

EE123 Spring 2019

Discussion Section 4

Michael Chen
based on slides by Nick Antipa (sp'18)

Overlap-Save from Midterm I Spring 2018

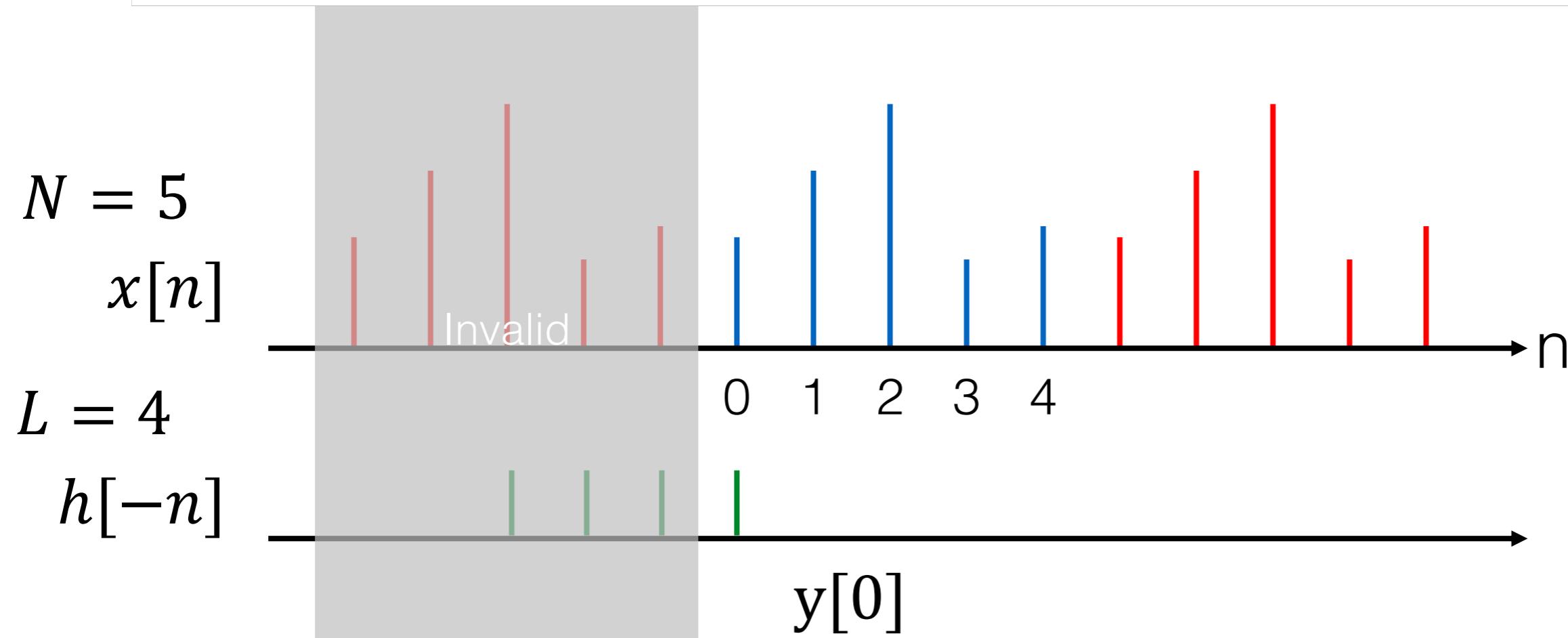
Let $h[n]$ be a **real-valued** length $L = 4$ sequence. You would like to compute the *linear* convolution

$$y[n] = x[n] * h[n],$$

using a function, which implements the overlap-and-save method.

Let $x_i[n]$ be the overlapped blocks, and let $y_i[n]$ be the saved blocks.

- a) If the block-size of each x_i block is N , what is M , the block size of each y_i block?



