the middle-ish area of the plot, sort of spread about 0.

> Take home: we can see all the numbers at once by putting them in a plot, and that is much easier and more helpful than looking at the raw numbers.

OK, so if these dots represent how happy 500 people are, what can we say about those people? First, the dots are kind of all over the place, so different people have different levels of happiness. Are there any trends? Are more people happy than unhappy, or vice-versa? It's hard to see that in the graph, so let's make a different one, called a **histogram**

2.2.2 Histograms

Making a histogram will be our first act of officially summarizing something about the data. We will no longer look at the individual bits of data, instead we will see how the numbers group together. Let's look at a histogram of the happiness data, and then explain it.



The dots have disappeared, and now we some bars. Each bar is a summary of the dots, representing the number of dots (frequency count) inside a particular range of happiness, also called **bins**. For example, how many people gave a happiness rating between 0 and 500? The fifth bar, the one between 0 and 500 on the x-axis, tells you how many. Look how tall that bar is. How tall is it? The height is shown on the y-axis, which provides a frequency count (the number of dots or data points). It looks like around 150 people said their happiness was between 0-500.

More generally, we see there are many bins on the x-axis. We have divided the data into bins of 500. Bin #1 goes from -2000 to -1500, bin #2 goes from -1500 to -1000, and so on until the last bin. To make the histogram, we just count up the number of data points falling inside each bin, then plot those frequency counts as a function of the bins. Voila, a histogram.

What does the histogram help us see about the data? First, we can see the **shape** of data. The shape of the histogram refers to how it goes up and down. The shape tells us where the data is. For example, when the bars are low we know there isn't much data there. When the bars are high, we know there is more data there. So, where is most of the data? It looks like it's mostly in the middle two bins, between -500 and 500. We can also see the **range** of the data. This tells us the minimums and the maximums of the data. Most of the data is between -1500 and +1500, so no infinite sadness or infinite happiness in our data-set.

When you make a histogram you get to choose how wide each bar will be. For example, below are four different histograms of the very same happiness data.

What changes is the width of the bins.

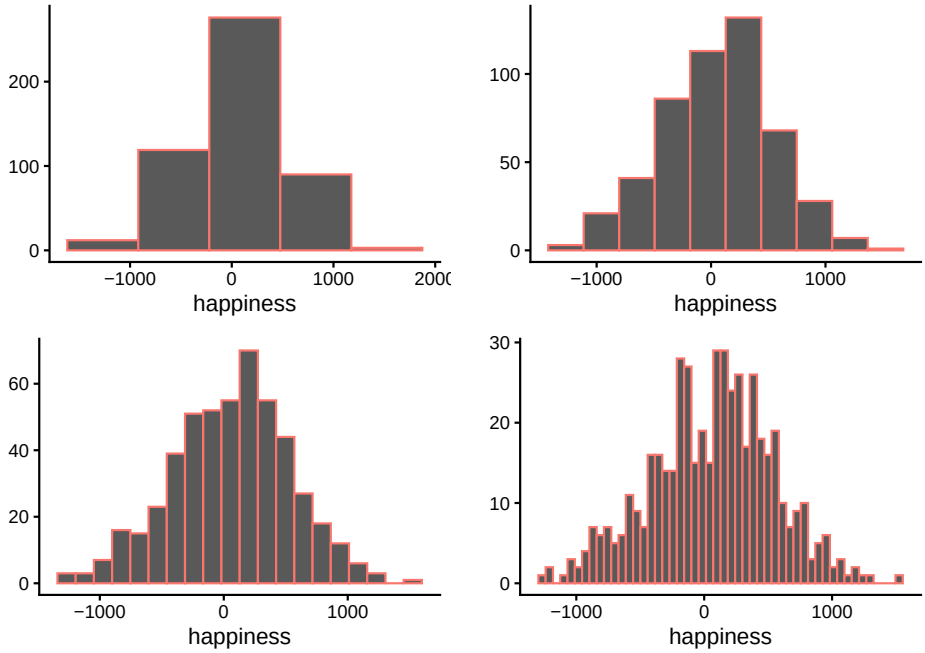


Figure 2.2: Four histograms of the same data using different bin widths

All of the histograms have roughly the same overall shape: From left to right, the bars start off small, then go up, then get small again. In other words, as the numbers get closer to zero, they start to occur more frequently. We see this general trend across all the histograms. But, some aspects of the trend fall apart when the bars get really narrow. For example, although the bars generally get taller when moving from -1000 to 0, there are some exceptions and the bars seem to fluctuate a little bit. When the bars are wider, there are less exceptions to the general trend. How wide or narrow should your histogram be? It's a Goldilocks question. Make it just right for your data.

2.3 Important Ideas: Distribution, Central Tendency, and Variance

Let's introduce three important terms we will use a lot, **distribution**, **central tendency**, and **variance**. These terms are similar to their everyday meanings (although I suspect most people don't say central tendency very often).

Distribution. When you order something from Amazon, where does it come from, and how does it get to your place? That stuff comes from one of Amazon's distribution centers. They distribute all sorts of things by spreading them

around to your doorstep. "To Distribute" " is to spread something. Notice, the data in the histogram is distributed, or spread across the bins. We can also talk about a distribution as a noun. The histogram is a distribution of the frequency counts across the bins. Distributions are **very, very, very, very, very** important. They can have many different shapes. They can describe data, like in the histogram above. And as we will learn in later chapters, they can **produce** data. Many times we will be asking questions about where our data came from, and this usually means asking what kind of distribution could have created our data (more on that later.)

Central Tendency is all about sameness: What is common about some numbers? For example, is there anything similar about all of the numbers in the histogram? Yes, we can say that most of them are near 0. There is a tendency for most of the numbers to be centered near 0. Notice we are being cautious about our generalization about the numbers. We are not saying they are all 0. We are saying there is a tendency for many of them to be near zero. There are lots of ways to talk about the central tendency of some numbers. There can even be more than one kind of tendency. For example, if lots of the numbers were around -1000, and a similar large amount of numbers were grouped around 1000, we could say there was two tendencies.

Variance is all about differentness: What is different about some numbers?. For example, is there anything different about all of the numbers in the histogram? YES!!! The numbers are not all the same! When the numbers are not all the same, they must vary. So, the variance in the numbers refers to how the numbers are different. There are many ways to summarize the amount of variance in the numbers, and we discuss these very soon.

2.4 Measures of Central Tendency (Sameness)

We've seen that we can get a sense of data by plotting dots in a graph, and by making a histogram. These tools show us what the numbers look like, approximately how big and small they are, and how similar and different they are from another. It is good to get a feeling about the numbers in this way. But, these visual sensitudes are not very precise. In addition to summarizing numbers with graphs, we can summarize numbers using numbers (NO, please not more numbers, we promise numbers can be your friend).

2.4.1 From many numbers to one

Measures of central tendency have one important summary goal: to reduce a pile of numbers to a single number that we can look at. We already know that looking at thousands of numbers is hopeless. Wouldn't it be nice if we could just look at one number instead? We think so. It turns out there are lots of ways to do this. Then, if your friend ever asks the frightening question, "hey, what are all these numbers like?". You can say they are like this one number

right here.

But, just like in Indiana Jones and the Last Crusade (highly recommended movie), you must choose your measure of central tendency wisely.

2.4.2 Mode

The **mode** is the most frequently occurring number in your measurement. That is it. How do you find it? You have to count the number of times each number appears in your measure, then whichever one occurs the most, is the mode.

Example: 1 1 1 2 3 4 5 6

The mode of the above set is 1, which occurs three times. Every other number only occurs once.

OK fine. What happens here:

Example: 1 1 1 2 2 2 3 4 5 6

Hmm, now 1 and 2 both occur three times each. What do we do? We say there are two modes, and they are 1 and 2.

Why is the mode a measure of central tendency? Well, when we ask, "what are my numbers like", we can say, "most of the number are, like a 1 (or whatever the mode is)".

Is the mode a good measure of central tendency? That depends on your numbers. For example, consider these numbers

1 1 2 3 4 5 6 7 8 9

Here, the mode is 1 again, because there are two 1s, and all of the other numbers occur once. But, are most of the numbers like, a 1. No, they are mostly not 1s.

"Argh, so should I or should I not use the mode? I thought this class was supposed to tell me what to do?". There is no telling you what to do. Every time you use a tool in statistics you have to think about what you are doing and justify why what you are doing makes sense. Sorry.

2.4.3 Median

The **median** is the exact middle of the data. After all, we are asking about central tendency, so why not go to the center of the data and see where we are. What do you mean middle of the data? Let's look at these numbers:

1 5 4 3 6 7 9

Umm, OK. So, three is in the middle? Isn't that kind of arbitrary. Yes. Before we can compute the median, we need to order the numbers from smallest to largest.

1 3 4 **5** 6 7 9

Now, 5 is in the middle. And, by middle we mean in the middle. There are three numbers to the left of 5, and three numbers to the right. So, five is definitely in the middle.

OK fine, but what happens when there aren't an even number of numbers? Then the middle will be missing right? Let's see:

1 2 3 4 5 6

There is no number between 3 and 4 in the data, the middle is empty. In this case, we compute the median by figuring out the number in between 3 and 4. So, the median would be 3.5.

Is the median a good measure of central tendency? Sure, it is often very useful. One property of the median is that it stays in the middle even when some of the other numbers get really weird. For example, consider these numbers:

1 2 3 4 4 4 **5** 6 6 6 7 7 1000

Most of these numbers are smallish, but the 1000 is a big old weird number, very different from the rest. The median is still 5, because it is in the middle of these ordered numbers. We can also see that five is pretty similar to most of the numbers (except for 1000). So, the median does a pretty good job of representing most of the numbers in the set, and it does so even if one or two of the numbers are very different from the others.

Finally, **outlier** is a term will we use to describe numbers that appear in data that are very different from the rest. 1000 is an outlier, because it lies way out there on the number line compared to the other numbers. What to do with outliers is another topic we discuss sometimes throughout this course.

2.4.4 Mean

Have you noticed this is a textbook about statistics that hasn't used a formula yet? That is about to change, but for those of you with formula anxiety, don't worry, we will do our best to explain them.

The **mean** is also called the average. And, we're guessing you might already now what the average of a bunch of numbers is? It's the sum of the numbers, divided by the number of number right? How do we express that idea in a formula? Just like this:

$$Mean = \bar{X} = \frac{\sum_{i=1}^{n} x_i}{N}$$

"That looks like Greek to me". Yup. The $\sum$ symbol is called **sigma**, and it stands for the operation of summing. The little "i" on the bottom, and the little "n" on the top refers to all of the numbers in the set, from the first number "i" to the last number "n". The letters are just arbitrary labels, called **variables** that we use for descriptive purposes. The x_i refers to individual numbers in the set. We sum up all of the numbers, then divide the sum by N, which is the

total number of numbers. Sometimes you will see $\bar{X}$ to refer to the mean of all of the numbers.

