EE 123 Discussion Section 6
Sampling

March 13, 2018

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Based on slides by Frank Ong

Announcements

- Submit your Lab 3 results in groups
- Please come to the lab sections next week to pick up your Radios.
- Questions?

Review of sampling

Continuous time signal

$$x_c(t) \longleftrightarrow X_c(j\Omega) = \int x_c(t) e^{-j\Omega t} dt$$

Continuous time sampling

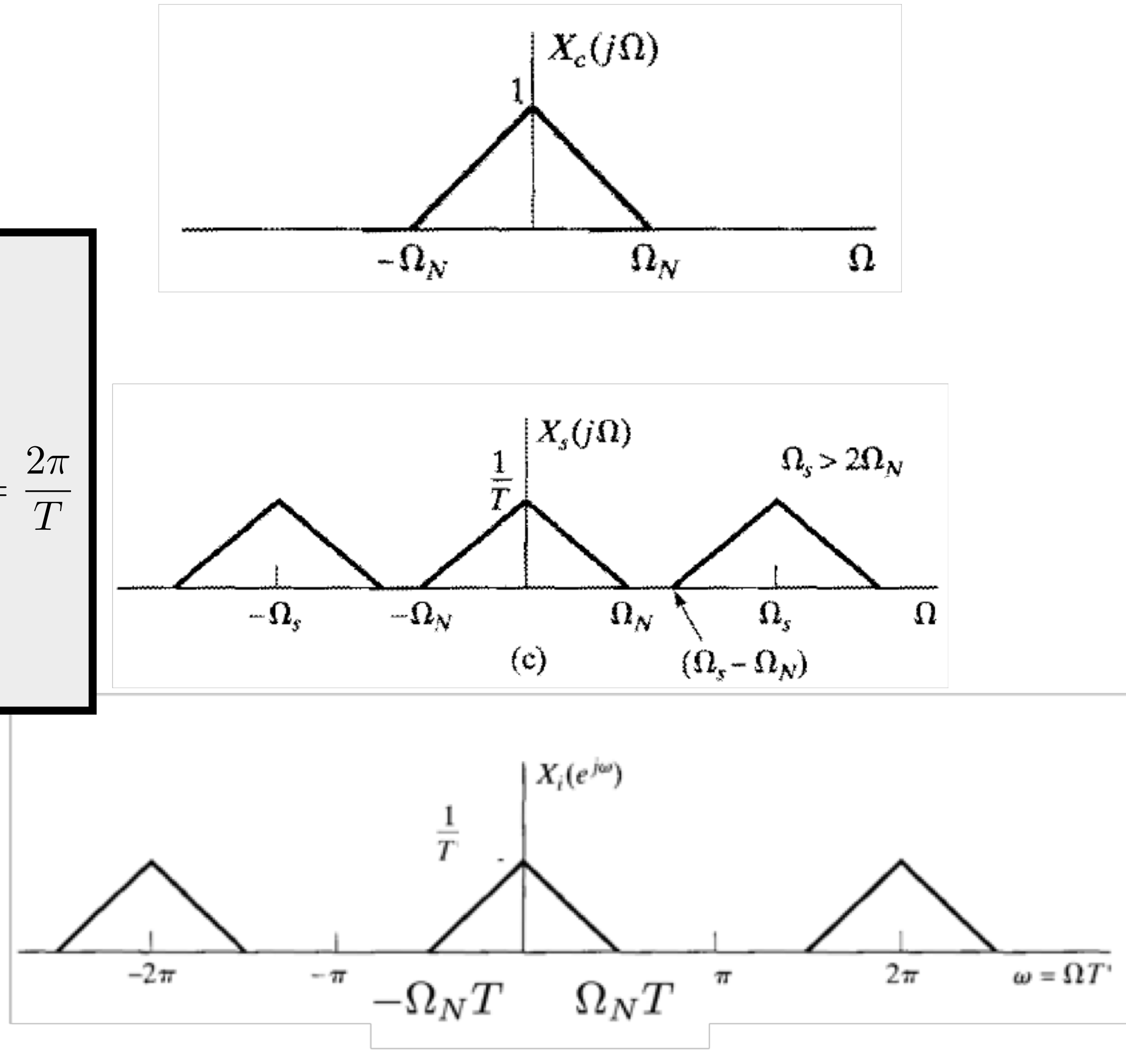
$$x_s(t) = x_c(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$X_s(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s)), \quad \Omega_s = \frac{2\pi}{T}$$

