EE123
Digital Signal Processing

Lecture 25
Generalized Linear Phase Systems

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Announcements

- Project
  - Teams and proposals by Friday
  - Take a look at the project page
- Radios
  - Pick your radios at the lab sessions Tue/Thu
- Midterm II next in 1 Week
  - Covers material up to, and including today

Generalized linear-phase systems

$$H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{Real, allow sign change}} e^{-j\alpha\omega + j\beta}$$

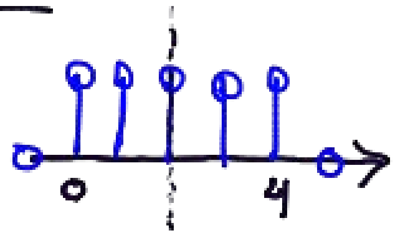
$$\mathrm{grd}\left[H(e^{j\omega})\right] = \alpha \quad \left(\text{except when } A(e^{j\omega}) \text{ changes sign}\right)$$

GLP for FIR → MUST have symmetry

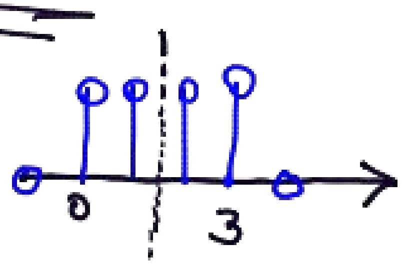
④

$h[n] = h[M-n]$:

Type I (M even)



Type II (M odd)



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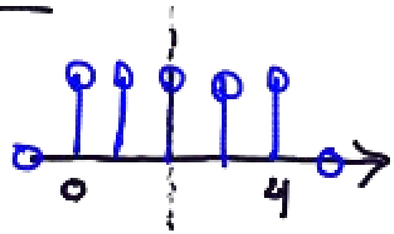

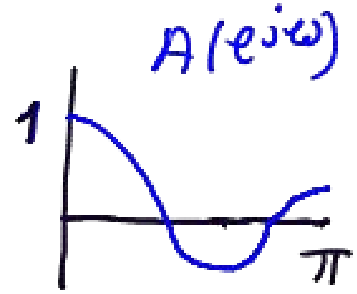

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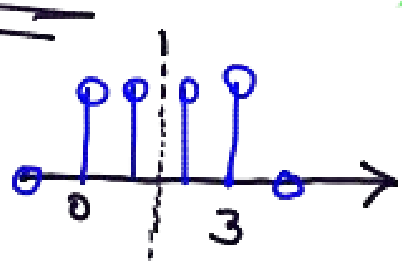

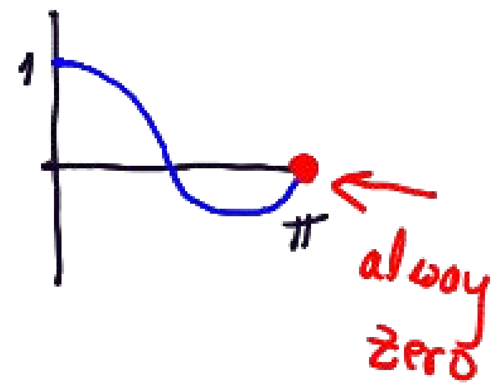

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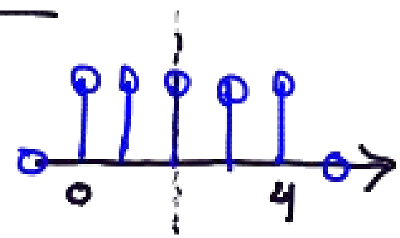
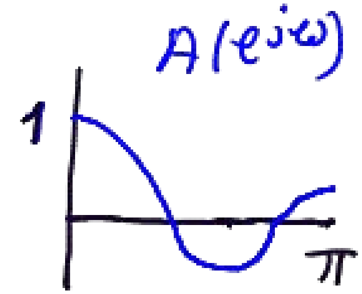
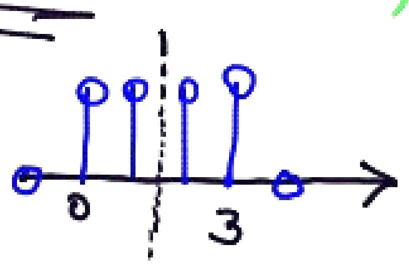
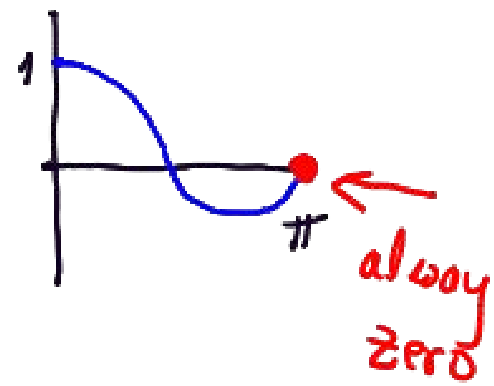




