EE 123 Discussion Section 1

Jan. 30, 2019

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Based on notes by Jon Tamir, Giulia Fanti and Frank Ong

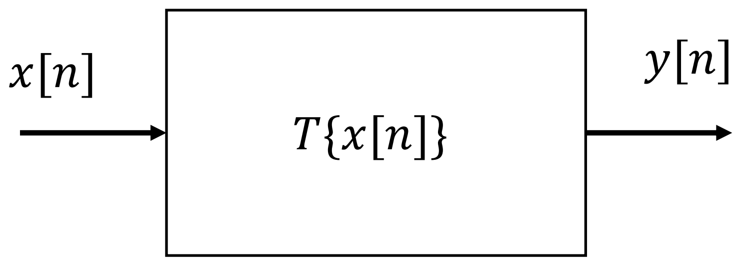
Announcements

- Office Hours
  - Miki: Wednesdays 4:15-5:15pm, Cory 506
  - Li-Hao: Monday 11am-12pm, Cory 504
  - Michael: Friday 3-4pm, Cory 504
  - Lab office hour: 10-11am, Cory 504
- Lab 0 – due Monday Feb. 4
- HW 1 – due Monday Feb. 4
- Questions?

About today

- Properties of systems
- Review on linear regression

Simple example

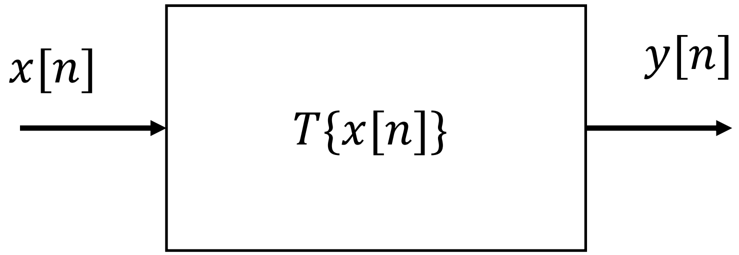


Consider a system below:

$$y[n] = \alpha x[n]$$

Is this system Linear/Time-invariant/Causal/BIBO stable?

Simple example



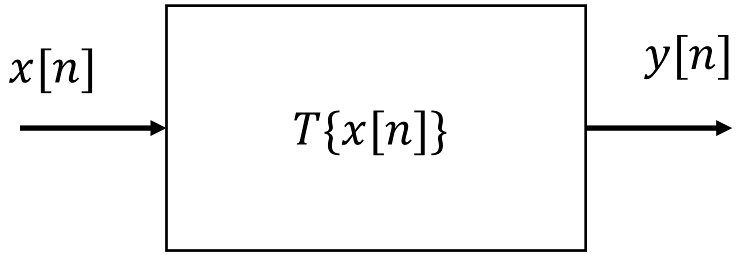
Consider a system below:

$$y[n] = \alpha x[n]$$

Is this system Linear/Time-invariant/Causal/BIBO stable?

YES to all

Simple example

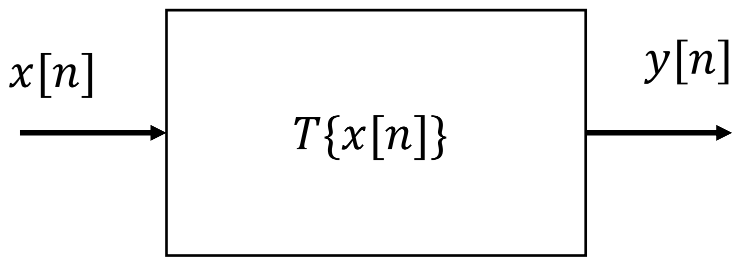


What about this modified system:

$$y[n] = \begin{cases} \alpha x[n], & x[n] \leq 1 \\ \alpha, & x[n] > 1 \end{cases}$$

Is this system Linear/Time-invariant/Causal/BIBO stable?

Simple example



What about this modified system:

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Is this system Linear/Time-invariant/Causal/BIBO stable?

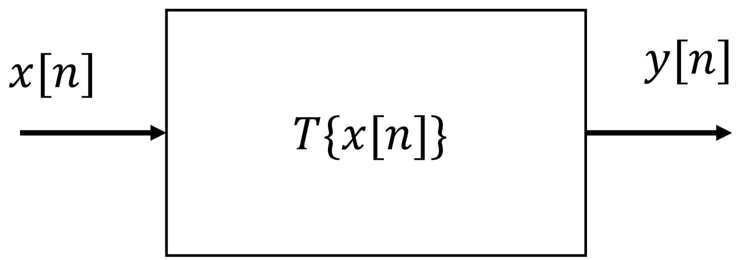
Not Linear → whenever x[n]>1, y[n] =α , this is not homogeneous.

Time invariant → Plug in x[n-1], the output is straight-up y[n-1]

Causal → It only depends on current x[n], which is memoryless/causal

stable → both α and x[n] are bounded then output y[n] is bounded

Simple example



What about this modified system:

$$y[n] = \begin{cases} \alpha x[n], & x[n] \leq 1 \\ \alpha, & x[n] > 1 \end{cases}$$

Any realistic system acts like this system?

