EE 123 Discussion Section 3

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Based on slides by Michael Lustig, Frank Ong, and Jon Tamir

Announcements

- Lab 1 – due next Wednesday Feb. 20 midnight
- Extra OH next Tuesday Feb. 19, 6-7pm, 531 Cory
- HW 3 (HW2 self-grade) – due next Monday Feb. 18
- Questions?

Discrete Fourier Transform Recap

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \qquad \text{[Analysis]}$$

$$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X[k] W_N^{-kn} \qquad \text{[Synthesis]}$$

- Sampling DTFT at $\omega = \frac{2\pi k}{N}$
- Periodic nature of the sequence from DFS
- Sampling z-transform at $z = e^{j\frac{2\pi k}{N}}$

DFT Question 1

$$x[n] = \{-3, 5, 4, -1, -9, -6, -8, 2\}$$

a) Evaluate $\sum_{k=0}^{7}(-1)^k X[k]$

b) Evaluate $\sum_{k=0}^{7} |X[k]|^2$

DFT solution 1

Part a)

$$(-1)^k = e^{-j\pi k} = e^{-j\frac{2\pi 4k}{8}} = W_8^{4k}$$

$$x[4] = \frac{1}{8}\sum_{k=0}^{7} X[k] W_8^{4k}$$

$$\Rightarrow \sum_{k=0}^{7} (-1)^k X[k] = 8x[4] = -72$$

Part b) by Parseval's theorem

$$\sum_{k=0}^{7} |X[k]|^2 = 8\sum_{n=0}^{7} |x[n]|^2 = 1888.$$

DFT Question - 2

Let $x[n]$ be N-point sequence. Let $X[k] = DFT\{x[n]\}$.

1. Express $x_2[n] = DFT\{X[k]\}$ in terms of $x[n]$
2. Express $x_3[k] = DFT\{x_2[n]\}$ in terms of $X[k]$
3. Express $x_4[n] = DFT\{x_3[k]\}$ in terms of $x[n]$

DFT solution - 2

$$
\begin{aligned}
x_2[n] &= DFT\{X[k]\} \\
&= \sum_{k=0}^{N-1} X[k] W_N^{kn} \\
&= \sum_{k=0}^{N-1} X[k]\, W_N^{-k(-n)} \\
&= Nx\left[\left((-n)\right)_N\right]
\end{aligned}
$$

DFT solution - 2

Using DFT Properties:

$$x_2[n] = Nx[((-n))_N]$$

$$x_3[k] = DFT\{x_2[n]\} = N\, X[((-k))_N]$$

$$x_4[n] = DFT\{x_3[k]\} = N^2\, x[n]$$

DFT Question 3 - DCT

- The Discrete Cosine Transform (DCT) is a DFT-related transform that decomposes a finite signal in terms of a sum of cosine functions

- The DCT is often used in compression schemes, such as MP3, JPEG, and MPEG

- One of the reasons is its energy **compactness**

DFT Question 3 - DCT

DEMO of DCT

DFT Question 3 - DCT

Definition of DCT (type II)

$$X_c[k] = 2\sum_{n=0}^{N-1} x[n]\cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

Question: Express $X_c[k]$ in terms of $X[k]$, the 2N-point DFT of $x[n]$

DFT Solution 3 - DCT

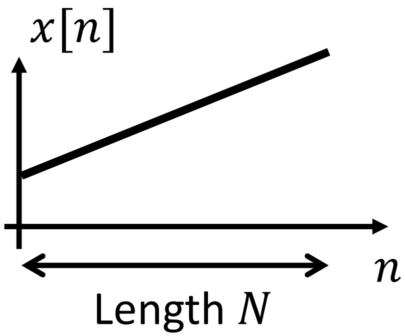
$$
\begin{aligned}
X_c[k] &= 2\sum_{n=0}^{N-1} x[n]\cos\left(\frac{\pi k(2n+1)}{2N}\right) \\
&= \sum_{n=0}^{N-1} x[n]\left(e^{-j\frac{\pi k(2n+1)}{2N}} + e^{j\frac{\pi k(2n+1)}{2N}}\right) \\
&= \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{2N}}e^{-j\frac{\pi k}{2N}} + \sum_{n=0}^{N-1} x[n]e^{j\frac{2\pi kn}{2N}}e^{j\frac{\pi k}{2N}} \\
&= \boxed{X[k]e^{-j\frac{\pi k}{2N}} + X[-k]e^{j\frac{\pi k}{2N}}}
\end{aligned}
$$

