EE 123 Discussion Section 7
Resampling

March 20, 2019

Li-Hao Yeh

Based on slides by Jon Tamir

Announcements

- Lab 3 Part I and II – due tomorrow March 21st
- HW 8 – due next Wednesday April 1st
- Questions?

Review of sampling

Continuous time signal

$$x_c(t) \longleftrightarrow X_c(j\Omega) = \int x_c(t) e^{-j\Omega t} dt$$

Continuous time sampling

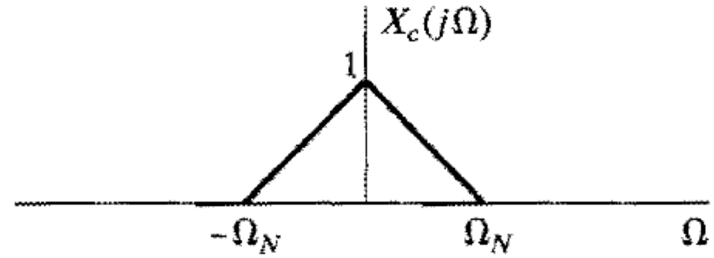
$$x_s(t) = x_c(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

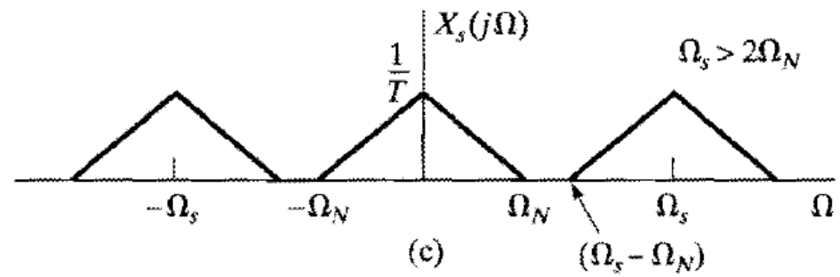
$$X_s(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s)), \quad \Omega_s = \frac{2\pi}{T}$$

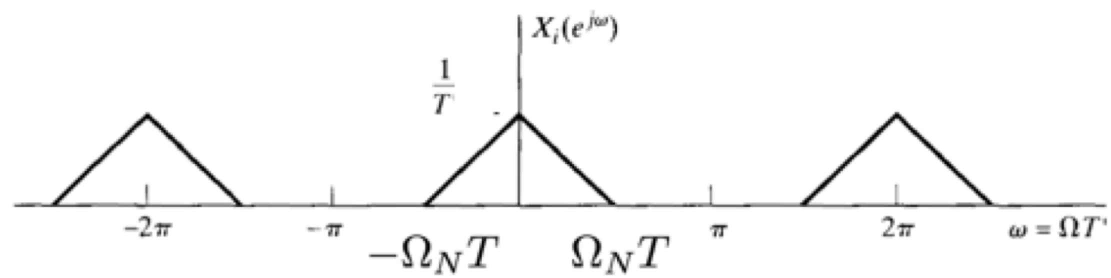
$$= \sum_{k=-\infty}^{\infty} x_c(nT) e^{-j\Omega T n}$$

Discrete time spectrum

$$X(e^{j\omega}) = X_s\left(j\left(\frac{\omega}{T}\right)\right)$$







Sampling question

4.23. Figure P4.23-1 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \le \omega \le \pi$ as

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi. \end{cases}$$

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.23-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

(i) $1/T_1 = 1/T_2 = 2 \times 10^4$
(ii) $1/T_1 = 4 \times 10^4$, $1/T_2 = 10^4$
(iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$.

(b) For $1/T_1 = 1/T_2 = 6 \times 10^3$, and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which the overall system is LTI? For this maximum choice of ω_c, specify $H_c(j\Omega)$.

