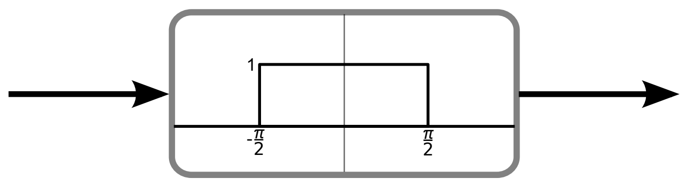
Assignment 1

Due February 4^{th} 2019

1. Read Chapter 2 Oppenheim and Schafer, 3rd ed.
2. Problem 2.51 , Oppenheim and Schafer, 3rd ed.
3. Problem 2.63 , Oppenheim and Schafer, 3rd ed.
4. Problem 2.77, Oppenheim and Schafer, 3rd ed.
5. Consider the following ideal low-pass system with cutoff frequency $\omega_0 = \pi/2$,



   (a) What is the impulse response of the system?
   (b) Is the system causal?
   (c) Is the system BIBO stable?
   (d) What if the cutoff frequency was $w_0 = 0.999999\pi$. What would be your answers to (b) and (c) if you were a mathematician? What would an engineer say?

6. Consider the moving average filter:

$$y[n] = \frac{1}{6}\sum_{k=0}^{5} x[n-k].$$

   (a) Calculate and draw the frequency response $|H(e^{j\omega})|$ and $\angle H(e^{j\omega})$ of this filter. Determine the zero-crossings of $H(e^{j\omega})$
   (b) Let $x[n] = cos(\omega_1 n)u[n]$ where ω_1 is the frequency of the first zero-crossing. Calculate the resulting output signal $y[n]$ for $n = 0, \cdots, 10$. Explain why $y[n]$ has a transient before settling to zero.

7. Consider the following filtering scheme where,

$$y[n] = \mathcal{P}\{x[n-2],\ x[n-1],\ x[n],\ x[n+1],\ x[n+2]\}.$$

   The function $\mathcal{P}(\cdot)$ performs a local quadratic polynomial regression $p[k] = a_0 + a_1 k + a_2 k^2$ $(-2 \leq k \leq 2)$ to the input and returns $p[0] = a_0$.

   (a) Find the solution for $y[0]$ for an arbitrary input $x[n]$. (HINT: a_0, a_1, and a_2 are a solution of a least-squares problem)

(b) Find the solution for $y[1]$ for an arbitrary input $x[n]$.

(c) What can you say about the properties of this filtering scheme?

- Is it linear? Is it shift-invariant? Is it stable?
- Does it have a frequency response?
- Do you really need to perform a polynomial regression for every n ?

(d) This filter is a form of the Savitzky - Golay smoothing filter. From Wikipedia:
"The SavitzkyGolay smoothing filter is a type of filter first described in 1964 by Abraham Savitzky and Marcel J. E. Golay. The Savitzky - Golay method essentially performs a local polynomial regression (of degree k) on a series of values (of at least k+1 points which are treated as being equally spaced in the series) to determine the smoothed value for each point...... **The main advantage of this approach is that it tends to preserve features of the distribution such as relative maxima, minima and width, which are usually 'flattened' by other adjacent averaging techniques (like moving averages, for example).** The paper that the filter appeared in is one of the most widely cited papers in the journal Analytical Chemistry[3] and is classed by that journal as one of its "10 seminal papers" saying "it can be argued that the dawn of the computer-controlled analytical instrument can be traced to this article".

Given your previous solutions, can you comment on the bolded text?