

### Assignment 8

Due April 1<sup>st</sup> 2019

1. Self-grade Homework 7.
2. Finish reading Chapter 4 Oppenheim and Schaffer, 3rd ed.
3. *Downsampling*

Downsampling a sequence by an integer factor  $N$  results in a sequence  $y[n] = x[nN]$ . Prove that in the Fourier domain

$$Y(e^{j\omega}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\omega-2k\pi)/N}), \quad (1)$$

by going back to the underlying time-domain signal and resampling it with an  $N$ -times longer sampling period. That is, consider  $x[n]$  and  $y[n] = x[nN]$  as two sampled versions of the same continuous-time signal, with sampling periods  $T$  and  $NT$ , respectively.

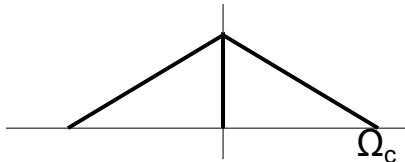
*Hint:* Recall that the discrete-time Fourier transform  $X(e^{j\omega})$  of  $x[n]$  is

$$X(e^{j\omega}) = X_T\left(\frac{\omega}{T}\right) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_C\left(\frac{\omega}{T} - k\frac{2\pi}{T}\right), \quad (2)$$

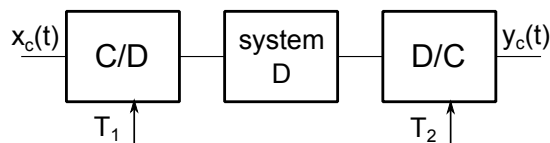
where  $T$  is the sampling period.

4. From Midterm II, sp'15: Digital Processing of Analog Signals

You are given a continuous signal  $x_c(t)$  with the following spectrum:



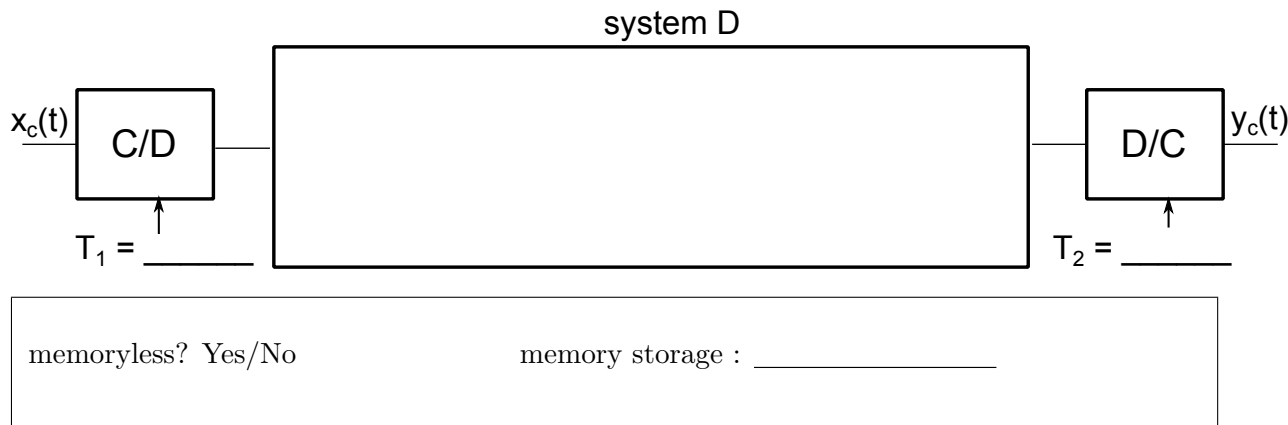
Also consider the typical digital processing system for analog signals that we covered in class where  $C/D$  and  $D/C$  are ideal continuous-to-Digital and digital-to-continuous that include the antialiasing filters. Note that  $C/D$  and  $D/C$  have no memory storage in them.