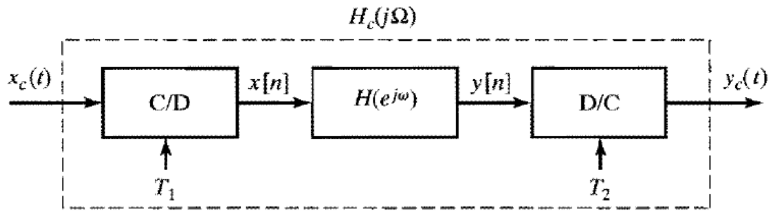


Figure P4.23-1

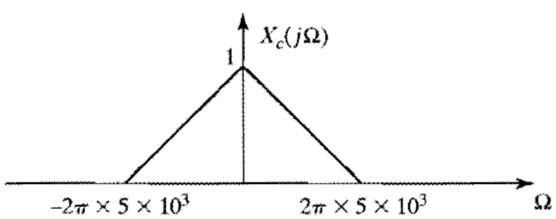
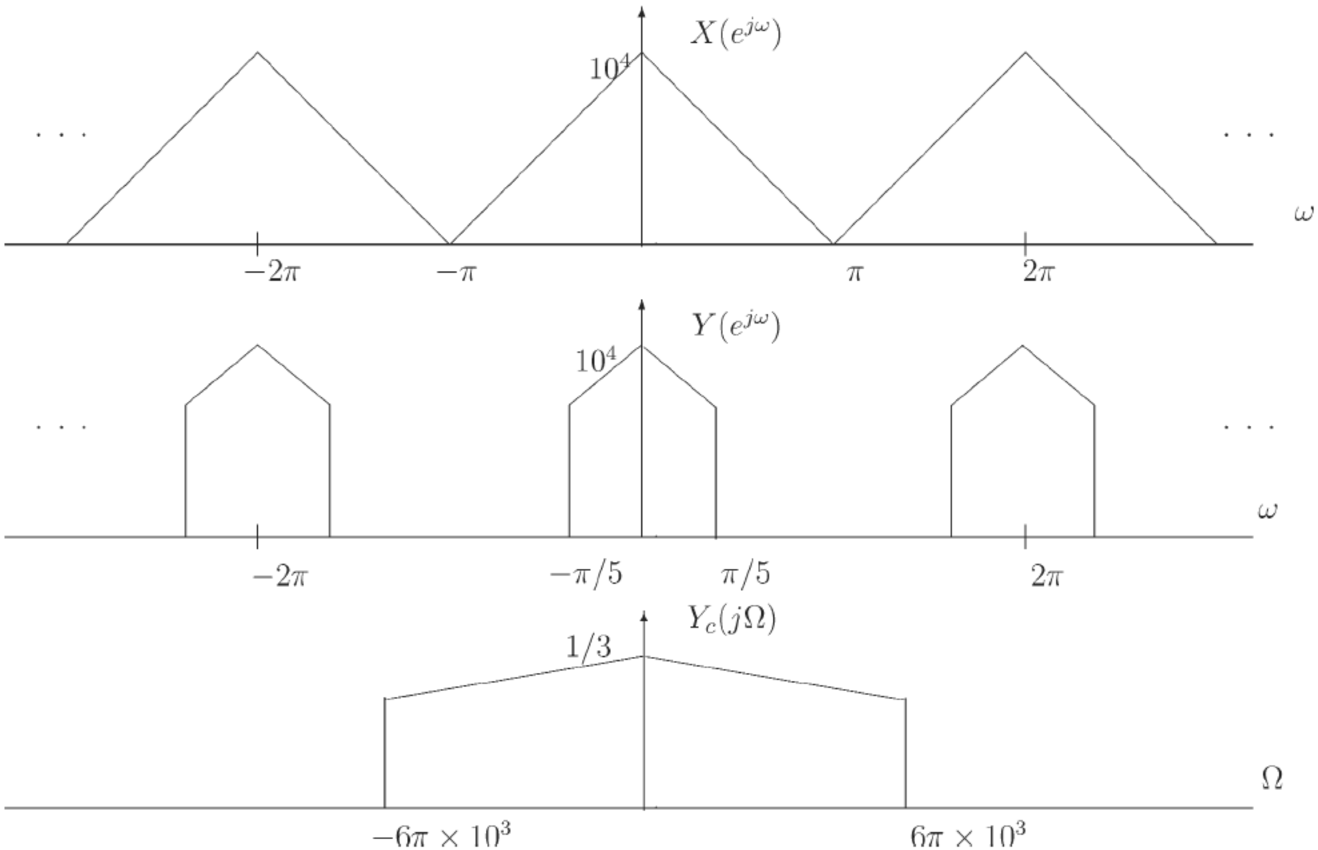