In plain English, the formula looks like:

$mean = \frac{\text{Sum of my numbers}}{\text{Count of my numbers}}$

"Well, why didn't you just say that?". We just did.

Let's compute the mean for these five numbers:

3 7 9 2 6

Add em up:

3+7+9+2+6 = 27

Count em up:

$i_1 = 3$, $i_2 = 7$, $i_3 = 9$, $i_4 = 2$, $i_5 = 6$; N=5, because i went from 1 to 5

Divide em:

mean = 27 / 5 = 5.4

Or, to put the numbers in the formula, it looks like this:

$Mean = \bar{X} = \frac{\sum_{i=1}^{n} x_i}{N} = \frac{3+7+9+2+6}{5} = \frac{27}{5} = 5.4$

OK fine, that is how to compute the mean. But, like we imagined, you probably already knew that, and if you didn't that's OK, now you do. What's next?

Is the mean a good measure of central tendency? By now, you should know: it depends.

2.4.5 What does the mean mean?

It is not enough to know the formula for the mean, or to be able to use the formula to compute a mean for a set of numbers. We believe in your ability to add and divide numbers. What you really need to know is what the mean really "means". This requires that you know what the mean does, and not just how to do it. Puzzled? Let's explain.

Can you answer this question: What happens when you divide a sum of numbers by the number of numbers? What are the consequences of doing this? What is the formula doing? What kind of properties does the result give us? FYI, the answer is not that we compute the mean.

OK, so what happens when you divide any number by another number? Of course, the key word here is divide. We literally carve the number up top in the numerator into pieces. How many times do we split the top number? That depends on the bottom number in the denominator. Watch:

$\frac{12}{3} = 4$

So, we know the answer is 4. But, what is really going on here is that we are slicing and dicing up 12 aren't we. Yes, and we slicing 12 into three parts. It turns out the size of those three parts is 4. So, now we are thinking of 12 as three different pieces $12 = 4 + 4 + 4$. I know this will be obvious, but what kind of properties do our pieces have? You mean the fours? Yup. Well, obviously they are all fours. Yes. The pieces are all the same size. They are all equal. So, division equalizes the numerator by the denominator…

"Umm, I think I learned this in elementary school, what does this have to do with the mean?". The number on top of the formula for the mean is just another numerator being divided by a denominator isn't it. In this case, the numerator is a sum of all the values in your data. What if it was the sum of all of the 500 happiness ratings? The sum of all of them would just be a single number adding up all the different ratings. If we split the sum up into equal parts representing one part for each person's happiness what would we get? We would get 500 identical and equal numbers for each person. It would be like taking all of the happiness in the world, then dividing it up equally, then to be fair, giving back the same equal amount of happiness to everyone in the world. This would make some people more happy than they were before, and some people less happy right. Of course, that's because it would be equalizing the distribution of happiness for everybody. This process of equalization by dividing something into equal parts is what the **mean** does. See, it's more than just a formula. It's an idea. This is just the beginning of thinking about these kinds of ideas. We will come back to this idea about the mean, and other ideas, in later chapters.

> Pro tip: The mean is the one and only number that can take the place of every number in the data, such that when you add up all the equal parts, you get back the original sum of the data.

2.4.6 All together now

Just to remind ourselves of the mode, median, and mean, take a look at the next histogram. We have overlaid the location of the mean (red), median (green), and mode (blue). For this dataset, the three measures of central tendency all give different answers. The mean is the largest because it is influenced by large numbers, even if they occur rarely. The mode and median are insensitive to large numbers that occur infrequently, so they have smaller values.

2.5 Measures of Variation (Differentness)

What did you do when you wrote essayss in high school about a book you read? Probably compare and contrast something right? When you summarize data, you do the same thing. Measures of central tendency give us something like comparing does, they tell us stuff about what is the same. Measures of

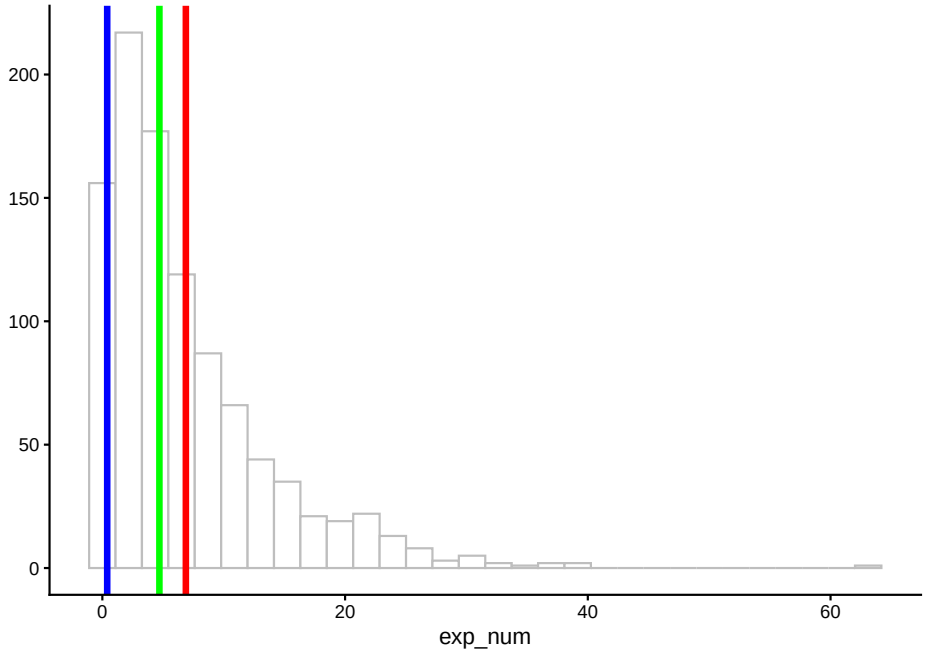


Figure 2.3: A histogram with the mean (red), the median (green), and the mode (blue)

variation give us something like contrasting does, they tell us stuff about what is different.

First, we note that whenever you see a bunch of numbers that aren't the same, you already know there are some differences. This means the numbers vary, and there is variation in the size of the numbers.

2.5.1 The Range

Consider these 10 numbers, that I already ordered from smallest to largest for you:

1 3 4 5 5 6 7 8 9 24

The numbers have variation, because they are not all the same. We can use the range to describe the width of the variation. The range refers to the **minimum** (smallest value) and **maximum** (largest value) in the set. So, the range would be 1 and 24.

The range is a good way to quickly summarize the boundaries of your data in just two numbers. By computing the range we know that none of the data is larger or smaller than the range. And, it can alert you to outliers. For example, if you are expecting your numbers to be between 1 and 7, but you find the range is 1 - 340,500, then you know you have some big numbers that shouldn't be there, and then you can try to figure out why those numbers occurred (and potentially remove them if something went wrong).

2.5.2 The Difference Scores

It would be nice to summarize the amount of differentness in the data. Here's why. If you thought that raw data (lots of numbers) is too big to look at, then you will be frightened to contemplate how many differences there are to look at. For example, these 10 numbers are easy to look at:

1 3 4 5 5 6 7 8 9 24

But, what about the difference between the numbers, what do those look like? We can compute the difference scores between each number, then put them in a matrix like the one below:

	1	3	4	5	5	6	7	8	9	24
1	0	2	3	4	4	5	6	7	8	23
3	-2	0	1	2	2	3	4	5	6	21
4	-3	-1	0	1	1	2	3	4	5	20
5	-4	-2	-1	0	0	1	2	3	4	19
5	-4	-2	-1	0	0	1	2	3	4	19
6	-5	-3	-2	-1	-1	0	1	2	3	18
7	-6	-4	-3	-2	-2	-1	0	1	2	17
8	-7	-5	-4	-3	-3	-2	-1	0	1	16
9	-8	-6	-5	-4	-4	-3	-2	-1	0	15
24	-23	-21	-20	-19	-19	-18	-17	-16	-15	0

We are looking at all of the possible differences between each number and every other number. So, in the top left, the difference between 1 and itself is 0. One column over to the right, the difference between 3 and 1 (3-1) is 2, etc. As you can see, this is a 10x10 matrix, which means there are 100 differences to look at. Not too bad, but if we had 500 numbers, then we would have 500*500 = 250,000 differences to look at (go for it if you like looking at that sort of thing).

Pause for a simple question. What would this matrix look like if all of the 10 numbers in our data were the same number? It should look like a bunch of 0s right? Good. In that case, we could easily see that the numbers have no variation.

But, when the numbers are different, we can see that there is a very large matrix of difference scores. How can we summarize that? How about we apply what we learned from the previous section on measures of central tendency. We have a lot of differences, so we could ask something like, what is the average difference that we have? So, we could just take all of our differences, and compute the mean difference right? What do you think would happen if we did that?

Let's try it out on these three numbers:

1 2 3

	1	2	3
1	0	1	2
2	-1	0	1
3	-2	-1	0

You might already guess what is going to happen. Let's compute the mean:

mean of difference scores = $\frac{0+1+2-1+0+1-2-1+0}{9} = \frac{0}{9} = 0$

Uh oh, we get zero for the mean of the difference scores. This will always happen whenever you take the mean of the difference scores. We can see that there are some differences between the numbers, so using 0 as the summary value for the variation in the numbers doesn't make much sense.

Furthermore, you might also notice that the matrices of difference scores are

redundant. The diagonal is always zero, and numbers on one side of the diagonal are the same as the numbers on the other side, except their signs are reversed. So, that's one reason why the difference scores add up to zero.

These are little problems that can be solved by computing the **variance** and the **standard deviation**. For now, the standard deviation is a just a trick that we use to avoid getting a zero. But, later we will see it has properties that are important for other reasons.

2.5.3 The Variance

Variability, variation, variance, vary, variable, varying, variety. Confused yet? Before we describe **the variance**, we want to you be OK with how this word is used. First, don't forget the big picture. We know that variability and variation refers to the big idea of differences between numbers. We can even use the word variance in the same way. When numbers are different, they have variance.

The formulas for variance and standard deviation depend on whether you think your data represents an entire population of numbers, or is sample from the population. We discuss this issue in later on. For now, we divide by N, later we discuss why you will often divide by N-1 instead.

The word **variance** also refers to a specific summary statistic, the sum of the squared deviations from the mean. Hold on what? Plain English please. The variance is the sum of the squared difference scores, where the difference scores are computed between each score and the mean. What are these scores? The scores are the numbers in the data set. Let's see the formula in English first:

$$variance = \frac{\text{Sum of squared difference scores}}{\text{Number of Scores}}$$

2.5.3.1 Deviations from the mean, Difference scores from the mean

We got a little bit complicated before when we computed the difference scores between all of the numbers in the data. Let's do it again, but in a more manageable way. This time, we calculate the difference between each score and the mean. The idea here is

1. We can figure out how similar our scores are by computing the mean
2. Then we can figure out how different our scores are from the mean

This could tell us, 1) something about whether our scores are really all very close to the mean (which could help us know if the mean is good representative number of the data), and 2) something about how much differences there are in the numbers.