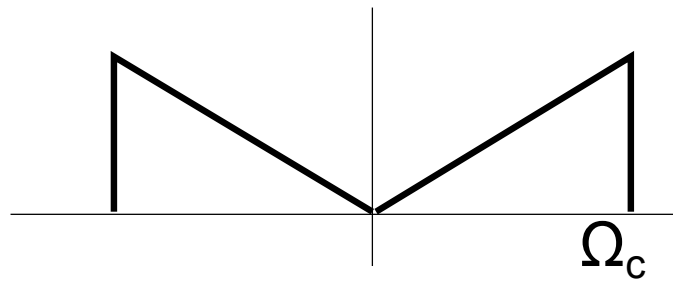
In this question you will be required to design several systems. (you can assume any system delays associated with the sampling are negligible)

- a) Design and sketch the **simplest** “system D” that will result in  $y_c(t) = x_c(t/2)$ . Use the **maximum possible** values for  $T_1$  and  $T_2$ .

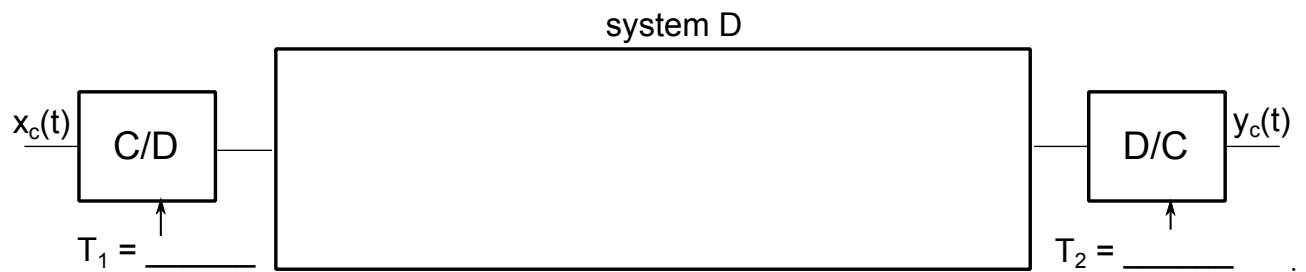
Is “system D” memoryless? if yes explain, if not how much memory storage it needs to process an infinite long signal  $x_c(t)$ ?



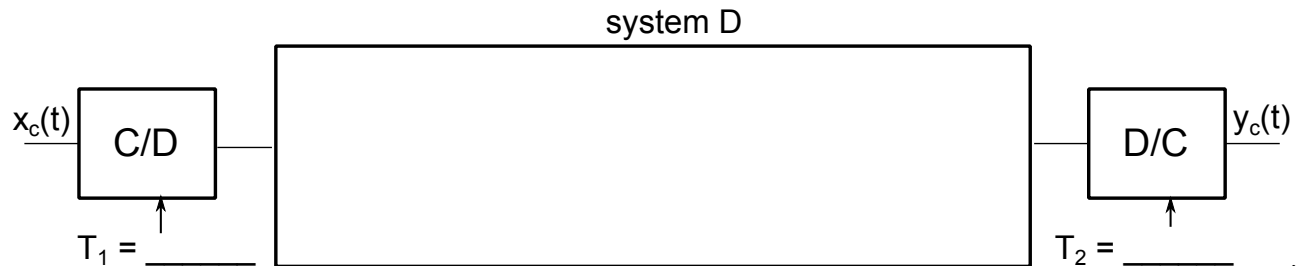
- b) Design and sketch the simplest “system D” that given an input  $x_c(t)$  above will swap the spectrum and result in  $y_c(t)$  for which the frequency response is:



