EE123

Digital Signal Processing

Lecture 20

Filter Design

M. Lustig, EECS UC Berkeley

Linear Filter Design

- Used to be an art
  - Now, lots of tools to design optimal filters
- For DSP there are two common classes
  - Infinite impulse response IIR
  - Finite impulse response FIR
- Both classes use finite order of parameters for design
- We will cover FIR designs, briefly mention IIR

What is a linear filter

- Attenuates certain frequencies
- Passes certain frequencies
- Effects both **phase** and **magnitude**
- IIR
  - Mostly non-linear phase response
  - Could be linear over a range of frequencies
- FIR
  - Much easier to control the phase
  - Both non-linear and linear phase

FIR Design by Windowing

- Given desired frequency response, $H_d(e^{j\omega})$, find an impulse response

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

ideal

- Obtain the Mth order causal FIR filter by truncating/windowing it

$$h[n] = \left\{ \begin{array}{ll} h_d[n]w[n] & 0 \le n \le M \\ 0 & \text{otherwise} \end{array} \right\}$$

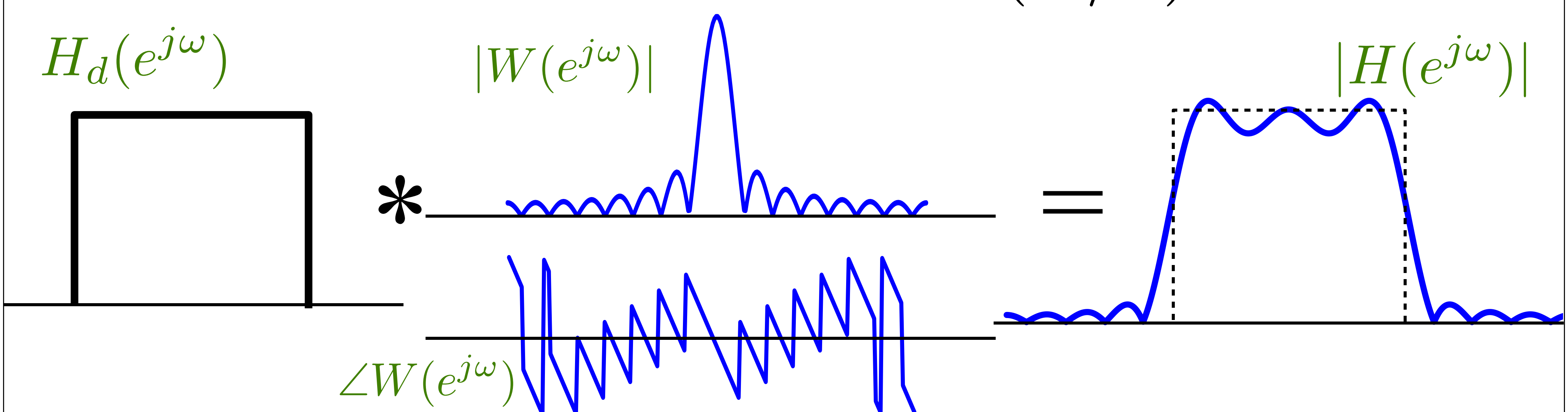
FIR Design by Windowing

- We already saw that,

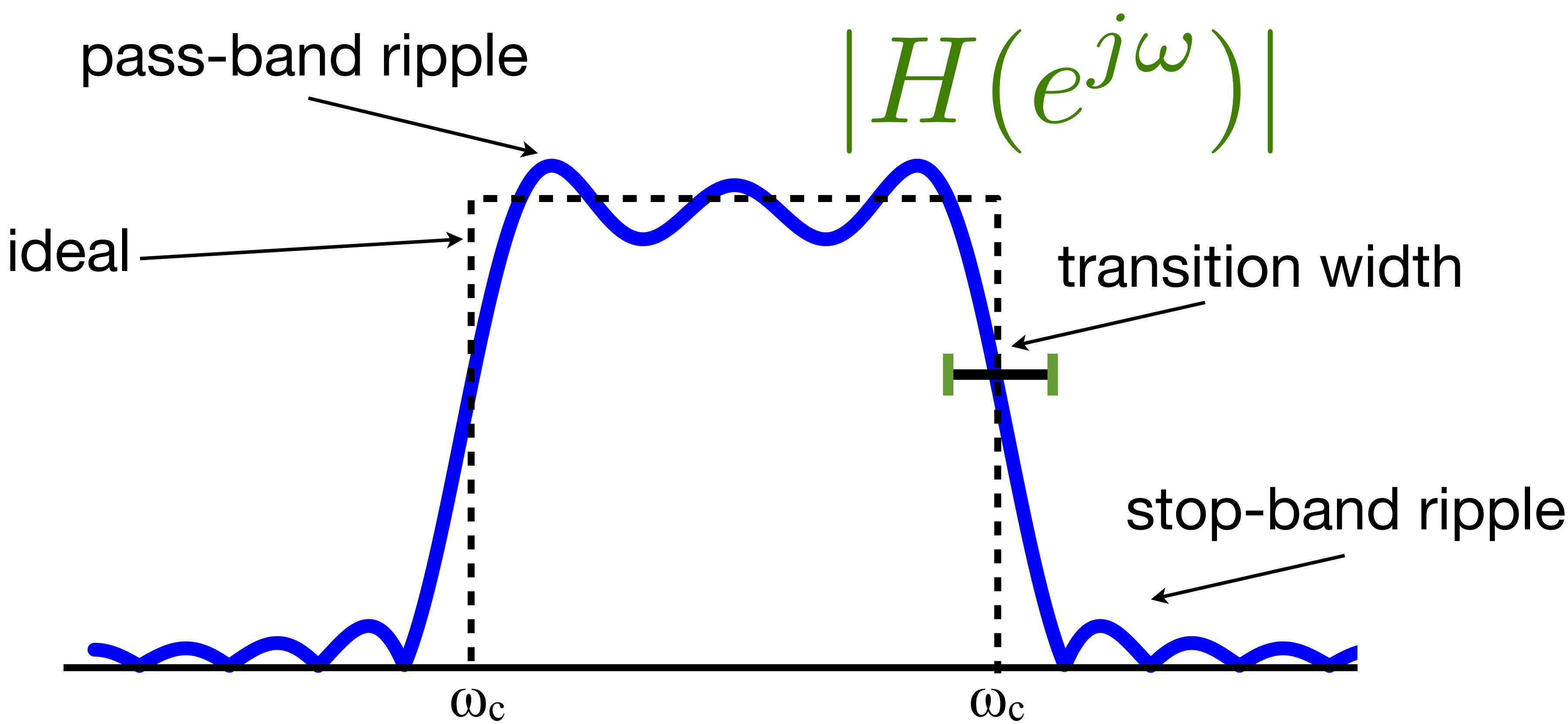
$$H(e^{j\omega}) = H_d(e^{j\omega}) \overset{\text{periodic}}{*} W(e^{j\omega})$$

- For Boxcar (rectangular) window

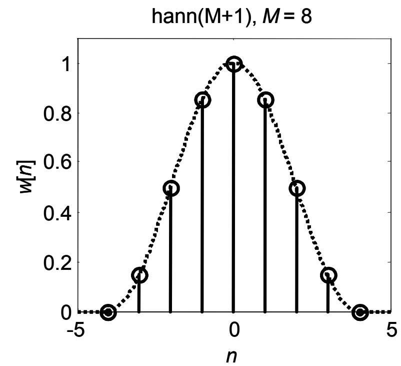
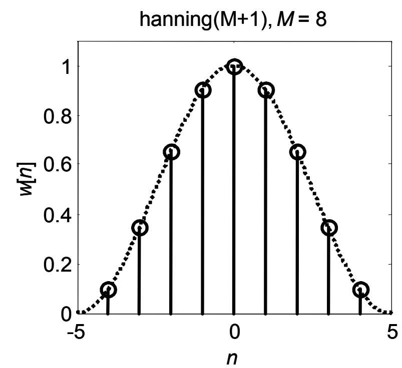
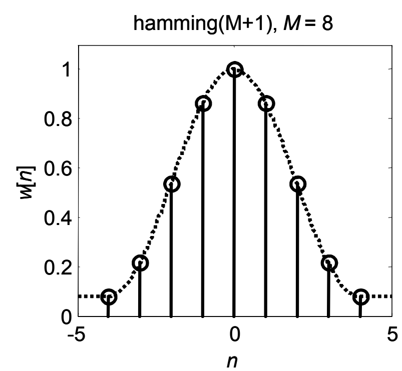
$$W(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \frac{\sin(w(M+1)/2)}{\sin(w/2)}$$



