EE123
Digital Signal Processing

Lecture 18
Filter Banks

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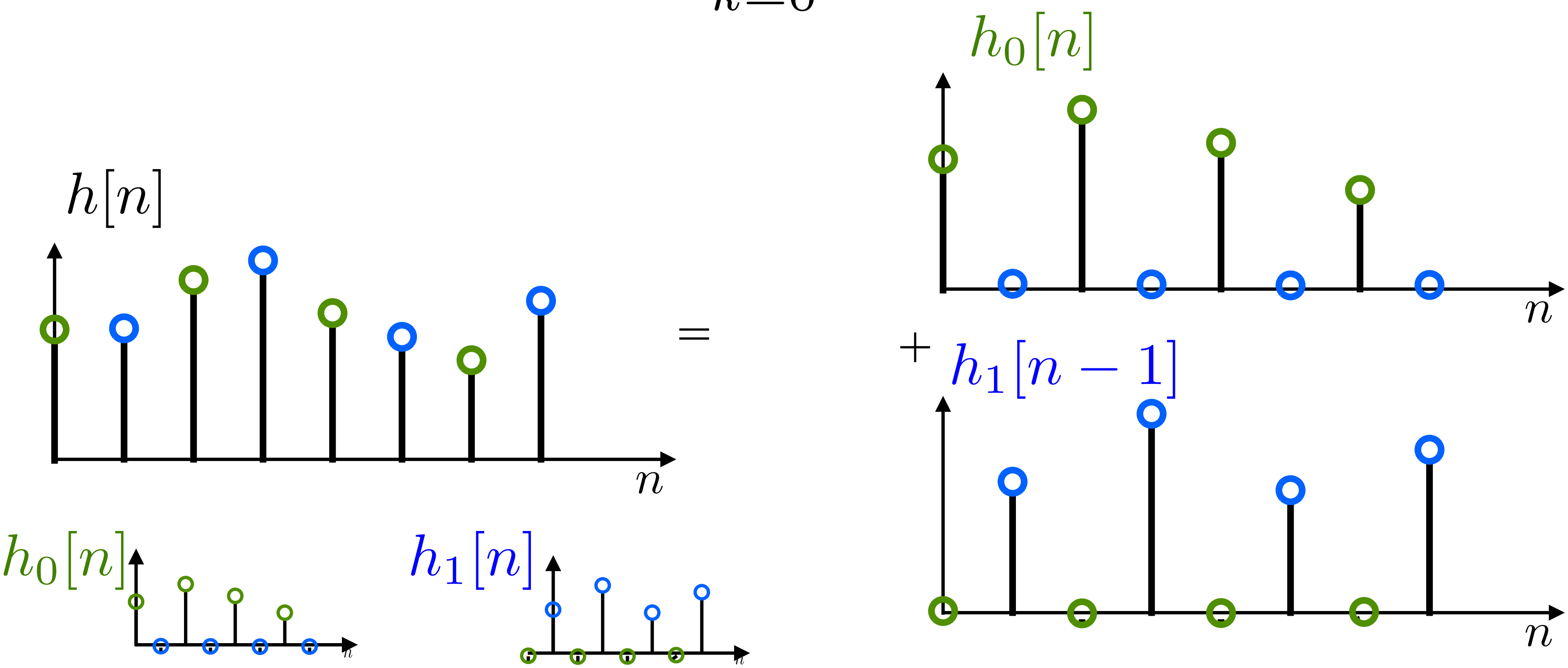
Last Time

- Exchange of filtering and expanders
- Today:
  - Exchange of filtering and compressors
  - Polyphase decomposition
  - Multi-rate Filter Banks
  - Subtleties in Time-Frequency tiling
  - Perfect reconstruction with non-ideal filters
  - Polyphase filter banks

Polyphase Decomposition

- We can decomposed an impulse response to:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

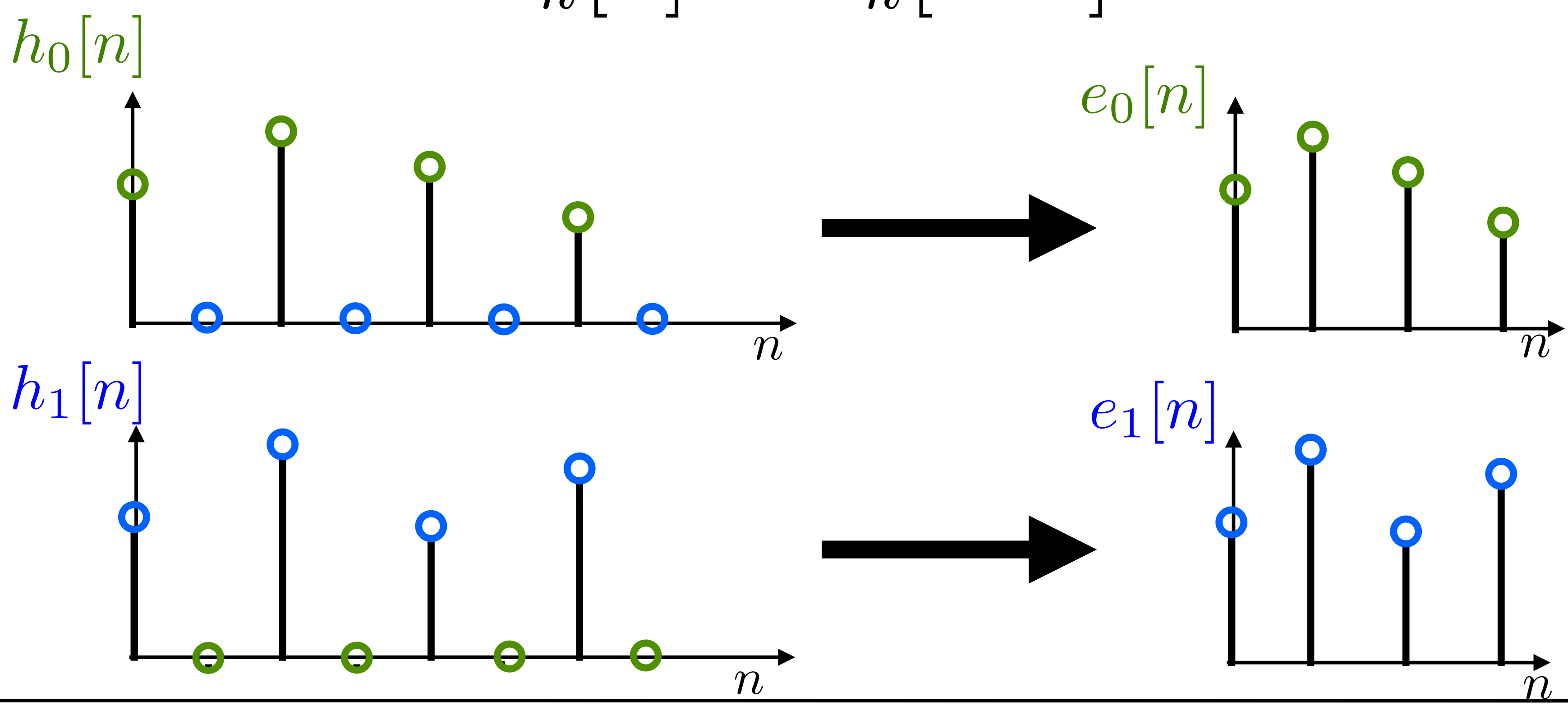


Polyphase Decomposition

- Define:

$$h_k[n] \longrightarrow \boxed{\downarrow\mathrm{M}} \longrightarrow e_k[n]$$

$$e_k[n] = h_k[nM]$$



Polyphase Decomposition

$$e_k[n] \longrightarrow \boxed{\uparrow \mathrm{M}} \longrightarrow h_k[n]$$

recall upsampling ⇒ scaling

$$H_k(z) = E_k(z^M)$$

Also, recall:

