EE123
Digital Signal Processing

Lecture 31
Tomography + Lab 5b

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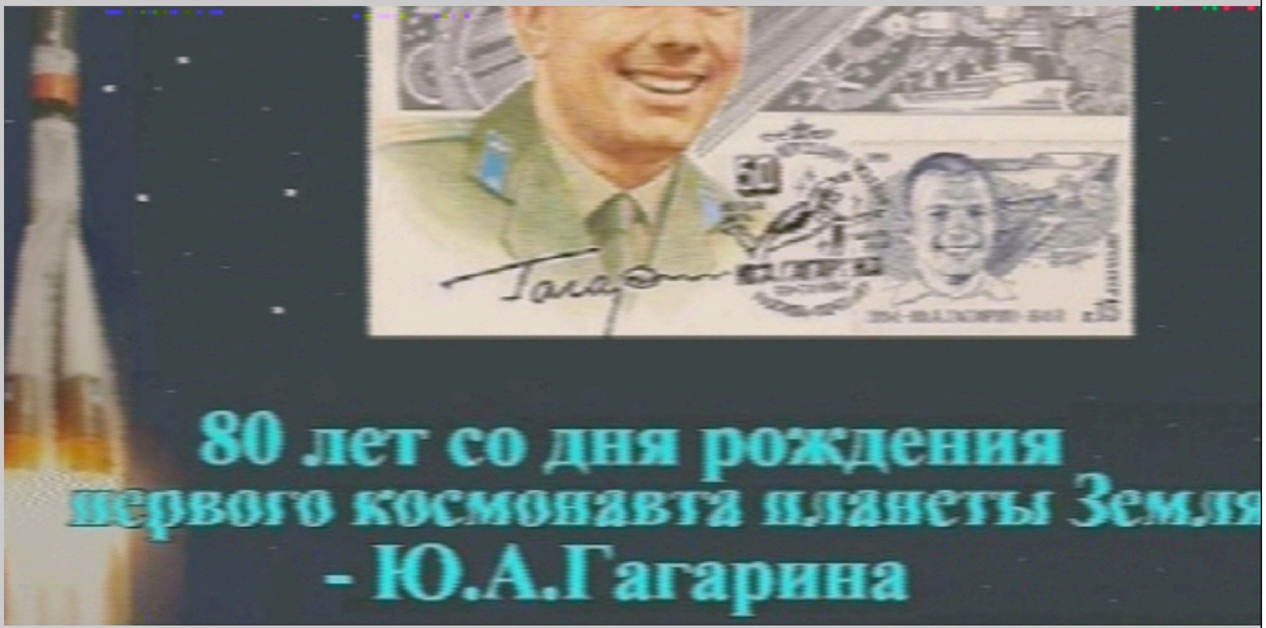
Projects

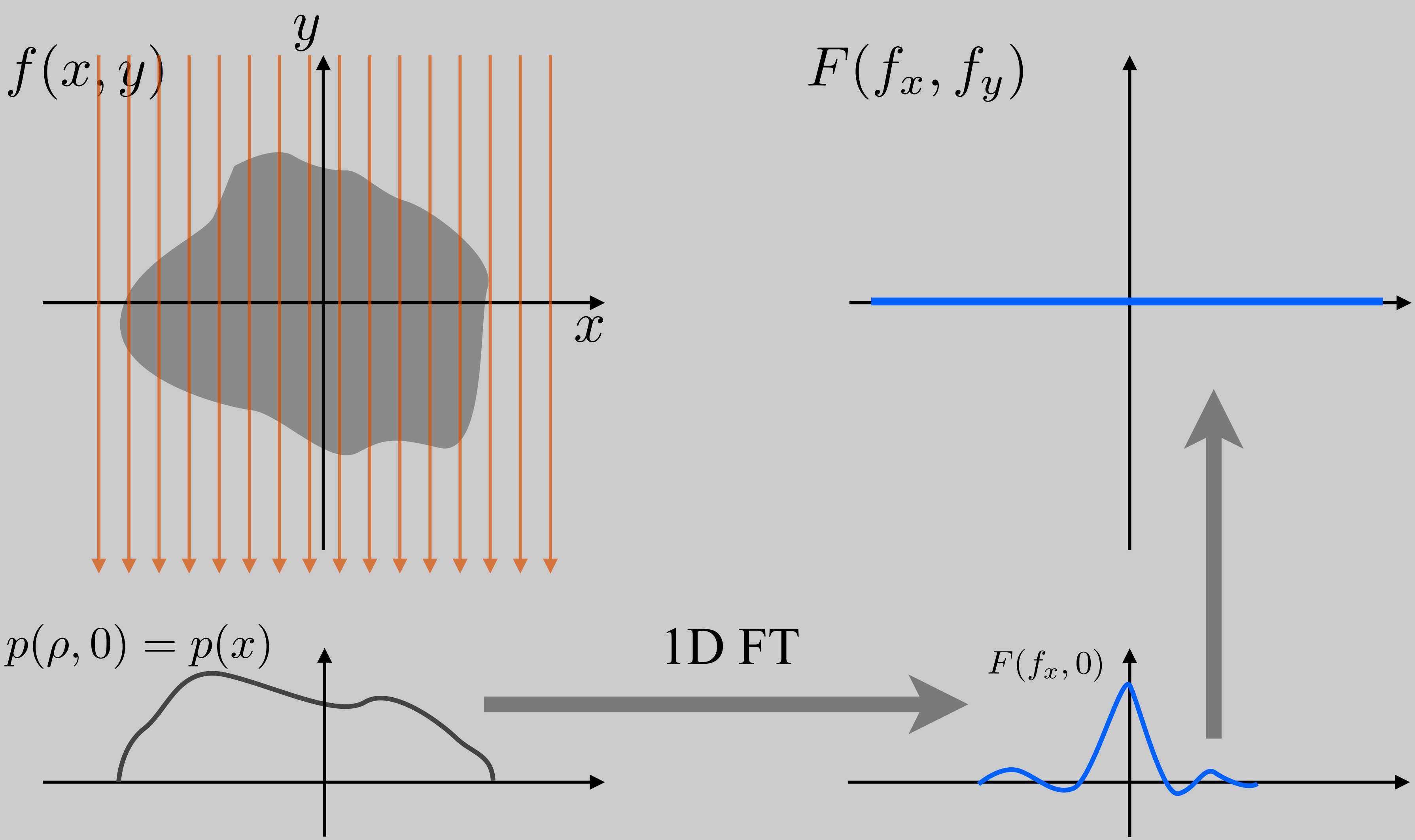
- Some no shows on Monday
- Today everyone has to meet with me -- I'll add more and post.



Projection Slice Theorem (Bracewell)

sine

$$\mathcal{F}_{1\mathrm{D}}\{p(\rho,\theta)\} = F(\rho\cos\theta, \rho\cos\theta)$$



