Digital Signal Processing
EE123

Miki Lustig, Spring 2019

Midterm Exam #2

NAME:

You have almost 2 hours for 3 problems.

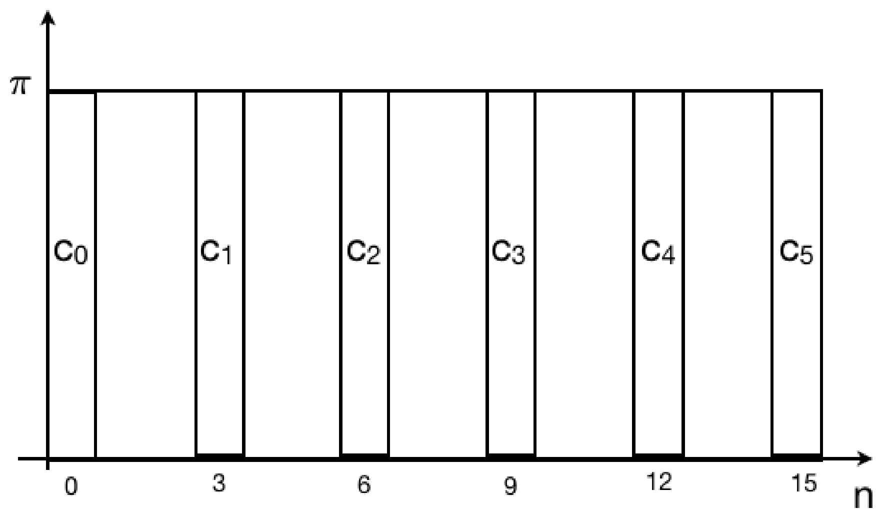
- Please do preliminary calculations on your own scratch paper. (Do not hand in.)
- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You can use your course notes, homework sets and solutions, calculator, and the text.

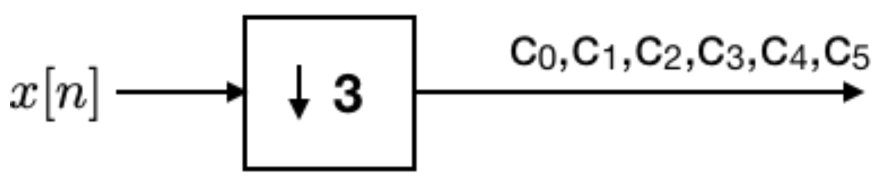
Problem	Score
1	
2	
3	
Total	

Problem 1: Time Frequency Tiling, Spectrum and Filter Banks *(80 points)*

Consider a non-causal Haar filter bank with $H_0(z) = 1 + z$, $H_1(z) = 1 - z$, and $\boxed{\downarrow M}$ operators. These are applied on an infinite sequence $x[n]$, that is nonzero between $0 \leq n < 16$. For each of the following time frequency tiling, construct a filter bank which matches it. For example:

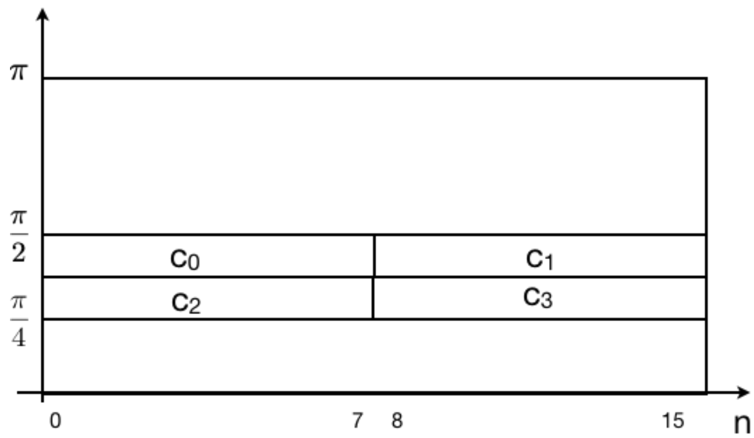


will be a result of



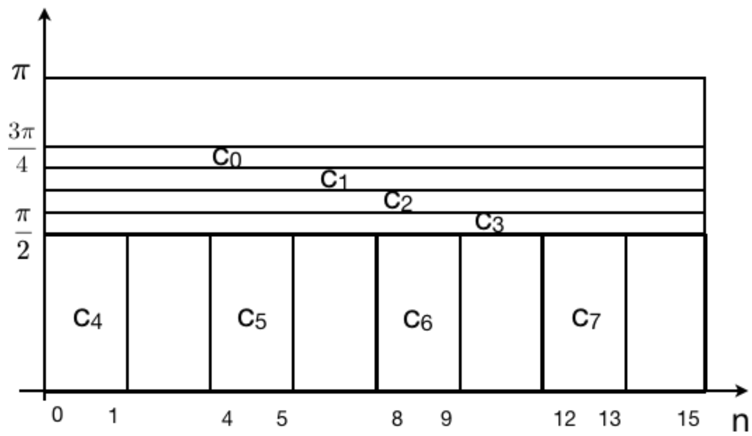
Make sure you show your work, including intermediate steps.

a) Sketch the filter bank which matches the following tiling:



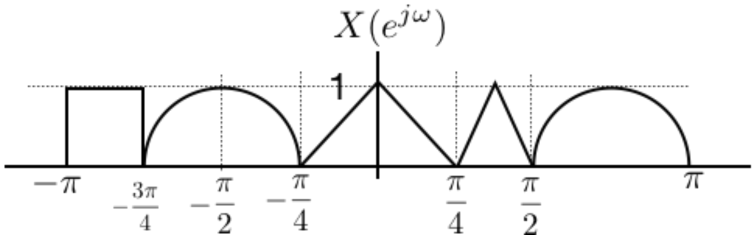
filter bank:

b) Sketch the filter bank which matches the following tiling:

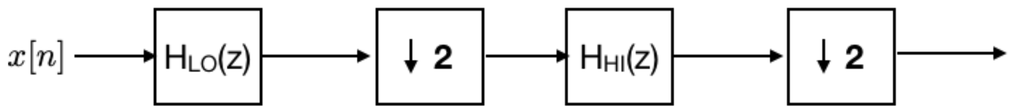


filter bank:

From here on, we replace the Haar filters with ideal low-pass and highpass filters, $H_{\text{LO}}(z)$ and $H_{\text{HI}}(z)$, with cutoff frequency of $\omega_c = \pi/2$. Also, consider a new input signal $x[n]$ with the spectrum $X(e^{j\omega})$:

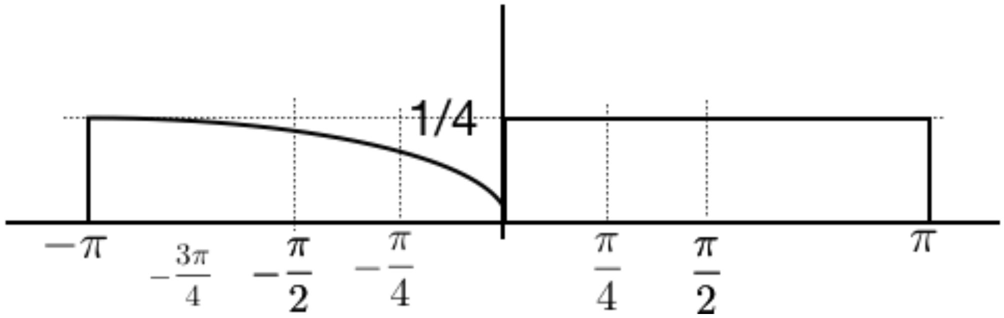


c) You pass $x[n]$ through the following filter-bank. Sketch the resulting spectrum.



spectrum:

d) Now, you pass the same $x[n]$ as in part (c) above through a new filter bank. The resulting spectrum is shown below.