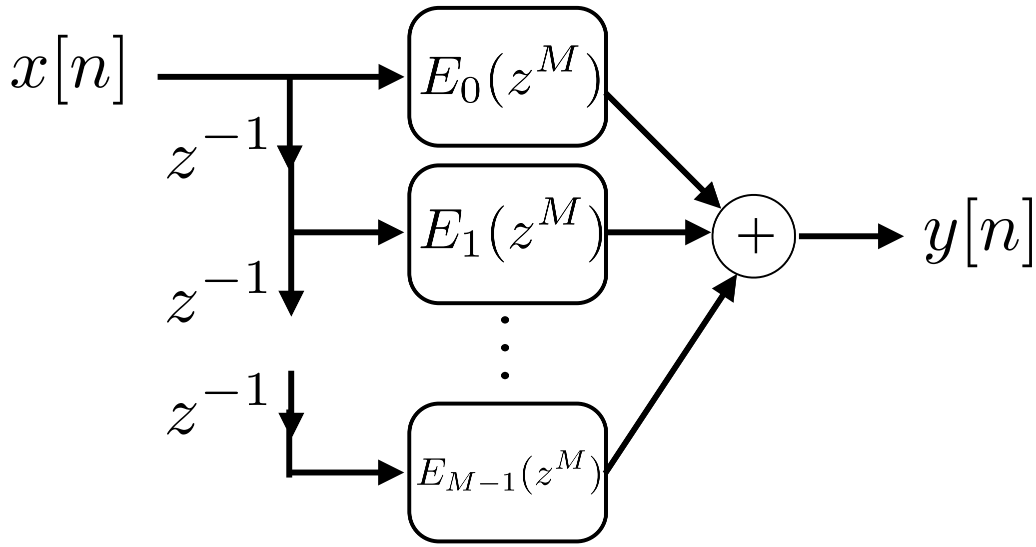
$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

So,

$$\boxed{H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}}$$

Polyphase Decomposition

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$



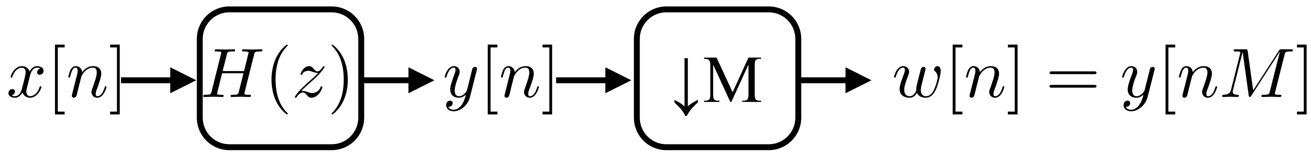
Why should you care?

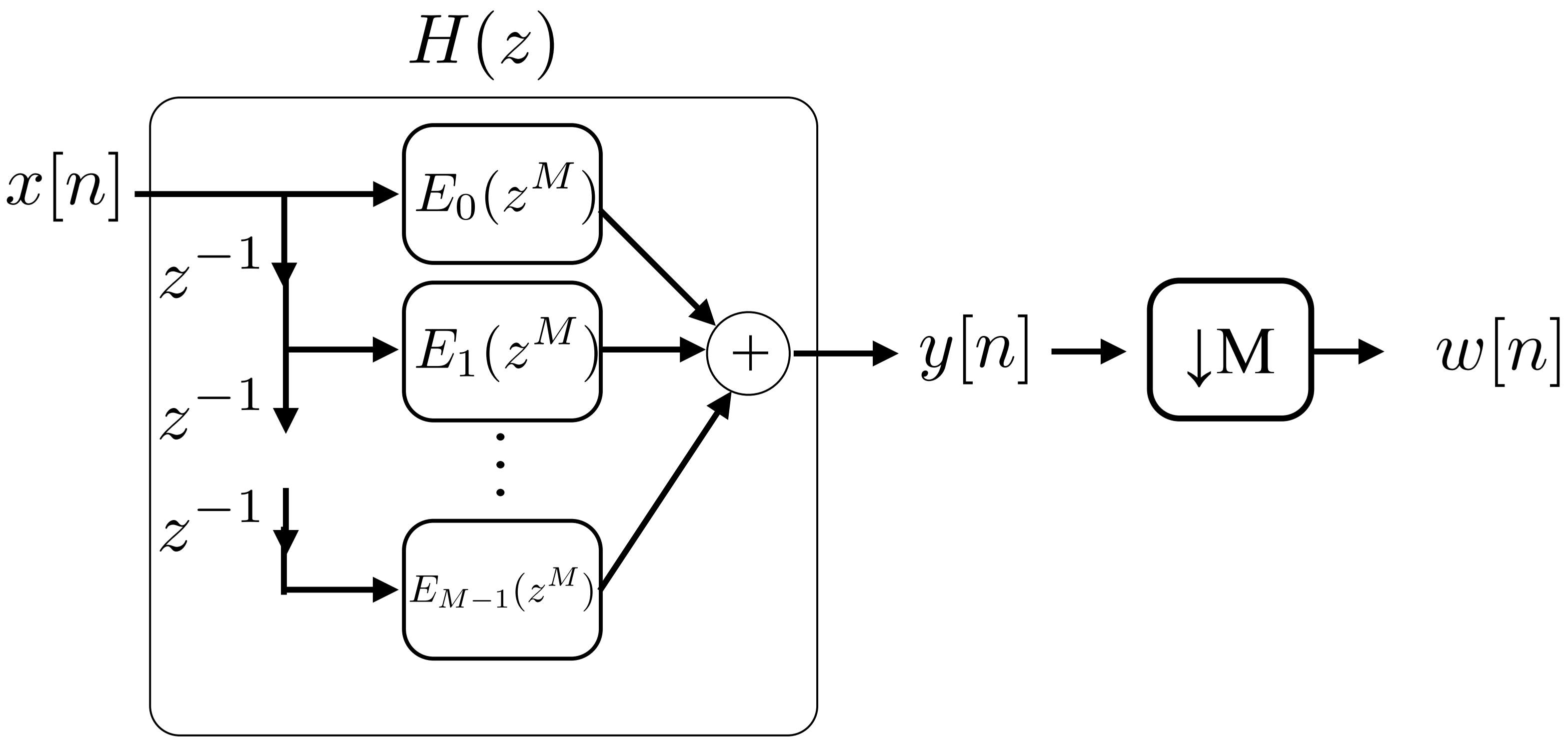
Polyphase Implementation of Decimation

$$x[n] \rightarrow \boxed{H(z)} \rightarrow y[n] \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow w[n] = y[nM]$$

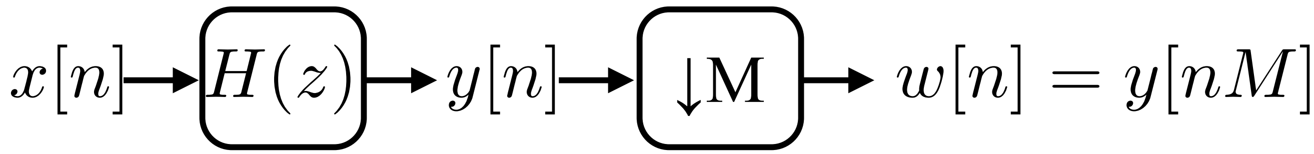
- Problem:
  - –Compute all y[n] and then throw away -- wasted computation!
    - For FIR length N ⇒ N mults/unit time
  - –Can interchange Filter with compressor?
    - Not in general!