Figure P4.23-2

Sampling question

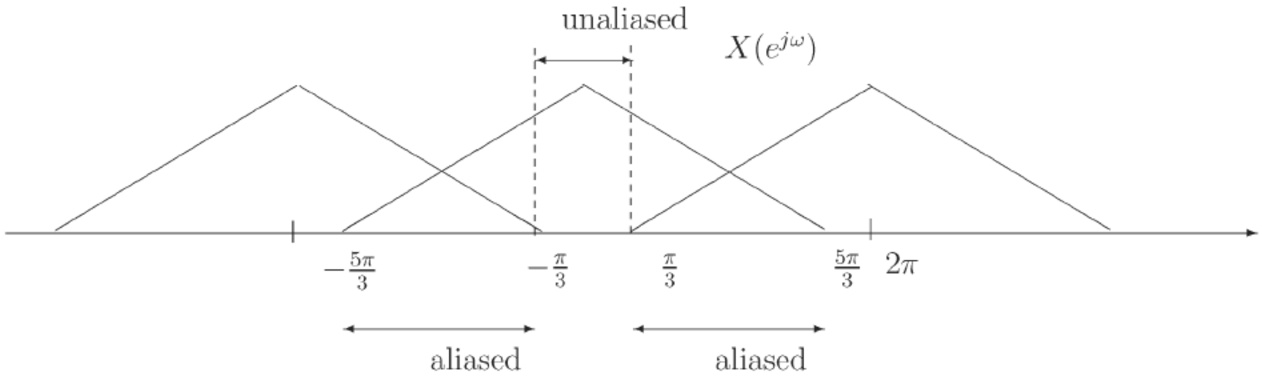
(iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$



Sampling question

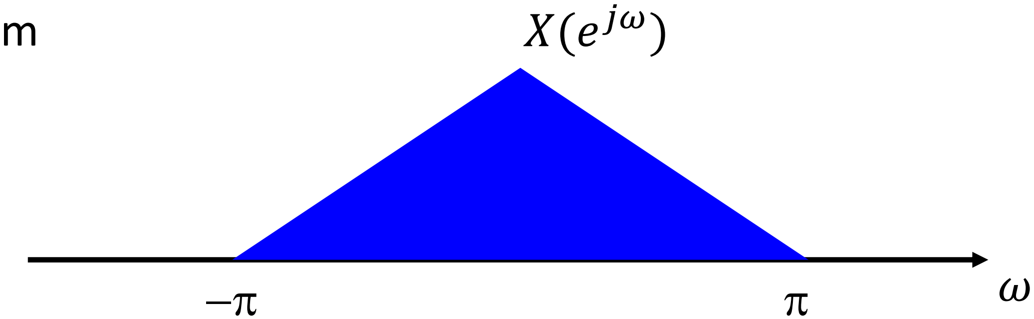
(b) From the figure below, it can be seen that the only portion of the spectrum which remains unaffected by the aliasing is $|\omega| < \pi/3$. So if we choose $\omega_c < \pi/3$, the overall system is LTI with a frequency response of

$$H_c(j\Omega) = \begin{cases} 1 & \text{for } |\Omega| < \omega_c \times 6 \times 10^3 \\ 0 & \text{otherwise.} \end{cases}$$