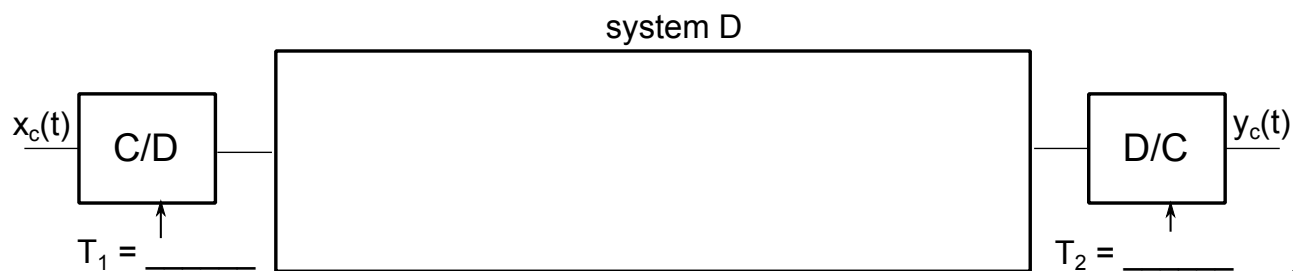
Use the maximum possible values for  $T_1$  and  $T_2$ .



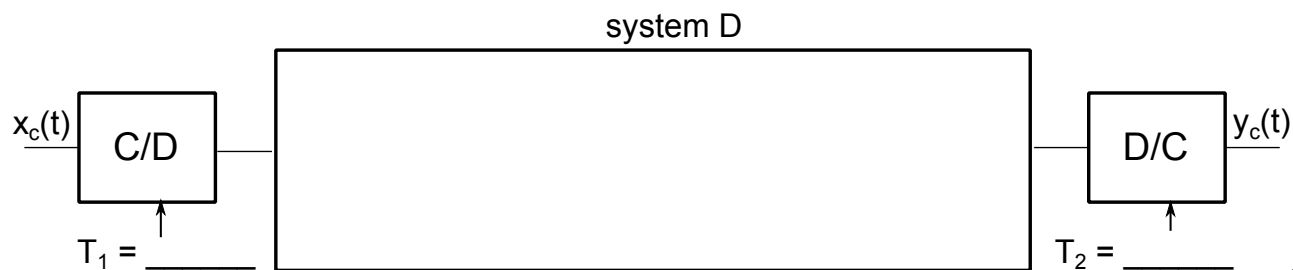
- c) Is “system D” in part (b) memoryless? if so, explain. If not, change the appropriate parameters and sketch a new system in which “system D” is memoryless.



- d) Design and sketch “system D” that results in the output  $y_c(t) = x_c^2(t)$ . Use the maximum possible values for  $T_1$  and  $T_2$ .

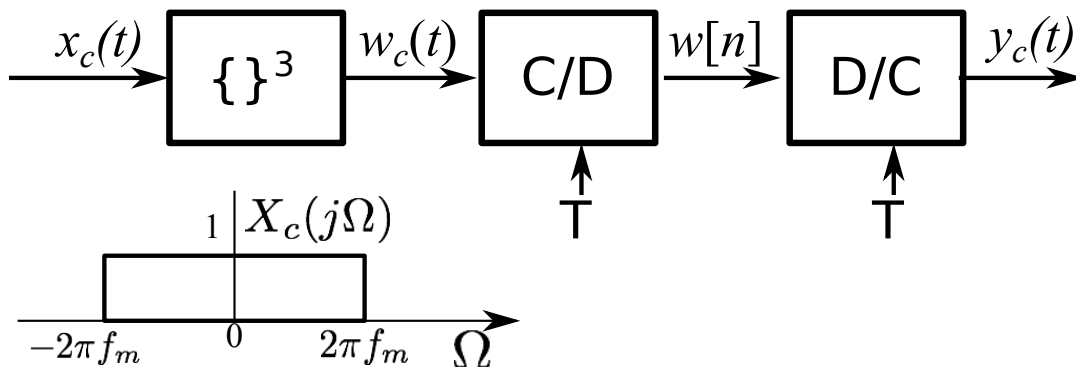


- e) Is “system D” in part (d) memoryless? if so, explain. If not, change the appropriate parameters and sketch a new system in which “system D” is memoryless.