Take a look at this table:

scores	values	mean	Difference_from_Mean
1	1	4.5	-3.5
2	6	4.5	1.5
3	4	4.5	-0.5
4	2	4.5	-2.5
5	6	4.5	1.5
6	8	4.5	3.5
Sums	27	27	0
Means	4.5	4.5	0

The first column shows we have 6 scores in the data set, and the `value` columns shows each score. The sum of the values, and the mean is presented on the last two rows. The sum and the mean were obtained by:

$\frac{1+6+4+2+6+8}{6} = \frac{27}{6} = 4.5$.

The third column `mean`, appears a bit silly. We are just listing the mean once for every score. If you think back to our discussion about the meaning of the mean, then you will remember that it equally distributes the total sum across each data point. We can see that here, if we treat each score as the mean, then every score is a 4.5. We can also see that adding up all of the means for each score gives us back 27, which is the sum of the original values. Also, we see that if we find the mean of the mean scores, we get back the mean (4.5 again).

All of the action is occurring in the fourth column, `Difference_from_Mean`. Here, we are showing the difference scores from the mean, using $X_i - \bar{X}$. In other words, we subtracted the mean from each score. So, the first score, 1, is -3.5 from the mean, the second score, 6, is +1.5 from the mean, and so on.

Now, we can look at our original scores and we can look at their differences from the mean. Notice, we don't have a matrix of raw difference scores, so it is much easier to look at. But, we still have a problem:

We can see that there are non-zero values in the difference scores, so we know there are a differences in the data. But, when we add them all up, we still get zero, which makes it seem like there are a total of zero differences in the data…Why does this happen…and what to do about it?

2.5.3.2 The mean is the balancing point in the data

One brief pause here to point out another wonderful property of the mean. It is the balancing point in the data. If you take a pen or pencil and try to balance it on your figure so it lays flat what are you doing? You need to find the center of mass in the pen, so that half of it is on one side, and the other half is on the other side. That's how balancing works. One side = the other side.

We can think of data as having mass or weight to it. If we put our data on our bathroom scale, we could figure out how heavy it was by summing it up. If we wanted to split the data down the middle so that half of the weight was equal

to the other half, then we could balance the data on top of a pin. The mean of the data tells you where to put the pin. It is the location in the data, where the numbers on the one side add up to the same sum as the numbers on the other side.

If we think this through, it means that the sum of the difference scores from the mean will always add up to zero. This is because the numbers on one side of the mean will always add up to -x (whatever the sum of those numbers is), and the numbers of the other side of the mean will always add up to +x (which will be the same value only positive). And:

$-x + x = 0$, right.

Right.

2.5.3.3 The squared deviations

Some devious someone divined a solution to the fact that differences scores from the mean always add to zero. Can you think of any solutions? For example, what could you do to the difference scores so that you could add them up, and they would weigh something useful, that is they would not be zero?

The devious solution is to square the numbers. Squaring numbers converts all the negative numbers to positive numbers. For example, $2^2 = 4$, and $-2^2 = 4$. Remember how squaring works, we multiply the number twice: $2^2 = 2 * 2 = 4$, and $-2^2 = -2 * -2 = 4$. We use the term **squared deviations** to refer to differences scores that have been squared. Deviations are things that move away from something. The difference scores move away from the mean, so we also call them **deviations**.

Let's look at our table again, but add the squared deviations.

scores	values	mean	Difference_from_Mean	Squared_Deviations
1	1	4.5	-3.5	12.25
2	6	4.5	1.5	2.25
3	4	4.5	-0.5	0.25
4	2	4.5	-2.5	6.25
5	6	4.5	1.5	2.25
6	8	4.5	3.5	12.25
Sums	27	27	0	35.5
Means	4.5	4.5	0	5.91666666666667

OK, now we have a new column called `squared_deviations`. These are just the difference scores squared. So, $-3.5^2 = 12.25$, etc. You can confirm for yourself with your cellphone calculator.

Now that all of the squared deviations are positive, we can add them up. When we do this we create something very special called the sum of squares (SS), also known as the sum of the squared deviations from the mean. We will talk at length about this SS later on in the ANOVA chapter. So, when you get there,

remember that you already know what it is, just some sums of some squared deviations, nothing fancy.

2.5.3.4 Finally, the variance

Guess what, we already computed the variance. It already happened, and maybe you didn't notice. "Wait, I missed that, what happened?".

First, see if you can remember what we are trying to do here. Take a pause, and see if you can tell yourself what problem we are trying solve.

pause

Without further ado, we are trying to get a summary of the differences in our data. There are just as many difference scores from the mean as there are data points, which can be a lot, so it would be nice to have a single number to look at, something like a mean, that would tell us about the average differences in the data.

If you look at the table, you can see we already computed the mean of the squared deviations. First, we found the sum (SS), then below that we calculated the mean = 5.916 repeating. This is **the variance**. The variance is the mean of the sum of the squared deviations:

$variance = \frac{SS}{N}$, where SS is the sum of the squared deviations, and N is the number of observations.

OK, now what. What do I do with the variance? What does this number mean? Good question. The variance is often an unhelpful number to look at. Why? Because it is not in the same scale as the original data. This is because we squared the difference scores before taking the mean. Squaring produces large numbers. For example, we see a 12.25 in there. That's a big difference, bigger than any difference between any two original values. What to do? How can we bring the numbers back down to their original unsquared size?

If you are thinking about taking the square root, that's a ding ding ding, correct answer for you. We can always unsquare anything by taking the square root. So, let's do that to 5.916. $\sqrt{5.916} = 2.4322829$.

2.5.4 The Standard Deviation

Oops, we did it again. We already computed the standard deviation, and we didn't tell you. The standard deviation is the square root of the variance...At least, it is right now, until we complicate matters for you in the next chapter.

Here is the formula for the standard deviation:

standard deviation $= \sqrt{Variance} = \sqrt{\frac{SS}{N}}$.

We could also expand this to say:

standard deviation $= \sqrt{\frac{\sum_i^n (x_i - \bar{x})^2}{N}}$

Don't let those big square root signs put you off. Now, you know what they are doing there. Just bringing our measure of the variance back down to the original size of the data. Let's look at our table again:

scores	values	mean	Difference_from_Mean	Squared_Deviations
1	1	4.5	-3.5	12.25
2	6	4.5	1.5	2.25
3	4	4.5	-0.5	0.25
4	2	4.5	-2.5	6.25
5	6	4.5	1.5	2.25
6	8	4.5	3.5	12.25
Sums	27	27	0	35.5
Means	4.5	4.5	0	5.91666666666667

We measured the standard deviation as 2.4322829. Notice this number fits right in the with differences scores from the mean. All of the scores are kind of in and around + or - 2.4322829. Whereas, if we looked at the variance, 5.916 is just too big, it doesn't summarize the actual differences very well.

What does all this mean? Well, if someone told they had some number with a mean of 4.5 (like the values in our table), and a standard deviation of 2.4322829, you would get a pretty good summary of the numbers. You would know that many of the numbers are around 4.5, and you would know that not all of the numbers are 4.5. You would know that the numbers spread around 4.5. You also know that the spread isn't super huge, it's only + or - 2.4322829 on average. That's a good starting point for describing numbers.

If you had loads of numbers, you could reduce them down to the mean and the standard deviation, and still be pretty well off in terms of getting a sense of those numbers.

2.6 Using Descriptive Statistics with data

Remember, you will be learning how to compute descriptive statistics using software in the labs. Check out the lab manual exercises for descriptives to see some examples of working with real data.

2.7 Rolling your own descriptive statistics

We spent many paragraphs talking about variation in numbers, and how to use calculate the **variance** and **standard deviation** to summarize the average differences between numbers in a data set. The basic process was to 1) calculate some measure of the differences, then 2) average the differences to create a summary. We found that we couldn't average the raw difference scores, because

we would always get a zero. So, we squared the differences from the mean, then averaged the squared differences differences. Finally, we square rooted our measure to bring the summary back down to the scale of the original numbers.

Perhaps you haven't heard, but there is more than one way to skin a cat, but we prefer to think of this in terms of petting cats, because some of us love cats. Jokes aside, perhaps you were also thinking that the problem of summing differences scores (so that they don't equal zero), can be solved in more than one way. Can you think of a different way, besides squaring?

2.7.1 Absolute deviations

How about just taking the absolute value of the difference scores. Remember, the absolute value converts any number to a positive value. Check out the following table:

scores	values	mean	Difference_from_Mean	Absolute_Deviations
1	1	4.5	-3.5	3.5
2	6	4.5	1.5	1.5
3	4	4.5	-0.5	0.5
4	2	4.5	-2.5	2.5
5	6	4.5	1.5	1.5
6	8	4.5	3.5	3.5
Sums	27	27	0	13
Means	4.5	4.5	0	2.16666666666667

This works pretty well too. By converting the difference scores from the mean to positive values, we can now add them up and get a non-zero value (if there are differences). Then, we can find the mean of the sum of the absolute deviations. If we were to map the terms sum of squares (SS), variance and standard deviation onto these new measures based off of the absolute deviation, how would the mapping go? For example, what value in the table corresponds to the SS? That would be the sum of absolute deviations in the last column. How about the variance and standard deviation, what do those correspond to? Remember that the variance is mean (SS/N), and the standard deviation is a square-rooted mean ($\sqrt{SS/N}$). In the table above we only have one corresponding mean, the mean of the sum of the absolute deviations. So, we have a **variance** measure that does not need to be square rooted. We might say the mean absolute deviation, is doing double-duty as a variance and a standard-deviation. Neat.

2.7.2 Other sign-inverting operations

In principle, we could create lots of different summary statistics for variance that solve the summing to zero problem. For example, we could raise every difference score to any even numbered power beyond 2 (which is the square). We could use, 4, 6, 8, 10, etc. There is an infinity of even numbers, so there is an infinity of possible variance statistics. We could also use odd numbers as powers,

and then take their absolute value. Many things are possible. The important aspect to any of this is to have a reason for what you are doing, and to choose a method that works for the data-analysis problem you are trying to solve. Note also, we bring up this general issue because we want you to understand that statistics is a creative exercise. We invent things when we need them, and we use things that have already been invented when they work for the problem at hand.