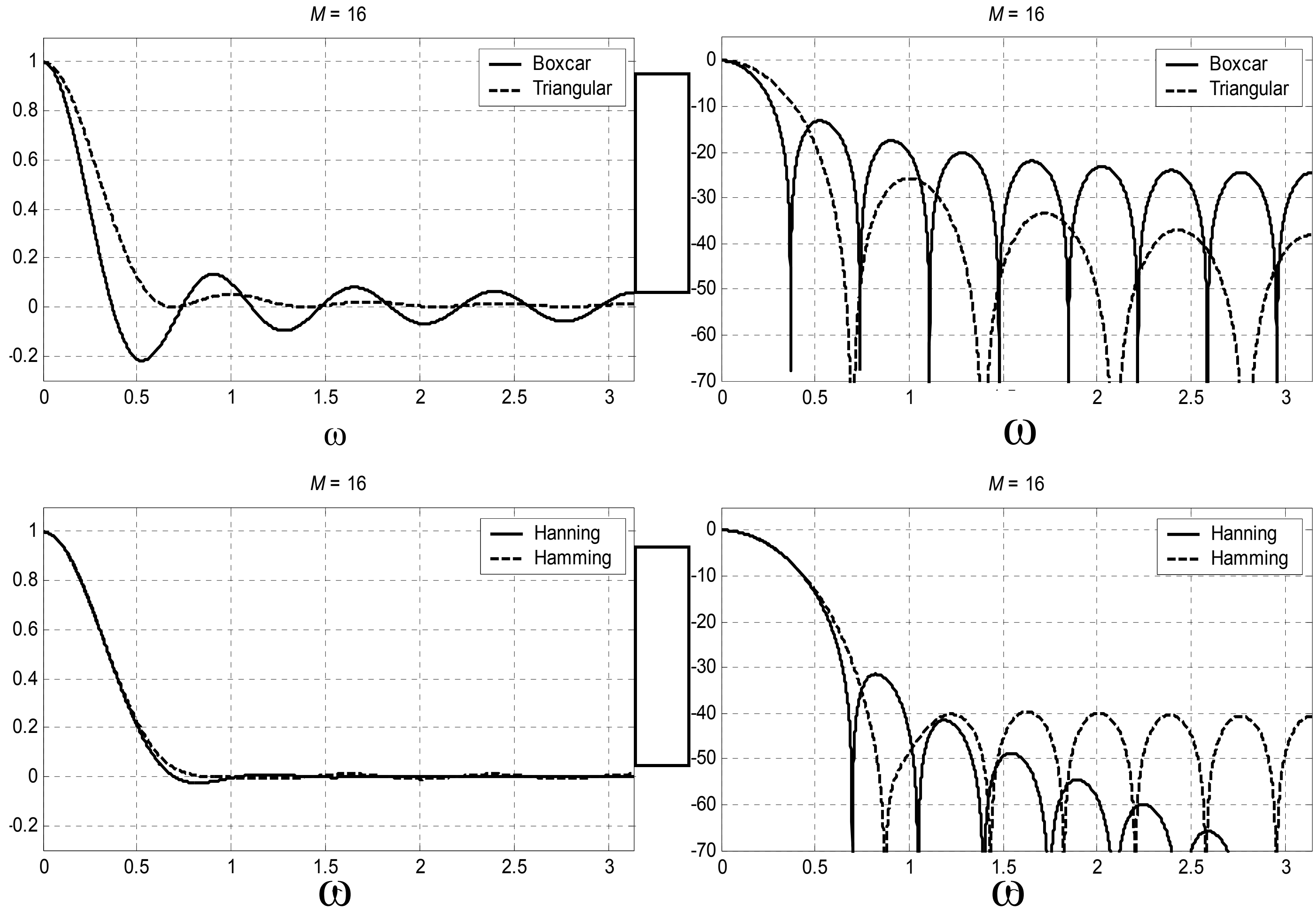
FIR Design by Windowing



Tapered Windows

Name(s)	Definition	MATLAB Command	Graph ($M = 8$)
Hann	$w[n] = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{\pi n}{M/2}\right)\right] & \lvert n\rvert \le M/2 \\ 0 & \lvert n\rvert > M/2 \end{cases}$	**`hann(M+1)`**	
Hanning	$w[n] = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{\pi n}{M/2+1}\right)\right] & \lvert n\rvert \le M/2 \\ 0 & \lvert n\rvert > M/2 \end{cases}$	**`hanning(M+1)`**	
Hamming	$w[n] = \begin{cases} 0.54 + 0.46\cos\left(\frac{\pi n}{M/2}\right) & \lvert n\rvert \le M/2 \\ 0 & \lvert n\rvert > M/2 \end{cases}$	**`hamming(M+1)`**	