Polyphase Implementation of Decimation

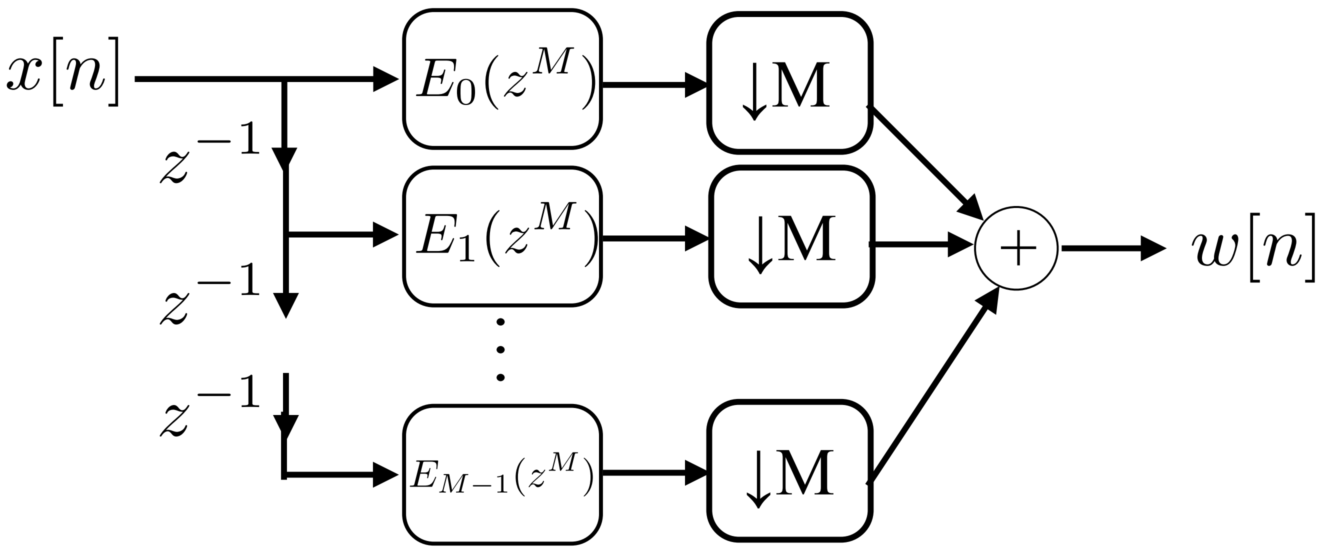




Polyphase Implementation of Decimation

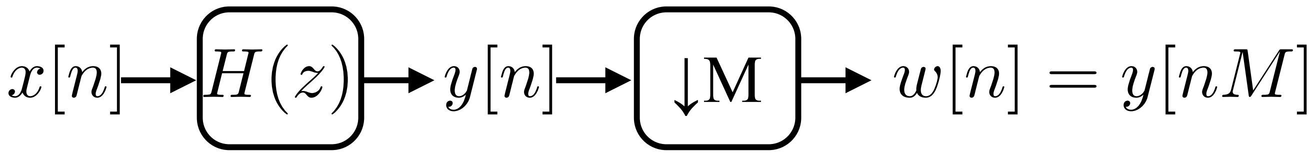


Interchange filter with decimation

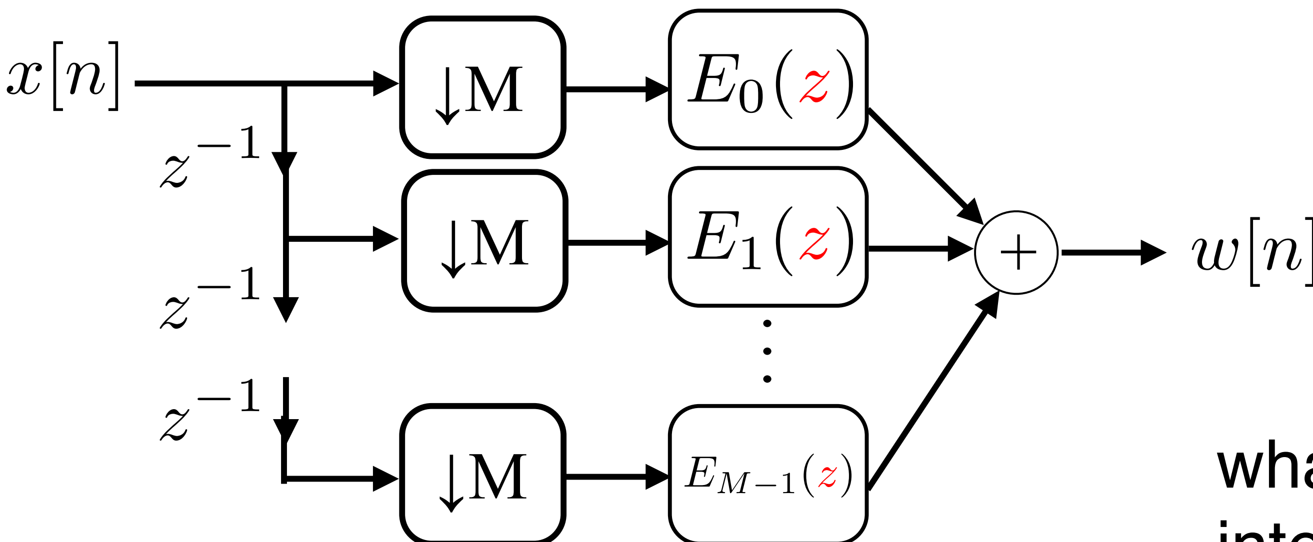


now, what can we do?

Polyphase Implementation of Decimation



Interchange filter with decimation



what about interpolation?

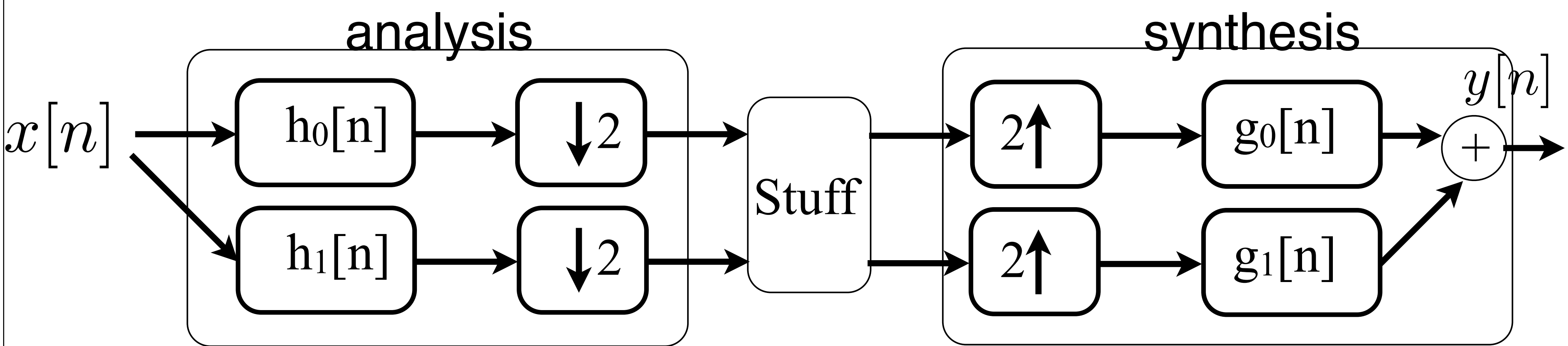
Computation:

Each Filter: N/M *(1/M) mult/unit time

Total: N/M mult/unit time

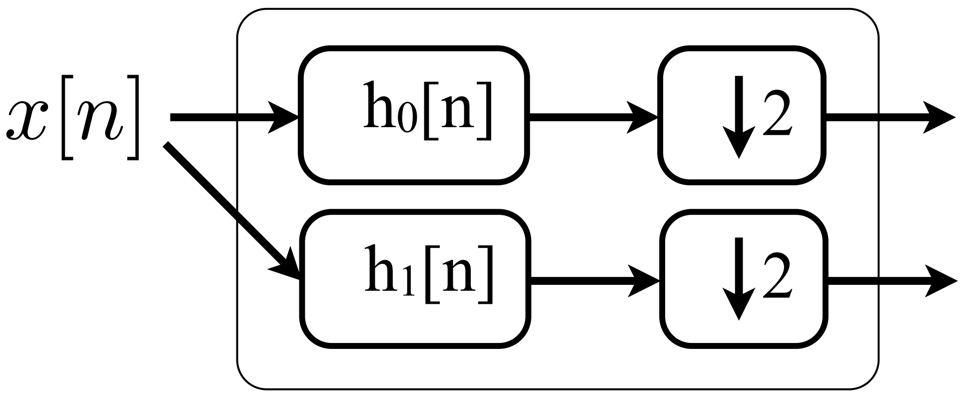
Multirate FilterBank

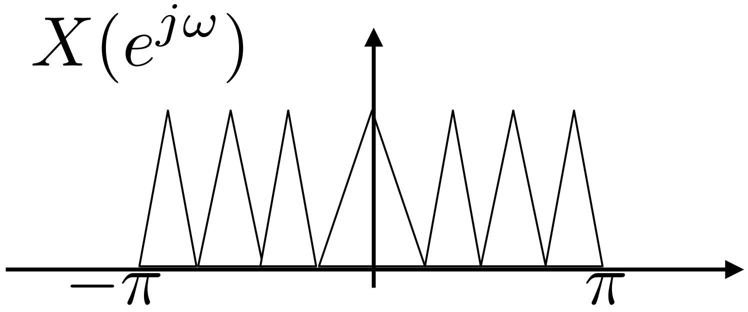
- $h_0[n]$ is low-pass, $h_1[n]$ is high-pass
- Often $h_1[n] = e^{j\pi n} h_0[n]$ or $H_1(e^{j\omega}) = H_0(e^{j(w-\pi)})$



