

Discussion 1

1. Consider the system

$$y[n] = \alpha x[n].$$

Is the system linear? Time invariant? Causal? BIBO stable?

2. Consider the system

$$y[n] = \begin{cases} \alpha x[n], & x[n] \leq 1, \\ \alpha, & x[n] > 1 \end{cases}.$$

Is the system linear? Time invariant? Causal? BIBO stable?

3. A discrete-time system H produces an output signal y that is the symmetric part of the input:

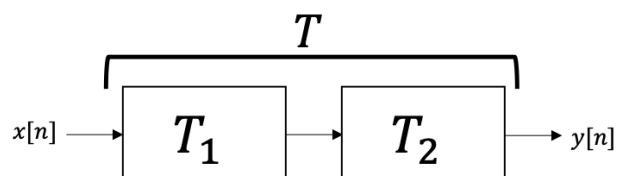
$$y[n] = \frac{x[n] + x[-n]}{2}$$

Which of the following is **true**?

- The system must be LTI
- The system cannot be LTI

4. Consider an LTI system with input $x[n]$ and output $y[n]$. When we input a signal $(1/3)^n u[n]$, where $u[n]$ is the unit step function, we observe an output $g[n]$. Can we express $y[n]$ in terms of $x[n]$ and $g[n]$?

5. Consider the following system:



Let T_1 and T_2 be separate systems and T be the cascaded system. **True or False:**

- (a) If T_1 is LTI and T_2 is not LTI, then T cannot be LTI
- (b) If T_1 is not LTI and T_2 is not LTI, then T cannot be LTI

6. Find $\beta \in \mathbb{R}^2$ which minimizes the mean squared error (MSE):

$$\frac{1}{2} \|\mathbf{x} - \mathbf{K}\beta\|_2^2$$

for known $\mathbf{x} \in \mathbb{R}^d$ and

$$\mathbf{K} = \begin{bmatrix} n & 1 \\ n-1 & 1 \\ \vdots & \vdots \\ -n & 1 \end{bmatrix}.$$