Tradeoff - Ripple vs Transition Width



Python: scipy.filter.firwin

FIR Filter Design

- Choose a desired frequency response $H_d(e^{j\omega})$
  - non causal (zero-delay), and infinite imp. response
  - If derived from C.T, choose T and use:

$$H_d(e^{j\omega}) = H_c(j\frac{\Omega}{T})$$

- Window:
  - Length M+1 ⇔ effect transition width
  - Type of window ⇔ transition-width/ ripple
  - Modulate to shift impulse response

$$H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$

FIR Filter Design

- Determine truncated impulse response $h_1[n]$

$$h_1[n] = \begin{cases} \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}e^{j\omega n} & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

- Apply window

$$h_w[n] = w[n]h_1[n]$$

- Check:
  - Compute $H_w(e^{j\omega})$, if does not meet specs increase M or change window

Example: FIR Low-Pass Filter Design

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Choose M ⇒ Window length and set

$$H_1(e^{j\omega}) = H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$

$$h_1[n] = \begin{cases} \frac{\sin(\omega_c(n-M/2))}{\pi(n-M/2)} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\omega_c}{\pi}\mathrm{sinc}(\frac{\omega_c}{\pi}(n - M/2))$$

Example: FIR Low-Pass Filter Design

- The result is a windowed sinc function

$$h_w[n] = w[n]h_1[n]$$

$$\frac{\omega_c}{\pi}\mathrm{sinc}(\frac{\omega_c}{\pi}(n - M/2))$$

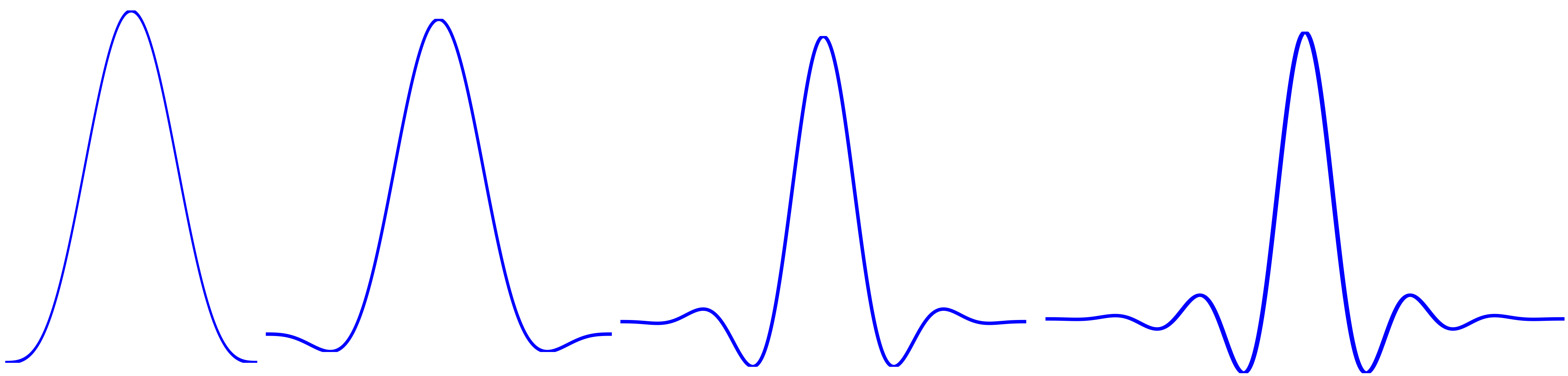
- High Pass Design:
  - Design low pass $h_w[n]$
  - Transform to $h_w[n](-1)^n$