$$A(e^{j\omega}) = h\left[\tfrac{M}{2}\right] + 2\sum_{k=1}^{M/2} h\left[\tfrac{M}{2}-k\right]\cos(\omega k)$$

Type II (M odd)





$A(e^{j\omega})$ = In the text

Least Squares

$$\text{argmin}_{\tilde{h}} \quad ||A\tilde{h} - b||^2$$

Solution:

$$\tilde{h} = (A^* A)^{-1} A^* b$$

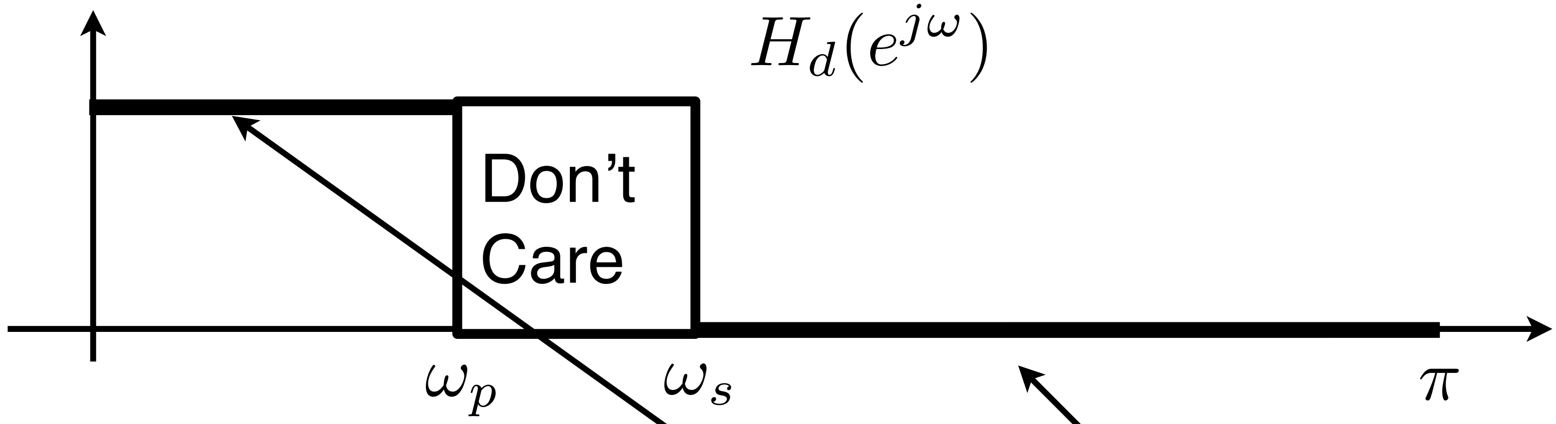
- Result will generally be non-symmetric and complex valued.
- However, if $\tilde{H}(e^{j\omega})$ is real, $\tilde{h}[n]$ should have symmetry!

Design of Linear-Phase L.P Filter

- Suppose:
  - $\tilde{H}(e^{j\omega})$ is real-symmetric
  - M is even (M+1 taps)
- Then:
  - $\tilde{h}[n]$ is real-symmetric around midpoint
- So:

$$
\begin{aligned}
\tilde{H}(e^{j\omega}) &= \tilde{h}[0] + \tilde{h}[1]e^{-j\omega} + \tilde{h}[-1]e^{+j\omega} \\
&\quad + \tilde{h}[2]e^{-j2\omega} + \tilde{h}[-2]e^{+j2\omega} \cdots \\
&= \tilde{h}[0] + 2\cos(\omega)\tilde{h}[1] + 2\cos(2\omega)\tilde{h}[2] + \cdots
\end{aligned}
$$