$$= \sum_{k=-\infty}^{\infty} x_c(nT) e^{-j\Omega T n}$$

Discrete time spectrum

$$X(e^{j\omega}) = X_s\left(j\left(\frac{\omega}{T}\right)\right)$$



Previous HW question 1

Each of the following continuous-time signals is used as the input $x_c(t)$ for an ideal C/D converter as shown in Figure 4.1 with the sampling period T specified. In each case, find the resulting discrete-time signal $x[n]$.

(a) $x_c(t) = \cos(2\pi(1000)t)$, $\quad T = (1/3000)$ sec

(b) $x_c(t) = \sin(2\pi(1000)t)$, $\quad T = (1/1500)$ sec

(c) $x_c(t) = \sin(2\pi(1000)t)\,/\,(\pi t)$, $\quad T = (1/5000)$ sec

Solution on piazza Resources

Previous HW question 2a

A continuous-time finite-duration signal $x_c(t)$ is sampled at a rate of 20,000 samples/s, yielding a 1000-point finite-length sequence $x[n]$ that is nonzero in the interval $0 \leq n \leq 999$. Assume for this problem that the continuous-time signal is also bandlimited such that $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(10{,}000)$; i.e., assume that negligible aliasing distortion occurs in sampling. Assume also that a device or program is available for computing 1000-point DFTs and inverse DFTs.

(a) If $X[k]$ denotes the 1000-point DFT of the sequence $x[n]$, how is $X[k]$ related to $X_c(j\Omega)$? What is the effective continuous-time frequency spacing between DFT samples?

Solution on piazza Resources

Previous HW question 2b

The following procedure is proposed for obtaining an expanded view of the Fourier transform $X_c(j\Omega)$ in the interval $|\Omega| \le 2\pi(5000)$, starting with the 1000-point DFT $X[k]$.

Step 1. Form the new 1000-point DFT

$$W[k] = \begin{cases} X[k], & 0 \le k \le 250, \\ 0, & 251 \le k \le 749, \\ X[k], & 750 \le k \le 999. \end{cases}$$

Step 2. Compute the inverse 1000-point DFT of $W[k]$, obtaining $w[n]$ for $n = 0, 1, \ldots, 999$.

Step 3. Decimate the sequence $w[n]$ by a factor of 2 and augment the result with 500 consecutive zero samples, obtaining the sequence

$$y[n] = \begin{cases} w[2n], & 0 \le n \le 499, \\ 0, & 500 \le n \le 999. \end{cases}$$

Step 4. Compute the 1000-point DFT of $y[n]$, obtaining $Y[k]$.

