HBC:

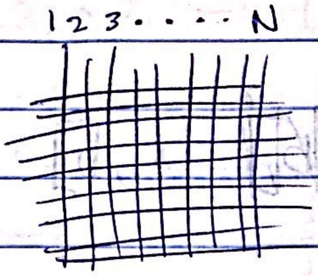
uses noise to escape lesser minima

Simulated Annealing

↳ traveling salesperson

(info traveling through a network/path, what's the fastest/best route)

N_{cities}



A_{ij} is the cost of traveling from city i to city j

$A_{ij} \neq A_{ji}$

Properties of Symmetric Matrices

<u>orthogonal vectors</u>

1) $u^T v = u \cdot v = 0$ (dot prod is 0)
2) $\{q_1, q_2, \ldots, q_n\}$ $q_i^T q_j = 0$, $q_i \in \mathbb{R}^n$

$$\vec{x} = \sum_{j=1}^{n} c_j q_j \quad \text{for any } \vec{x} \in \mathbb{R}^n$$

diagonals must be real

ex $A = \begin{bmatrix} 11 & 2+i \\ 2-i & 1 \end{bmatrix}$

$A \in \mathbb{R}^{N\times N}$ and <u>symmetric</u> OR $A \in \mathbb{C}^{N\times N}$ and Hermetian

$A = A^* = \overline{(A^T)}$ (transpose conjugate)

1) A has real eigenvalues
2) all of A's eigenvectors are orthogonal

Assume $A \in \mathbb{C}^{N \times N}$ and Hermetian $(A = A^*)$

Prove (1) $Av = \lambda v$

$v^T A v = \lambda v^T v$ left multiply by v^T

$v^* A v = \lambda v^* v$ (or v^* b/c v could have complex #s)

$$\lambda = \frac{v^* A v}{v^* v}$$

(& transpose)

* try to conjugate both sides → if it's the same, λ is real! *

$$\lambda^* = \left(\frac{v^* A v}{v^* v}\right)^*$$

☆ same Transpose rules apply to * ☆ $(DC)^T = C^T D^T$

$$\lambda^* = \frac{v^* A^* v^{**}}{v^* v^{**}} = \frac{v^* A^* v}{v^* v}$$

b/c $A^* = A$ (A is Hermetian),

$$\lambda^* = \frac{v^* A v}{v^* v}$$

So, λ is real! □

Prove (2) $Av = \lambda v$ Assume ① $\lambda \neq \xi$, ② $v^* x \neq 0$ ③ $|x| = 1$, $|v| = 1$

$Ax = \xi x$ so $v \neq x$ b/c e-vec are LI

$A(v - x) = \lambda v - \xi x$

$(v-x)^* A (v-x) = (v-x)^* (\lambda v - \xi x)$ b/c normalized assumption

$\underbrace{v^* A v}_{\lambda} - v^* A x - x^* A v + \underbrace{x^* A x}_{+\xi} = \lambda v^* v - \lambda x^* v - \xi v^* x + \xi x^* x$

$= \lambda$ $+ \xi$

$-v^* A x - x^* A v = -\lambda x^* v - \xi v^* x$

change A to A^* $\quad -v^* A^* x - x^* A^* v = -\lambda x^* v - \xi v^* x$

$+\lambda v^* x + \xi x^* v = +\lambda x^* v + \xi v^* x$

since $(Av)^* = (\lambda v)^*$, $v^* A^* = \lambda v^*$ (same for 2nd eqn)

Since $\lambda \neq \xi$, $v^* x = 0$. Contradiction □

Scanned with CamScanner

Krylov Subspace:

"find the best soln of problem in a reduced subspace & keep building the space up"

- Lanczos Algorithm: eigenvalue
- GMRES: $Ax=b$
- Conjugate Gradient