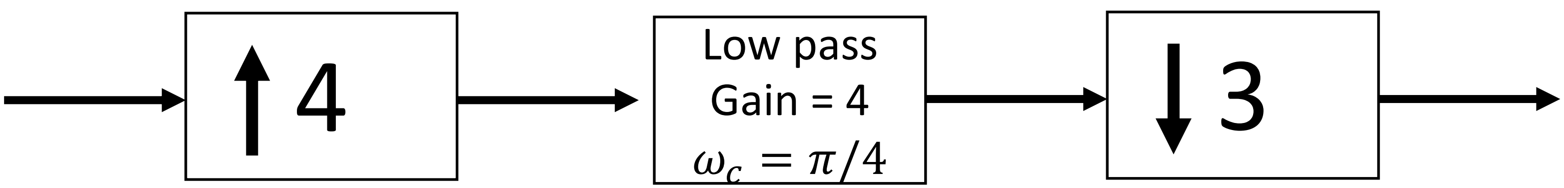


Review on resampling

Given spectrum

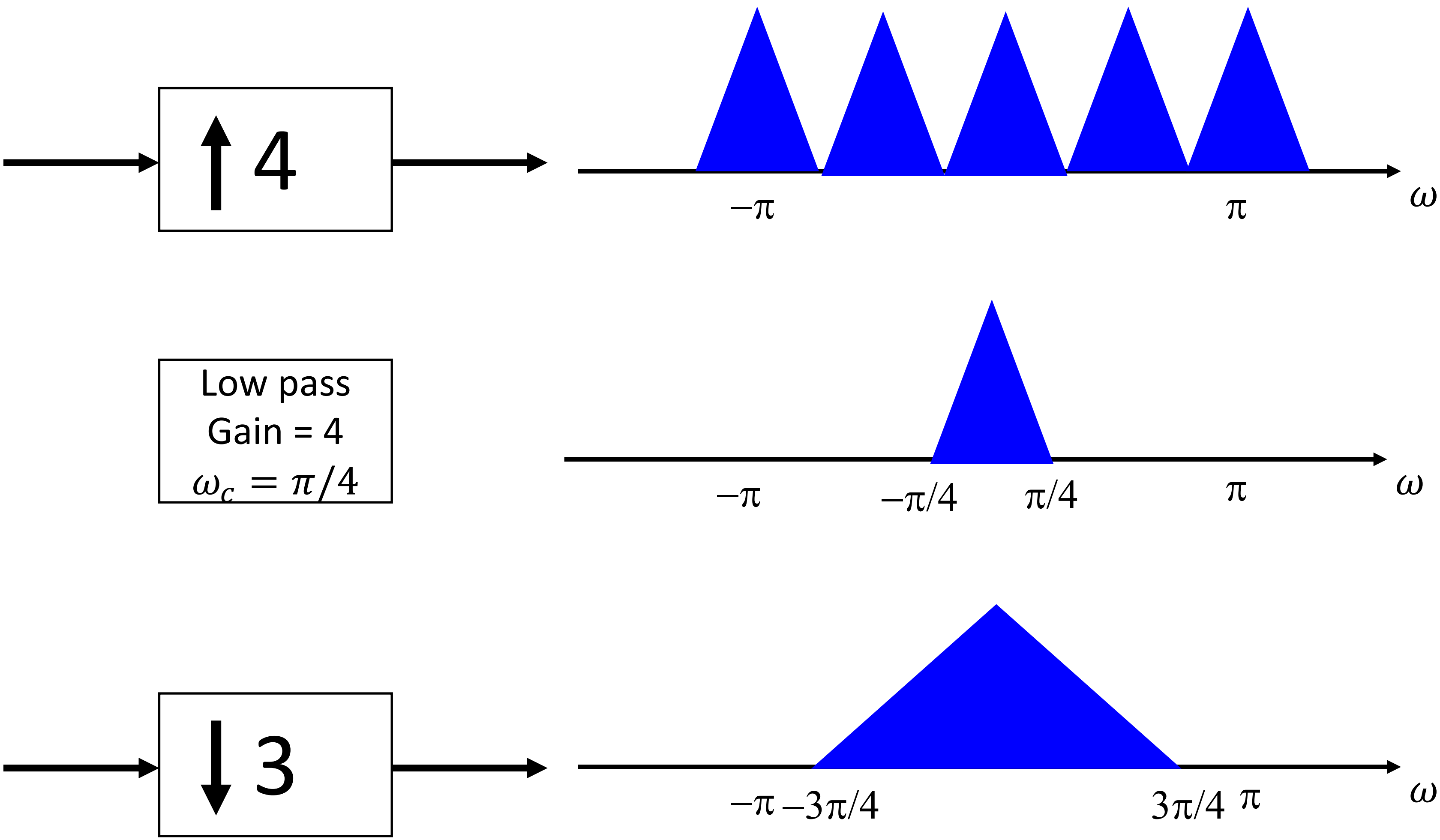


What will you do if I want to resample the signal with a period of 3T/4?



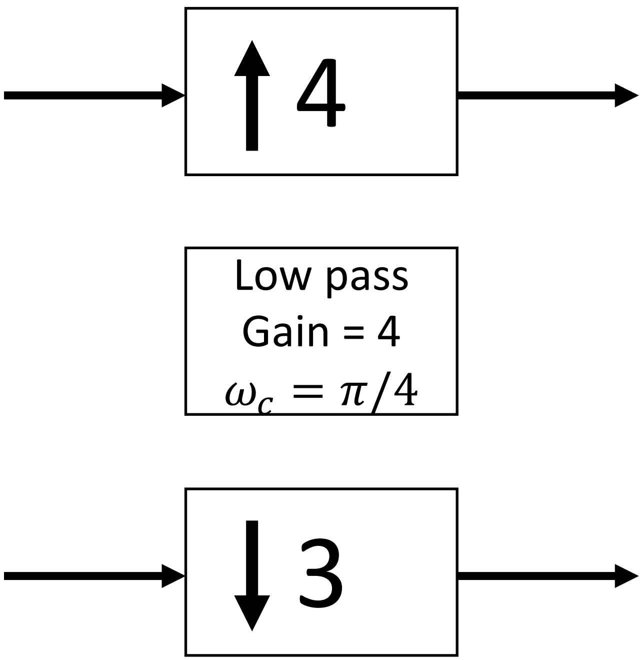
Assuming the low-pass filter is an ideal low pass filter,
how would you draw the spectrum at each stage?

Review on resampling



Assuming the filter is $H\left(e^{j\omega}\right)$, can you write up the expression of spectrum for each stage?