(b) The designer of this procedure asserts that

$$Y[k] = \alpha X_c(j2\pi \cdot 10 \cdot k), \qquad k = 0, 1, \ldots, 500,$$

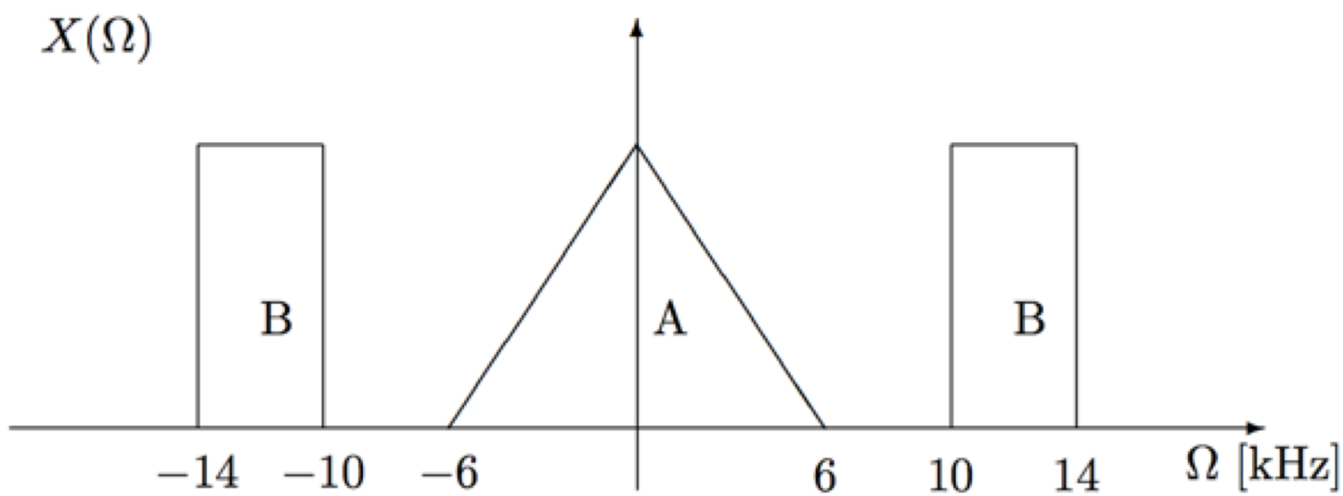
where α is a constant of proportionality. Is this assertion correct? If not, explain why not.

