**Final**—by Erin Grimes, Mar. 19, 2020—MATH 435 Spring 2020—Western Washington University

1. Minimize , subject to . Given and and with as an inequality constraint, the Karush-Kuhn-Tucker Theorem applies. If an optimizer exists, then there exists such that

If then implies .

On the other hand, if then is inactive, and forces and , or and , a contradiction. Therefore, .

1. Let . Using , let , , , . Then …

The Amijo condition requires

1. Minimize . The associated Lagrangian is . Since depends on and , Beltrami’s identity does not apply. Note the following:

Then the Euler-Lagrange equations become

a homogeneous second-order ODE.

4. Let , , . First note that , and , are continuous and differentiable on . Then whenever ; in particular, when so for all . Also, for all , hence for all . Therefore, minimizes .

10. Let and . We seek which minimizes subject to . Then by the KKT theorem, if a solution exists, there exists such that

If is an inactive contraint, which is to say , then and ; so is a candidate.

On the other hand, if then so we have , or and . Then is defined only when , which is to say , a contradiction. Thus, is infeasible and there is only one minimizer.

11. Let , , , , . Then

We seek such that the Armijo condition,

is satisfied, as well as the weak Wolfe condition,

which simplifies to

but not the strong Wolfe condition,

The term for all . Equality in all three conditions are satisfied whenever , since for all ; thus, assume .

Let us assume , so . Then the strong Wolfe condition becomes

Let be some positive angle in the 4th quadrant of the unit circle, say . Then the strong Wolfe condition is false, while the weak Wolfe condition is satisfied.

12. Find which minimizes subject to and . By the KKT theorem, if a solution exists, there exists such that

If , then and .

Otherwise, if then , which contradicts to . Thus, is infeasible.

Otherwise, if then , hence is infeasible. Therefore, is the only minimizer.