Subtleties in Time-Freq Tiling

- Assume h_0, h_1 are ideal low,high pass filters





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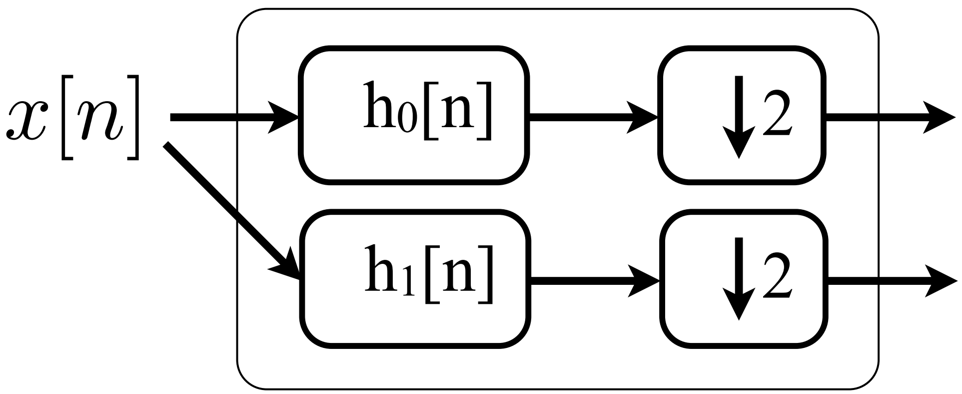

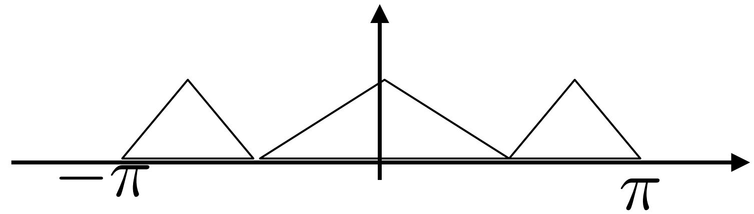

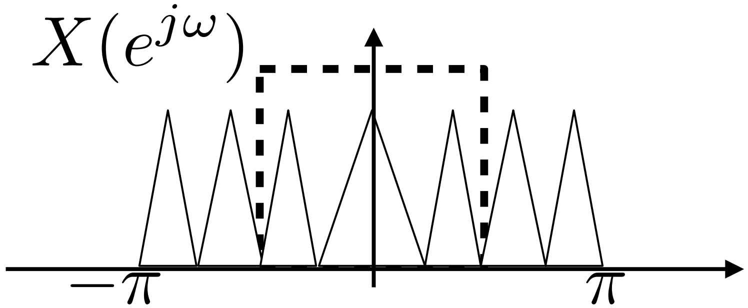

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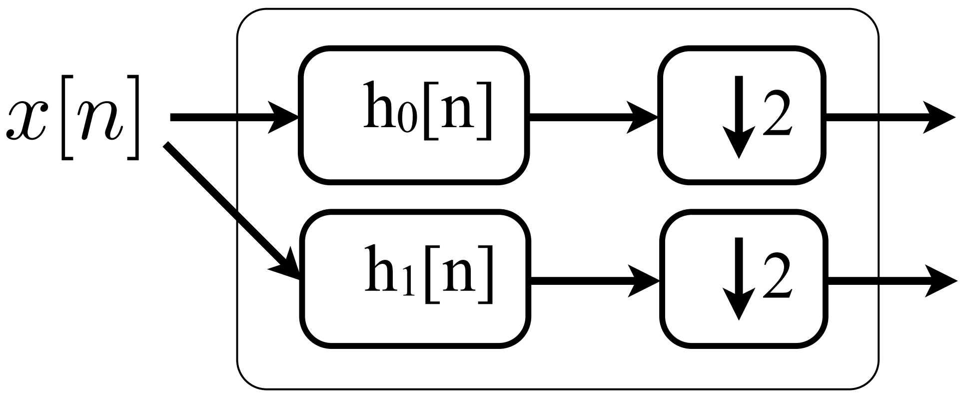