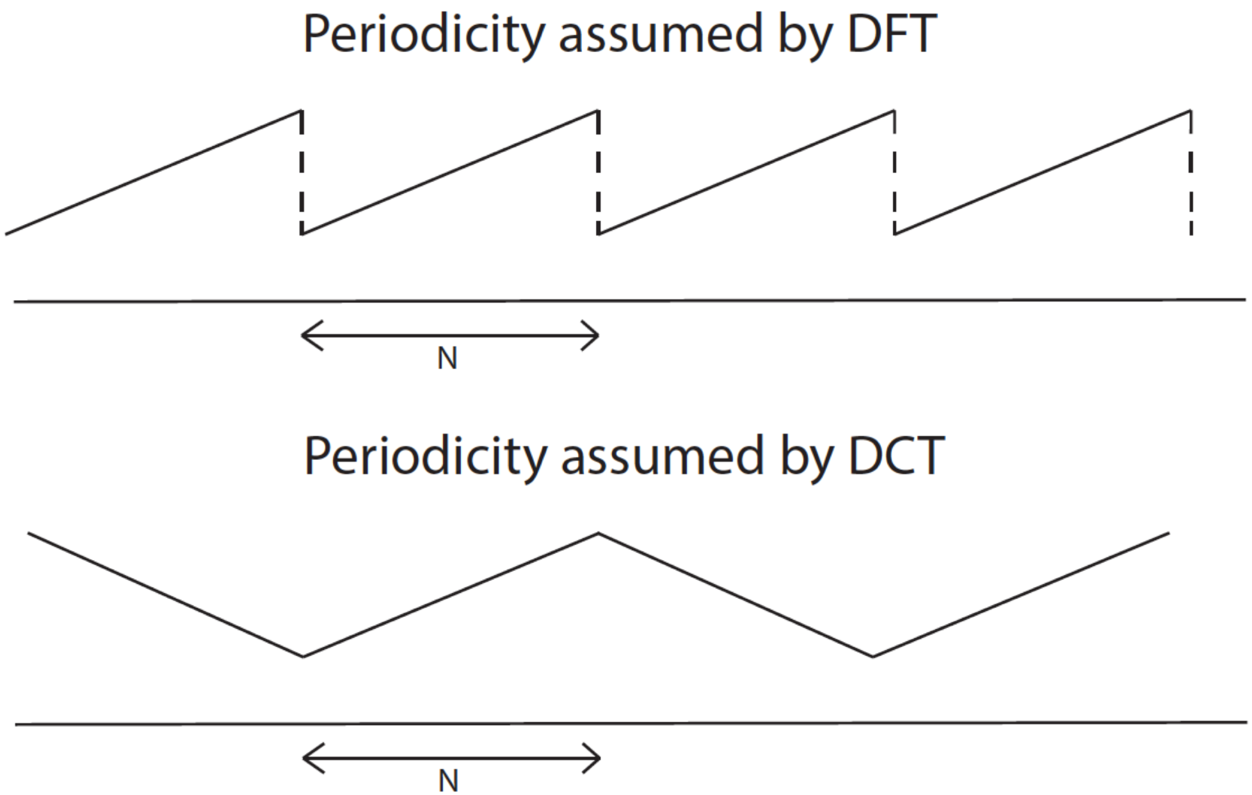
DFT Solution 3 - DCT

$$
\begin{aligned}
X_c[k] &= X[k]e^{-j\frac{\pi k}{2N}} + X[-k]e^{j\frac{\pi k}{2N}} \\
X_c[k] &= e^{-j\frac{\pi k}{2N}}\left(X[k] + X[-k]e^{j\frac{2\pi k}{2N}}\right)
\end{aligned}
$$

$$
x_c[n] = \mathrm{Shift}_{\frac{1}{2}}\{x[((n))_{2N}] + x[((-n-1))_{2N}]\}
$$

DFT Solution 3 - DCT





DCT symmetric extension is better because
sharp transitions require many coefficients to represent