Review on resampling



$$X_e\left(e^{j\omega}\right) = X(e^{j4\omega})$$

$$X_u\left(e^{j\omega}\right) = H\left(e^{j\omega}\right)X(e^{j4\omega})$$

$$X_d\left(e^{j\omega}\right) = \sum_{i=0}^{2} X_u(e^{j(\frac{\omega}{3}-\frac{2\pi i}{3})})$$

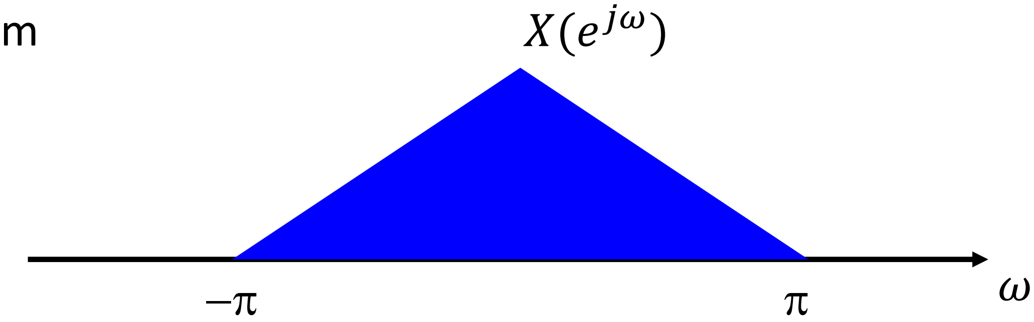
If we only want to express $X_d(e^{j\omega})$, how many terms do we need to consider?

How do these signals look like in the sequence space?

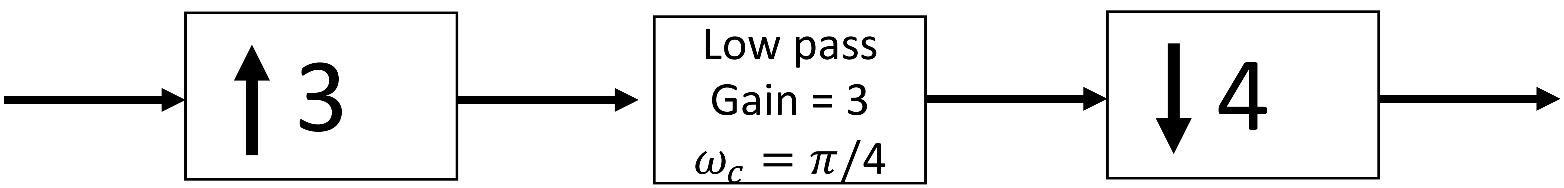
How would you reconstruct this signal with linear interpolation with period 3T/4?