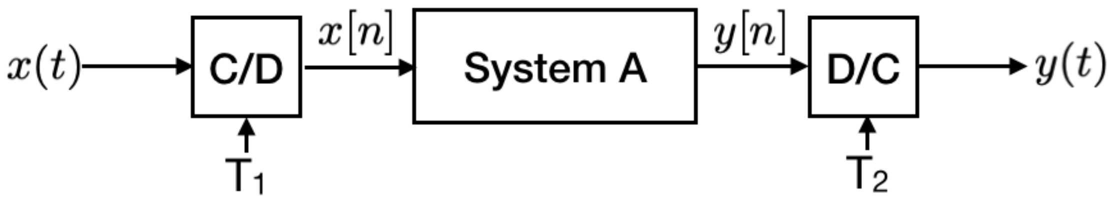


Sketch a filter bank which will result in the above spectrum.

filterbank:

Problem 2: Sampling, reconstruction and discrete systems *(80 points)*

Consider following system:



a) Let $y[n] = x[n]$, $T_1 = T_2 = \frac{1}{3}$, $x(t) = \cos(2\pi t)$. Find the expression for $y(t)$. Explain the result, and how you got to it.

$y(t) =$

b) Let System A be a $\boxed{\downarrow 2}$ compressor, $T_1 = \frac{1}{2}$, $T_2 = 1$, $x(t) = \cos(2\pi t)$. Find the expression for $y(t)$. Explain the result, and how you got to it.

$y(t) =$

c) Let System A be a $\boxed{\uparrow 2}$ expander, $T_1 = T_2 = \frac{3}{4}$, $x(t) = \cos(2\pi t)$. Find the expression for $y(t)$. Explain the result, and how you got to it.

$y(t) =$

d) Let System A be $\boxed{\uparrow 2}$ followed by an ideal low-pass filter with gain=2 for $|\omega| \leq \pi/2$ and zero elsewhere. Let $T_1 = \frac{1}{2}$, and $T_2 = \frac{1}{4}$ and $x(t) = \cos(2\pi t + \pi/3)$. Find the expression for $y(t)$. Explain the result, and how you got to it.

$y(t) =$

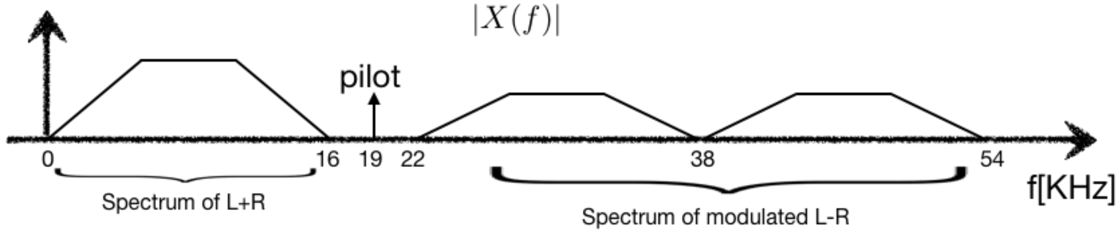
Problem 3: Filter design for Broadcast Stereo FM *(60 points)*

The baseband signal of broadcast FM-stereo as discussed in Lab-III is given by

$$x(t) = L(t) + R(t) + 0.1\cos(2\pi f_p t) + \cos(2\pi \cdot 2f_p t)\big(L(t) - R(t)\big),$$

where $L(t)$ and $R(t)$ at the left and right channels respectively, and $f_p = 19000$Hz.