Least-Squares Linear-Phase Filter



Given M, ω_P, ω_s find the best LS filter:

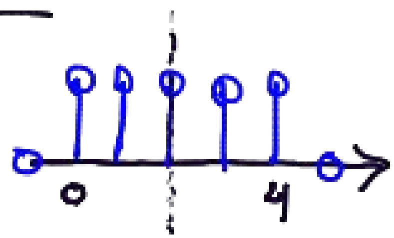
$A =$

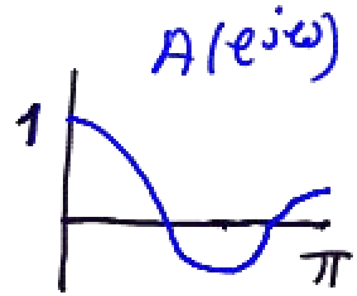
$b = [\quad , \quad]^T$

GLP for FIR $\rightarrow$ must have symmetry

$h[n] = h[M-n]$:

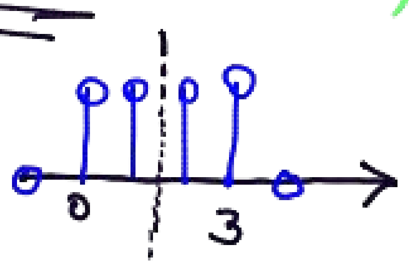
Type I (M even)

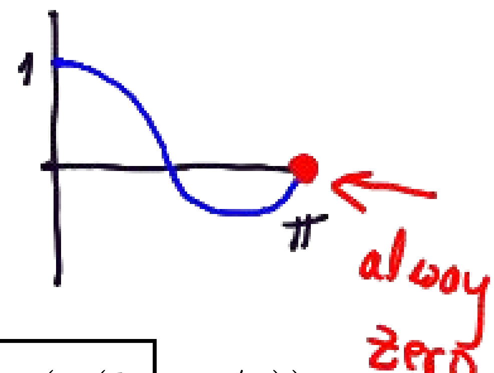




$$A(e^{j\omega}) = h[\tfrac{M}{2}] + 2\sum_{k=1}^{M/2} h[\tfrac{M}{2}-k]\cos(\omega k)$$

Type II (M odd)





$$\boxed{A(e^{j\omega}) = \sum_{k=1}^{(M+1)/2} 2h[(M+1)/2-k]\cos(\omega(k-1/2))}$$

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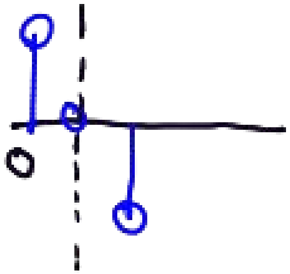
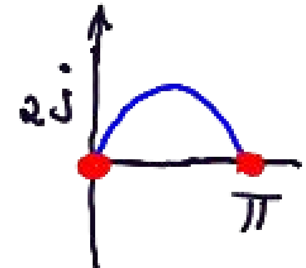
$$h[n] = -h[M-n]$$

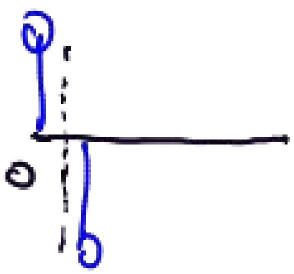
Type III (M even)

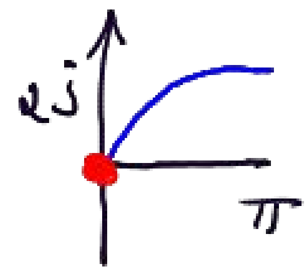




$$A(e^{j\omega}) = j\,2\sum_{k=1}^{M/2} h\left[\tfrac{M}{2}-k\right]\sin(\omega k)$$

Type IV (M odd)





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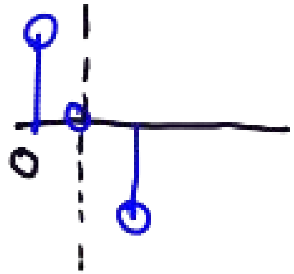