Review on resampling

Given spectrum

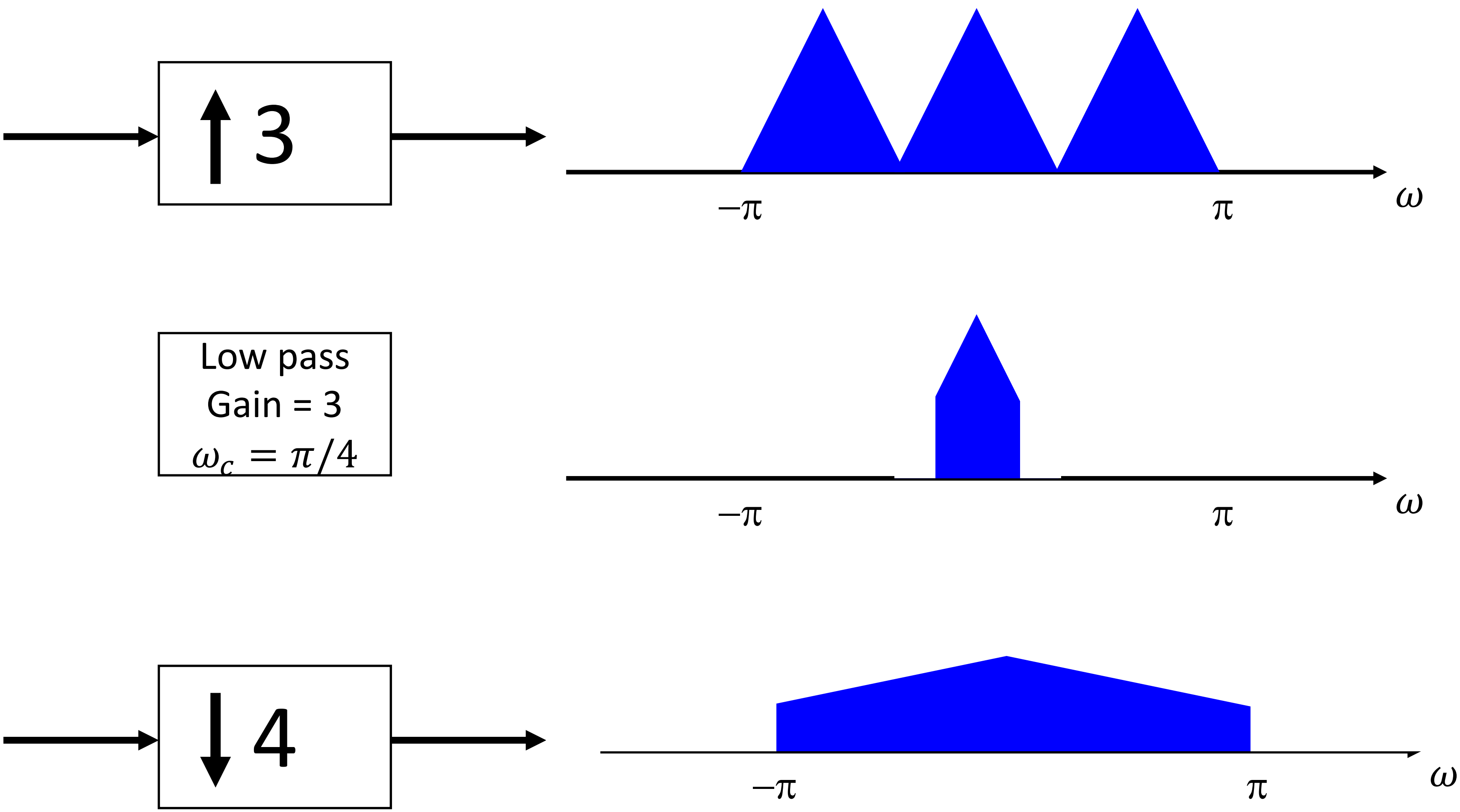


What will you do if I want to resample the signal with a period of 4T/3?



Assuming the low-pass filter is an ideal low pass filter,
how would you draw the spectrum at each stage?

Review on resampling



Sampling question

4.32. Consider the discrete-time system shown in Figure P4.32-1

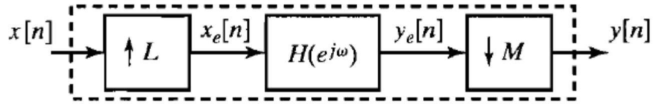


Figure P4.32-1

where

(i) L and M are positive integers.

(ii) $x_e[n] = \begin{cases} x[n/L] & n = kL, \quad k \text{ is any integer} \\ 0 & \text{otherwise.} \end{cases}$

(iii) $y[n] = y_e[nM]$.

(iv) $H(e^{j\omega}) = \begin{cases} M & |\omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| \le \pi \end{cases}$.

(a) Assume that $L = 2$ and $M = 4$, and that $X(e^{j\omega})$, the DTFT of $x[n]$, is real and is as shown in Figure P4.32-2. Make an appropriately labeled sketch of $X_e(e^{j\omega})$, $Y_e(e^{j\omega})$, and $Y(e^{j\omega})$, the DTFTs of $x_e[n]$, $y_e[n]$, and $y[n]$, respectively. Be sure to clearly label salient amplitudes and frequencies.

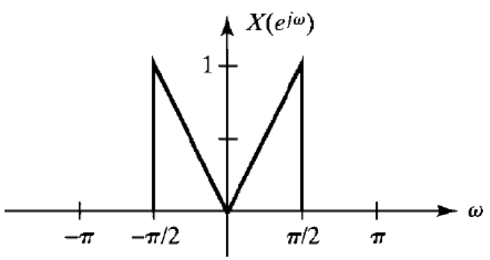
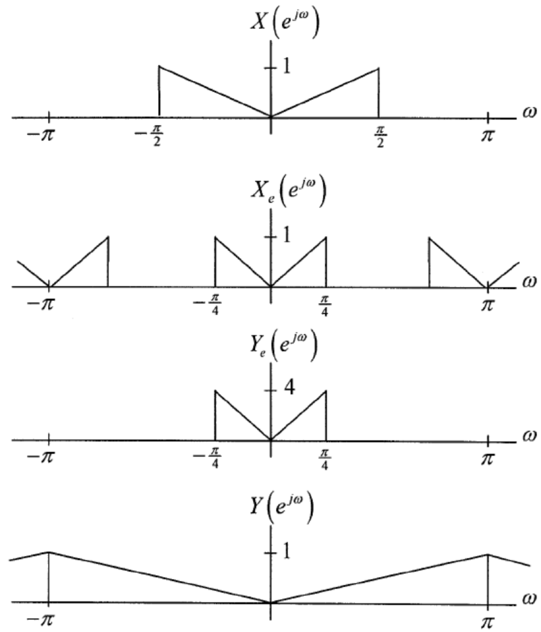