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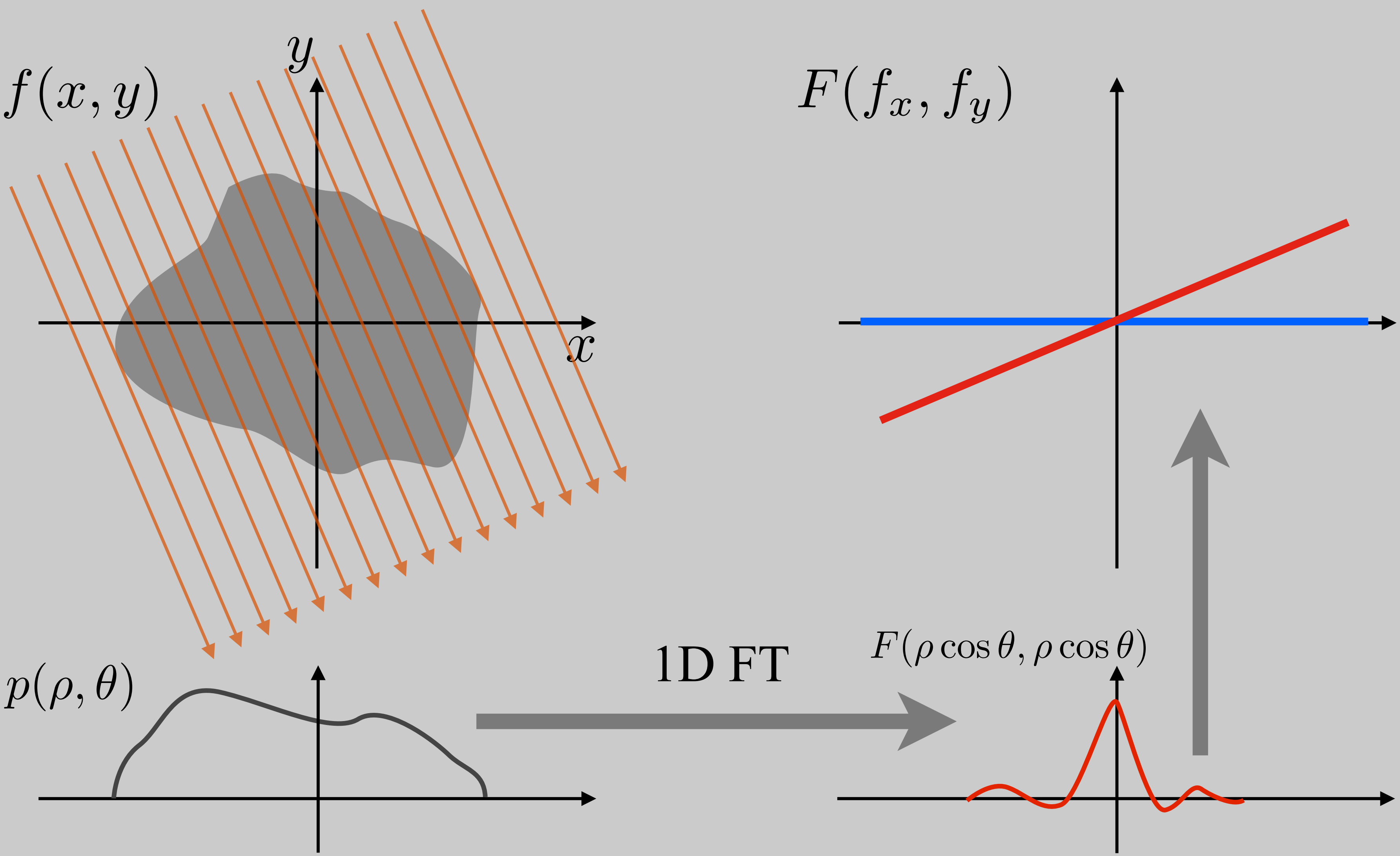

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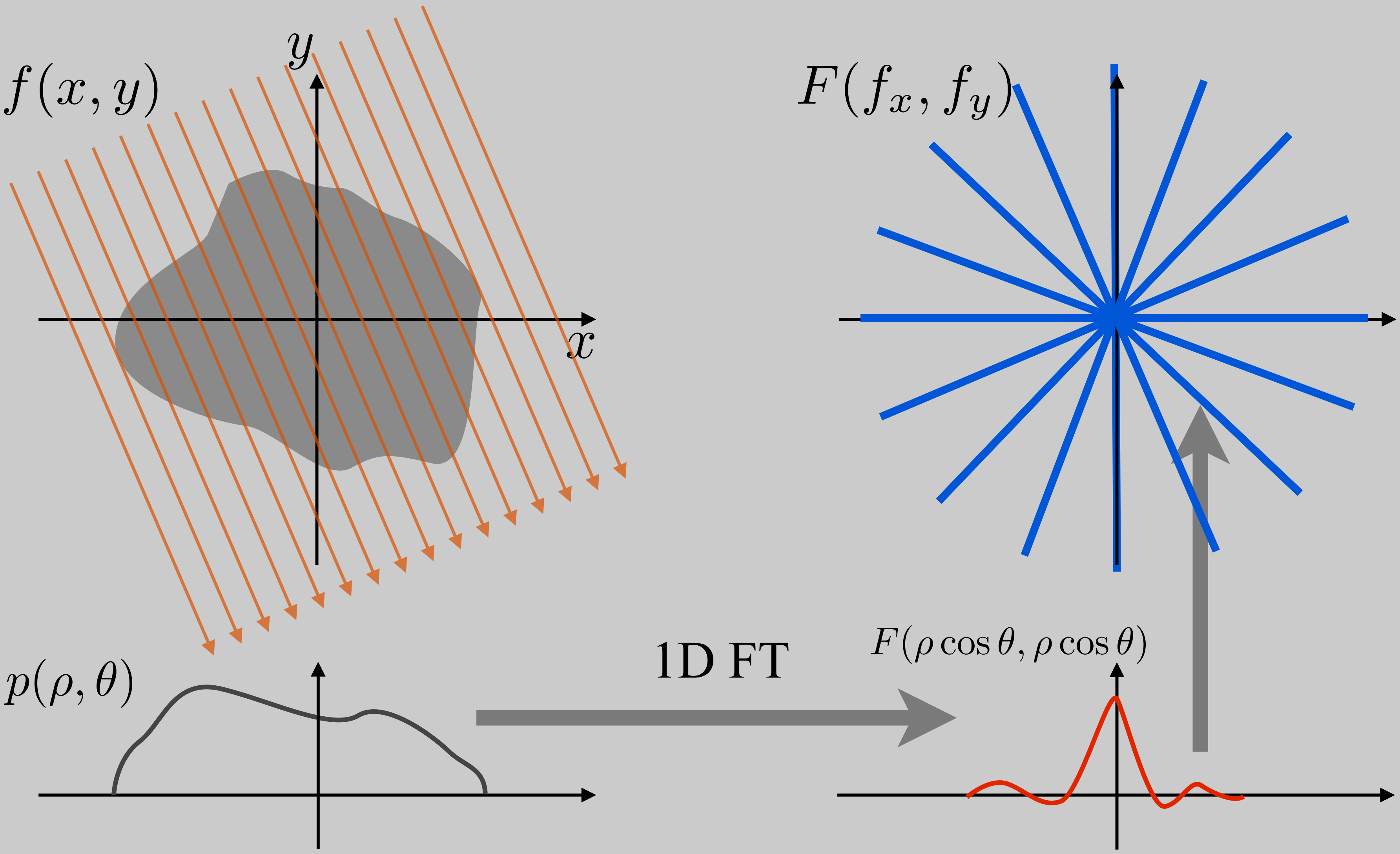


Projection Slice Theorem (Bracewell)

Proof (for $\theta = 0$)

$$p(x) = \int_{-\infty}^{\infty} m(x,y)\,dy$$

$$M(k_x, 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\, e^{-i2\pi(k_x x + k_y y)}\,dx\,dy =$$

