EE 123 Discussion Section 2

Feb. 6, 2019
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(original slides from Li-Hao Yeh)

Announcements

- Office Hours
  - Miki: Wednesdays 4:15-5:15pm, Cory 506
  - Li-Hao: Mondays 11-12pm, Cory 504
  - Michael: Fridays 11-12pm, Cory 504
- Lab 1 – due next Friday Feb. 15
- HW 2 – due next Monday Feb. 11
- Questions?

Z-transform recap

z-transform (always associated with a ROC)

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Inverse z-transform

- Inspection method

1) Properties of z-transform
2) Partial fraction expansion/long division

Example of inverse z-transform

$$X(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

$x[n]$ is absolutely summable

Strategies:

1. Assuming numerator is in order of M, the denominator is in order of N
2. If $M \geq N$, long division to make $M < N$
3. Partial fraction expansion
4. Locate region of convergence
5. Inspection method

Example of inverse z-transform

$$X(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

$x[n]$ is absolutely summable

1. Long division

$$\begin{array}{r|l} & \qquad\qquad -2 \\ \cline{2-2} 1 - \frac{13}{6}z^{-1} + z^{-2} & 1 + z^{-1} - 2z^{-2} \\ & -2 + \frac{13}{3}z^{-1} - 2z^{-2} \\ \cline{2-2} & 3 - \frac{10}{3}z^{-1} \end{array}$$

Example of inverse z-transform

$$X(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

$x[n]$ is absolutely summable

2. Partial expansion method

$$X(z) = -2 + \frac{3 - \frac{10}{3}z^{-1}}{1 - \frac{13}{6}z^{-1} + z^{-2}} = -2 + \frac{3 - \frac{10}{3}z^{-1}}{\left(1 - \frac{3}{2}z^{-1}\right)\left(1 - \frac{2}{3}z^{-1}\right)}$$

$$= -2 + \frac{A}{\left(1 - \frac{3}{2}z^{-1}\right)} + \frac{B}{\left(1 - \frac{2}{3}z^{-1}\right)}$$

Example of inverse z-transform

$$X(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

$x[n]$ is absolutely summable

2. Partial expansion method

$$3 - \frac{10}{3}z^{-1} = A\left(1 - \frac{2}{3}z^{-1}\right) + B\left(1 - \frac{3}{2}z^{-1}\right)$$

$$z^{-1} = \frac{3}{2} \quad \Rightarrow \quad -2 = B \cdot \frac{-5}{4}$$

$$z^{-1} = \frac{2}{3} \quad \Rightarrow \quad \frac{7}{9} = A \cdot \frac{5}{9}$$

Example of inverse z-transform

$$X(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

$x[n]$ is absolutely summable

3. Locate region of convergence

$x[n]$ is absolutely summable ➔ $x[n]$ has Fourier transform

➔ ROC should contain |z| = 1 circle ➔ $\frac{2}{3} < |z| < \frac{3}{2}$

Example of inverse z-transform

$$X(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

$x[n]$ is absolutely summable

4. Inspection method

Left-sided Right-sided

$$X(z) = -2 + \frac{A}{\left(1 - \frac{3}{2}z^{-1}\right)} + \frac{B}{\left(1 - \frac{2}{3}z^{-1}\right)}$$

$$\Rightarrow \quad x[n] = -2\delta[n] - A\left(\frac{3}{2}\right)^n u[-n-1] + B\left(\frac{2}{3}\right)^n u[n]$$

Discrete Fourier transform (DFT)

Important concept:

1. DFT comes from discrete Fourier series → the periodic boundary
2. DFT is related to DTFT and z transform in the following ways

$$X[k] = X(e^{j\omega})\Big|_{\omega=\frac{2k\pi}{N}} = X(z)\Big|_{z=e^{j\frac{2k\pi}{N}}}$$

DFT example question

Express the following signal with x[n] – a 8-point sequence
x[n] = 0, for n<0 or n>7
X[k] – 8-point DFT of x[n]

$$\frac{1}{8}\sum_{k=0}^{7} X[k]e^{j(2\pi/8)kn}\,|_{n=9} \;=?$$

DFT example question

Express the following signal with x[n] – a 8-point sequence
x[n] = 0, for n<0 or n>7
X[k] – 8-point DFT of x[n]

$$\frac{1}{8}\sum_{k=0}^{7} X[k]e^{j(2\pi/8)kn}\,|_{n=9}\;=?$$

Solution:

$$\frac{1}{8}\sum_{k=0}^{7} X[k]e^{j(2\pi/8)kn} = x[((n))_8]$$

$$x[((9))_8] = x[1]$$

DFT example question

Express the following signal with x[n] – a 8-point sequence
x[n] = 0, for n<0 or n>7
X[k] – 8-point DFT of x[n]

v[n] – a 8-point sequence, v[n] = 0 for n<0 or n>7, V[k] – 8-point DFT of v[n]

$$V[k] = X(z)\,|_{z=2e^{j(2\pi k+\pi)/8}}$$

$$v[n] = ?$$

DFT example question

Express the following signal with x[n] – a 8-point sequence

x[n] = 0, for n<0 or n>7

X[k] – 8-point DFT of x[n]

Solution:

$$V[k] = V(z)\,|_{z=e^{j(2\pi k)/8}} = X(z)\,|_{z=2e^{j(2\pi k+\pi)/8}}$$

$$V(z) = X(z \cdot 2e^{j\pi/8})$$

$$v[n] \cdot (2e^{j\pi/8})^n \xleftrightarrow{\mathcal{Z}} V(z/2e^{j\pi/8}) = X(z)$$

$$v[n] = x[n] \cdot (2e^{j\pi/8})^{-n}$$