Figure P4.32-2

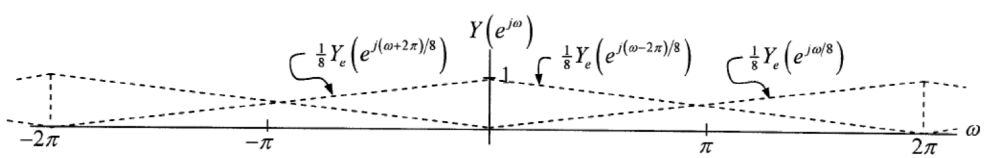
(b) Now assume $L = 2$ and $M = 8$. Determine $y[n]$ in this case.
Hint: See which diagrams in your answer to part (a) change.

Sampling question

4.32. A. With $L=2$ and $M=4$,



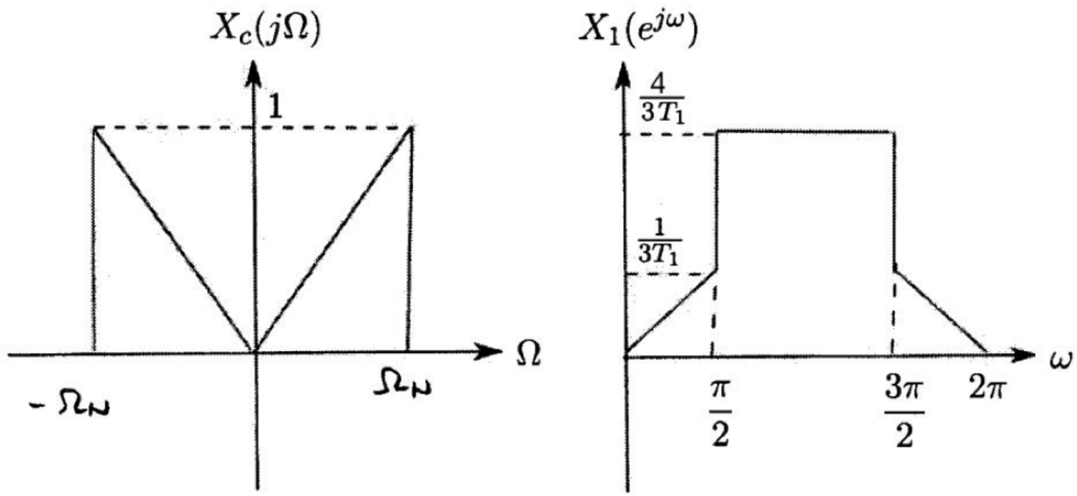
B. With $L=2$ and $M=8$, $X_e(e^{j\omega})$ and $Y_e(e^{j\omega})$ remain as in part A, except that $Y_e(e^{j\omega})$ now has a peak value of 8. After expanding we have



We see that $Y(e^{j\omega})=1$ for all ω. Inverse transforming gives $y[n]=\delta[n]$ in this case.

Sampling question

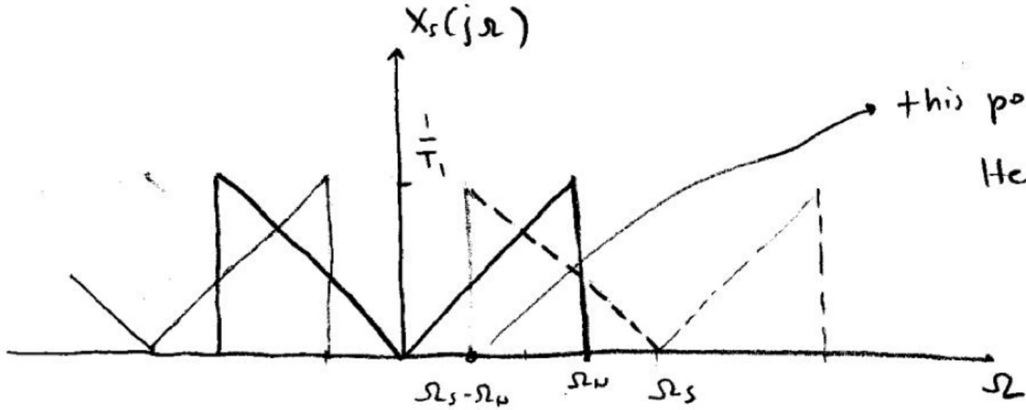
2. A continuous time signal $x_c(t)$ with the spectrum $X_c(j\Omega)$ depicted below is sampled with period T_1, resulting in a discrete sequence $x_1[n]$ with the DTFT $X_1(e^{j\omega})$ below.