- General bandpass
  - Transform to $2h_w[n]cos(\omega_0 n)$

Characterization of Filter Shape

Time-Bandwidth Product, a unitless measure

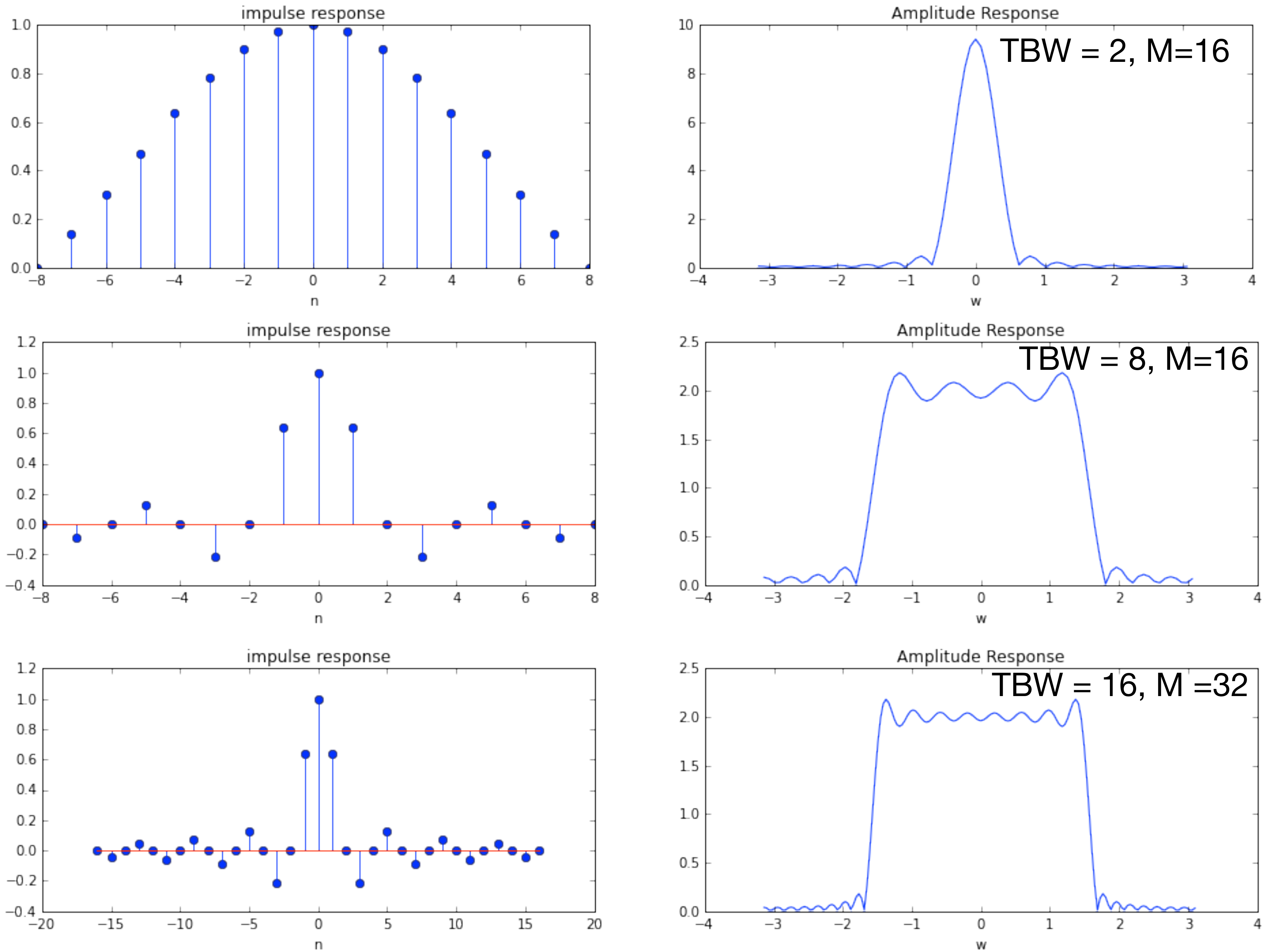
$T(BW) = (M+1)\omega/2\pi$ ⇒ also, total # of zero crossings