5. From Midterm II, fall '12: Non-linearity, sampling, and reconstruction

Consider the following system:



where the spectrum of  $X_c(j\Omega)$  is given above,  $1/T = 3f_m$  Hz and the operator  $\{\}^n$  is a memoryless non-linear system that takes the  $n^{\text{th}}$  power of its input.

- a) Sketch the spectra of  $w_c(t)$ ,  $w[n]$  and  $y_c(t)$ . Label the axes correctly. Does  $y_c(t) = x_c^3(t)$ ?

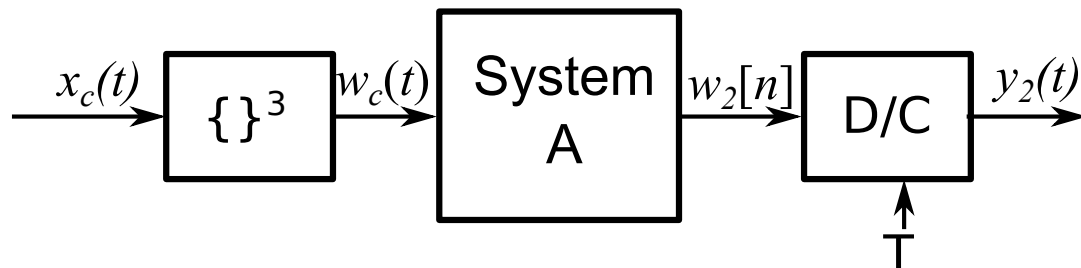
$W_c(j\Omega)$ :

$W(e^{j\omega})$ :

$Y(j\Omega)$ :

$y_c(t) = x_c^3(t)$ ?: yes/no

b) Consider the following case:



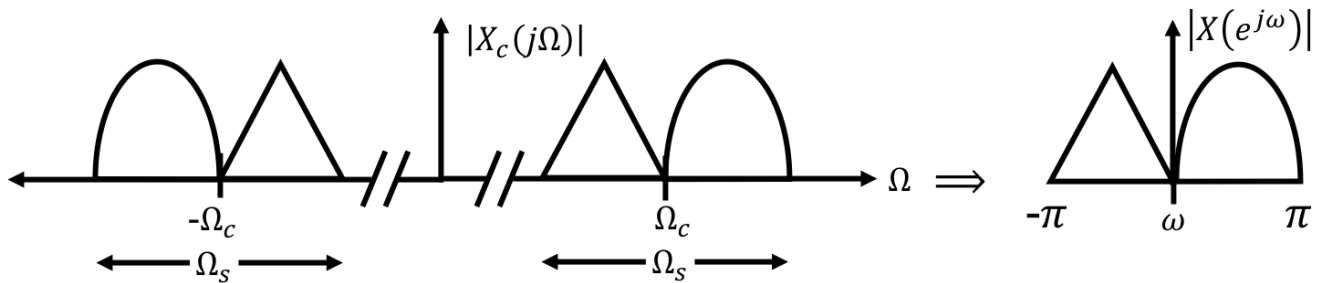
where the signal  $x_c(t)$  is the band limited signal from part (a), and  $1/T = 3f_m$ .

Your task is to design a continuous-time to discrete-time signal-recovery system ("system A"), for which  $y_2(t) = x_c(t)$ . You can use any (linear or not linear) components you need in discrete-time, but apart from a C/D, you are **not allowed to process anything in continuous time**.