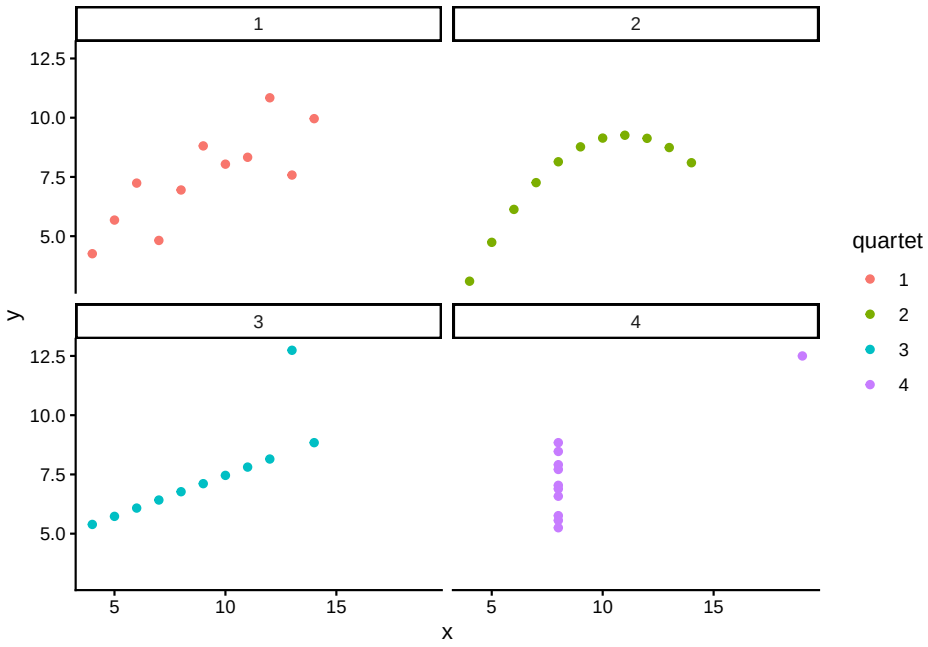
2.8 Remember to look at your data

Descriptive statistics are great and we will use them a lot in the course to describe data. You may suspect that descriptive statistics also have some shortcomings. This is very true. They are compressed summaries of large piles of numbers. They will almost always be unable to represent all of the numbers fairly. There are also different kinds of descriptive statistics that you could use, and it sometimes not clear which one's you should use.

Perhaps the most important thing you can do when using descriptives is to use them in combination with looking at the data in a graph form. This can help you see whether or not your descriptives are doing a good job of representing the data.

2.8.1 Anscombe's Quartet

To hit this point home, and to get you thinking about the issues we discuss in the next chapter, check this out. It's called Anscombe's Quartet, because these interesting graphs and numbers and numbers were produced by **?**. You are looking at pairs of measurements. Each graph has an X and Y axis, and each point represents two measurements. Each of the graphs looks very different, right?



Well, would you be surprised if I told that the descriptive statistics for the numbers in these graphs are exactly the same? It turns out they do have the same descriptive statistics. In the table below I present the mean and variance for the x-values in each graph, and the mean and the variance for the y-values in each graph.

quartet	mean_x	var_x	mean_y	var_y
1	9	11	7.500909	4.127269
2	9	11	7.500909	4.127629
3	9	11	7.500000	4.122620
4	9	11	7.500909	4.123249

The descriptives are all the same! Anscombe put these special numbers together to illustrate the point of graphing your numbers. If you only look at your descriptives, you don't know what patterns in the data they are hiding. If you look at the graph, then you can get a better understanding.

2.8.2 Datasaurus Dozen

If you thought that Anscombe's quartet was neat, you should take a look at the Datasaurus Dozen (**?**). Scroll down to see the examples. You will be looking at dot plots. The dot plots show many different patterns, including dinosaurs! What's amazing is that all of the dots have very nearly the same descriptive statistics. Just another reminder to look at your data, it might look like a dinosaur!

2.9 Videos

2.9.1 Measures of center: Mode

2.9.2 Measures of center: Median and Mean

2.9.3 Standard deviation part I

2.9.4 Standard deviation part II

Chapter 3

Probability, Sampling, and Estimation

I have studied many languages-French, Spanish and a little Italian, but no one told me that Statistics was a foreign language. —Charmaine J. Forde

Sections 3.1 & 3.9 - Adapted text by Danielle Navarro Section 3.10 - 3.11 & 3.13 - Mix of Matthew Crump & Danielle Navarro Section 3.12-3.13 - Adapted text by Danielle Navarro

Up to this point in the book, we've discussed some of the key ideas in experimental design, and we've talked a little about how you can summarize a data set. To a lot of people, this is all there is to statistics: it's about calculating averages, collecting all the numbers, drawing pictures, and putting them all in a report somewhere. Kind of like stamp collecting, but with numbers. However, statistics covers much more than that. In fact, descriptive statistics is one of the smallest parts of statistics, and one of the least powerful. The bigger and more useful part of statistics is that it provides tools **that let you make inferences about data**.

Once you start thinking about statistics in these terms – that statistics is there to help us draw inferences from data – you start seeing examples of it everywhere. For instance, here's a tiny extract from a newspaper article in the Sydney Morning Herald (30 Oct 2010):

> "I have a tough job," the Premier said in response to a poll which found her government is now the most unpopular Labor administration in polling history, with a primary vote of just 23 per cent.

This kind of remark is entirely unremarkable in the papers or in everyday life, but let's have a think about what it entails. A polling company has conducted a survey, usually a pretty big one because they can afford it. I'm too lazy to

track down the original survey, so let's just imagine that they called 1000 voters at random, and 230 (23%) of those claimed that they intended to vote for the party. For the 2010 Federal election, the Australian Electoral Commission reported 4,610,795 enrolled voters in New South Whales; so the opinions of the remaining 4,609,795 voters (about 99.98% of voters) remain unknown to us. Even assuming that no-one lied to the polling company the only thing we can say with 100% confidence is that the true primary vote is somewhere between 230/4610795 (about 0.005%) and 4610025/4610795 (about 99.83%). So, on what basis is it legitimate for the polling company, the newspaper, and the readership to conclude that the ALP primary vote is only about 23%?

The answer to the question is pretty obvious: if I call 1000 people at random, and 230 of them say they intend to vote for the ALP, then it seems very unlikely that these are the **only** 230 people out of the entire voting public who actually intend to do so. In other words, we assume that the data collected by the polling company is pretty representative of the population at large. But how representative? Would we be surprised to discover that the true ALP primary vote is actually 24%? 29%? 37%? At this point everyday intuition starts to break down a bit. No-one would be surprised by 24%, and everybody would be surprised by 37%, but it's a bit hard to say whether 29% is plausible. We need some more powerful tools than just looking at the numbers and guessing.

Inferential statistics provides the tools that we need to answer these sorts of questions, and since these kinds of questions lie at the heart of the scientific enterprise, they take up the lions share of every introductory course on statistics and research methods. However, our tools for making statistical inferences are 1) built on top of **probability theory**, and 2) require an understanding of how samples behave when you take them from distributions (defined by probability theory…). So, this chapter has two main parts. A brief introduction to probability theory, and an introduction to sampling from distributions.

3.1 How are probability and statistics different?

Before we start talking about probability theory, it's helpful to spend a moment thinking about the relationship between probability and statistics. The two disciplines are closely related but they're not identical. Probability theory is "the doctrine of chances". It's a branch of mathematics that tells you how often different kinds of events will happen. For example, all of these questions are things you can answer using probability theory:

- What are the chances of a fair coin coming up heads 10 times in a row?
- If I roll two six sided dice, how likely is it that I'll roll two sixes?
- How likely is it that five cards drawn from a perfectly shuffled deck will all be hearts?
- What are the chances that I'll win the lottery?

Notice that all of these questions have something in common. In each case the "truth of the world" is known, and my question relates to the "what kind of events" will happen. In the first question I **know** that the coin is fair, so there's a 50% chance that any individual coin flip will come up heads. In the second question, I **know** that the chance of rolling a 6 on a single die is 1 in 6. In the third question I **know** that the deck is shuffled properly. And in the fourth question, I **know** that the lottery follows specific rules. You get the idea. The critical point is that probabilistic questions start with a known *model* of the world, and we use that model to do some calculations.

The underlying model can be quite simple. For instance, in the coin flipping example, we can write down the model like this: $P(\text{heads}) = 0.5$ which you can read as "the probability of heads is 0.5".

As we'll see later, in the same way that percentages are numbers that range from 0% to 100%, probabilities are just numbers that range from 0 to 1. When using this probability model to answer the first question, I don't actually know exactly what's going to happen. Maybe I'll get 10 heads, like the question says. But maybe I'll get three heads. That's the key thing: in probability theory, the **model** is known, but the **data** are not.

So that's probability. What about statistics? Statistical questions work the other way around. In statistics, we know the truth about the world. All we have is the data, and it is from the data that we want to **learn** the truth about the world. Statistical questions tend to look more like these:

- If my friend flips a coin 10 times and gets 10 heads, are they playing a trick on me?
- If five cards off the top of the deck are all hearts, how likely is it that the deck was shuffled?
- If the lottery commissioner's spouse wins the lottery, how likely is it that the lottery was rigged?

This time around, the only thing we have are data. What I **know** is that I saw my friend flip the coin 10 times and it came up heads every time. And what I want to *infer* is whether or not I should conclude that what I just saw was actually a fair coin being flipped 10 times in a row, or whether I should suspect that my friend is playing a trick on me. The data I have look like this:

```
H H H H H H H H H H
```

and what I'm trying to do is work out which "model of the world" I should put my trust in. If the coin is fair, then the model I should adopt is one that says that the probability of heads is 0.5; that is, $P(\text{heads}) = 0.5$. If the coin is not fair, then I should conclude that the probability of heads is **not** 0.5, which we would write as $P(\text{heads}) \neq 0.5$. In other words, the statistical inference problem is to figure out which of these probability models is right. Clearly, the statistical question isn't the same as the probability question, but they're deeply connected

to one another. Because of this, a good introduction to statistical theory will start with a discussion of what probability is and how it works.

3.2 What does probability mean?

Let's start with the first of these questions. What is "probability"? It might seem surprising to you, but while statisticians and mathematicians (mostly) agree on what the **rules** of probability are, there's much less of a consensus on what the word really **means**. It seems weird because we're all very comfortable using words like "chance", "likely", "possible" and "probable", and it doesn't seem like it should be a very difficult question to answer. If you had to explain "probability" to a five year old, you could do a pretty good job. But if you've ever had that experience in real life, you might walk away from the conversation feeling like you didn't quite get it right, and that (like many everyday concepts) it turns out that you don't **really** know what it's all about.

So I'll have a go at it. Let's suppose I want to bet on a soccer game between two teams of robots, **Arduino Arsenal** and **C Milan**. After thinking about it, I decide that there is an 80% probability that **Arduino Arsenal** winning. What do I mean by that? Here are three possibilities…

- They're robot teams, so I can make them play over and over again, and if I did that, **Arduino Arsenal** would win 8 out of every 10 games on average.
- For any given game, I would only agree that betting on this game is only "fair" if a $1 bet on **C Milan** gives a $5 payoff (i.e. I get my $1 back plus a $4 reward for being correct), as would a $4 bet on **Arduino Arsenal** (i.e., my $4 bet plus a $1 reward).
- My subjective "belief" or "confidence" in an **Arduino Arsenal** victory is four times as strong as my belief in a **C Milan** victory.