TBW=2 TBW=4 TBW=8 TBW=12

Larger TBW ⇒ More of the “sinc” function

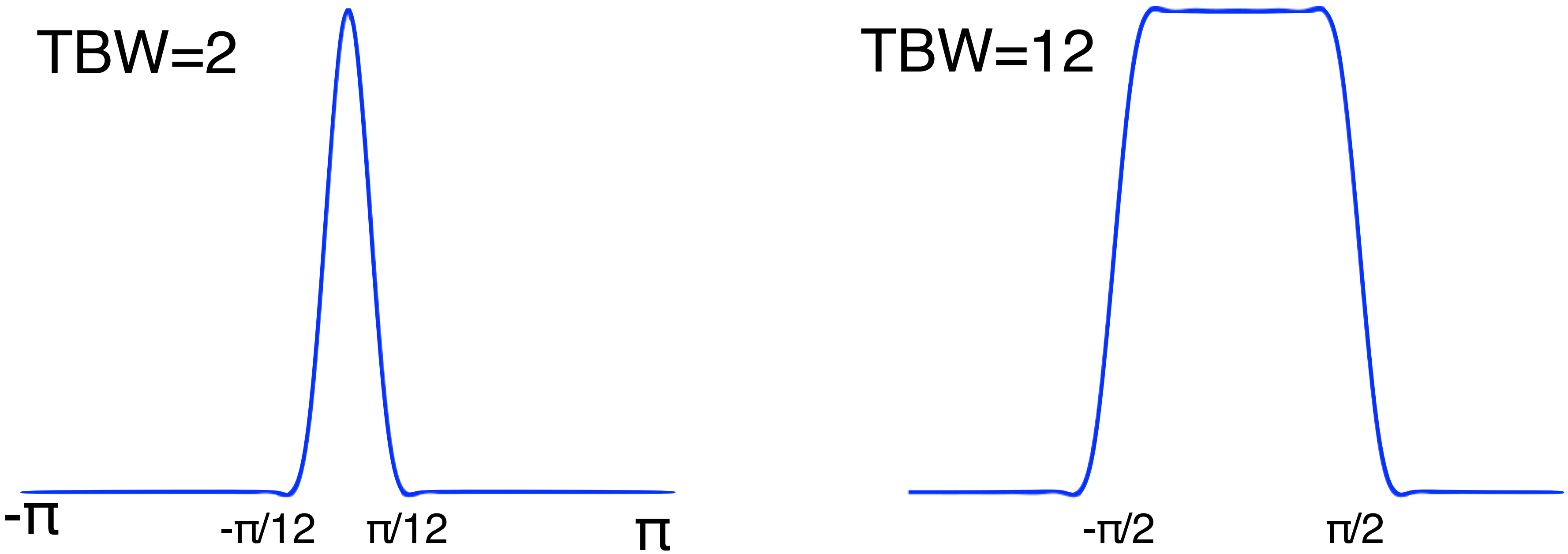
hence, frequency response looks more like a rect function



M. Lustig, EECS UC Berkeley

Frequency Response Profile

Q: What are the lengths of these filters in samples?



$2 = (M+1)*(\pi/6) \,/\, (2\pi) \Rightarrow M=23$

$12= (M+1)*(\pi) \,/\, (2\pi) \Rightarrow M=23$

Note that transition is the same!

Alternative Design Through FFT

- To design order M filter:
- Over-Sample/discretize the frequency response at P points where P >> M (P=15M is good)