1. MOS/BJT amplifier input/output
2. Hooke's law
3. Hysteresis

They all have limited linear region

Another system (from old exam)

A discrete-time system H produces an output signal y that is the symmetric part of the input:

$$y[n] = \frac{x[n] + x[-n]}{2}$$

Which of the following are true?

- The system must be LTI
- The system cannot be LTI

Another system (from old exam)

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$$y[n] = \frac{x[n] + x[-n]}{2}$$

Which of the following are true?

- The system must be LTI
- **The system cannot be LTI**

Another system (from old exam)

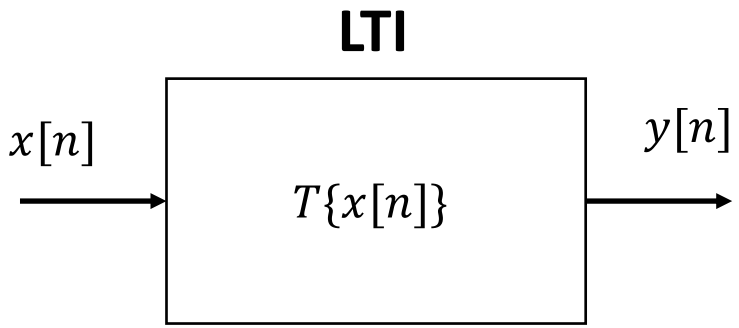
Not time invariant:

- For $x_1[n] = \delta[n]$, then $y_1[n] = \delta[n]$
- For $x_2[n] = \delta[n-1]$, then $y_2[n] = \frac{\delta[n-1]+\delta[n+1]}{2}$
- $y_1[0] = 1$ but $y_2[1] = \frac{1}{2}$

$\rightarrow$Not time invariant

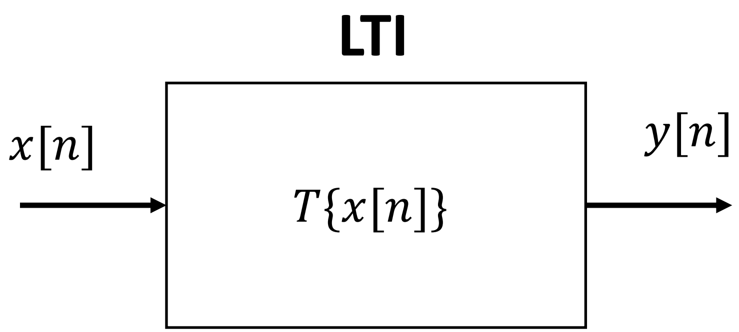
(however, the system is linear)

LTI system example (Problem 2.43)



Consider an LTI system with input $x[n]$ and output $y[n]$, when we input a signal $\left(\frac{1}{3}\right)^n u[n]$, where $u[n]$ is unit step function, we observe an output $g[n]$. Can we express $y[n]$ in terms of $x[n]$ and $g[n]$?

LTI system example (Problem 2.43)



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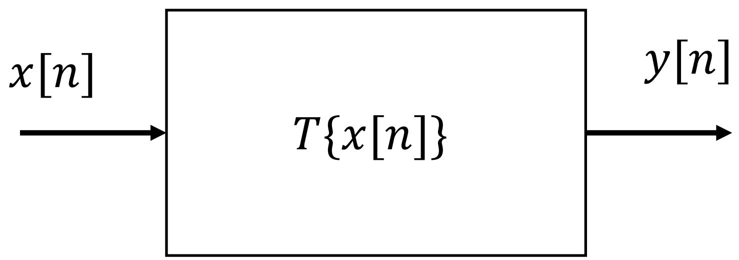
The key is to massage the input into $\boldsymbol{\delta[n]}$

We know $\boldsymbol{u[n] - u[n-1] = \delta[n]}$

LTI system example (Problem 2.43)

LTI



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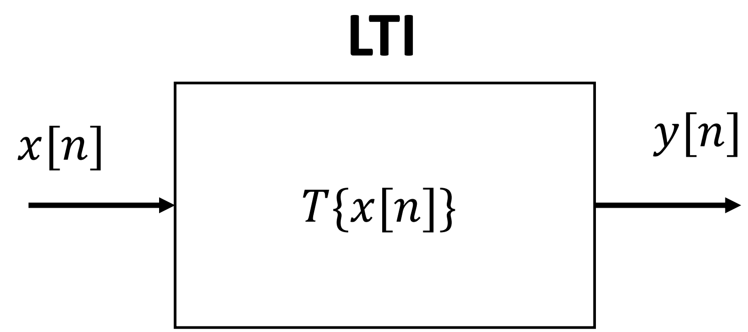
By time-invariance: input $\left(\frac{1}{3}\right)^{n-1} u[n-1]$, output $g[n-1]$