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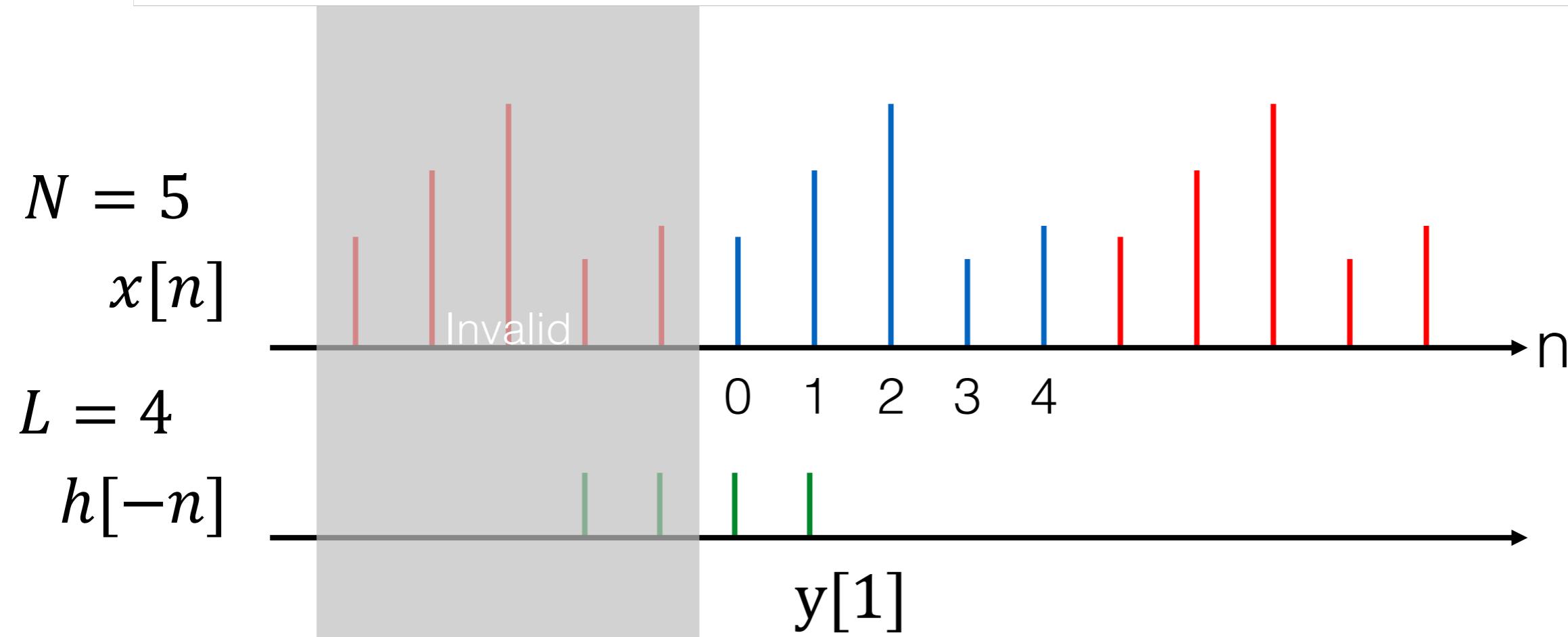
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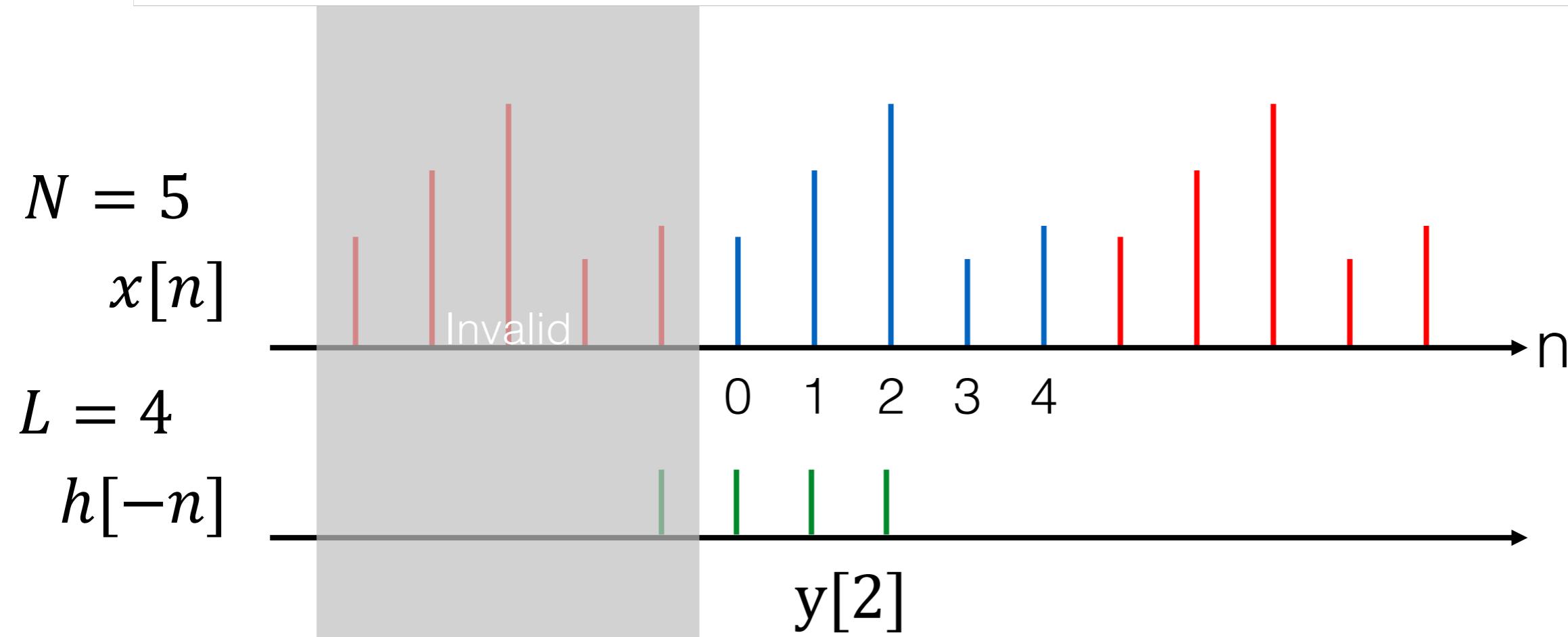
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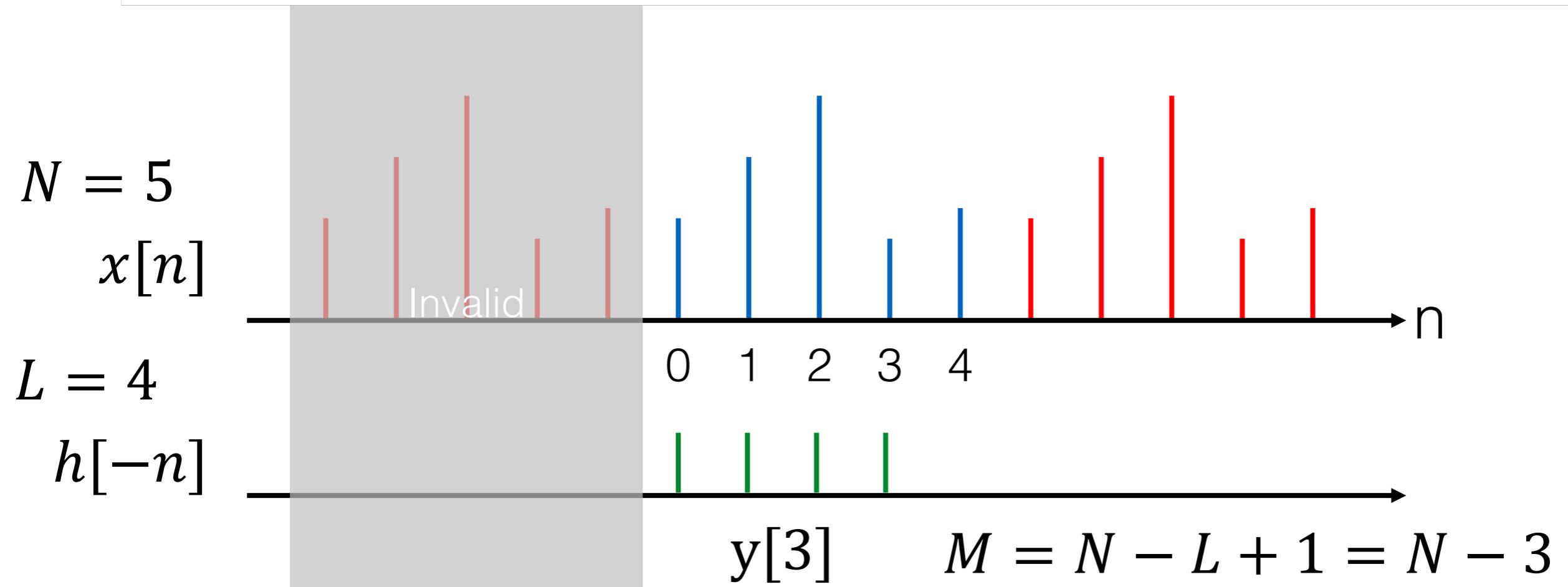
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- b) You decide to use a radix-2 FFT in the overlap-and-save method. Samples of $x[n]$ are arriving at a rate of 1000 samples/sec. What is the maximum block size N you could use when the latency of your filtering system should not exceed 1 second?
(You can assume negligible time for computing $O(N \log N)$ FFTs and $O(N)$ multiplications)