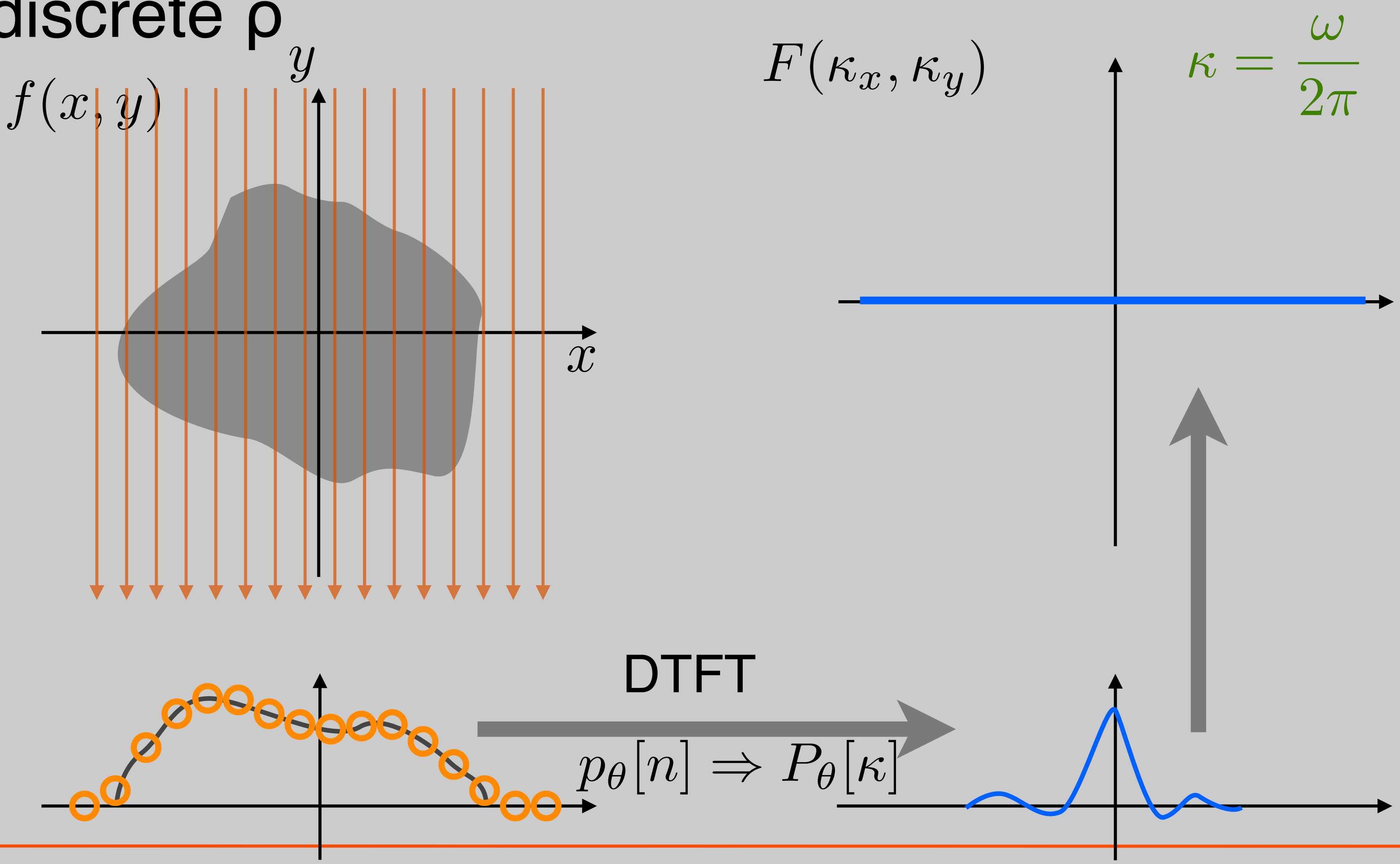
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\, e^{-i2\pi k_x x}\,dx\,dy =$$

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} m(x,y)\,dy\right] e^{-i2\pi k_x x}\,dx =$$

$$= \int_{-\infty}^{\infty} p(x)\, e^{-i2\pi k_x x}\,dx = \mathcal{F}\{p(x)\}$$

Partly Discrete Reconstruction

- Let's assume continuous angle Θ, discrete ρ



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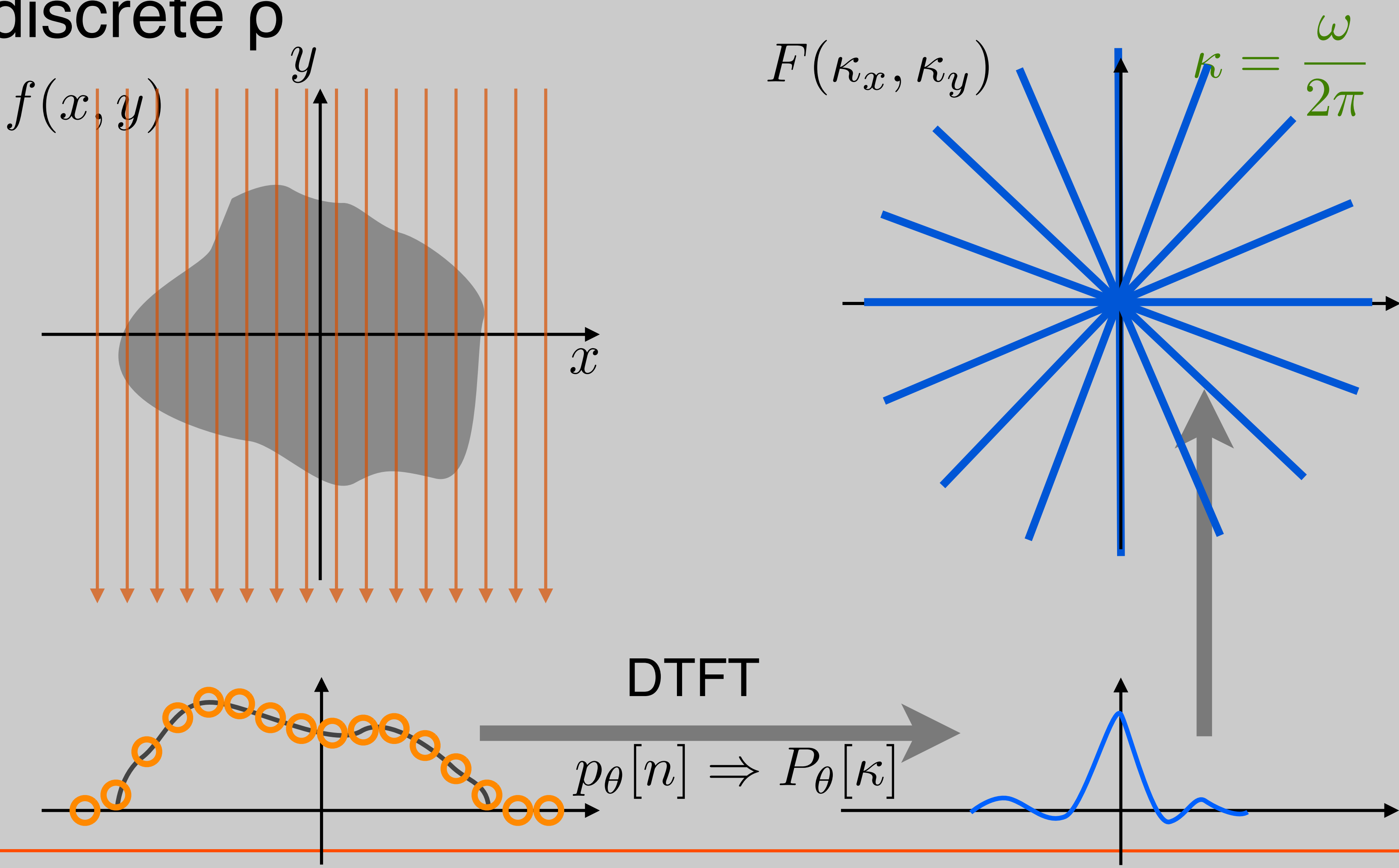