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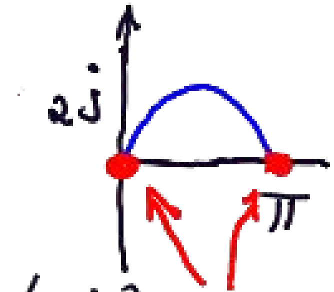
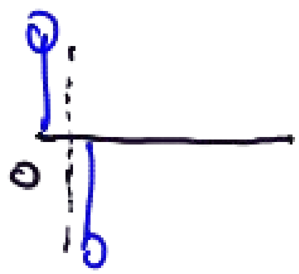
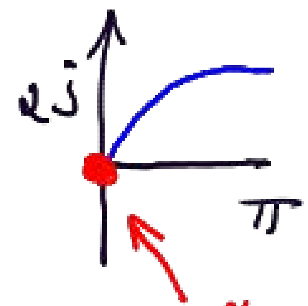


always $=0$

Type IV (M odd)





always $=0$

$A(e^{j\omega}) =$ See text

Zeroes of GLP system

Type I, II: $h[n] = h[\boxed{M} - \boxed{n}]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

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$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} \underbrace{h[M-n]}_{\triangleq k} z^{\overbrace{M-n}^{\triangleq k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

Zeroes of GLP system

⑥

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$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

$$\Longrightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

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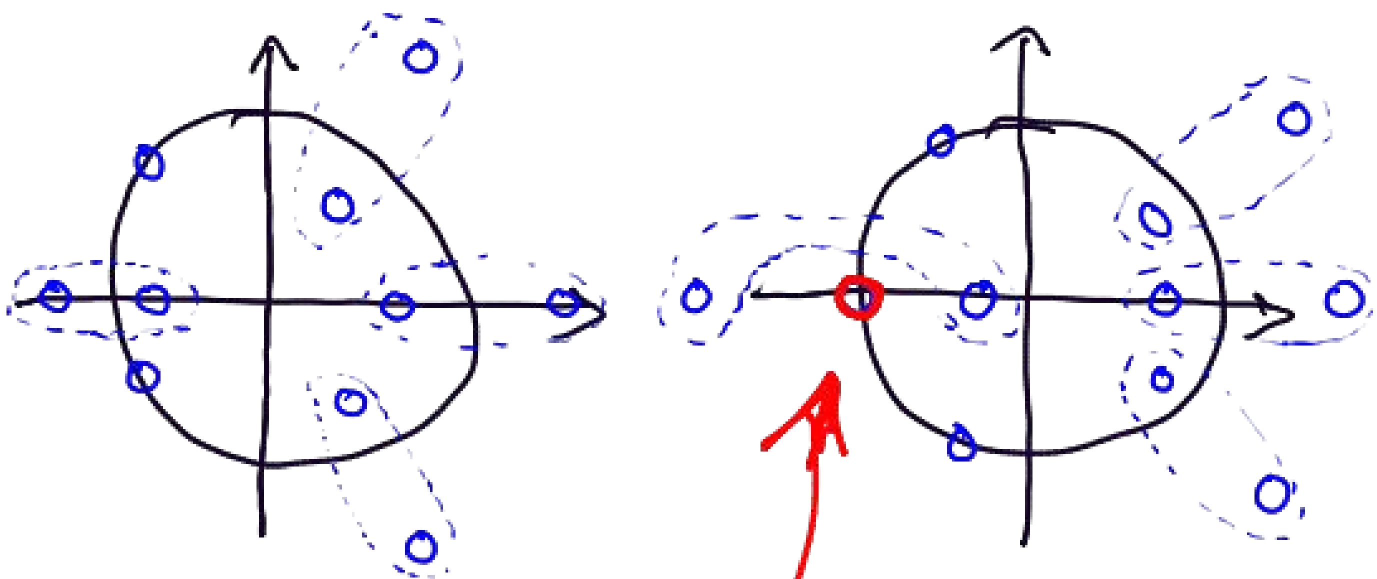
$H(z) = z^{-M} H(z^{-1})$ Type I, II



$H(-1) = 0$ Type II (Never high-pass)