$$N = 2^i \leq 1000 \rightarrow \text{maximum block size } N = 512$$

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- c) For computing the overlap-and-save method you precompute $H[k] = \text{DFT}_N\{h[n]\}$. Unfortunately due to type-casting programming error the algorithm uses only the real-part $\tilde{H}[k] = \text{Real}\{H[k]\}$ and outputs the result: $\tilde{y}[n]$. Note, that the overlap-and-save function remains unchanged and "unaware" of the bug.

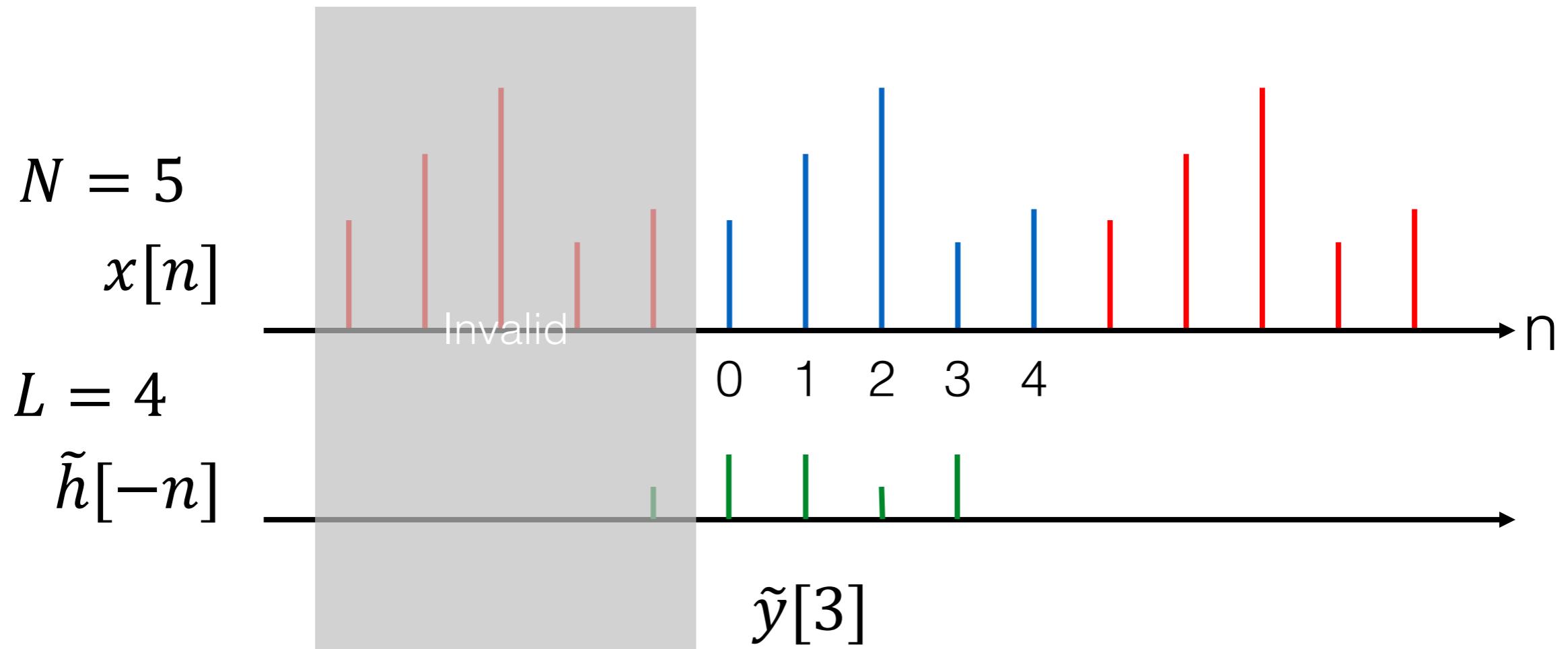
What is $\tilde{h}[n]$ the inverse DFT of $\tilde{H}[k]$? Express the solution in terms of $h_{zp}[n]$, the zero-padded original $h[n]$. (Hint: You might find symmetry properties of the DFT useful)