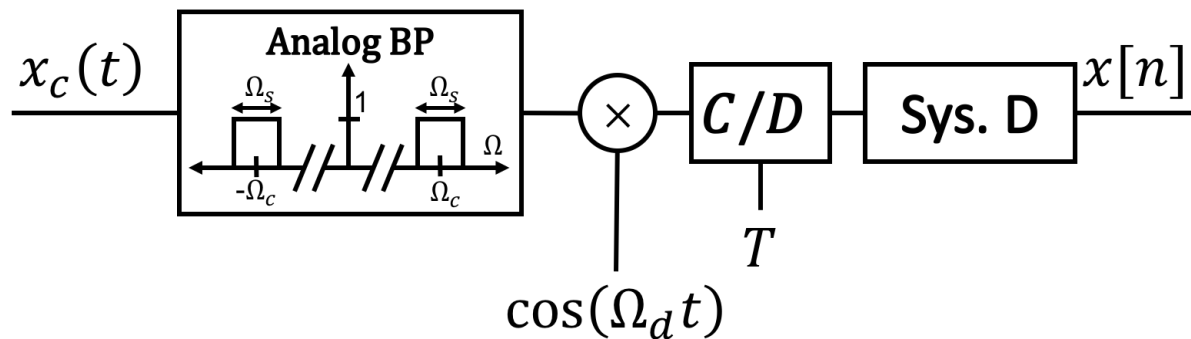
sketch system A:

6. From Midterm II, Fall '18: Software Radio

Given a **real-valued** continuous time signal,  $x_c(t)$ , your goal is to obtain samples  $x[n]$  which represent a bandwidth of  $\Omega_s$  around a center frequency  $\Omega_c$ , as shown here:



You would like to implement this using a single mixer and C/D as shown in the diagram below.



Draw a diagram of System D, and specify  $\Omega_d$  and  $T$ . Be sure to explain how you got to your solution by reasoning and showing intermediate results at various stages within the system.

Cont. here....

$\Omega_d =$

$T =$

System D diagram



## 7. Nonuniform Sampling

A student has an assignment to monitor the level of the Hetch-Hetchy reservoir in Yosemite National Park. In order to meet the Nyquist rate, he has to drive and sample the water level at least once a day. Driving every day in the bay area traffic is no fun, so he decides instead to visit the reservoir every 2 days. He arrives at Yosemite early in the day and records the level of the reservoir. Then, he goes on a 6 hours hike and before heading back he records the level again. The samples he collects are  $g_1[n] = g(nT)$  and  $g_2[n] = g(nT + 1/4)$ , where  $g(t)$  is the continuous water level in the lake and  $T = 2$  [days]. The corresponding continuous, sampled signals are

$$\begin{aligned}\hat{g}_1(t) &= g(t)\text{III}_T(t) \\ \hat{g}_2(t) &= g(t)\text{III}_T(t - \Delta_T)\end{aligned}$$

where  $\Delta_T = 1/4$  [days], and  $\text{III}_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$  denotes an impulse train. We will show in this question that it is possible to reconstruct the continuous signal, even though the Nyquist rate appears to be violated. We will solve the problem in the frequency domain.

- (a)  $g(t)$  is band limited. If  $T = 1$  [days] is the Nyquist rate, what is its highest frequency  $f_{nyq}$ ?
- (b) For the interval  $0 \leq f \leq \frac{1}{2}$  [days<sup>-1</sup>], express  $\hat{G}_1(f)$  as a function of  $G(f)$ .
- (c) For the interval  $0 \leq f \leq \frac{1}{2}$  [days<sup>-1</sup>] express  $\hat{G}_2(f)$  as a function of  $G(f)$ .
- (d) Use the result from (b)+(c) to find  $G(f)$  as a function of  $\hat{G}_1(f)$  and  $\hat{G}_2(f)$  for  $0 \leq f \leq \frac{1}{2}$  [days<sup>-1</sup>].
- (e) Repeat (b)-(d) for the interval  $-\frac{1}{2} \leq f \leq 0$  [days<sup>-1</sup>].
- (f) In order to recover  $G(f)$  you had to invert a set of linear equations. Express the set of equations in (b)-(c) in matrix form. Plot the condition number (use the `numpy.linalg.cond` function in Python) for  $0 < \Delta_T < \frac{1}{2}$ . The condition number is the ratio between the highest and lowest eigen-values and indicates how much noise amplification occurs when inverting the matrix. From the plot, what can you conclude about the cost of nonuniform sampling?