→ FOR GLP, IF $a = re^{j\theta}$ is a zero

$\frac{1}{a^*}$ is also a zero

Zeroes of GLP system

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} \underbrace{h[M-n]}_{\hat{=}k} z^{\overbrace{M-n}^{\hat{=}k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

$$\Rightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

for type II:

$$H(-1) = (-1)^{\overset{\text{odd}}{M}} H(-1) = -H(-1) \Rightarrow \boxed{H(-1) = 0}$$

similarly, can show for ⑧

type III, IV

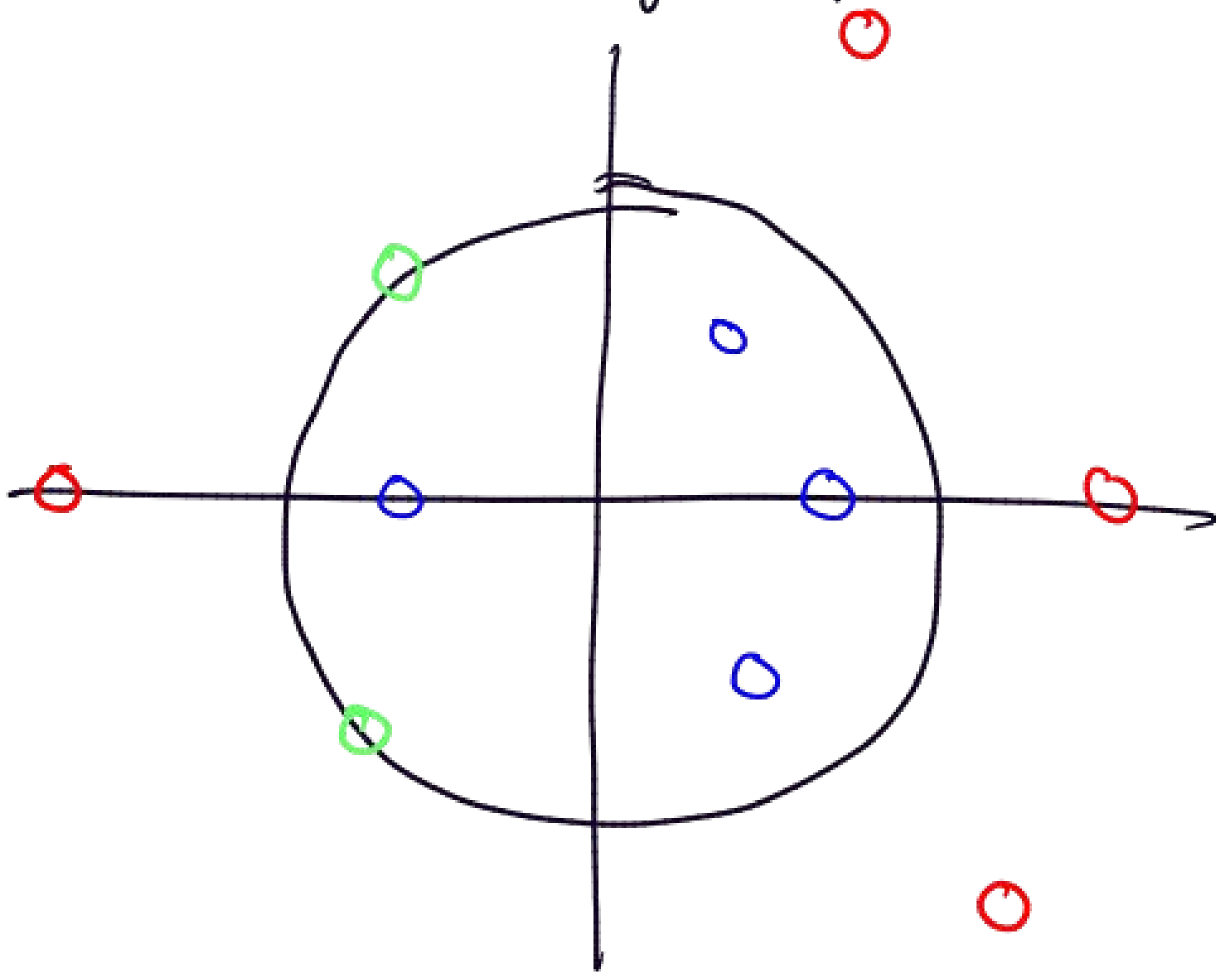
$$\boxed{H(z) = -z^{-M} H(z^{-1})}$$

$$\boxed{H(1) = 0} \rightarrow \text{Never low-pass}$$

for type III

$$\boxed{H(-1) = 0} \quad \text{only band pass}$$

Relation of FIR GLP to min-phase systems



$$H(z) = H_{min}(z)\, H_{max}(z)\, H_{uc}(z)$$

minimum phase

maximum phase