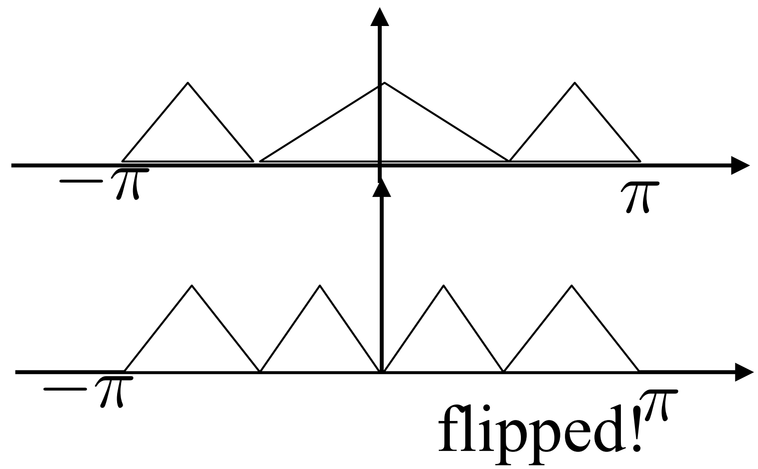

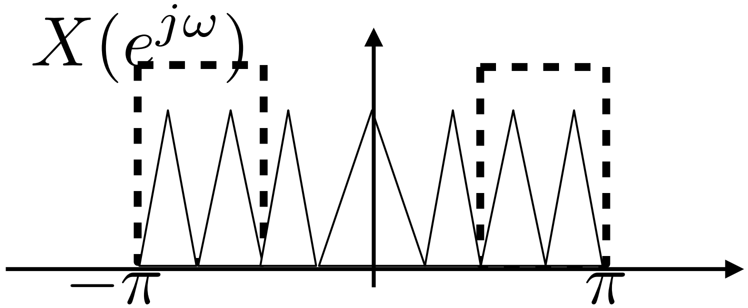

Subtleties in Time-Freq Tiling

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