Solution on piazza Resources

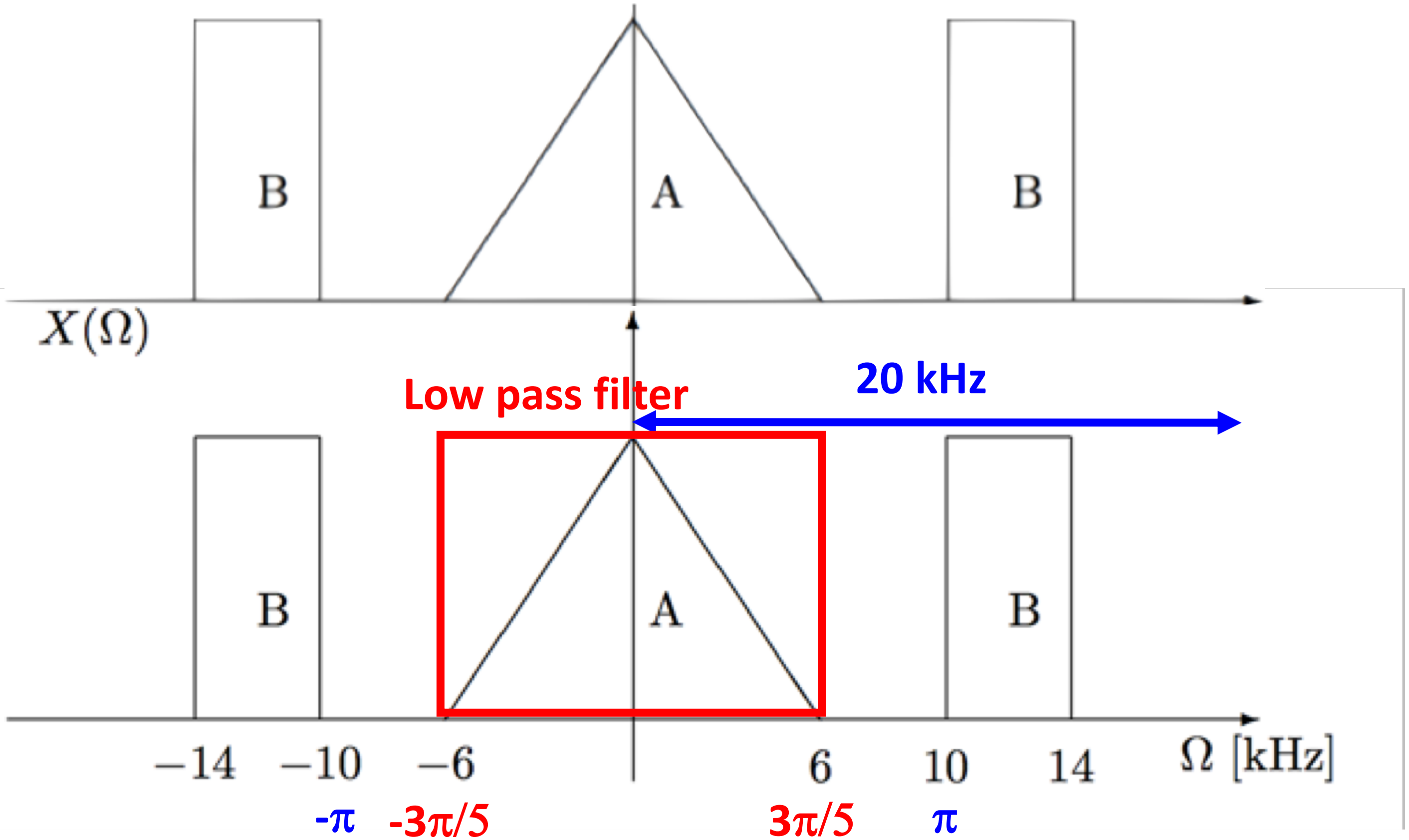
Sampling question 1

4. An analog signal, whose spectrum is shown below, is to be processed with a digital filter using ideal C/D and D/C converters (with no analog anti-aliasing filters).

a) (8 points) What is the minimum sufficient sampling rate to extract portion A of the signal? Sketch the magnitude of the digital filter that would be used at this sampling rate.