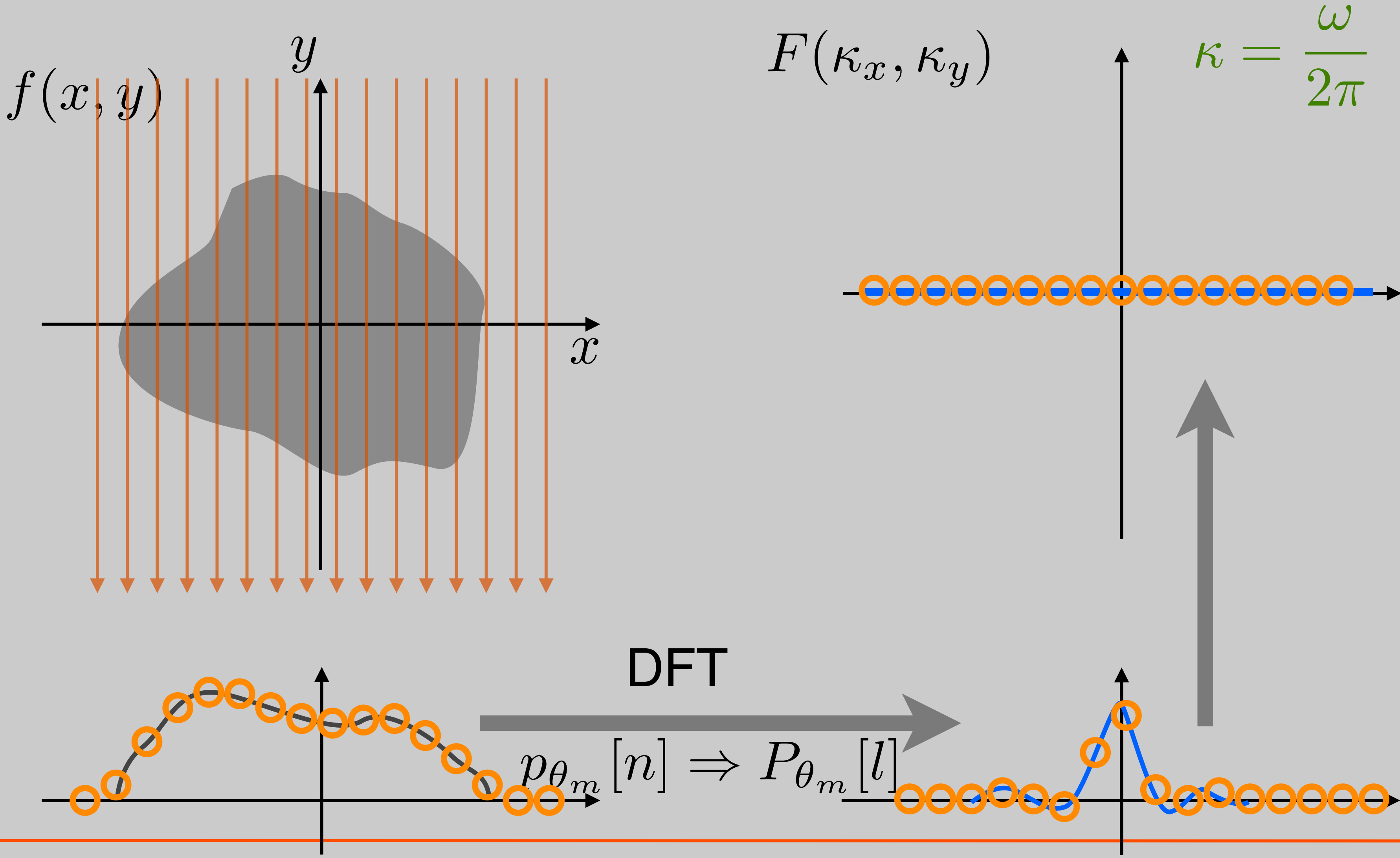
Reconstruction From Polar Coordinates

$$
\begin{aligned}
f[n,m] &= \int_{-0.5}^{0.5}\int_{-0.5}^{0.5} F(\kappa_x,\kappa_y)e^{2\pi j(\kappa_x n+\kappa_y m)}d\kappa_x d\kappa_y \\
&= \int_{0}^{\pi}\int_{\text{-0.5}}^{0.5} F(\rho,\theta)e^{2\pi j(\rho\cos(\theta)n+\rho\sin(\theta)m)}|\rho|d\rho d\theta
\end{aligned}
$$

- Polar frequency data must be multiplied by $|\rho|$
- Also called a rho filter

Discrete Reconstruction

- Let's assume discrete angle Θ_m, discrete ρ



Discrete Reconstruction

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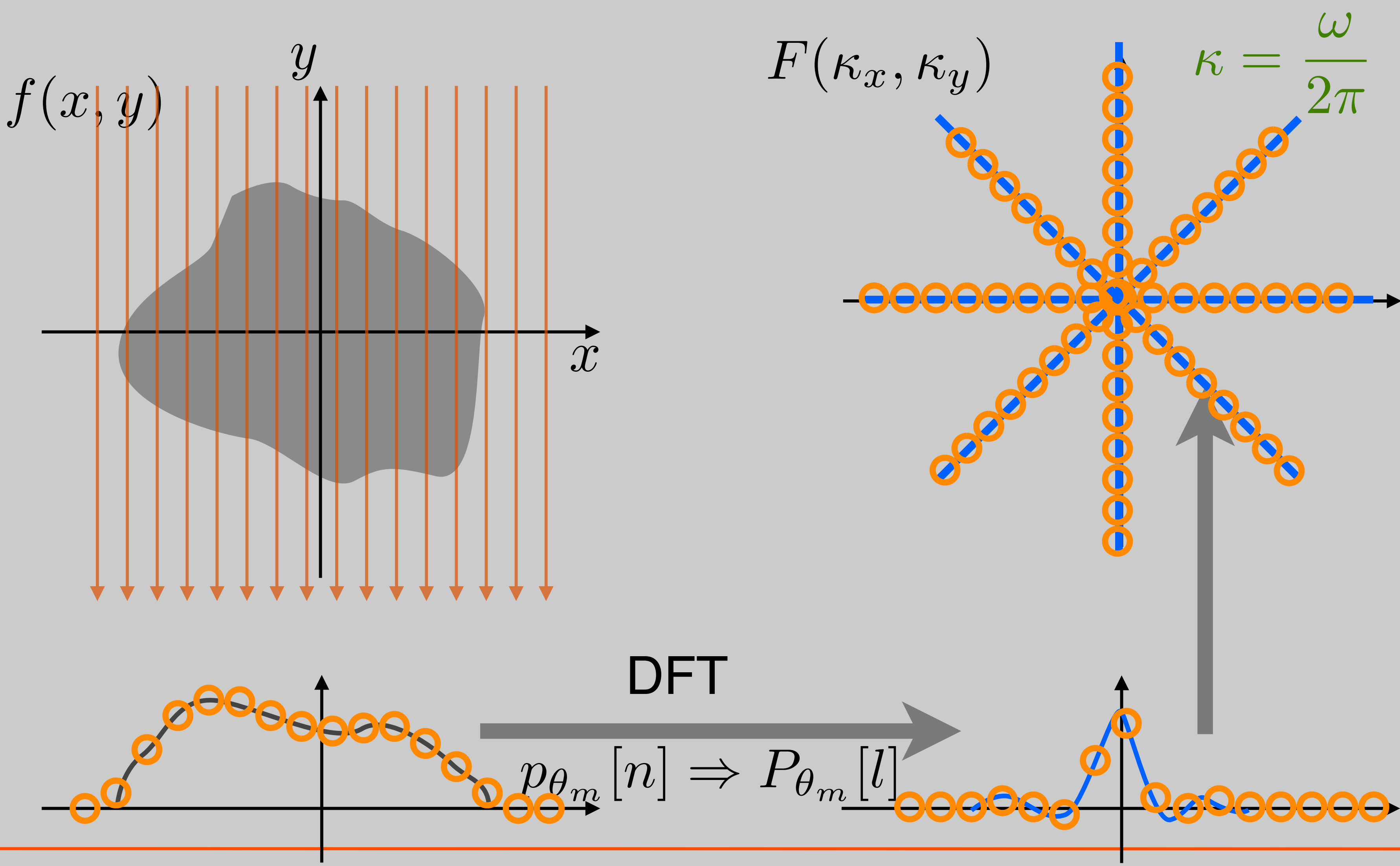