Each of these seems sensible. However they're not identical, and not every statistician would endorse all of them. The reason is that there are different statistical ideologies (yes, really!) and depending on which one you subscribe to, you might say that some of those statements are meaningless or irrelevant. In this section, I give a brief introduction the two main approaches that exist in the literature. These are by no means the only approaches, but they're the two big ones.

3.2.1 The frequentist view

The first of the two major approaches to probability, and the more dominant one in statistics, is referred to as the *frequentist view*, and it defines probability as a *long-run frequency*. Suppose we were to try flipping a fair coin, over and over again. By definition, this is a coin that has $P(H) = 0.5$. What might we observe? One possibility is that the first 20 flips might look like this:

```
T,H,H,H,H,T,T,H,H,H,H,T,H,H,T,T,T,T,T,H
```

In this case 11 of these 20 coin flips (55%) came up heads. Now suppose that I'd been keeping a running tally of the number of heads (which I'll call N_H) that I've seen, across the first N flips, and calculate the proportion of heads N_H/N every time. Here's what I'd get (I did literally flip coins to produce this!):

number of flips	1	2	3	4	5	6	7	8	9	10
number of heads	0	1	2	3	4	4	4	5	6	7
proportion	.00	.50	.67	.75	.80	.67	.57	.63	.67	.70

number of flips	11	12	13	14	15	16	17	18	19	20
number of heads	8	8	9	10	10	10	10	10	10	11
proportion	.73	.67	.69	.71	.67	.63	.59	.56	.53	.55

Notice that at the start of the sequence, the **proportion** of heads fluctuates wildly, starting at .00 and rising as high as .80. Later on, one gets the impression that it dampens out a bit, with more and more of the values actually being pretty close to the "right" answer of .50. This is the frequentist definition of probability in a nutshell: flip a fair coin over and over again, and as N grows large (approaches infinity, denoted $N \rightarrow \infty$), the proportion of heads will converge to 50%. There are some subtle technicalities that the mathematicians care about, but qualitatively speaking, that's how the frequentists define probability. Unfortunately, I don't have an infinite number of coins, or the infinite patience required to flip a coin an infinite number of times. However, I do have a computer, and computers excel at mindless repetitive tasks. So I asked my computer to simulate flipping a coin 1000 times, and then drew a picture of what happens to the proportion N_H/N as N increases. Actually, I did it four times, just to make sure it wasn't a fluke. The results are shown in Figure **??**. As you can see, the **proportion of observed heads** eventually stops fluctuating, and settles down; when it does, the number at which it finally settles is the true probability of heads.

The frequentist definition of probability has some desirable characteristics. First, it is objective: the probability of an event is **necessarily** grounded in the world. The only way that probability statements can make sense is if they refer to (a sequence of) events that occur in the physical universe. Second, it is unambiguous: any two people watching the same sequence of events unfold, trying to calculate the probability of an event, must inevitably come up with the same answer.

However, it also has undesirable characteristics. Infinite sequences don't exist in the physical world. Suppose you picked up a coin from your pocket and started to flip it. Every time it lands, it impacts on the ground. Each impact wears the coin down a bit; eventually, the coin will be destroyed. So, one might ask

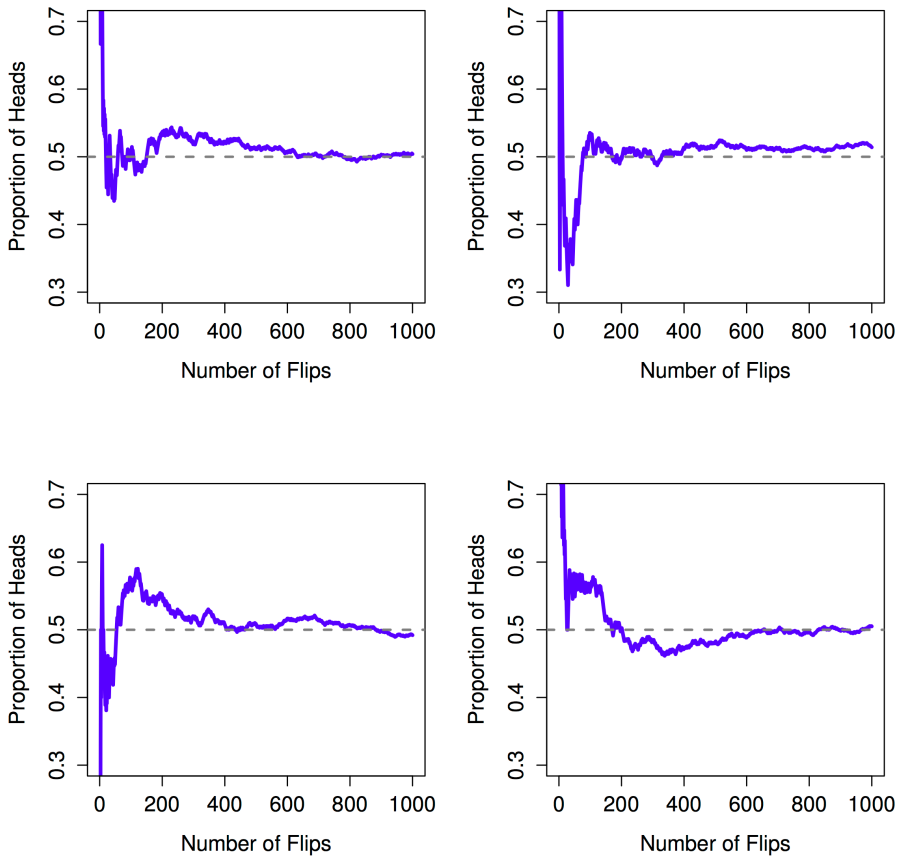


Figure 3.1: An illustration of how frequentist probability works. If you flip a fair coin over and over again, the proportion of heads that you've seen eventually settles down, and converges to the true probability of 0.5. Each panel shows four different simulated experiments: in each case, we pretend we flipped a coin 1000 times, and kept track of the proportion of flips that were heads as we went along. Although none of these sequences actually ended up with an exact value of .5, if we'd extended the experiment for an infinite number of coin flips they would have.

whether it really makes sense to pretend that an "infinite" sequence of coin flips is even a meaningful concept, or an objective one. We can't say that an "infinite sequence" of events is a real thing in the physical universe, because the physical universe doesn't allow infinite anything.

More seriously, the frequentist definition has a narrow scope. There are lots of things out there that human beings are happy to assign probability to in everyday language, but cannot (even in theory) be mapped onto a hypothetical sequence of events. For instance, if a meteorologist comes on TV and says, "the probability of rain in Adelaide on 2 November 2048 is 60%" we humans are happy to accept this. But it's not clear how to define this in frequentist terms. There's only one city of Adelaide, and only 2 November 2048. There's no infinite sequence of events here, just a once-off thing. Frequentist probability genuinely **forbids** us from making probability statements about a single event. From the frequentist perspective, it will either rain tomorrow or it will not; there is no "probability" that attaches to a single non-repeatable event. Now, it should be said that there are some very clever tricks that frequentists can use to get around this. One possibility is that what the meteorologist means is something like this: "There is a category of days for which I predict a 60% chance of rain; if we look only across those days for which I make this prediction, then on 60% of those days it will actually rain". It's very weird and counterintuitive to think of it this way, but you do see frequentists do this sometimes.

3.2.2 The Bayesian view

The **Bayesian view** of probability is often called the subjectivist view, and it is a minority view among statisticians, but one that has been steadily gaining traction for the last several decades. There are many flavours of Bayesianism, making hard to say exactly what "the" Bayesian view is. The most common way of thinking about subjective probability is to define the probability of an event as the **degree of belief** that an intelligent and rational agent assigns to that truth of that event. From that perspective, probabilities don't exist in the world, but rather in the thoughts and assumptions of people and other intelligent beings. However, in order for this approach to work, we need some way of operationalising "degree of belief". One way that you can do this is to formalise it in terms of "rational gambling", though there are many other ways. Suppose that I believe that there's a 60% probability of rain tomorrow. If someone offers me a bet: if it rains tomorrow, then I win $5, but if it doesn't rain then I lose $5. Clearly, from my perspective, this is a pretty good bet. On the other hand, if I think that the probability of rain is only 40%, then it's a bad bet to take. Thus, we can operationalise the notion of a "subjective probability" in terms of what bets I'm willing to accept.

What are the advantages and disadvantages to the Bayesian approach? The main advantage is that it allows you to assign probabilities to any event you want to. You don't need to be limited to those events that are repeatable. The main disadvantage (to many people) is that we can't be purely objective

– specifying a probability requires us to specify an entity that has the relevant degree of belief. This entity might be a human, an alien, a robot, or even a statistician, but there has to be an intelligent agent out there that believes in things. To many people this is uncomfortable: it seems to make probability arbitrary. While the Bayesian approach does require that the agent in question be rational (i.e., obey the rules of probability), it does allow everyone to have their own beliefs; I can believe the coin is fair and you don't have to, even though we're both rational. The frequentist view doesn't allow any two observers to attribute different probabilities to the same event: when that happens, then at least one of them must be wrong. The Bayesian view does not prevent this from occurring. Two observers with different background knowledge can legitimately hold different beliefs about the same event. In short, where the frequentist view is sometimes considered to be too narrow (forbids lots of things that that we want to assign probabilities to), the Bayesian view is sometimes thought to be too broad (allows too many differences between observers).

3.2.3 What's the difference? And who is right?

Now that you've seen each of these two views independently, it's useful to make sure you can compare the two. Go back to the hypothetical robot soccer game at the start of the section. What do you think a frequentist and a Bayesian would say about these three statements? Which statement would a frequentist say is the correct definition of probability? Which one would a Bayesian do? Would some of these statements be meaningless to a frequentist or a Bayesian? If you've understood the two perspectives, you should have some sense of how to answer those questions.

Okay, assuming you understand the different, you might be wondering which of them is **right**? Honestly, I don't know that there is a right answer. As far as I can tell there's nothing mathematically incorrect about the way frequentists think about sequences of events, and there's nothing mathematically incorrect about the way that Bayesians define the beliefs of a rational agent. In fact, when you dig down into the details, Bayesians and frequentists actually agree about a lot of things. Many frequentist methods lead to decisions that Bayesians agree a rational agent would make. Many Bayesian methods have very good frequentist properties.

For the most part, I'm a pragmatist so I'll use any statistical method that I trust. As it turns out, that makes me prefer Bayesian methods, for reasons I'll explain towards the end of the book, but I'm not fundamentally opposed to frequentist methods. Not everyone is quite so relaxed. For instance, consider Sir Ronald Fisher, one of the towering figures of 20th century statistics and a vehement opponent to all things Bayesian, whose paper on the mathematical foundations of statistics referred to Bayesian probability as "an impenetrable jungle [that] arrests progress towards precision of statistical concepts" **?**, p. 311. Or the psychologist Paul Meehl, who suggests that relying on frequentist methods could turn you into "a potent but sterile intellectual rake who leaves in his merry path

a long train of ravished maidens but no viable scientific offspring" **?**, p. 114. The history of statistics, as you might gather, is not devoid of entertainment.