$$\tilde{H}[k] = \text{Real}\{H[k]\} = \frac{1}{2}(H[k] + H^*[k])$$

$$\tilde{h}[n] = \text{IDFT} \left\{ \frac{1}{2}(H[k] + H^*[k]) \right\} = \frac{1}{2} \left(h_{zp}[n] + h_{zp} [((-n))_N] \right)$$

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d) Is the modified system in part (c) linear? Is it time-invariant?



still perform a circular convolution \rightarrow linear

some saved values now depends on shifted $x[n]$ \rightarrow not time-invariant

STFT

$$X[r, k] = \sum_{m=?}^{?} x[rR + m]w[m]e^{-j2\pi km/N}$$

What is the variable of summation?

STFT

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And the length of the sum?

STFT

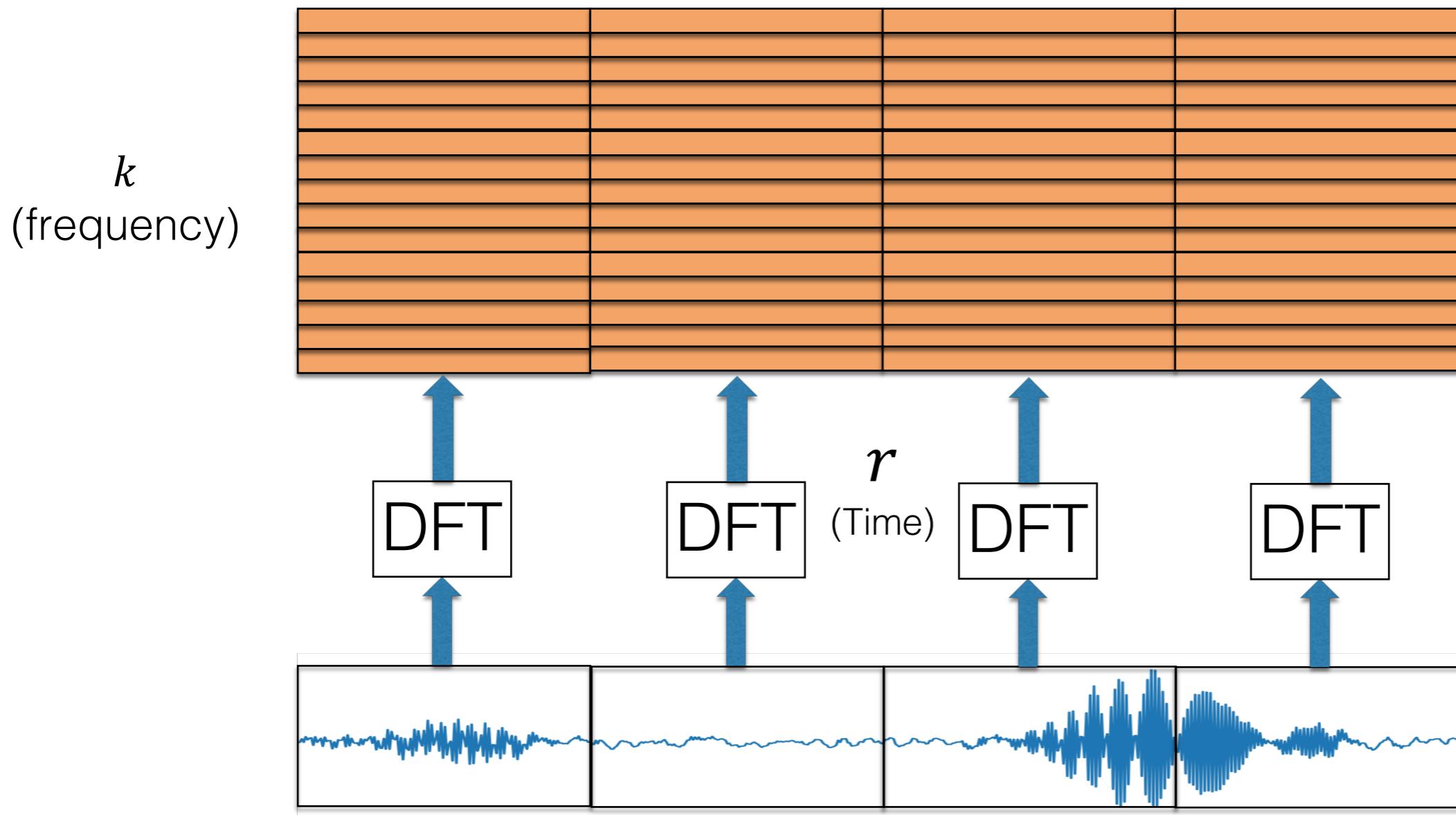
$$X[r, k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