Sampling solution 1a

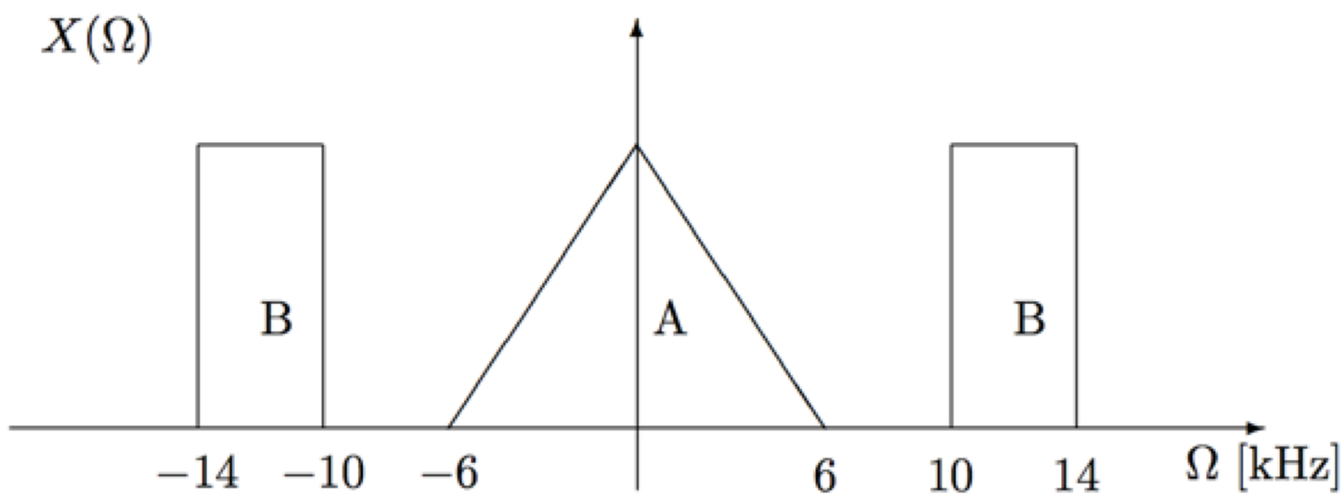


Sampling question 1

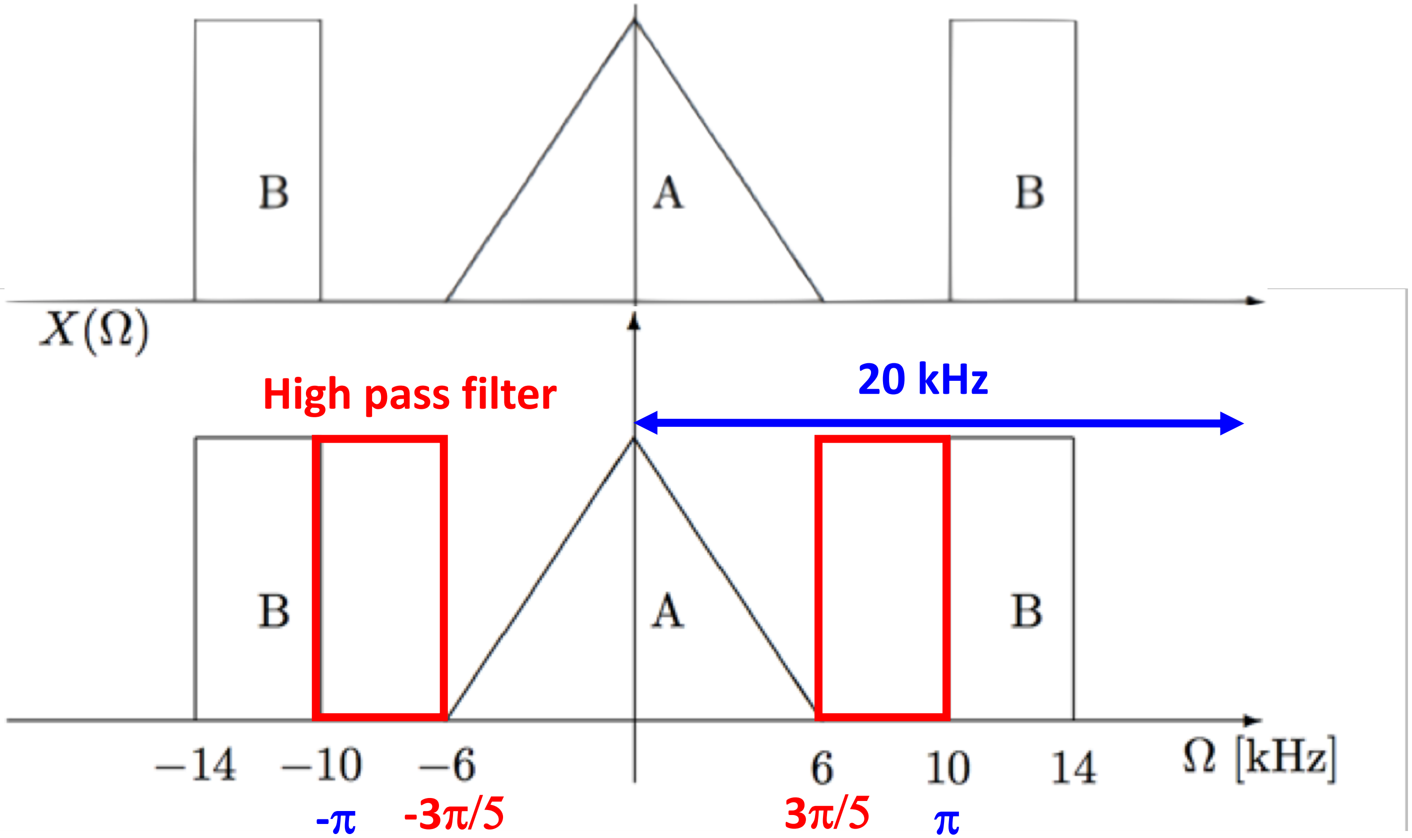
4. An analog signal, whose spectrum is shown below, is to be processed with a digital filter using ideal C/D and D/C converters (with no analog anti-aliasing filters).

a) (8 points) What is the minimum sufficient sampling rate to extract portion A of the signal? Sketch the magnitude of the digital filter that would be used at this sampling rate.

b) (7 points) Repeat for portion B of the signal.