The associated magnitude spectrum of $x(t)$ is given by:



Similarly to Lab-III, you perform FM demodulation on the IQ data you got from the SDR, and obtain samples, $x[n]$ of $x(t)$ at a sampling rate of $f_s = 240$KHz. In the lab, you reconstructed the mono $L(t) + R(t)$ signal, and the subcarriers (not shown here). In this question we will design a stereo reconstruction to separate $L(t)$ and $R(t)$.

a) The following system recovers $L[n] + R[n]$:

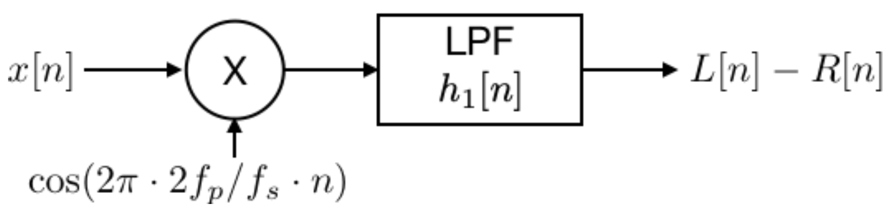
You would like to design a causal, linear-phase M^{th} order low-pass filter using the window method. Other parameters are TBW=8, M even and Hamming window function $W[n]$ which is nonzero for $0 \leq n \leq M$.

Write an expression for the impulse response, $h_1[n]$. What is M? (round up to the closest even integer). By design, the system is linear phase. What is the group delay?

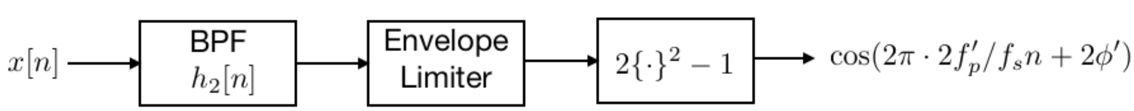
more space.....

$h_1[n] =$	$M =$	group-delay=

b) Ideally, if the frequency source at the FM station and your receiver would be synchronized in frequency and phase, the following system would recover $L[n] - R[n]$:



Unfortunately, this is not the case. Therefore, we would like to leverage the pilot tone around $\omega = 2\pi f_p/f_s$ as a frequency reference for the demodulation. Leveraging the trigonometric relation $\cos(2\alpha) = 2\cos^2(\alpha) - 1$, the frequency recovery system which recovers the transmitting station pilot frequency f_p' and ϕ' is given by:

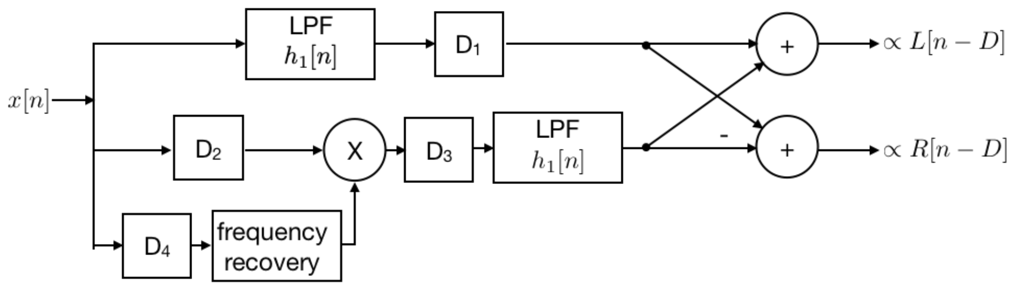


Design a TBW=2, Hamming-windowed, even P^{th} order low pass filter with equivalent BW=240Hz (cutoff frequency 120Hz). Use $W[n]$ to express the window function. Modulate the filter to the appropriate frequency to obtain the desired bandpass filter. Write an expression for the impulse response $h_2[n]$. What is P? What is the group delay?

more space.....

$h_2[n] =$	$P =$	group-delay=

c) In order to separate the result into the $L[n]$ and $R[n]$ we must compensate for all the different delays in the system. Consider the full system:



What are the delay values for D_1, D_2, D_3, and D_4 that would result in the minimum total delay, D, between the input and the outputs? What is D?

$D_1 =$	$D_2 =$	$D_3 =$	$D_4 =$	$D =$