3.3 Basic probability theory

Ideological arguments between Bayesians and frequentists notwithstanding, it turns out that people mostly agree on the rules that probabilities should obey. There are lots of different ways of arriving at these rules. The most commonly used approach is based on the work of Andrey Kolmogorov, one of the great Soviet mathematicians of the 20th century. I won't go into a lot of detail, but I'll try to give you a bit of a sense of how it works. And in order to do so, I'm going to have to talk about my pants.

3.3.1 Introducing probability distributions

One of the disturbing truths about my life is that I only own 5 pairs of pants: three pairs of jeans, the bottom half of a suit, and a pair of tracksuit pants. Even sadder, I've given them names: I call them X_1, X_2, X_3, X_4 and X_5. I really do: that's why they call me Mister Imaginative. Now, on any given day, I pick out exactly one of pair of pants to wear. Not even I'm so stupid as to try to wear two pairs of pants, and thanks to years of training I never go outside without wearing pants anymore. If I were to describe this situation using the language of probability theory, I would refer to each pair of pants (i.e., each X) as an *elementary event*. The key characteristic of elementary events is that every time we make an observation (e.g., every time I put on a pair of pants), then the outcome will be one and only one of these events. Like I said, these days I always wear exactly one pair of pants, so my pants satisfy this constraint. Similarly, the set of all possible events is called a *sample space*. Granted, some people would call it a "wardrobe", but that's because they're refusing to think about my pants in probabilistic terms. Sad.

Okay, now that we have a sample space (a wardrobe), which is built from lots of possible elementary events (pants), what we want to do is assign a *probability* of one of these elementary events. For an event X, the probability of that event $P(X)$ is a number that lies between 0 and 1. The bigger the value of $P(X)$, the more likely the event is to occur. So, for example, if $P(X) = 0$, it means the event X is impossible (i.e., I never wear those pants). On the other hand, if $P(X) = 1$ it means that event X is certain to occur (i.e., I always wear those pants). For probability values in the middle, it means that I sometimes wear those pants. For instance, if $P(X) = 0.5$ it means that I wear those pants half of the time.

At this point, we're almost done. The last thing we need to recognise is that "something always happens". Every time I put on pants, I really do end up wearing pants (crazy, right?). What this somewhat trite statement means, in probabilistic terms, is that the probabilities of the elementary events need to add

up to 1. This is known as the *law of total probability*, not that any of us really care. More importantly, if these requirements are satisfied, then what we have is a *probability distribution*. For example, this is an example of a probability distribution

Which pants?	Label	Probability
Blue jeans	X_1	$P(X_1) = .5$
Grey jeans	X_2	$P(X_2) = .3$
Black jeans	X_3	$P(X_3) = .1$
Black suit	X_4	$P(X_4) = 0$
Blue tracksuit	X_5	$P(X_5) = .1$

Each of the events has a probability that lies between 0 and 1, and if we add up the probability of all events, they sum to 1. Awesome. We can even draw a nice bar graph to visualise this distribution, as shown in Figure **??**. And at this point, we've all achieved something. You've learned what a probability distribution is, and I've finally managed to find a way to create a graph that focuses entirely on my pants. Everyone wins!

The only other thing that I need to point out is that probability theory allows you to talk about *non elementary events* as well as elementary ones. The easiest way to illustrate the concept is with an example. In the pants example, it's perfectly legitimate to refer to the probability that I wear jeans. In this scenario, the "Dan wears jeans" event said to have happened as long as the elementary event that actually did occur is one of the appropriate ones; in this case "blue jeans", "black jeans" or "grey jeans". In mathematical terms, we defined the "jeans" event E to correspond to the set of elementary events (X_1, X_2, X_3). If any of these elementary events occurs, then E is also said to have occurred. Having decided to write down the definition of the E this way, it's pretty straightforward to state what the probability $P(E)$ is: we just add everything up. In this particular case

$$P(E) = P(X_1) + P(X_2) + P(X_3)$$

and, since the probabilities of blue, grey and black jeans respectively are .5, .3 and .1, the probability that I wear jeans is equal to .9.

At this point you might be thinking that this is all terribly obvious and simple and you'd be right. All we've really done is wrap some basic mathematics around a few common sense intuitions. However, from these simple beginnings it's possible to construct some extremely powerful mathematical tools. I'm definitely not going to go into the details in this book, but what I will do is list some of the other rules that probabilities satisfy. These rules can be derived from the simple assumptions that I've outlined above, but since we don't actually use these rules for anything in this book, I won't do so here.

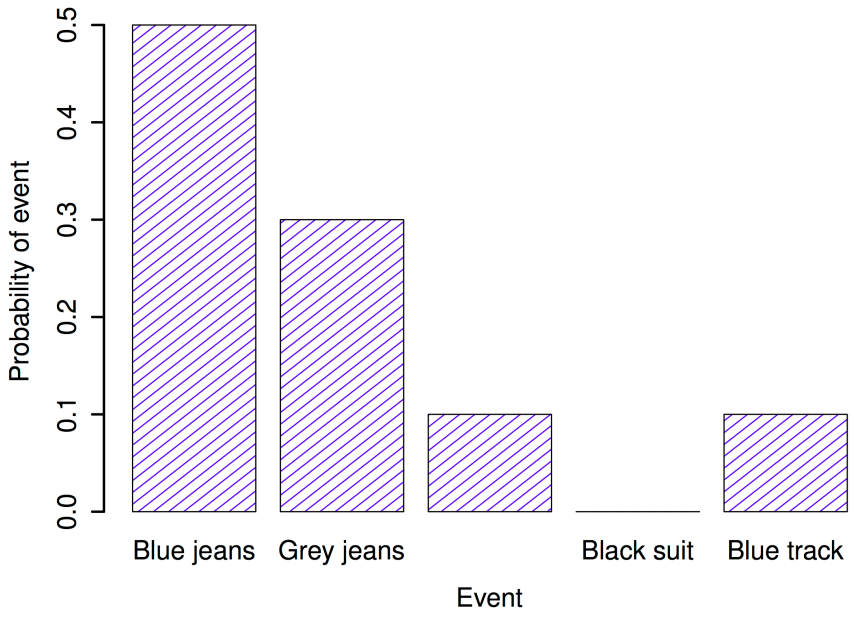


Figure 3.2: A visual depiction of the pants probability distribution. There are five elementary events, corresponding to the five pairs of pants that I own. Each event has some probability of occurring: this probability is a number between 0 to 1. The sum of these probabilities is 1

Table 3.4: Some basic rules that probabilities must satisfy. You don't really need to know these rules in order to understand the analyses that we'll talk about later in the book, but they are important if you want to understand probability theory a bit more deeply.

English	Notation	Formula
not A	$P(\neg A)$	$= 1 - P(A)$
A or B	$P(A \cup B)$	$= P(A) + P(B) - P(A \cap B)$
A and B	$P(A \cap B)$	$= P(A\|B)P(B)$

Now that we have the ability to "define" non-elementary events in terms of elementary ones, we can actually use this to construct (or, if you want to be all mathematicallish, "derive") some of the other rules of probability. These rules are listed above, and while I'm pretty confident that very few of my readers actually care about how these rules are constructed, I'm going to show you anyway: even though it's boring and you'll probably never have a lot of use for these derivations, if you read through it once or twice and try to see how it works, you'll find that probability starts to feel a bit less mysterious, and with any luck a lot less daunting. So here goes. Firstly, in order to construct the rules I'm going to need a sample space X that consists of a bunch of elementary events x, and two non-elementary events, which I'll call A and B. Let's say:

$$\begin{aligned} X &= (x_1, x_2, x_3, x_4, x_5) \\ A &= (x_1, x_2, x_3) \\ B &= (x_3, x_4) \end{aligned}$$

To make this a bit more concrete, let's suppose that we're still talking about the pants distribution. If so, A corresponds to the event "jeans", and B corresponds to the event "black":

$$\begin{aligned} \text{"jeans"} &= (\text{"blue jeans"}, \text{"grey jeans"}, \text{"black jeans"}) \\ \text{"black"} &= (\text{"black jeans"}, \text{"black suit"}) \end{aligned}$$

So now let's start checking the rules that I've listed in the table.

In the first line, the table says that

$$P(\neg A) = 1 - P(A)$$

and what it **means** is that the probability of "not A" is equal to 1 minus the probability of A. A moment's thought (and a tedious example) make it obvious why this must be true. If A coresponds to the even that I wear jeans (i.e., one of x_1 or x_2 or x_3 happens), then the only meaningful definitionof "not A" (which is mathematically denoted as $\neg A$) is to say that $\neg A$ consists of **all** elementary events that don't belong to A. In the case of the pants distribution it means that $\neg A = (x_4, x_5)$, or, to say it in English: "not jeans" consists of all pairs of pants

that aren't jeans (i.e., the black suit and the blue tracksuit). Consequently, every single elementary event belongs to either A or $\neg A$, but not both. Okay, so now let's rearrange our statement above:

$$P(\neg A) + P(A) = 1$$

which is a trite way of saying either I do wear jeans or I don't wear jeans: the probability of "not jeans" plus the probability of "jeans" is 1. Mathematically:

$$\begin{aligned} P(\neg A) &= P(x_4) + P(x_5) \\ P(A) &= P(x_1) + P(x_2) + P(x_3) \end{aligned}$$

so therefore

$$\begin{aligned} P(\neg A) + P(A) &= P(x_1) + P(x_2) + P(x_3) + P(x_4) + P(x_5) \\ &= \textstyle\sum_{x \in X} P(x) \\ &= 1 \end{aligned}$$

Excellent. It all seems to work.

Wow, I can hear you saying. That's a lot of xs to tell me the freaking obvious. And you're right: this **is** freaking obvious. The whole **point** of probability theory to to formalise and mathematise a few very basic common sense intuitions. So let's carry this line of thought forward a bit further. In the last section I defined an event corresponding to **not** A, which I denoted $\neg A$. Let's now define two new events that correspond to important everyday concepts: A **and** B, and A **or** B. To be precise:

English statement:	Mathematical notation:
"A and B" both happen	$A \cap B$
at least one of "A or B" happens	$A \cup B$

Since A and B are both defined in terms of our elementary events (the xs) we're going to need to try to describe $A \cap B$ and $A \cup B$ in terms of our elementary events too. Can we do this? Yes we can The only way that both A and B can occur is if the elementary event that we observe turns out to belong to both A and B. Thus "$A \cap B$" includes only those elementary events that belong to both A and B...

$$\begin{aligned} A &= (x_1, x_2, x_3) \\ B &= (x_3, x_4) \\ A \cap B &= (x_3) \end{aligned}$$

So, um, the only way that I can wear "jeans" (x_1, x_2, x_3) and "black pants" (x_3, x_4) is if I wear "black jeans" (x_3). Another victory for the bloody obvious.

