**Midterm Exam 3** *by Erin Grimes—11.24.2019—MATH 473, Fall 2019, Western Washington University*

1. Let be a nonzero vector. is orthogonal is to say

since and are symmetrical

since

since is a scalar

since

since .

1. By definition, a Householder transformation where . Let where , which is to say .

where and .

Check

1. Let be a matrix, be an orthogonal matrix and assume is defined.

Given ,

1. Let and where .

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and the first column of is so

1. By Bjorck’s theorem, . Hence,

. For the residue, assume that : let and . Then where , and , which is minimized when . Therefore, is the corresponding residue.

1. First note that inasmuch as is nearly rank one since , the condition number is large. It can be seen that exactly, so the residue and the first problem is perfectly conditioned. On the other hand, and . So ; thus we should expect high sensitivity in the case of the second problem and the problem is likely ill conditioned.
2. Let and .
3. Recall that . Then for matrices and and matrix , .
4. From part (i), let where . Then

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1. Let . Let . Note that . Perform the following computations sequentially.

* .

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* .

Thus, the above computations correspond to the Givens transformation

Computing this transformation using the obvious way is likely to induce cancellation error when or . On the other hand, the non-obvious computation produces only overflow error when , which is to say ; which is less intrinsically likely to occur since this implies is a near multiple of .

1. Let and assume .
2. .

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1. Let , and note that

and since .

We seek a mutually orthogonal vector to both and , so let . Let and and . Then it is clear that is orthogonal and . Then

* Solve for : .

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1. Supposing is very small compared to then the condition number is very large since the columns of are nearly linearly dependent. Then the solution of will be inaccurate since , where here . On the other hand, using the method solves exactly the perturbed least squares system where here and , where for a slowly growing function .
2. Let . By Bjorck’s theorem,

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Now is a rank-1 matrix, hence its decomposition has rank-1 and .

Solution: .