Filtered Back Projection

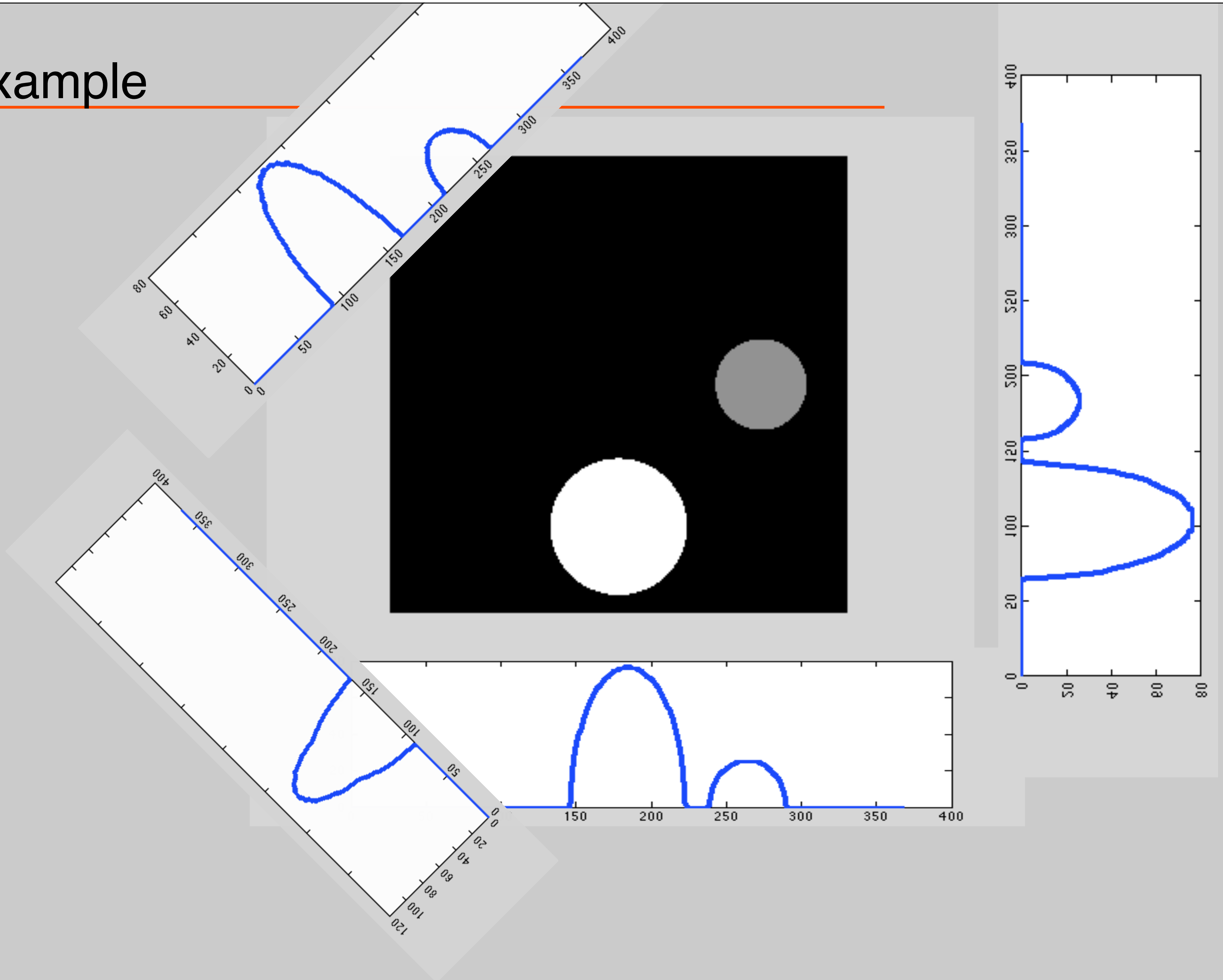
- Replace integrals with sums. Sum over radius and angle
- Define a (filtered) backprojection:

$$C_{\theta_m}[n_x, n_y] = \sum_{l=-N/2}^{(N/2)-1} F[l, \theta_m] e^{2\pi j (l/N \cos(\theta_m) n_x + l/N \sin(\theta_m) n_y)} \underbrace{|l/N|}_{=\rho}$$

