EE123 Spring 2019 Miki Lustig
Digital Signal Processing

Assignment 3

Due February 18^{th} 2019

1. Self-grade Homework 2
2. Read Chapters 8,9.1-9.5 Oppenheim and Schafer, 3rd ed.
3. Problem 8.37, Oppenheim and Schafer, 3rd ed.
4. Problem 8.67 Oppenheim and Schafer, 3rd ed. (Not easy!)
5. *Symmetries of the DFT*

   Let $f[n]$ be an 8-sample sequence with the $DFT\{f[n])\} = F[k]$. We know some of the values of $f[n]$:

   $$f[n] = \{1 \quad (1+i) \quad -1 \quad (1-i) \quad 1 \quad ? \quad ? \quad ?\}$$

   Other values, indicated by "?", are unknown. We want to find these unknown values based on the what we know about the symmetry of the signal or the symmetry of its DFT.

   Determine the rest of $f[n]$ for each of the following cases. If $F[k]$ cannot possess the given property, explain.

   (a) $F[k]$ is even (i.e. $F[k] = F[((-k))_8]$).
   (b) $F[k]$ is odd (i.e. $F[k] = -F[((-k))_8]$).
   (c) $F[k]$ is imaginary-valued.
   (d) $F[k]$ is real-valued.

6. *Faster DFT's?*

   Let $f[n]$ and $g[n]$ be N-point real-valued sequences with DFT's of $F[k]$ and $G[k]$ respectively. Consider the possibility of computing both of their DFT's simultaneously by using an N-point complex DFT.

   One idea is to construct $h[n] = f[n] + jg[n]$. If $DFT\{h[n]\} = H[k]$, find separate expressions (if possible) for $F[k]$ and $G[k]$ in terms of $H[k]$. If it is not possible to separate $F[k]$ and $G[k]$ from $H[k]$, explain why.

7. *Diagonalizing circulant matrices* A circulant matrix is a matrix of the form

   $$H_m = \begin{bmatrix} c_1 & c_2 & \dots & c_{n-1} & c_n \\ c_n & c_1 & \dots & c_{n-2} & c_{n-1} \\ & & \dots & & \\ c_2 & c_3 & \dots & c_n & c_1 \end{bmatrix},$$

   i.e. each row is a circular shift of the row above it. Show that the DFT matrix diagonalizes all circulant matrices. (Hint: recall the shift and modulation properties of the DFT)

8. *Adapted from Midterm I fall'12:* Hadamard Transform

Consider a new transform which is defined recursively in matrix form as:

$$H_m = \begin{bmatrix} H_{m-1} & H_{m-1} \\ H_{m-1} & -H_{m-1} \end{bmatrix},$$

where $H_0 = 1$. (This transform is called the Hadamard Transform)

a) This transform can be used to perform (somewhat OK) frequency analysis. Compute H_3 which is an 8×8 matrix. What is the order of the basis functions that represent functions with increasing frequency content?

b) Much like the DFT, the Hadamard matrix has structure which can be exploited for rapid computation. Draw a flow diagram (similarly to the FFT) for calculating the 8×8 Hadamard transform (H_3) in $O(N\mathrm{Log}_2 N)$. Note, that there are several options, much like the decimation-in-time and decimation-in-frequency FFT's.

x_0	X_0
x_1	X_1
x_2	X_2
x_3	X_3
x_4	X_4
x_5	X_5
x_6	X_6
x_7	X_7