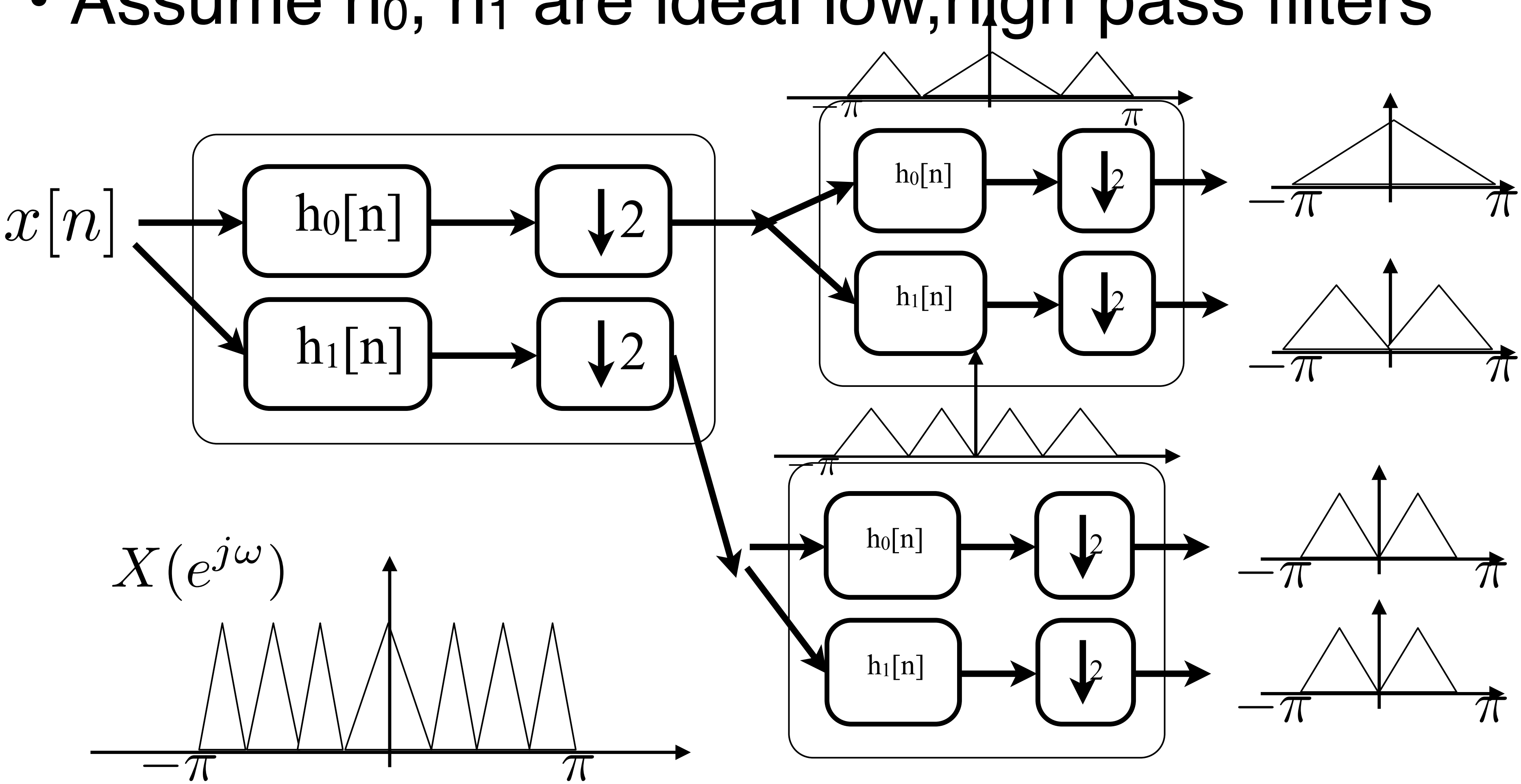
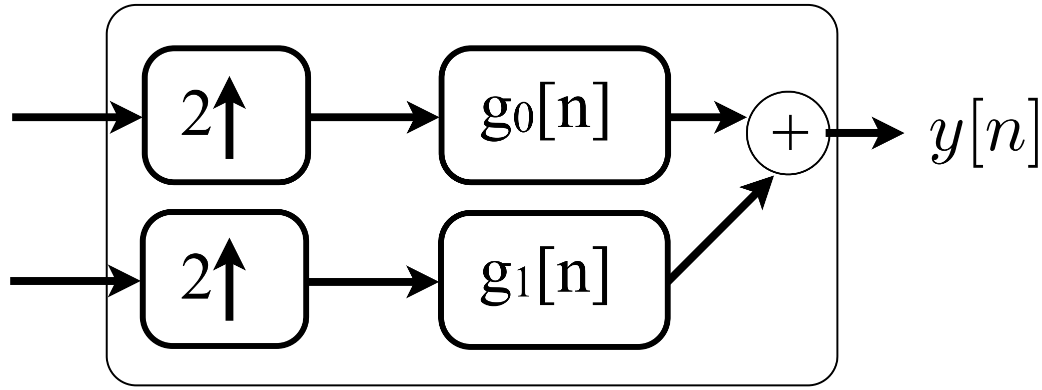
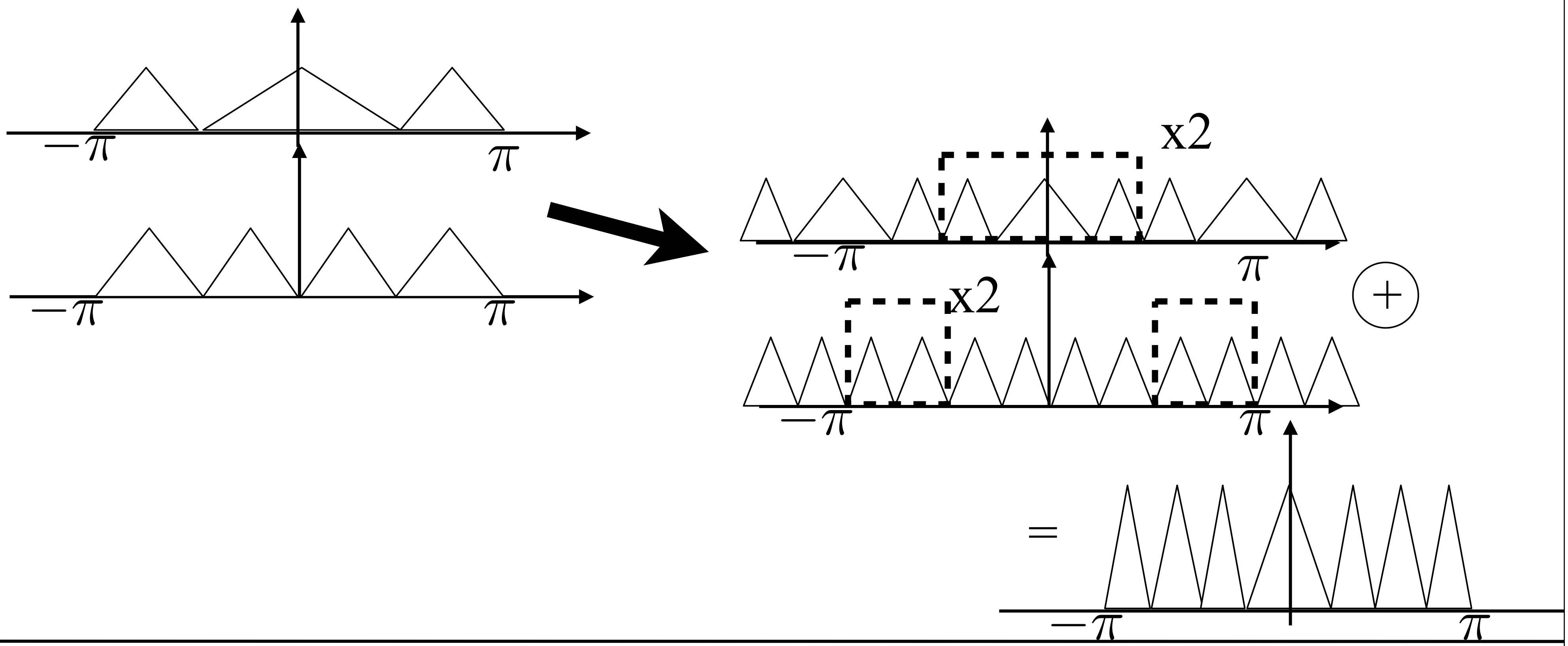
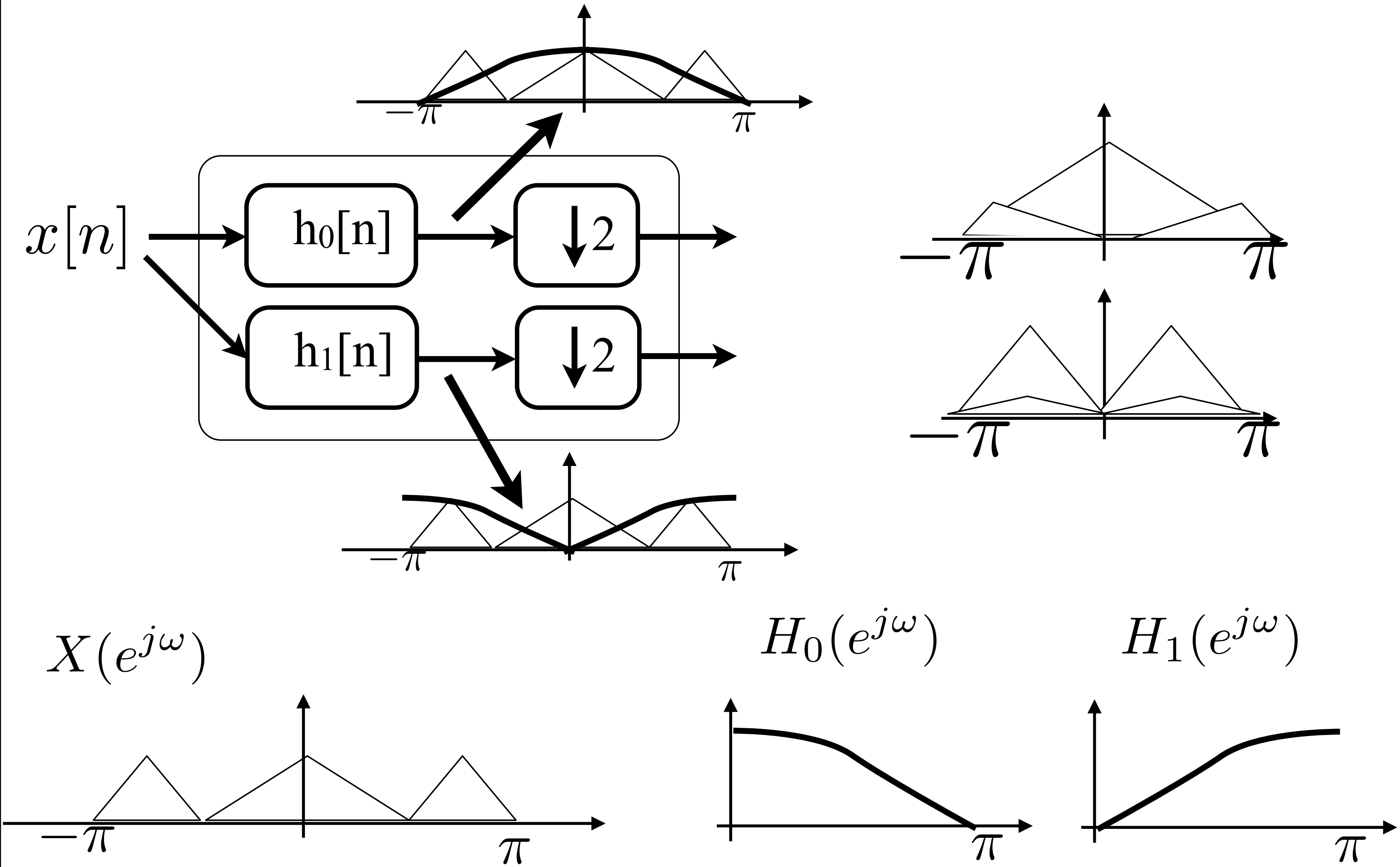