So,

$$f[n_x, n_y] = \sum_m C_{\theta_m}[n_x, n_y]$$

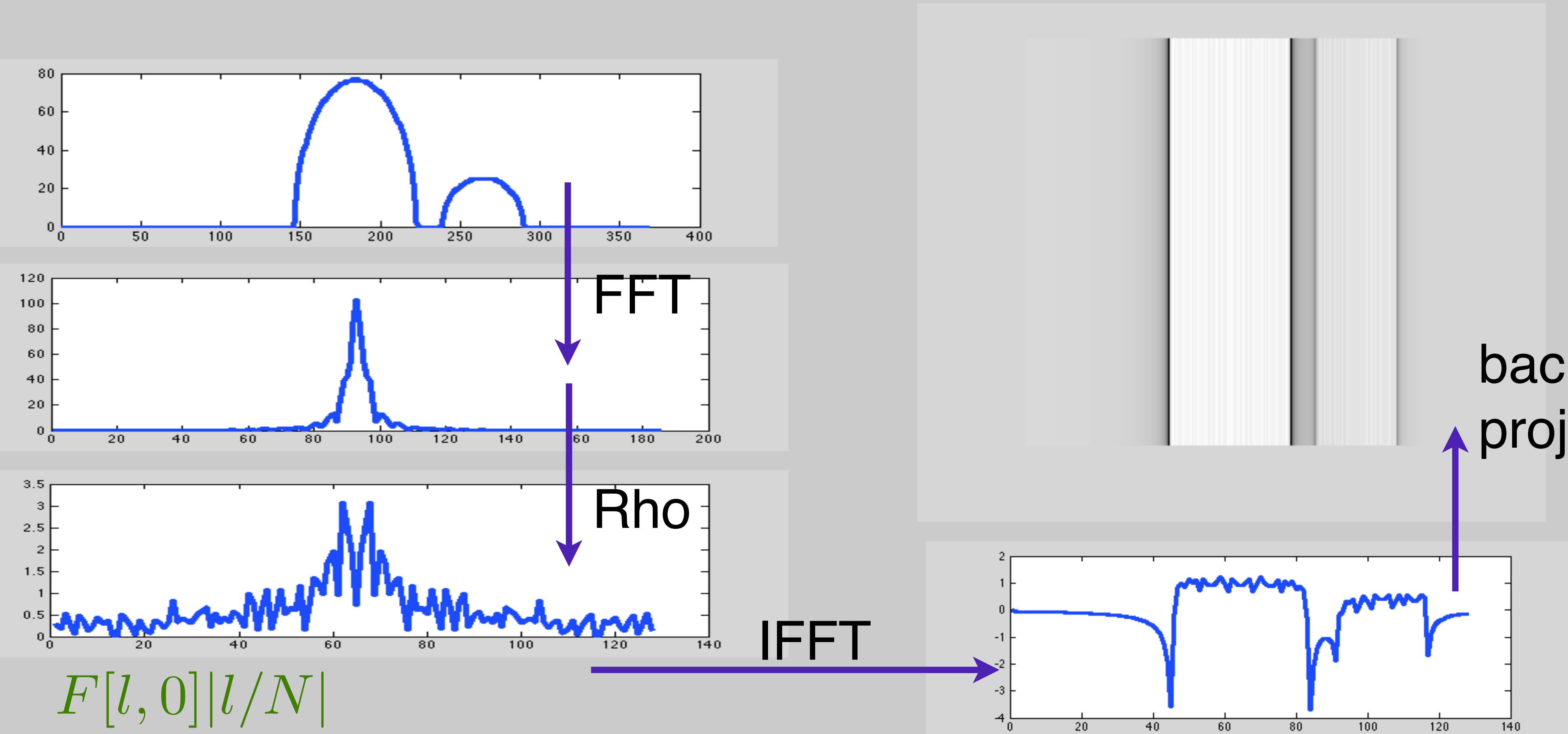
Example



Example Convolution Back Projection

- For Θ=0

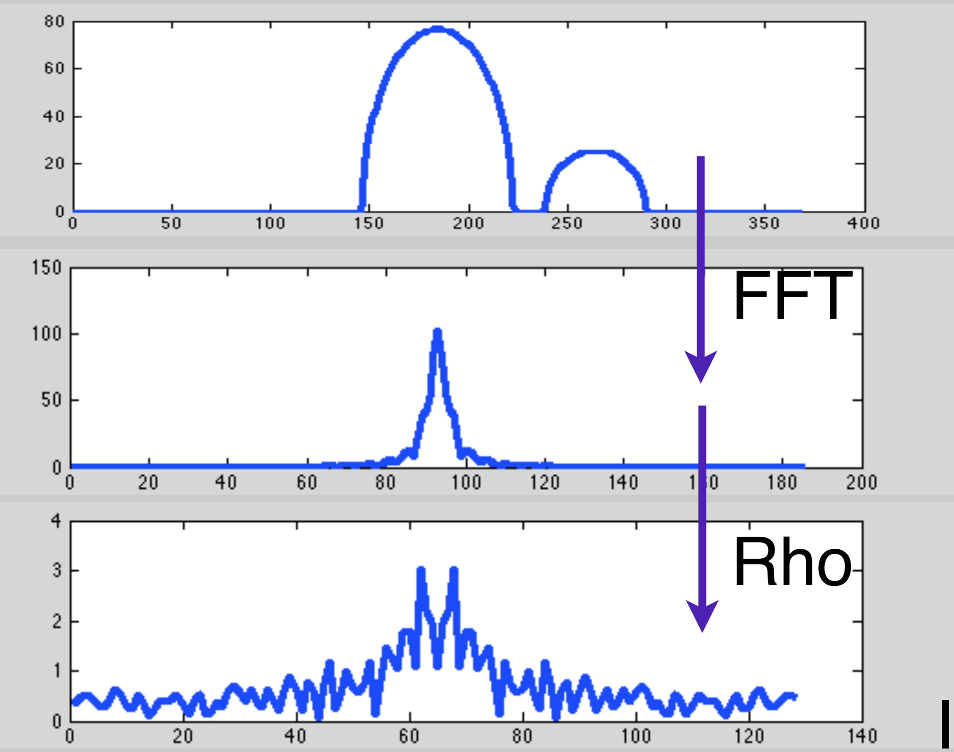
$$C_0[n_x, n_y] = \sum_{l=-N/2}^{(N/2)-1} F[l,0]|l/N|e^{2\pi j(l/N n_x)}$$



$F[l,0]|l/N|$

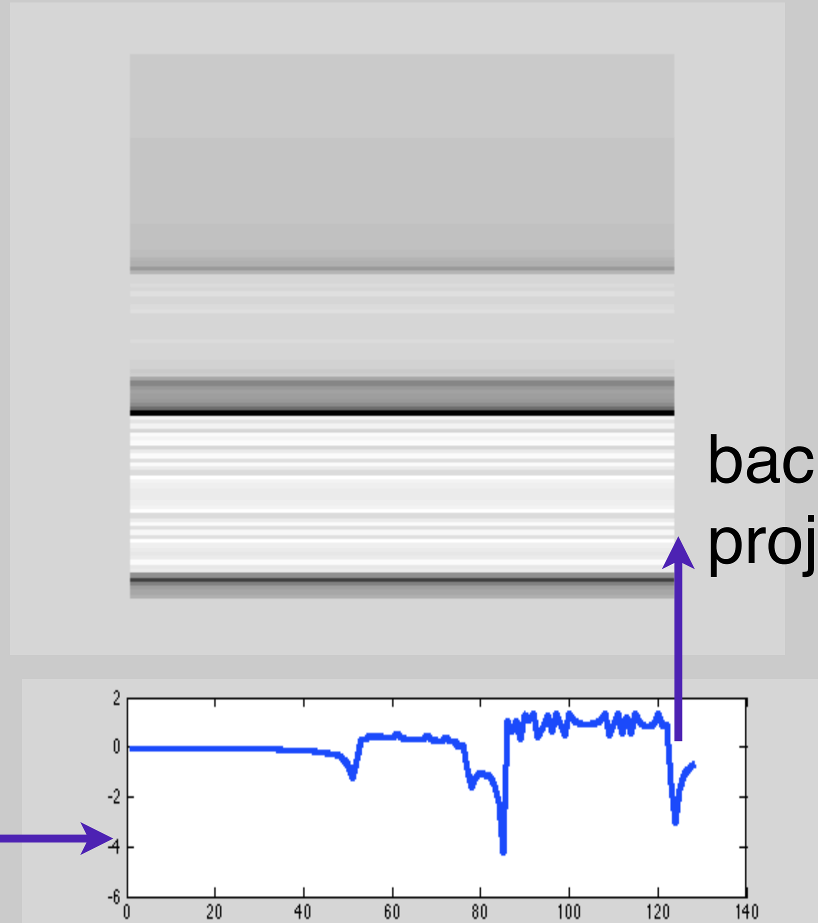
Example Convolution Back Projection

- For Θ=π/2 $$C_{\pi/2}[n_x, n_y] = \sum_{l=-N/2}^{(N/2)-1} F[l, \pi/2]|l/N|e^{2\pi j(l/N n_y)}$$