At this point, you're not going to be at all shocked by the definition of $A \cup B$, though you're probably going to be extremely bored by it. The only way that I

can wear "jeans" or "black pants" is if the elementary pants that I actually do wear belongs to A or to B, or to both. So…

$$\begin{aligned} A &= (x_1, x_2, x_3) \\ B &= (x_3, x_4) \\ A \cup B &= (x_1, x_2, x_3, x_4) \end{aligned}$$

Oh yeah baby. Mathematics at its finest.

So, we've defined what we mean by $A \cap B$ and $A \cup B$. Now let's assign probabilities to these events. More specifically, let's start by verifying the rule that claims that:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Using our definitions earlier, we know that $A \cup B = (x_1, x_2, x_3, x_4)$, so

$$P(A \cup B) = P(x_1) + P(x_2) + P(x_3) + P(x_4)$$

and making similar use of the fact that we know what elementary events belong to A, B and $A \cap B$….

$$\begin{aligned} P(A) &= P(x_1) + P(x_2) + P(x_3) \\ P(B) &= P(x_3) + P(x_4) \\ P(A \cap B) &= P(x_3) \end{aligned}$$

and therefore

$$\begin{aligned} P(A) + P(B) - P(A \cap B) &= P(x_1) + P(x_2) + P(x_3) + P(x_3) + P(x_4) - P(x_3) \\ &= P(x_1) + P(x_2) + P(x_3) + P(x_4) \\ &= P(A \cup B) \end{aligned}$$

Done.

The next concept we need to define is the notion of "B given A", which is typically written $B|A$. Here's what I mean: suppose that I get up one morning, and put on a pair of pants. An elementary event x has occurred. Suppose further I yell out to my wife (who is in the other room, and so cannot see my pants) "I'm wearing jeans today!". Assuming that she believes that I'm telling the truth, she knows that A is true. **Given** that she knows that A has happened, what is the **conditional probability** that B is also true? Well, let's think about what she knows. Here are the facts:

- **The non-jeans events are impossible**. If A is true, then we know that the only possible elementary events that could have occurred are x_1, x_2 and x_3 (i.e.,the jeans). The non-jeans events x_4 and x_5 are now impossible, and must be assigned probability zero. In other words, our **sample space** has been restricted to the jeans events. But it's still the case that the probabilities of these these events **must** sum to 1: we know for sure that I'm wearing jeans.

- **She's learned nothing about which jeans I'm wearing**. Before I made my announcement that I was wearing jeans, she already knew that I was five times as likely to be wearing blue jeans ($P(x_1) = 0.5$) than to be wearing black jeans ($P(x_3) = 0.1$). My announcement doesn't change this... I said **nothing** about what colour my jeans were, so it must remain the case that $P(x_1)/P(x_3)$ stays the same, at a value of 5.

There's only one way to satisfy these constraints: set the impossible events to have zero probability (i.e., $P(x|A) = 0$ if x is not in A), and then divide the probabilities of all the others by $P(A)$. In this case, since $P(A) = 0.9$, we divide by 0.9. This gives:

which pants?	elementary event	old prob, $P(x)$	new prob, $P(x\|A)$
blue jeans	x_1	0.5	0.556
grey jeans	x_2	0.3	0.333
black jeans	x_3	0.1	0.111
black suit	x_4	0	0
blue tracksuit	x_5	0.1	0

In mathematical terms, we say that

$$P(x|A) = \frac{P(x)}{P(A)}$$

if $x \in A$, and $P(x|A) = 0$ otherwise. And therefore...

$$\begin{aligned} P(B|A) &= P(x_3|A) + P(x_4|A) \\ &= \frac{P(x_3)}{P(A)} + 0 \\ &= \frac{P(x_3)}{P(A)} \end{aligned}$$

Now, recalling that $A \cap B = (x_3)$, we can write this as

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

and if we multiply both sides by $P(A)$ we obtain:

$$P(A \cap B) = P(B|A)P(A)$$

which is the third rule that we had listed in the table.

3.4 The binomial distribution

As you might imagine, probability distributions vary enormously, and there's an enormous range of distributions out there. However, they aren't all equally important. In fact, the vast majority of the content in this book relies on one of five distributions: the binomial distribution, the normal distribution, the t distribution, the χ^2 ("chi-square") distribution and the F distribution. Given this, what I'll do over the next few sections is provide a brief introduction to all five of these, paying special attention to the binomial and the normal. I'll start with the binomial distribution, since it's the simplest of the five.

3.4.1 Introducing the binomial

The theory of probability originated in the attempt to describe how games of chance work, so it seems fitting that our discussion of the *binomial distribution* should involve a discussion of rolling dice and flipping coins. Let's imagine a simple "experiment": in my hot little hand I'm holding 20 identical six-sided dice. On one face of each die there's a picture of a skull; the other five faces are all blank. If I proceed to roll all 20 dice, what's the probability that I'll get exactly 4 skulls? Assuming that the dice are fair, we know that the chance of any one die coming up skulls is 1 in 6; to say this another way, the skull probability for a single die is approximately .167. This is enough information to answer our question, so let's have a look at how it's done.

As usual, we'll want to introduce some names and some notation. We'll let N denote the number of dice rolls in our experiment; which is often referred to as the *size parameter* of our binomial distribution. We'll also use θ to refer to the the probability that a single die comes up skulls, a quantity that is usually called the *success probability* of the binomial. Finally, we'll use X to refer to the results of our experiment, namely the number of skulls I get when I roll the dice. Since the actual value of X is due to chance, we refer to it as a *random variable*. In any case, now that we have all this terminology and notation, we can use it to state the problem a little more precisely. The quantity that we want to calculate is the probability that $X = 4$ given that we know that $\theta = .167$ and $N = 20$. The general "form" of the thing I'm interested in calculating could be written as

$$P(X \mid \theta, N)$$

and we're interested in the special case where $X = 4$, $\theta = .167$ and $N = 20$. There's only one more piece of notation I want to refer to before moving on to discuss the solution to the problem. If I want to say that X is generated randomly from a binomial distribution with parameters θ and N, the notation I would use is as follows:

$$X \sim \text{Binomial}(\theta, N)$$

Yeah, yeah. I know what you're thinking: notation, notation, notation. Really, who cares? Very few readers of this book are here for the notation, so I should

probably move on and talk about how to use the binomial distribution. I've included the formula for the binomial distribution in Table [tab:distformulas], since some readers may want to play with it themselves, but since most people probably don't care that much and because we don't need the formula in this book, I won't talk about it in any detail. Instead, I just want to show you what the binomial distribution looks like. To that end, Figure **??** plots the binomial probabilities for all possible values of X for our dice rolling experiment, from $X = 0$ (no skulls) all the way up to $X = 20$ (all skulls). Note that this is basically a bar chart, and is no different to the "pants probability" plot I drew in Figure **??**. On the horizontal axis we have all the possible events, and on the vertical axis we can read off the probability of each of those events. So, the probability of rolling 4 skulls out of 20 times is about 0.20 (the actual answer is 0.2022036, as we'll see in a moment). In other words, you'd expect that to happen about 20% of the times you repeated this experiment.

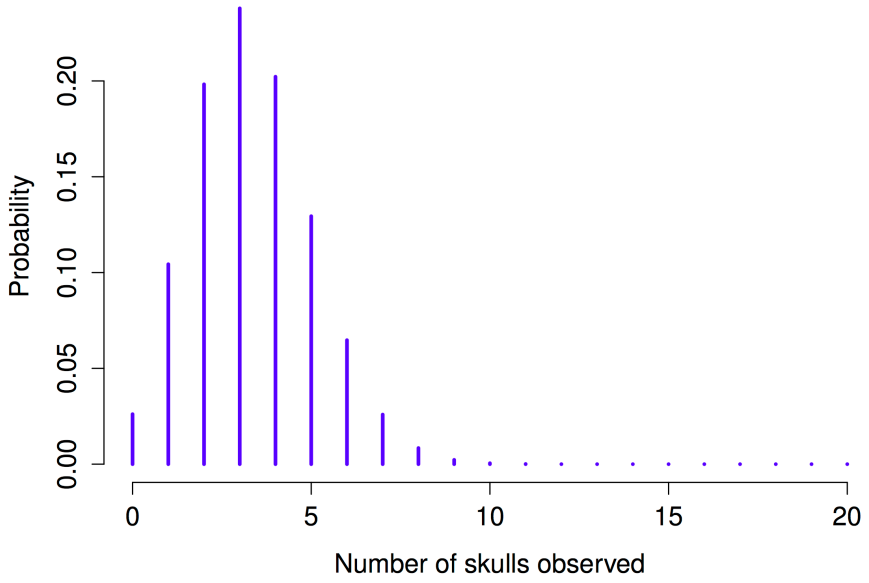


Figure 3.3: The binomial distribution with size parameter of N =20 and an underlying success probability of 1/6. Each vertical bar depicts the probability of one specific outcome (i.e., one possible value of X). Because this is a probability distribution, each of the probabilities must be a number between 0 and 1, and the heights of the bars must sum to 1 as well.

3.4.2 Working with the binomial distribution in R

R has a function called `dbinom` that calculates binomial probabilities for us. The main arguments to the function are

- `x` This is a number, or vector of numbers, specifying the outcomes whose probability you're trying to calculate.
- `size` This is a number telling R the size of the experiment.
- `prob` This is the success probability for any one trial in the experiment.

So, in order to calculate the probability of getting skulls, from an experiment of trials, in which the probability of getting a skull on any one trial is … well, the command I would use is simply this:

```
dbinom( x = 4, size = 20, prob = 1/6 )
```

```
## [1] 0.2022036
```

To give you a feel for how the binomial distribution changes when we alter the values of θ and N, let's suppose that instead of rolling dice, I'm actually flipping coins. This time around, my experiment involves flipping a fair coin repeatedly, and the outcome that I'm interested in is the number of heads that I observe. In this scenario, the success probability is now $\theta = 1/2$. Suppose I were to flip the coin $N = 20$ times. In this example, I've changed the success probability, but kept the size of the experiment the same. What does this do to our binomial distribution?

Well, as Figure **??**a shows, the main effect of this is to shift the whole distribution, as you'd expect. Okay, what if we flipped a coin $N = 100$ times? Well, in that case, we get Figure **??**b. The distribution stays roughly in the middle, but there's a bit more variability in the possible outcomes.

At this point, I should probably explain the name of the `dbinom` function. Obviously, the "binom" part comes from the fact that we're working with the binomial distribution, but the "d" prefix is probably a bit of a mystery. In this section I'll give a partial explanation: specifically, I'll explain why there is a prefix. As for why it's a "d" specifically, you'll have to wait until the next section. What's going on here is that R actually provides **four** functions in relation to the binomial distribution. These four functions are `dbinom`, `pbinom`, `rbinom` and `qbinom`, and each one calculates a different quantity of interest. Not only that, R does the same thing for **every** probability distribution that it implements. No matter what distribution you're talking about, there's a `d` function, a `p` function, `r` a function and a `q` function.