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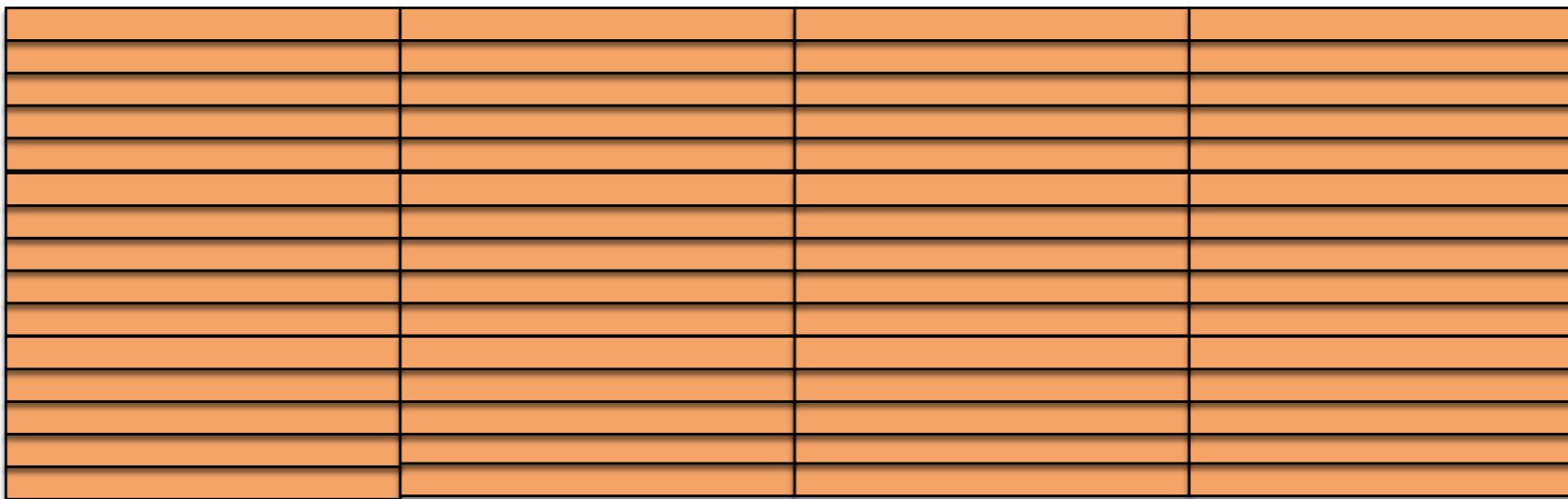
What is R for?

Non-overlapping windows



$$X[r, k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}$$

Non-overlapping rect windows



$$\begin{aligned} X[r, k] &= \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N} \\ &= \underbrace{\sum_{m=0}^{L-1} \text{rect}\left(\frac{m - L/2}{L}\right)}_{\text{Window Function}} x[rR + m]e^{-j2\pi km/N} \end{aligned}$$

What will this do to frequency space?

It will blur frequency space

Question: if I want to display a real-time equalizer of my music, what would a reasonable STFT window length be? Assume I am sampling at 48 kHz, and want to refresh my spectrum analyzer at 24 Hz.

$$A: 48000/24 = 2000 \text{ samples}$$

What is my frequency resolution, in Radians and Hertz?

$$\Delta t = L = 2000$$

$$\Delta t \Delta \omega = 2\pi$$

$$\Delta \omega = \frac{2\pi}{2000}$$

$$\Delta f = \frac{48,000}{2000} = 24 \text{ Hz}$$

Python demo