Perfect Reconstruction Ideal Filters

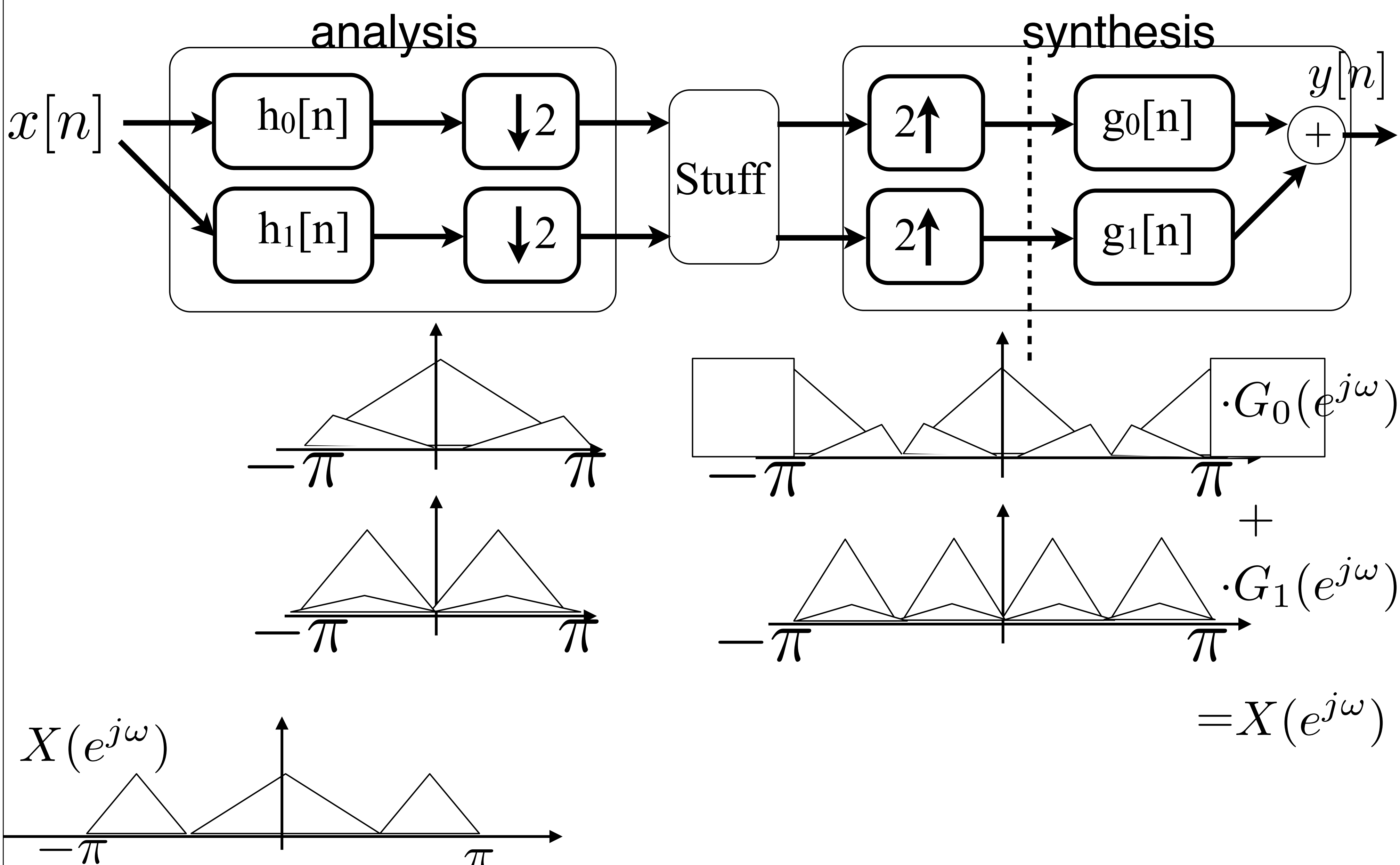




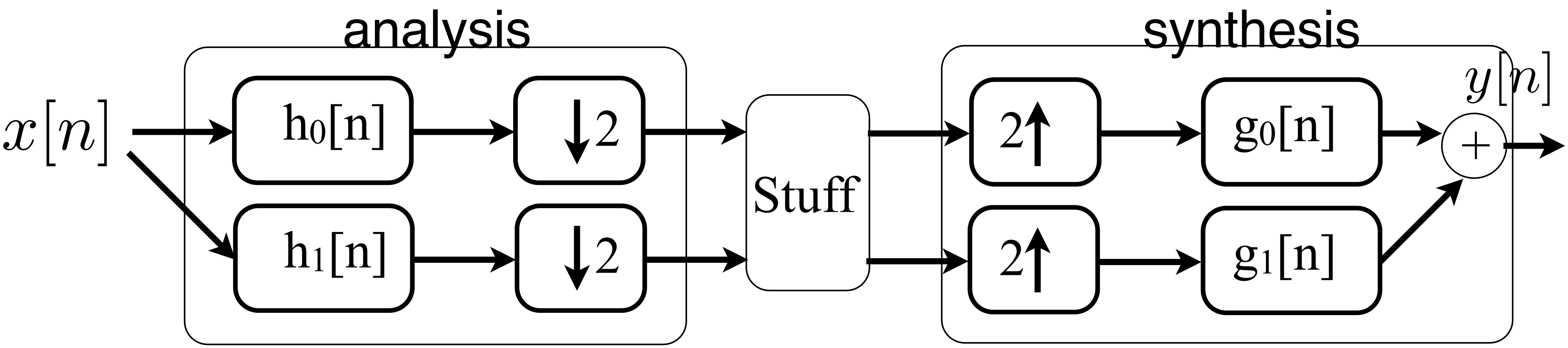
Non ideal LP and HP Filters



Perfect Reconstruction non-Ideal Filters



Perfect Reconstruction non-Ideal Filters

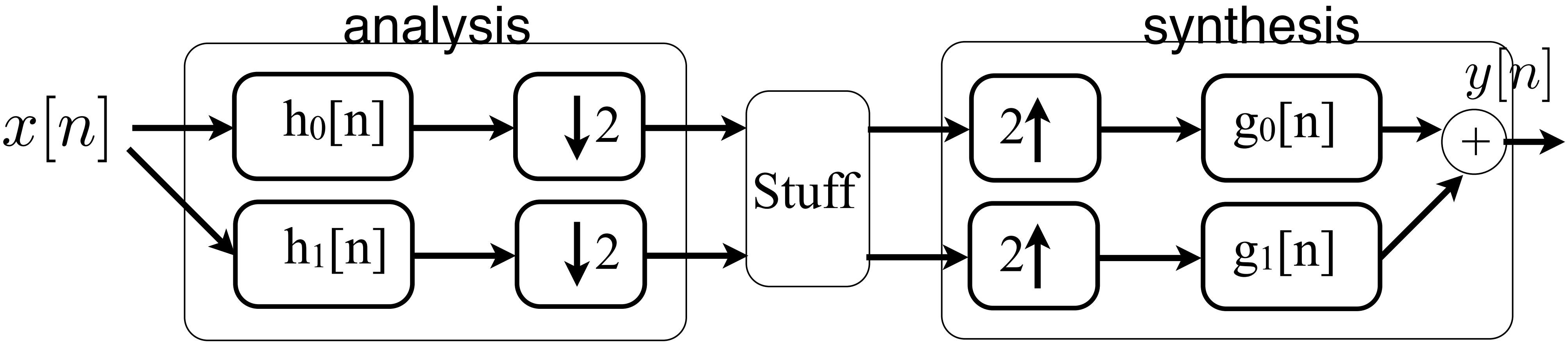


$$\begin{aligned} Y(e^{j\omega}) &= \frac{1}{2}\left[G_0(e^{j\omega})H_0(e^{j\omega}) + G_1(e^{j\omega})H_1(e^{j\omega})\right] X(e^{j\omega}) \\ &+ \frac{1}{2}\left[G_0(e^{j\omega})H_0(e^{j(\omega-\pi)}) + G_1(e^{j\omega})H_1(e^{j(\omega-\pi)})\right] X(e^{j(\omega-\pi)}) \end{aligned}$$

aliasing

need to cancel!

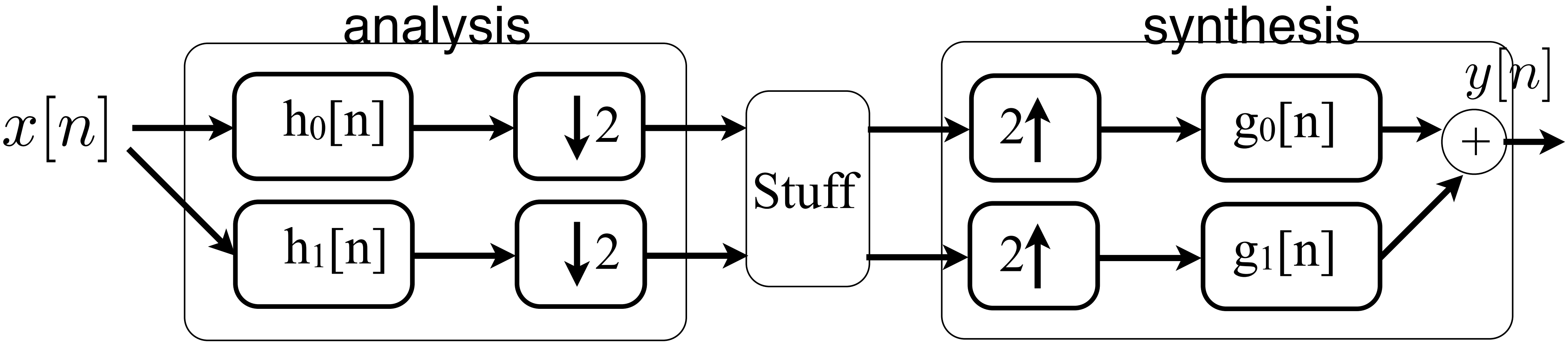
Quadrature Mirror Filters - perfect recon



QMF - mirror around pi/2

$$\begin{aligned} H_1(e^{j\omega}) &= H_0(e^{j(\omega-\pi)}) \\ G_0(e^{j\omega}) &= 2H_0(e^{j\omega}) \\ G_1(e^{j\omega}) &= -2H_1(e^{j\omega}) \end{aligned}$$

Quadrature Mirror Filters - perfect recon



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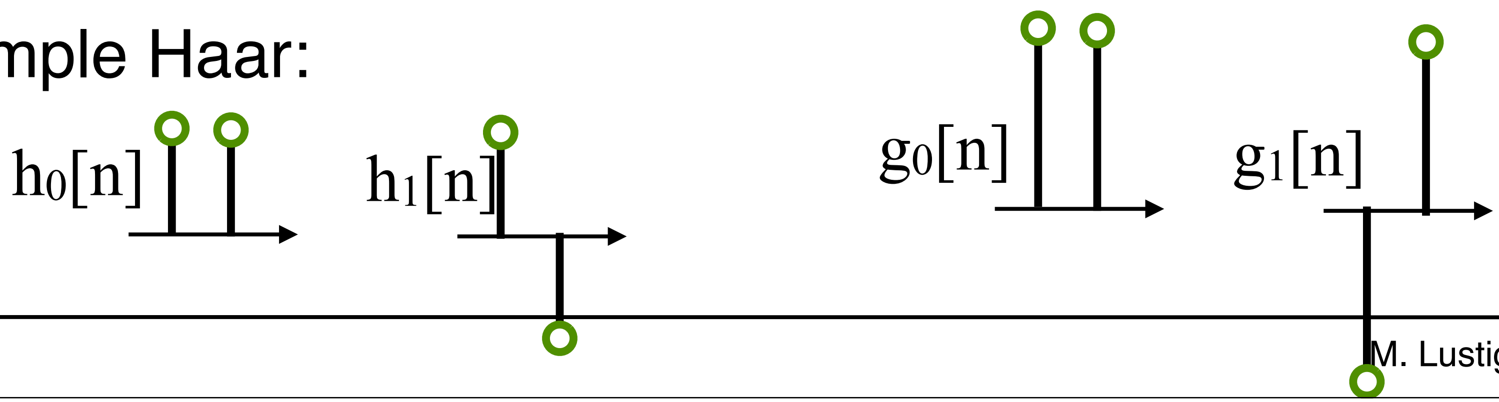
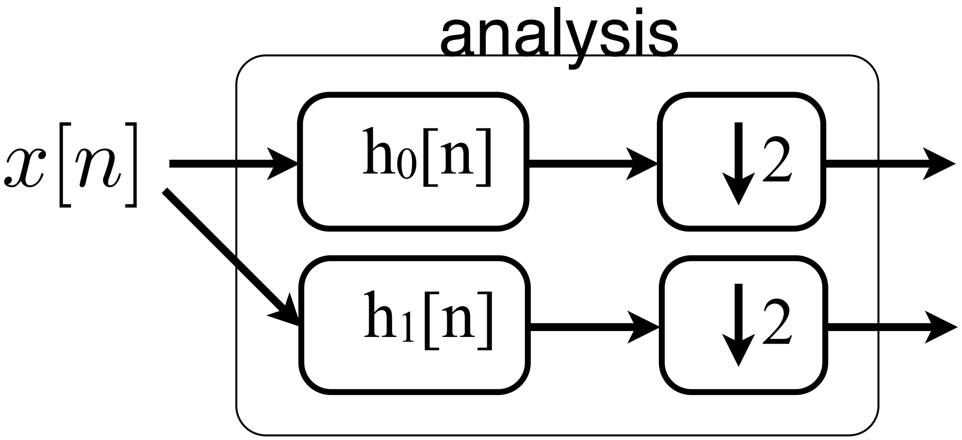
G_1(e^{j\omega}) &= -2H_1(e^{j\omega})
\end{aligned}
$$