a) (15 points) Determine the largest sampling period T_2 that would avoid aliasing, and express it in terms of T_1. Sketch the DTFT of the sequence $x_2[n]$, sampled with period T_2.

b) (15 points) Draw the block diagram of a post-processing unit that down-samples $x_2[n]$ by a factor of T_2/T_1. Sketch the DTFT of the output and compare it with $X_1(e^{j\omega})$ above.

Sampling question



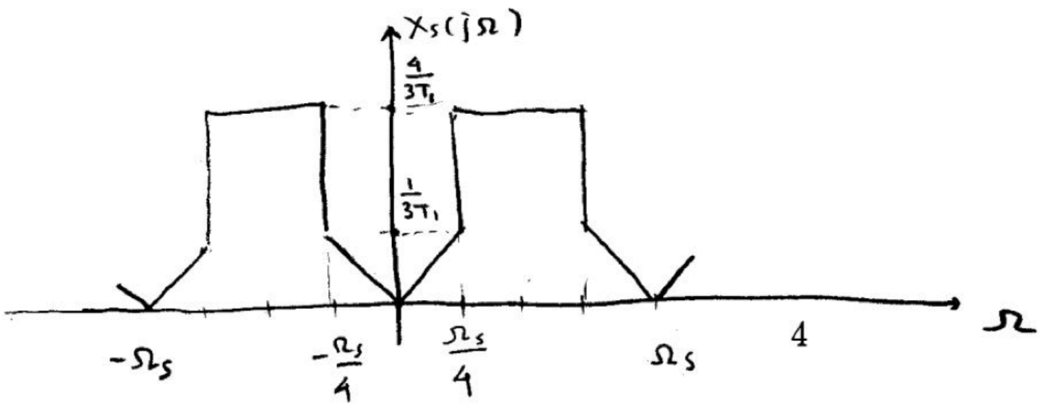
this point maps to $\frac{\pi}{2}$.

Hence, $\Omega_s - \Omega_M = \frac{\Omega_s}{4}$

$$\Rightarrow \Omega_s = \frac{4}{3}\Omega_M$$

$$\Omega_s = \frac{2\pi}{T_1}$$

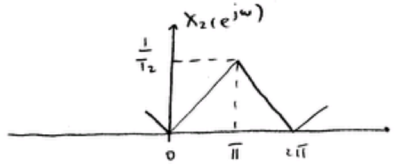
$$\Rightarrow \boxed{\Omega_M = \frac{3}{4} \cdot \frac{2\pi}{T_1}}$$



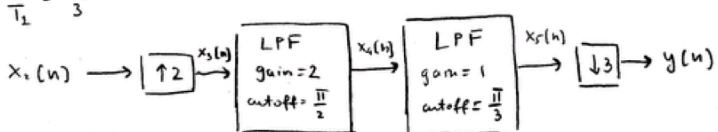
Sampling question

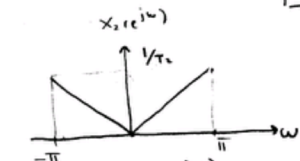
To avoid aliasing, we need

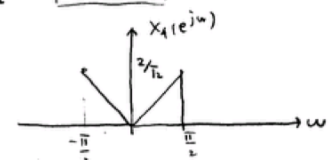
$$\frac{2\pi}{T_2} = 2\Omega_N = \frac{3\pi}{T_1} \Rightarrow \boxed{T_2 = \frac{2}{3}\cdot T_1}$$

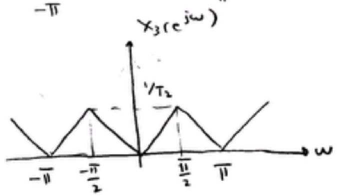


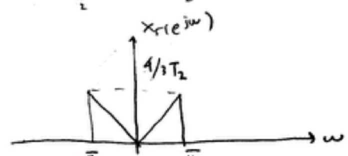
b) $\frac{T_2}{T_1} = \frac{2}{3}$



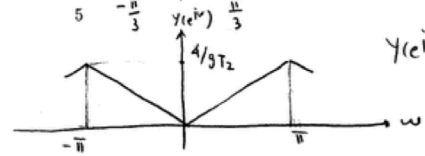








5



$Y(e^{j\omega}) \neq X_1(e^{j\omega})$