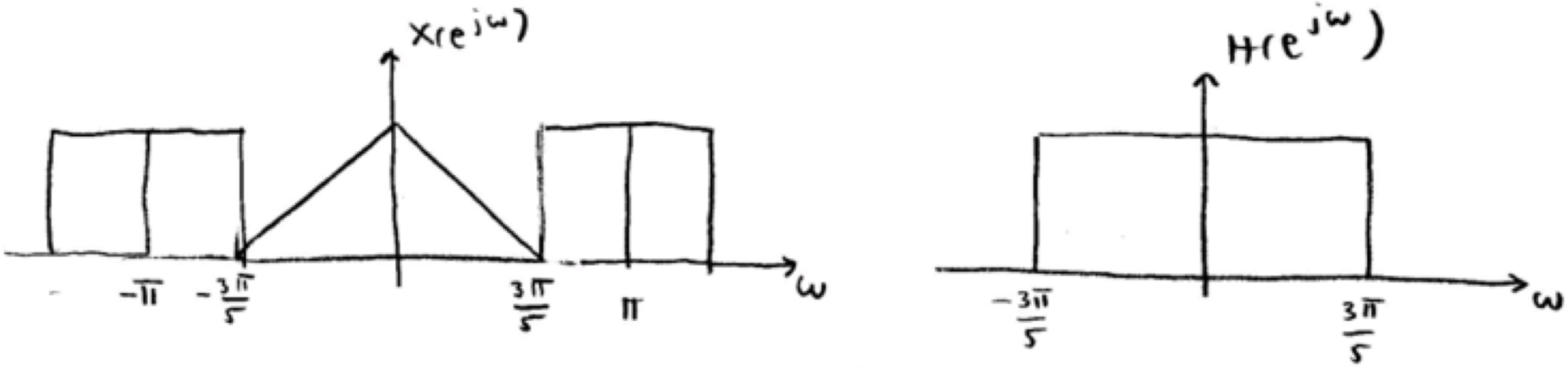


Sampling solution 1b

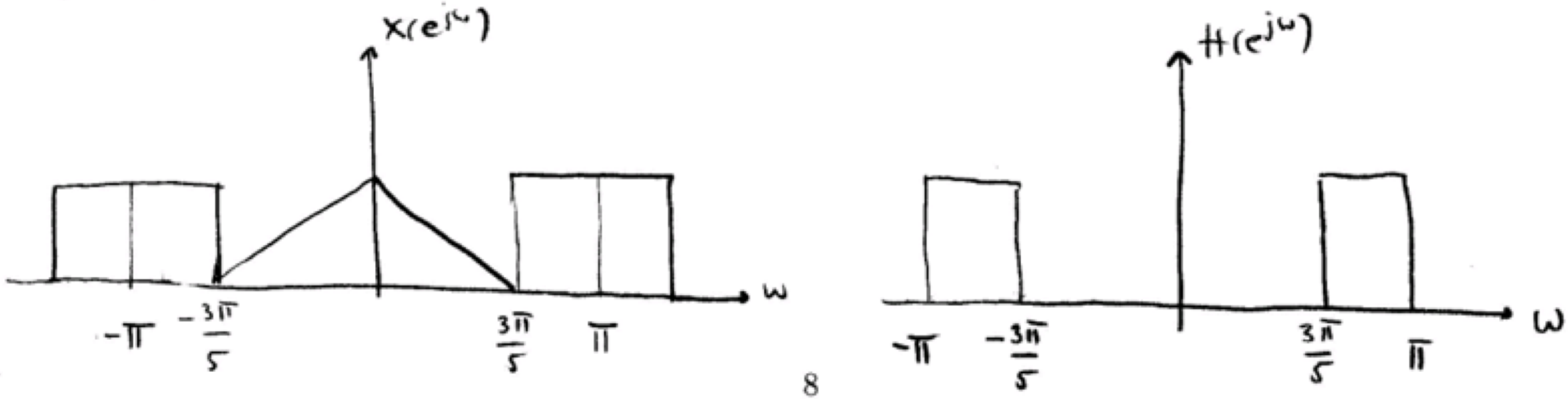


Sampling solution 1

a) $f_s = 20$ kHz



b) $f_s = 20$ kHz



Why do we care sampling

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