Let's have a look at what all four functions do. Firstly, all four versions of the function require you to specify the `size` and `prob` arguments: no matter what you're trying to get R to calculate, it needs to know what the parameters are. However, they differ in terms of what the other argument is, and what the output is. So let's look at them one at a time.

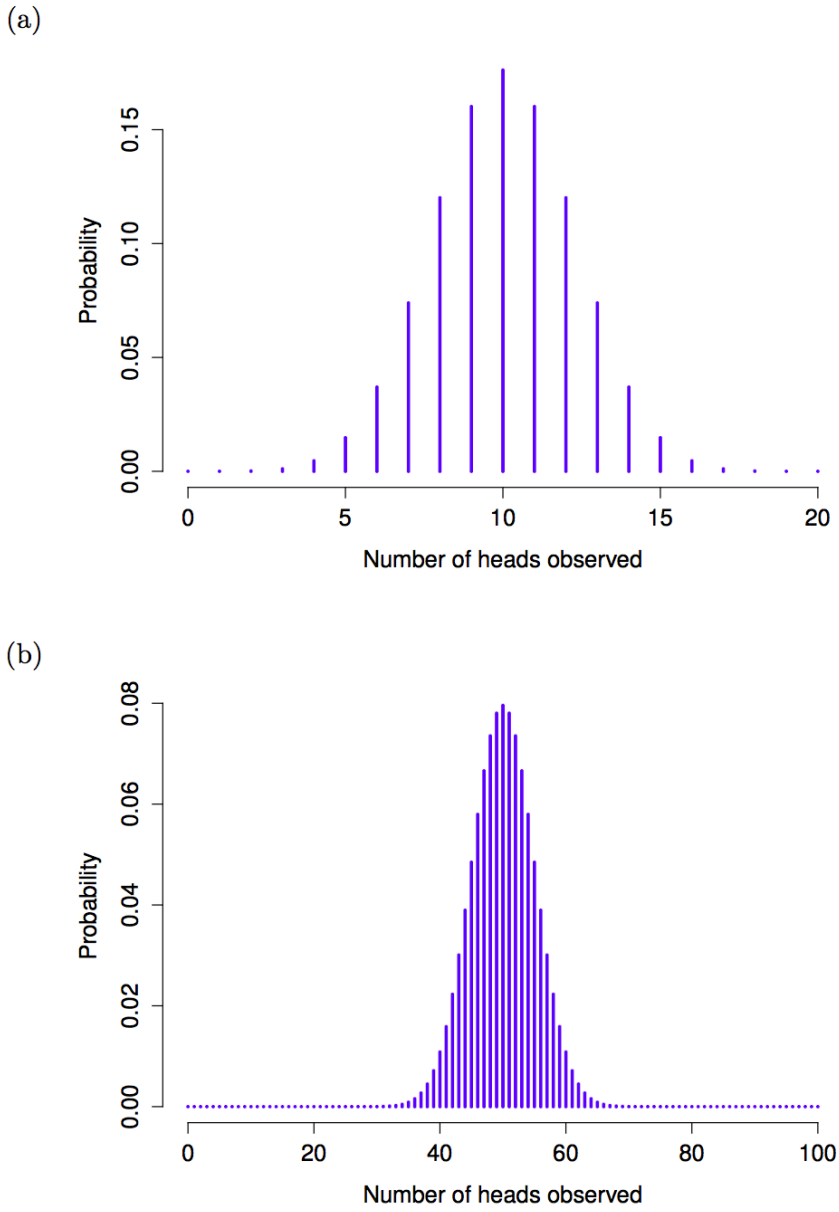


Figure 3.4: Two binomial distributions, involving a scenario in which I'm flipping a fair coin, so the underlying success probability is 1/2. In panel (a), we assume I'm flipping the coin N = 20 times. In panel (b) we assume that the coin is flipped N = 100 times.

- The `d` form we've already seen: you specify a particular outcome `x`, and the output is the probability of obtaining exactly that outcome. (the "d" is short for *density*, but ignore that for now).
- The `p` form calculates the *cumulative probability*. You specify a particular quantile `q` , and it tells you the probability of obtaining an outcome **smaller than or equal to** `q`.
- The `q` form calculates the *quantiles* of the distribution. You specify a probability value `p`, and it gives you the corresponding percentile. That is, the value of the variable for which there's a probability `p` of obtaining an outcome lower than that value.
- The `r` form is a *random number generator*: specifically, it generates `n` random outcomes from the distribution.

This is a little abstract, so let's look at some concrete examples. Again, we've already covered `dbinom` so let's focus on the other three versions. We'll start with `pbinom`, and we'll go back to the skull-dice example. Again, I'm rolling 20 dice, and each die has a 1 in 6 chance of coming up skulls. Suppose, however, that I want to know the probability of rolling 4 **or fewer** skulls. If I wanted to, I could use the `dbinom` function to calculate the exact probability of rolling 0 skulls, 1 skull, 2 skulls, 3 skulls and 4 skulls and then add these up, but there's a faster way. Instead, I can calculate this using the `pbinom` function. Here's the command:

```
pbinom( q= 4, size = 20, prob = 1/6)
```

```
## [1] 0.7687492
```

In other words, there is a 76.9% chance that I will roll 4 or fewer skulls. Or, to put it another way, R is telling us that a value of 4 is actually the 76.9th percentile of this binomial distribution.

Next, let's consider the `qbinom` function. Let's say I want to calculate the 75th percentile of the binomial distribution. If we're sticking with our skulls example, I would use the following command to do this:

```
qbinom( p = 0.75, size = 20, prob = 1/6 )
```

```
## [1] 4
```

Hm. There's something odd going on here. Let's think this through. What the `qbinom` function appears to be telling us is that the 75th percentile of the binomial distribution is 4, even though we saw from the function that 4 is **actually** the 76.9th percentile. And it's definitely the `pbinom` function that is correct. I promise. The weirdness here comes from the fact that our binomial distribution doesn't really **have** a 75th percentile. Not really. Why not? Well, there's a 56.7% chance of rolling 3 or fewer skulls (you can type `pbinom(3, 20, 1/6)` to confirm this if you want), and a 76.9% chance of rolling 4 or fewer skulls.

So there's a sense in which the 75th percentile should lie "in between" 3 and 4 skulls. But that makes no sense at all! You can't roll 20 dice and get 3.9 of them come up skulls. This issue can be handled in different ways: you could report an in between value (or **interpolated** value, to use the technical name) like 3.9, you could round down (to 3) or you could round up (to 4).

The `qbinom` function rounds upwards: if you ask for a percentile that doesn't actually exist (like the 75th in this example), R finds the smallest value for which the the percentile rank is **at least** what you asked for. In this case, since the "true" 75th percentile (whatever that would mean) lies somewhere between 3 and 4 skulls, Rrounds up and gives you an answer of 4. This subtlety is tedious, I admit, but thankfully it's only an issue for discrete distributions like the binomial. The other distributions that I'll talk about (normal, t, χ^2 and F) are all continuous, and so R can always return an exact quantile whenever you ask for it.

Finally, we have the random number generator. To use the `rbinom` function, you specify how many times R should "simulate" the experiment using the `n` argument, and it will generate random outcomes from the binomial distribution. So, for instance, suppose I were to repeat my die rolling experiment 100 times. I could get R to simulate the results of these experiments by using the following command:

```
rbinom( n = 100, size = 20, prob = 1/6 )
```

```
##   [1] 4 2 3 1 4 3 5 3 2 3 6 2 2 5 7 2 4 0 4 4 6 8 3 2 3 5 4 4 0 4 5 4 2 6 4 2 4
##  [38] 7 4 4 3 2 3 3 4 2 2 1 5 5 3 0 5 4 5 3 6 2 0 4 1 3 4 5 4 4 2 3 6 3 4 6 3 7
##  [75] 3 1 3 5 5 4 6 4 3 1 3 1 2 4 4 3 4 2 3 5 0 5 3 0 3 3
```

As you can see, these numbers are pretty much what you'd expect given the distribution shown in Figure **??**. Most of the time I roll somewhere between 1 to 5 skulls. There are a lot of subtleties associated with random number generation using a computer, but for the purposes of this book we don't need to worry too much about them.

3.5 The normal distribution

While the binomial distribution is conceptually the simplest distribution to understand, it's not the most important one. That particular honour goes to the *normal distribution*, which is also referred to as "the bell curve" or a "Gaussian distribution".

A normal distribution is described using two parameters, the mean of the distribution μ and the standard deviation of the distribution σ. The notation that we sometimes use to say that a variable X is normally distributed is as follows:

$$X \sim \text{Normal}(\mu, \sigma)$$

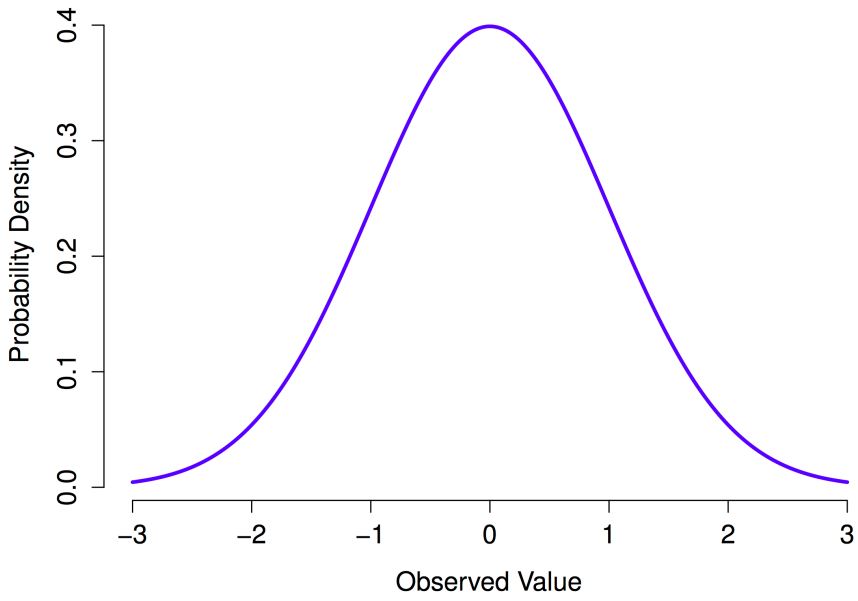


Figure 3.5: The normal distribution with mean = 0 and standard deviation = 1. The x-axis corresponds to the value of some variable, and the y-axis tells us something about how likely we are to observe that value. However, notice that the y-axis is labelled Probability Density and not Probability. There is a subtle and somewhat frustrating characteristic of continuous distributions that makes the y axis behave a bit oddly: the height of the curve here isn't actually the probability of observing a particular x value. On the other hand, it is true that the heights of the curve tells you which x values are more likely (the higher ones!).