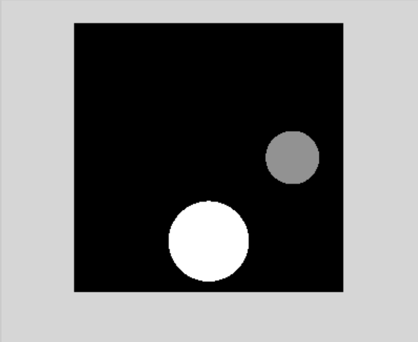


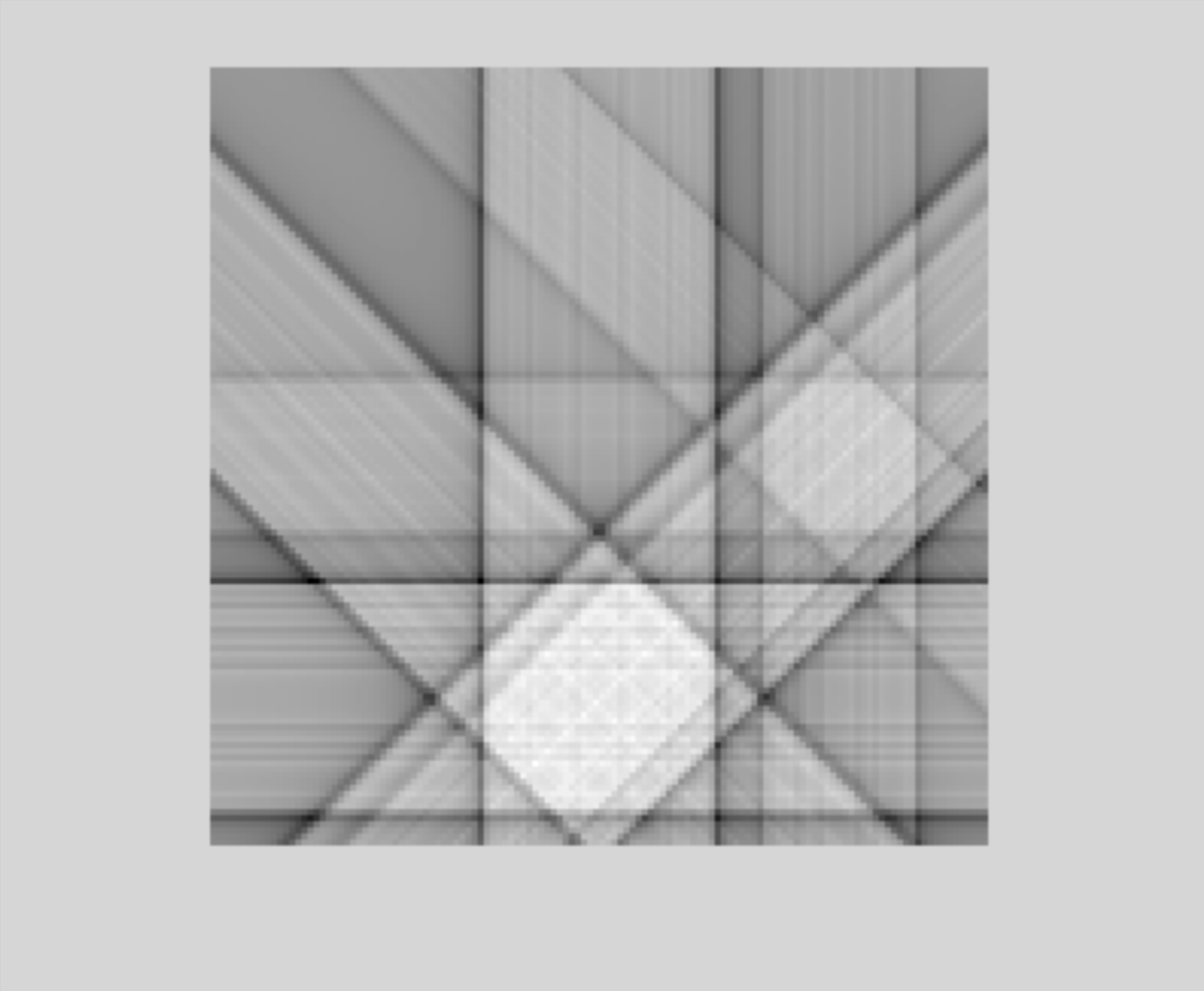
$F[l, \pi/2]|l/N|$

IFFT



Convolution Back Projection

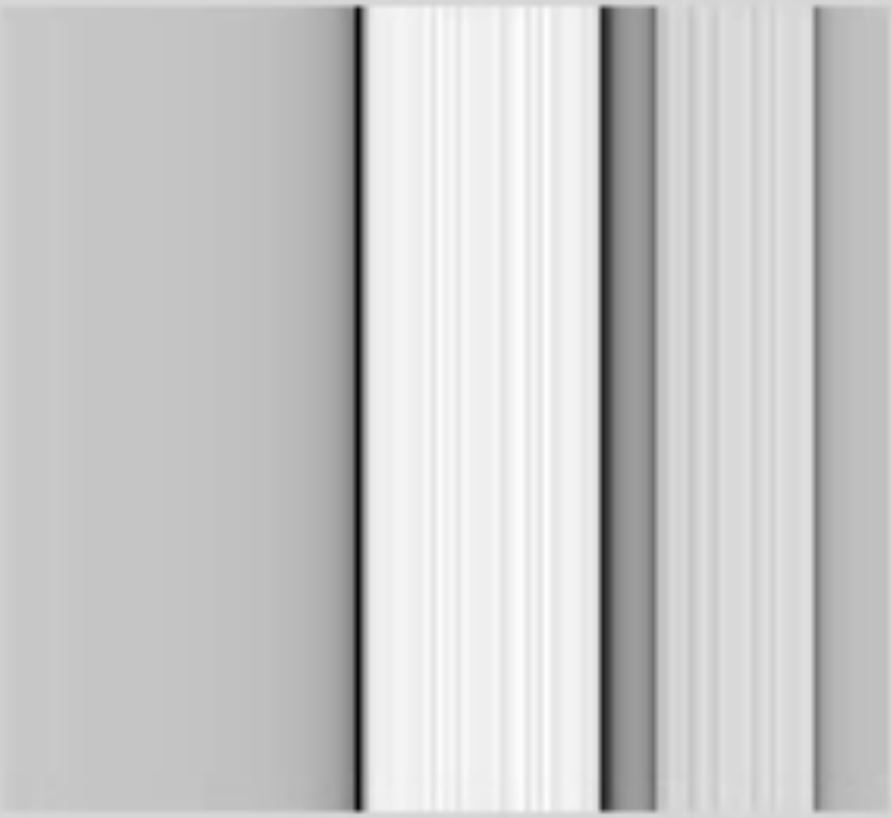




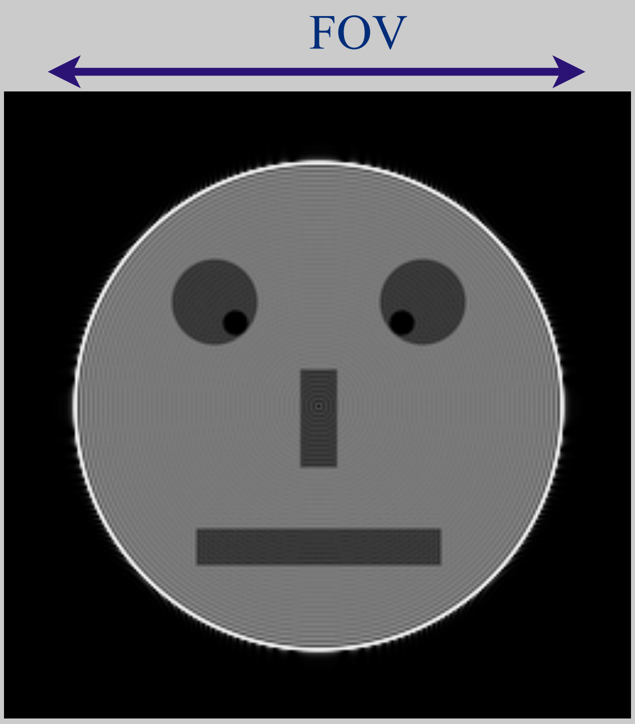
Filtered Back Projection

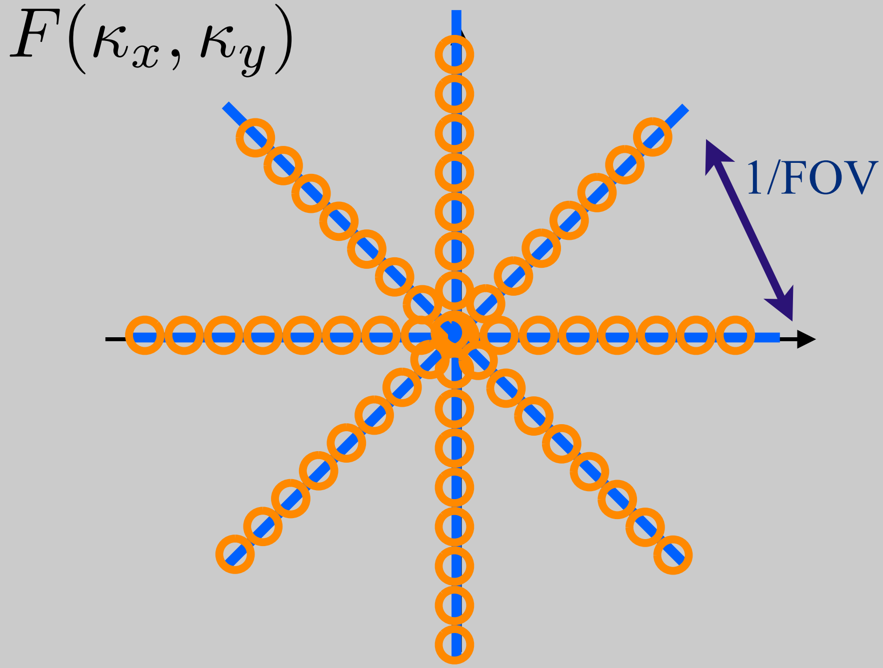
Back projection

Filtered Back projection



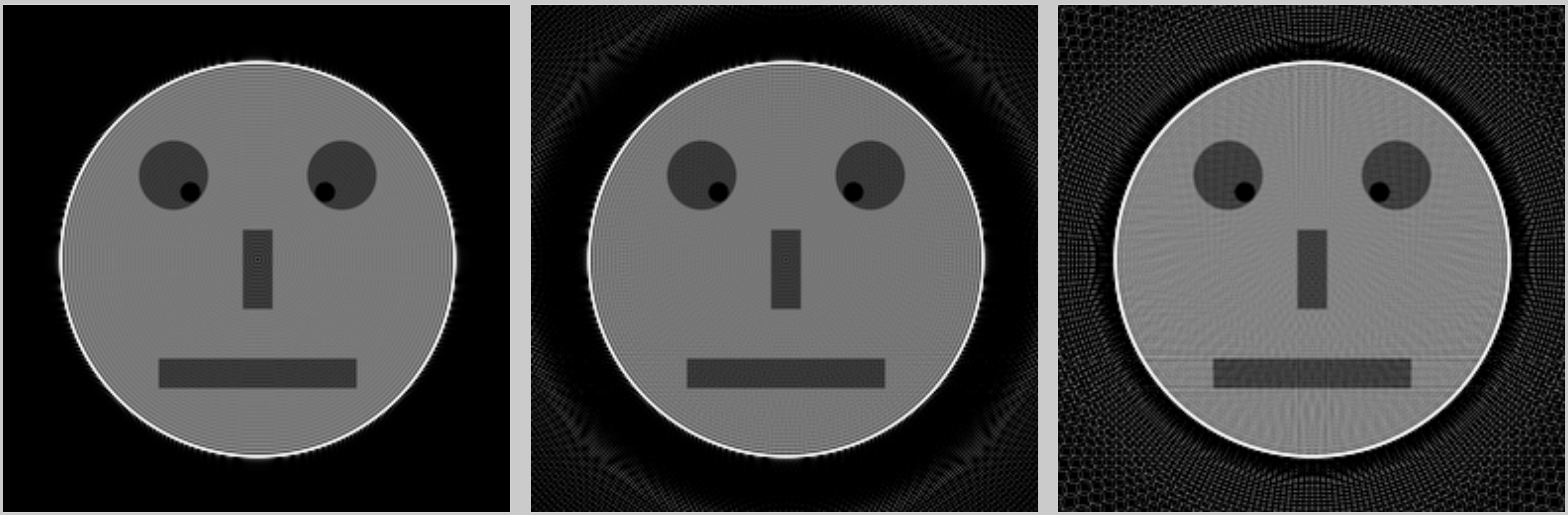