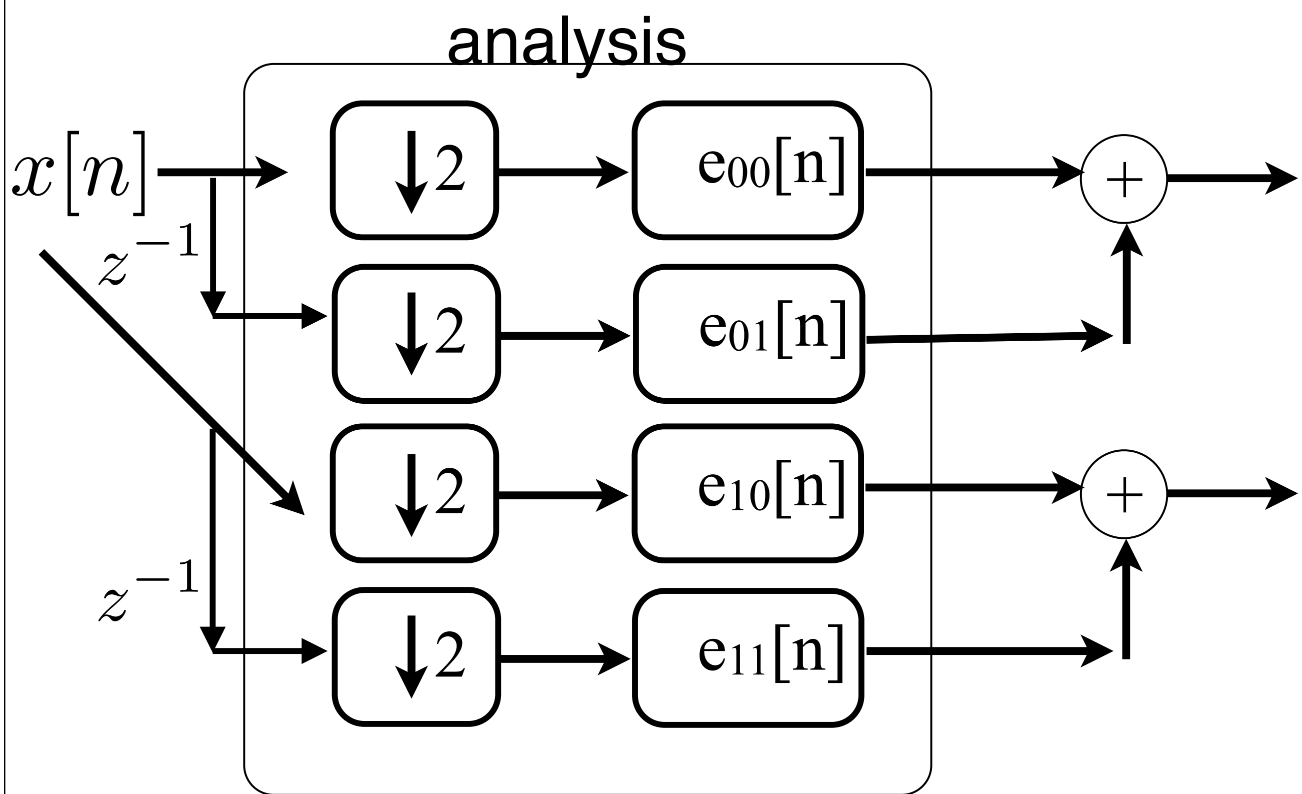
Example Haar:





$$
\begin{aligned}
e_{00} &= h_0[2n] \\
e_{01} &= h_0[2n+1] \\
e_{10} &= h_1[2n] = e^{j2\pi n} h_0[2n] = e_{00}[n] \\
e_{11} &= h_1[2n+1] = e^{j2\pi n} e^{j\pi} h_0[2n+1] = -e_{01}[n]
\end{aligned}
$$

Polyphase Filter-Bank



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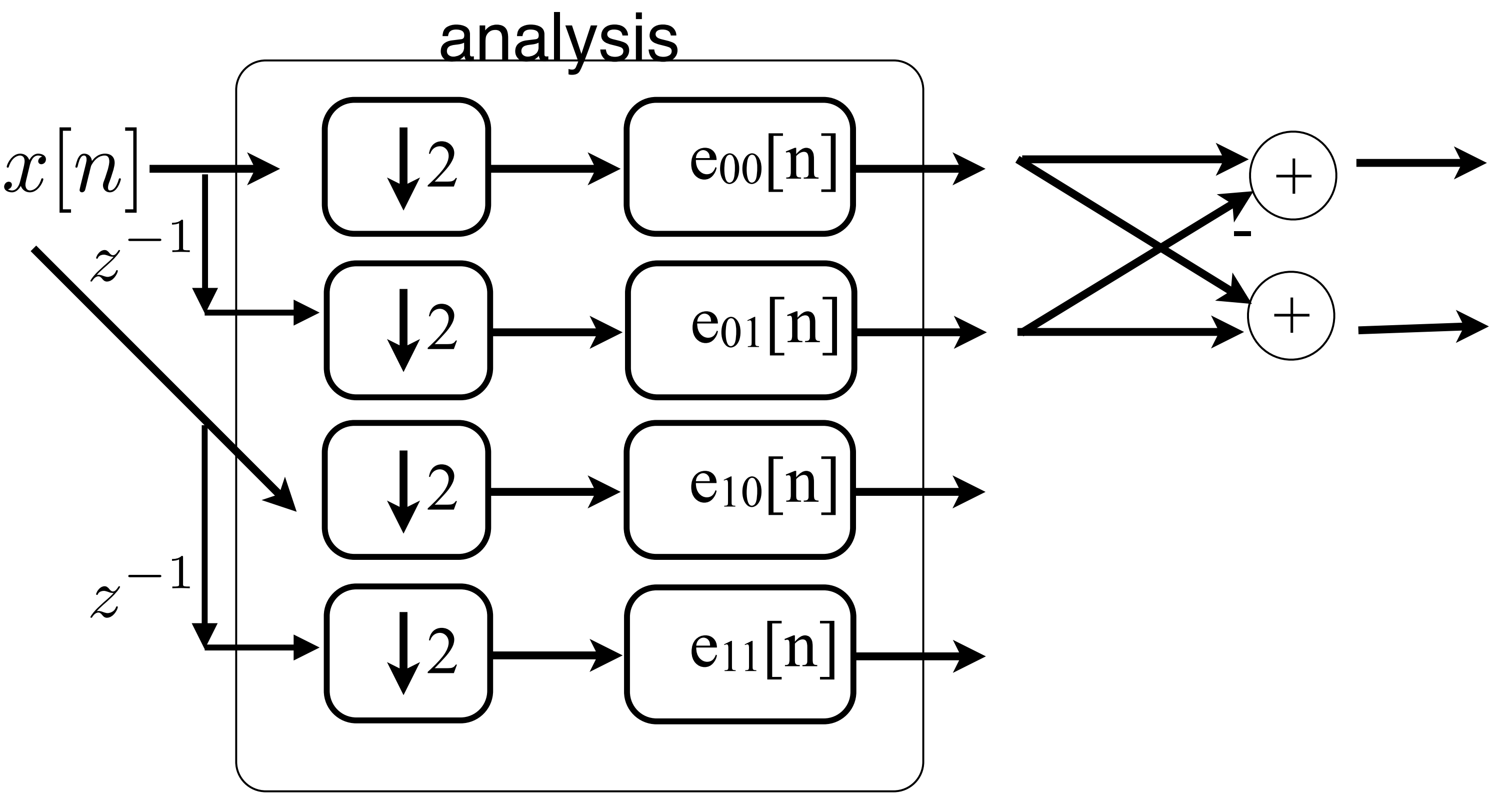
Polyphase Filter-Bank



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