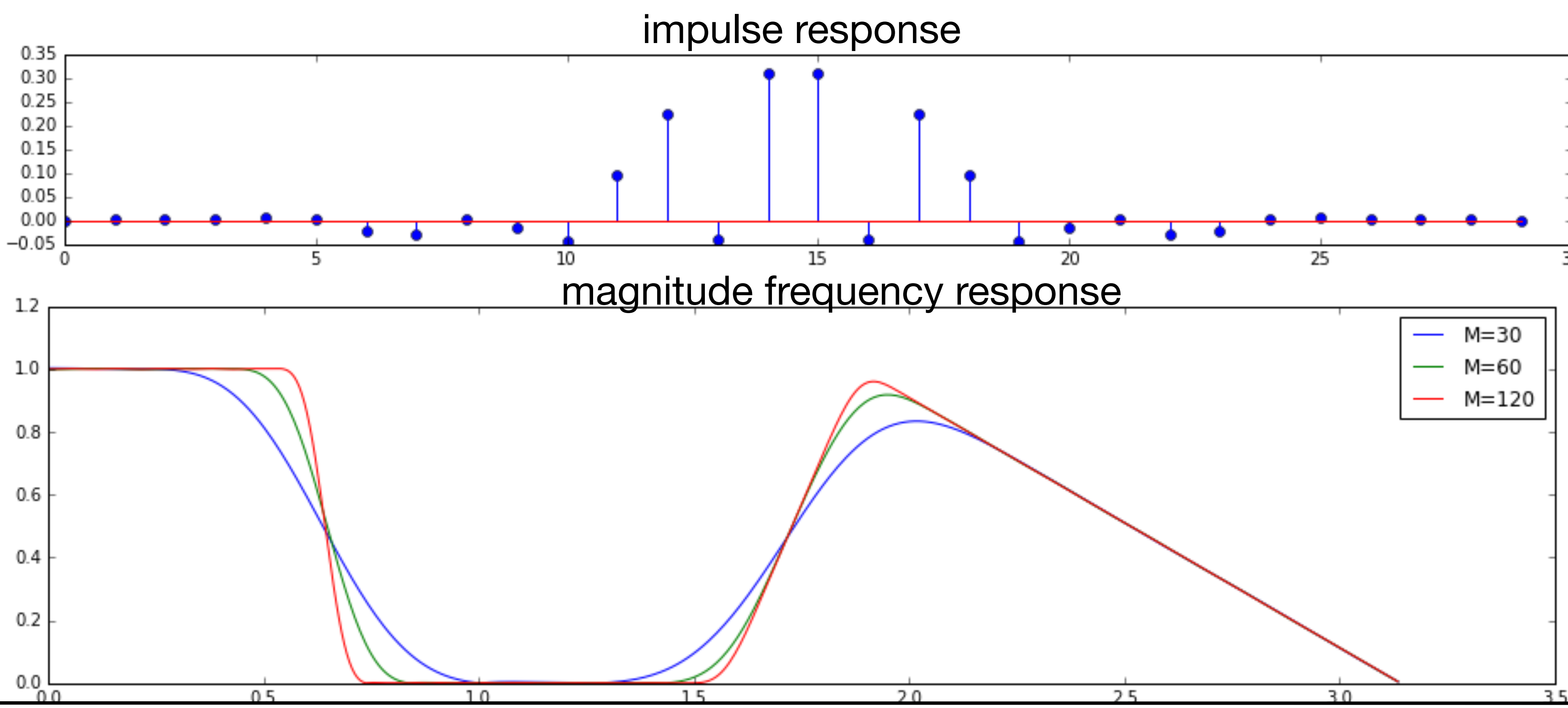
$$H_1(e^{j\omega_k}) = H_d(e^{j\omega_k})e^{-j\omega_k \frac{M}{2}}$$

  - Sampled at: $\omega_k = k\frac{2\pi}{P}$ $\quad |k = [0, \cdots, P-1]$
  - Compute $h_1[n] = \mathrm{IDFT}_P(H_1[k])$
  - Apply M+1 length window:

$$h_w[n] = w[n]h_1[n]$$

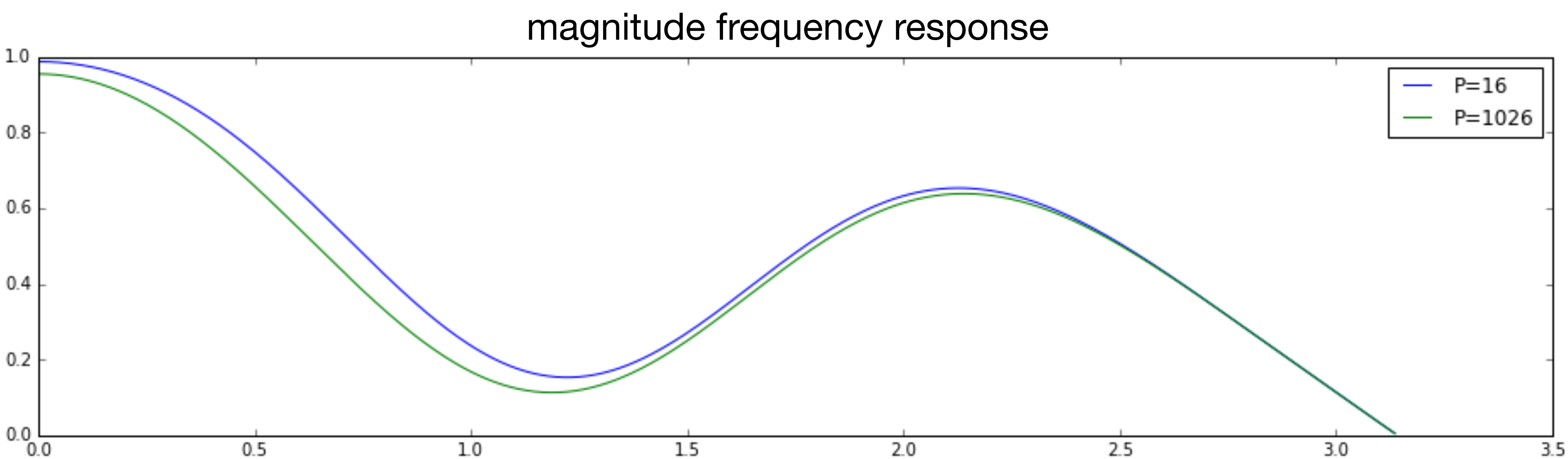
Example: signal.firwin2

- signal.firwin2(M+1,omega_vec/pi, amp_vec)
- taps1 = signal.firwin2(30, [0.0,0.2,0.21,0.5, 0.6, 1.0], [1.0, 1.0, 0.0,0.0,1.0,0.0])



Example: Design using FFT

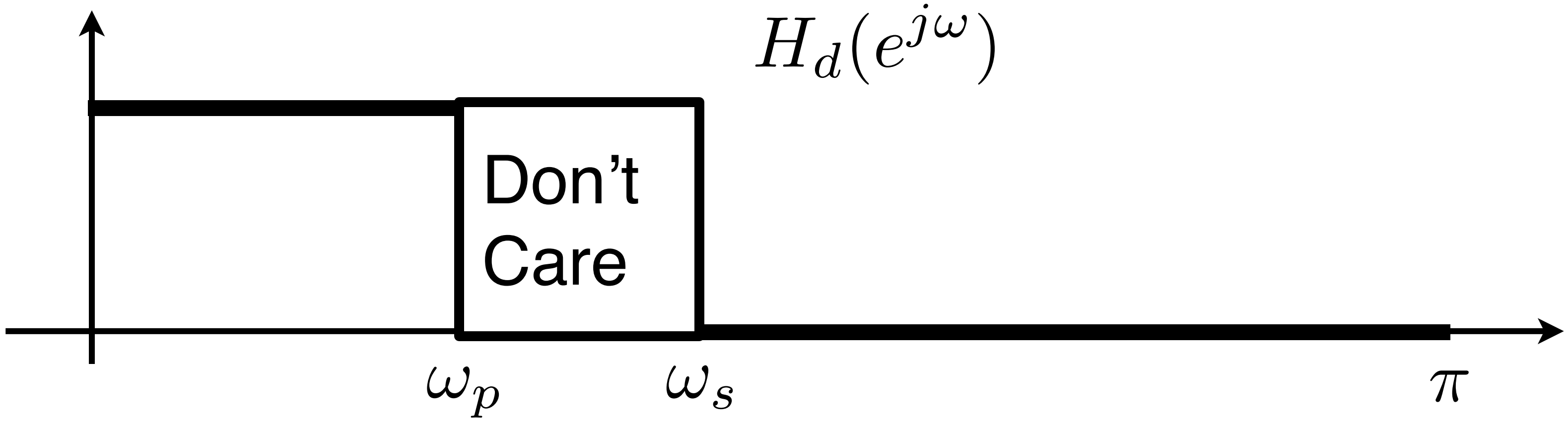
- For M+1=14
  - P = 16 and P = 1026



Optimal Filter Design

- Window method
  - Design Filters heuristically using windowed sinc functions
- Optimal design
  - Design a filter h[n] with $H(e^{j\omega})$
  - Approximate $H_d(e^{j\omega})$ with some optimality criteria - or satisfies specs.

Optimality



- Least Squares:

$$\text{minimize} \quad \int_{\omega \in \text{care}} |H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$$

Variation: weighted least-squares

$$\text{minimize} \quad \int_{-\pi}^{\pi} W(\omega) |H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$$

Optimality

- Chebychev Design (min-max)

$$\text{minimize}_{\omega \in \text{care}} \quad \max \left| H(e^{j\omega}) - H_d(e^{j\omega}) \right|$$

  - Parks-McClellan algorithm - equi-ripple
  - Also known as Remez exchange algorithms (signal.remez)

Example of Complex Filter

Larson et. al, "Multiband Excitation Pulses for Hyperpolarized 13C Dynamic Chemical Shift Imaging" JMR 2008;194(1):121-127

Need to design 11 taps filter with following frequency response: