Signals and Systems: Energy and Power Signals
信号与系统：能量和功率信号

Electronics & Electrical | Electron | Digital Electronics

电子与电气 电子 | 数字 | 电子

Energy Signal 能量信号

A signal is said to be an energy signal if and only if its total energy E is finite, i.e., $0 < E < \infty$. For an energy signal, the average power P = 0. The nonperiodic signals are the examples of energy signals.

当且仅当信号的总能量 E 是有限的，即 $0 < E < \infty$ 时，信号才被称为能量信号。对于能量信号，平均功率P=0。非周期信号是能量信号的例子。

Power Signal 电源信号

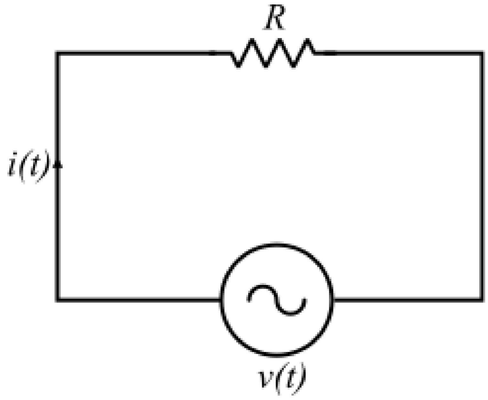
A signal is said to be a power signal if its average power P is finite, i.e., $0 < P < \infty$. For a power signal, the total energy E = ∞. The periodic signals are the examples of power signals.

如果信号的平均功率 P 是有限的，即 $0 < P < \infty$，则称该信号为功率信号。对于功率信号，总能量 E = ∞。周期信号是功率信号的示例。

Continuous Time Case 连续时间情况

In electric circuits, the signals may represent current or voltage. Consider a voltage v(t) applied across a resistance R and i(t) is the current flowing through it as shown in the figure.

在电路中，信号可以代表电流或电压。考虑施加在电阻 R 上的电压 v(t)，i(t) 是流过电阻 R 的电流，如图所示。



The instantaneous power in the resistance R is given by,

电阻 R 中的瞬时功率由下式给出：

$$p(t) = v(t) \cdot i(t) \ldots (1)$$

By Ohm’s law, 根据欧姆定律，

$$\mathrm{p(t) = v(t)\frac{v(t)}{R} = \frac{v^2(t)}{R}} \quad \cdots (2)$$

Also, 还，

$$p(t) = i(t)R \cdot i(t) = i^2(t)R \quad \ldots (3)$$

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When the values of the resistance R = 1Ω, then the power dissipated in it is known as normalised power. Hence,

当电阻 R 的值 = 1Ω 时，其中消耗的功率称为归一化功率。因此，

Normalised power, $p(t) = v^2(t) = i^2(t) \quad \ldots (4)$

标准化幂，$p(t) = v^2(t) = i^2(t) \ldots (4)$

If v(t) or i(t) is denoted by a continuous-time signal x(t), then the instantaneous power is equal to the square of the amplitude of the signal, i.e.,

如果 v(t) 或 i(t) 用连续时间信号 x(t) 表示，则瞬时功率等于信号幅度的平方，即

$$p(t) = |x(t)|^2 \quad \ldots (5)$$

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Therefore, the average power or normalised power of a continuous time signal x(t) is given by,

因此，连续时间信号 x(t) 的平均功率或归一化功率由下式给出：

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-(T/2)}^{(T/2)} |x(t)|^2 \, dt \quad \text{Watts} \qquad \cdots (6)$$

The total energy or normalised energy of a continuous time signal is defined as,

连续时间信号的总能量或归一化能量定义为，

$$E = \lim_{T\to\infty} \int_{-(T/2)}^{(T/2)} |x(t)|^2 \, dt \quad \text{Joules} \qquad \cdots (7)$$

Discrete Time Case 离散时间案例

For the discrete time signal x(n), the integrals are replaced by summations. Hence, the total energy of the discrete time signal x(n) is defined as

对于离散时间信号 x(n)，积分由求和代替。因此，离散时间信号 x(n) 的总能量定义为

$$E = \sum_{n=-\infty}^{\infty} |x(t)|^2$$

The average power of a discrete time signal x(t) is defined as

离散时间信号 x(t) 的平均功率定义为

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(t)|^2$$

Important Points 要点

- Both energy and power signals are mutually exclusive, i.e., no signal can be both power signal and energy signal.

  能量和功率信号是互斥的，即没有信号可以同时是功率信号和能量信号。

- A signal is neither energy nor power signal if both energy and power of the signal are equal to infinity.

  如果信号的能量和功率都等于无穷大，则信号既不是能量信号也不是功率信号。

- All practical signals have finite energy; thus they are energy signals.

  所有实际信号的能量都是有限的；因此它们是能量信号。

- In practice, the physical generation of power signal is impossible since its requires infinite duration and infinite energy.

  在实践中，功率信号的物理生成是不可能的，因为它需要无限的持续时间和无限的能量。

- All finite duration signals of finite amplitude are energy signals.

  所有有限幅度的有限持续时间信号都是能量信号。

- Sum of an energy signal and power signal is a power signal.

  能量信号和功率信号之和是功率信号。

- A signal whose amplitude is constant over infinite duration is a power signal.

  幅度在无限持续时间内恒定的信号是功率信号。

- The energy of a signal is not affected by the **time shifting** and time inversion. It is only affected by the **time scaling**.

  信号的能量不受**时移**和时间反转的影响。它仅受**时间缩放**的影响。

Numerical Example 数值例子

Determine the power and energy of the signal $x(t) = A\ \sin(\omega_0 t + \varphi)$.

确定信号的功率和能量 $x(t) = A\ \sin(\omega_0 t + \varphi)$。

Solution 解决方案

Given signal is, 给定信号是，

$$x(t) = A\ \sin(\omega_0 t + \varphi)$$

$$x(t) = A\ \sin(\omega_0 t + \varphi)$$

Average Power of the Signal
信号平均功率

$$\mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_{-(\mathrm{T}/2)}^{(\mathrm{T}/2)} |\mathrm{x}(\mathrm{t})|^2\ \mathrm{dt}$$

$$\Rightarrow \mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_{-(\mathrm{T}/2)}^{(\mathrm{T}/2)} |\mathrm{A}\ \sin(\omega_0 \mathrm{t} + \varphi)|^2\ \mathrm{dt}$$

$$\Rightarrow \mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{\mathrm{A}^2}{\mathrm{T}} \int_{-(\mathrm{T}/2)}^{(\mathrm{T}/2)} \left| \frac{1-\cos(2\omega_0 \mathrm{t} + 2\varphi)}{2} \right| \mathrm{dt}$$

$$\Rightarrow \mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{\mathrm{A}^2}{2\mathrm{T}} \int_{-(\mathrm{T}/2)}^{(\mathrm{T}/2)} \mathrm{dt} - \frac{\mathrm{A}^2}{2\mathrm{T}} \int_{-(\mathrm{T}/2)}^{(\mathrm{T}/2)} \cos(2\omega_0 \mathrm{t} + 2\varphi)\, \mathrm{dt}$$

$$\Rightarrow \mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{\mathrm{A}^2}{2\mathrm{T}} \int_{-(\mathrm{T}/2)}^{(\mathrm{T}/2)} \mathrm{dt} - 0 = \lim_{\mathrm{T}\to\infty} \frac{\mathrm{A}^2}{2\mathrm{T}} \left[\frac{\mathrm{T}}{2} + \frac{\mathrm{T}}{2} \right] = \frac{\mathrm{A}^2}{2}$$

Normalised Energy of the Signal
信号的归一化能量

$$\mathrm{E} = \int_{-\infty}^{\infty} |\mathrm{x}(\mathrm{t})|^2\, \mathrm{dt} = \int_{-\infty}^{\infty} |\mathrm{A}\ \sin(\omega_0 \mathrm{t} + \varphi)|^2\, \mathrm{dt}$$

$$\Rightarrow \mathrm{E} = \mathrm{A}^2 \int_{-\infty}^{\infty} \left[\frac{1-\cos(2\omega_0 \mathrm{t} + 2\varphi)}{2} \right] \mathrm{dt}$$

$$\Rightarrow \mathrm{E} = \frac{\mathrm{A}^2}{2} \int_{-\infty}^{\infty} \mathrm{dt} - \frac{\mathrm{A}^2}{2} \int_{-\infty}^{\infty} \cos(2\omega_0 \mathrm{t} + 2\varphi)\, \mathrm{dt}$$

$$\Rightarrow \mathrm{E} = \frac{\mathrm{A}^2}{2} [\mathrm{t}]_{-\infty}^{\infty} - 0 = \infty$$