By linearity: input $\left(\frac{1}{3}\right)^{n} (u[n]-u[n-1]) = \delta[n]$,

output $g[n] - \frac{1}{3} g[n-1]$ (impulse response)

LTI system example (Problem 2.43)



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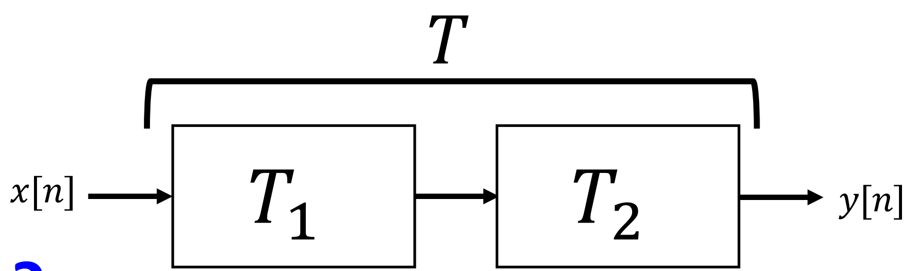
$$\boldsymbol{y[n] = x[n] * \left(g[n] - \frac{1}{3} g[n-1]\right)}$$

Cascaded system problem

Let T1 and T2 be two separate systems and T be the cascaded system:



True or False?

- If T1 is LTI and T2 is not LTI, then T cannot be LTI

False

Consider the system T1=0. Then T=0

- If T1 is not LTI and T2 is not LTI, then T cannot be LTI

False

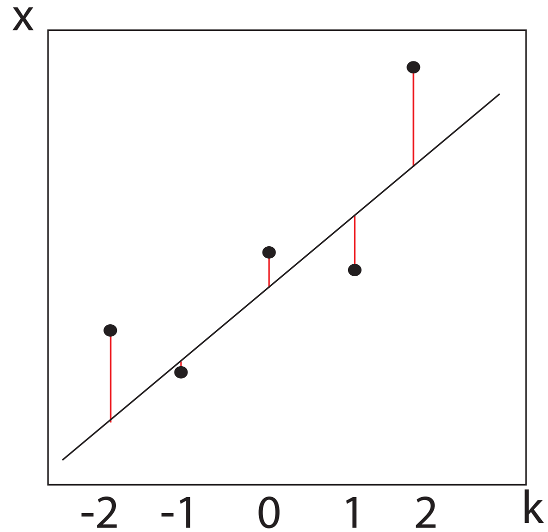
Consider the system $T_1\{x\} = x^3$ and $T_2\{x\} = x^{\frac{1}{3}}$. Then $T\{x\} = x$

Linear regression primer

Many signal processing problems can be formulated as a **least squares**, where we try to find model parameters that best fit the observed data. We will see this many, many times

Linear regression primer

<u>Example:</u> Linear regression. Suppose we observe five data points $x[k]$, where $k = \{-2, -1, 0, 1, 2\}$. We want to fit a line $x = mk + b$ by minimizing the squared distance between the line and the data points:



Linear regression primer

For each value of k, we have a linear equation for our model:
Example, $k = 2$: $x[2] = 2m + b$

And we have a squared error with our data:
Example, $k = 2$: $\left(x[2] - (b + 2m)\right)^2$

Sum of squared errors: $\sum_k \left(x[k] - (mk + b)\right)^2$
→ In matrix form, Error = $\frac{1}{2}\|\mathbf{x} - \mathbf{K}\boldsymbol{\beta}\|_2^2$

Error = $\frac{1}{2}\left\| \begin{pmatrix} x_{-2} \\ x_{-1} \\ x_0 \\ x_1 \\ x_2 \end{pmatrix} - \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right\|_2^2$

Linear regression primer

To find the best fit from a least squares sense, minimize the sum of squared errors:

$$\underset{m,b}{\text{minimize}}\,\frac{1}{2}\left\|\begin{pmatrix} x_{-2} \\ x_{-1} \\ x_0 \\ x_1 \\ x_2 \end{pmatrix} - \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{pmatrix} m \\ b \end{pmatrix}\right\|_2^2 = \underset{\boldsymbol{\beta}}{\text{minimize}}\,\frac{1}{2}\|\mathbf{x} - \mathbf{K}\boldsymbol{\beta}\|_2^2$$

Linear regression primer

To solve for b and m, take the derivative (gradient) with respect to b and to m, and set to zero:

$$\underset{\boldsymbol{\beta}}{\text{minimize}} \frac{1}{2} \|\mathbf{x} - \mathbf{K}\boldsymbol{\beta}\|_2^2$$

$$\mathbf{K}^{\mathbf{T}}\mathbf{K}\boldsymbol{\beta} - \mathbf{K}^{\mathbf{T}}\mathbf{x} = 0 \qquad \Rightarrow \boldsymbol{\beta} = \left(\mathbf{K}^{\mathbf{T}}\mathbf{K}\right)^{-1}\mathbf{K}^{\mathbf{T}}\mathbf{x}$$

In Python,

```
K = np.array( […] )
x = np.array( […])
beta = np.linalg.solve(K, x)
```