How Many Projections?





How Many Projections?

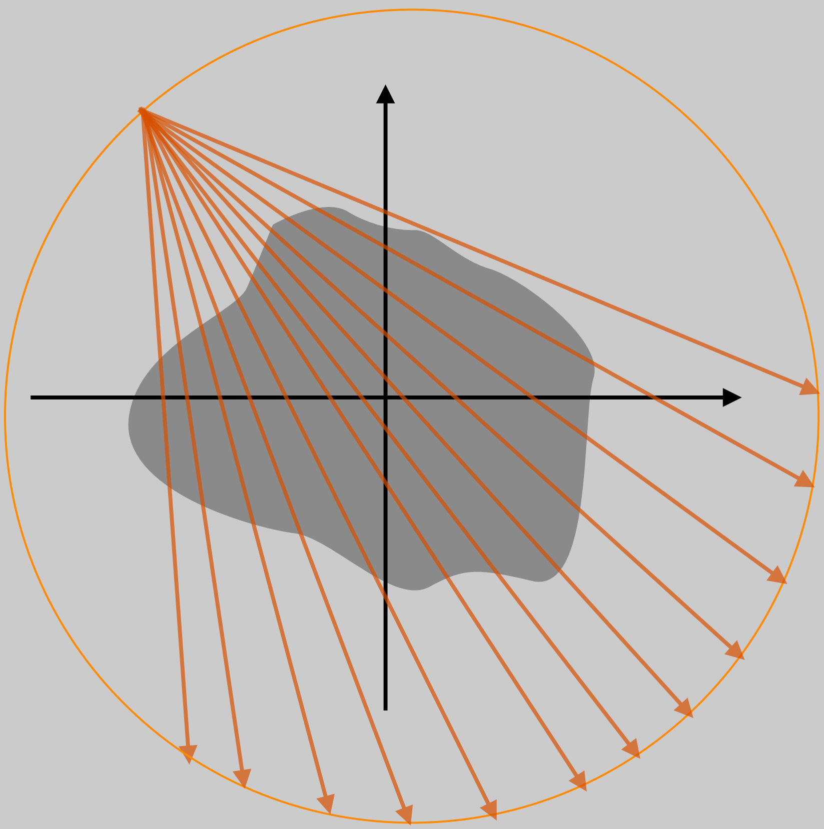
256 Proj. | 128 Proj | 64 Proj



Fan Beam CT

- Single Source
- Many detectors

- How to reconstruct?



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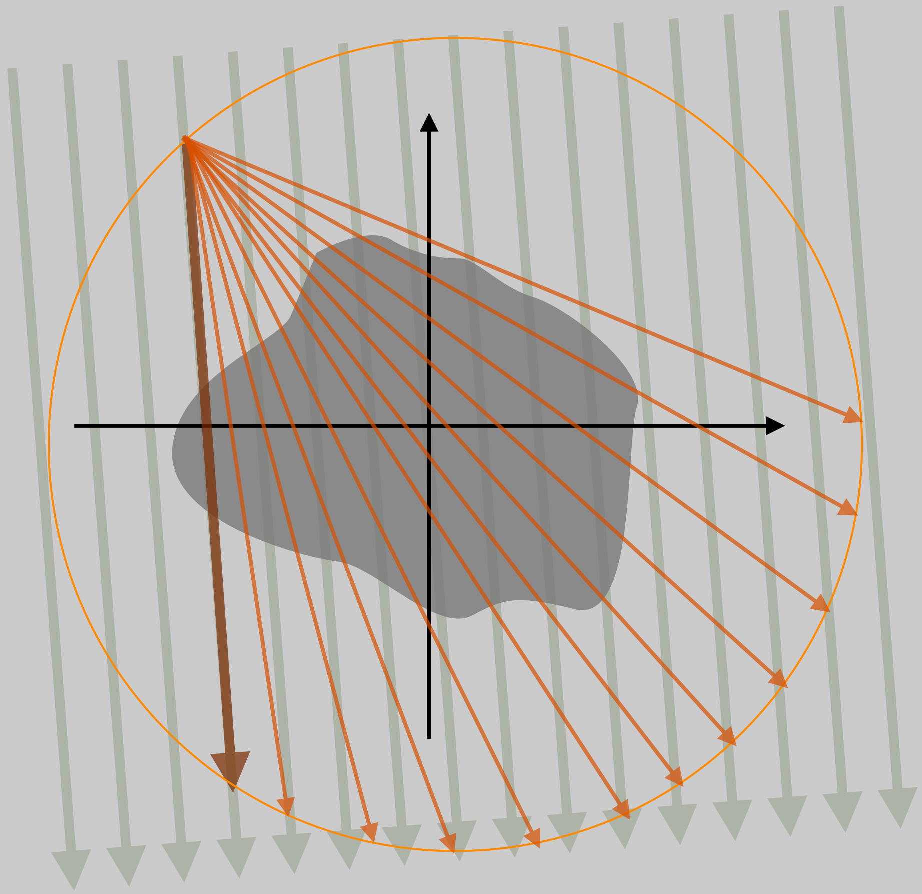
- Re-binning!



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