

# NON-LINEAR LEAST SQUARES

## Pseudocode

① function perform-Nonlinear-Least-Squares

• initialize:

[1 0.5 0.3]

-  $\vec{\beta}^P$  = initial guess (3x1 column vector)

- error,  $\vec{y}$  - N (# iterations)

- tolerance (1e-5)

• read in data  $\{(x_j, y_j)\}_{j=1}^N$

- create xVec & yVec

while err > tol

2 different functions

→ Form "Jacobian-like" matrix,  $J$ , for  $\vec{\beta}^P$  and data (function ②)

→ Form  $\Delta \vec{y}$  for  $\vec{\beta}^P$  and data (function ②)

→ form pseudo-inverse and get:  
$$\Delta \vec{\beta}^K = (J^T J)^{-1} J^T \Delta \vec{y}^K$$

$\vec{\beta}^N = \vec{\beta}^P + \Delta \vec{\beta}^K$  % New approx

err =  $\|\vec{\beta}^N - \vec{\beta}^P\|_2$  % compute error (L<sup>2</sup>-Norm)

$\vec{\beta}^P = \vec{\beta}^N$  % redefine for next iter

N = N + 1 % update # of iters

end



② function J=compute\_Jacobian

• input ( $\vec{\beta}^p, xVec$ )

• output J

for i=1:length(xVec) % iterate thru data pts

$$J(i, 1) = f_{\beta_1}(xVec(i), \vec{\beta}^k)$$

% partial of f wRT  $\beta_1$

$$J(i, 2) = f_{\beta_2}(xVec(i), \vec{\beta}^k)$$

% partial of f wRT  $\beta_2$

$$J(i, 3) = f_{\beta_3}(xVec(i), \vec{\beta}^k)$$

% partial of f wRT  $\beta_3$

end

computing partials:

$$f(x; \vec{\beta}) = \beta_1 e^{\frac{-(x-\beta_2)^2}{2\beta_3^2}}$$

$$\frac{\partial f}{\partial \beta_1}(x; \vec{\beta}) = e^{\frac{-(x-\beta_2)^2}{2\beta_3^2}}$$

$$\frac{\partial f}{\partial \beta_2}(x; \vec{\beta}) = \frac{\beta_1(x-\beta_2)}{\beta_3^2} e^{\frac{-(x-\beta_2)^2}{2\beta_3^2}}$$

$$\frac{\partial f}{\partial \beta_3}(x; \vec{\beta}) = \frac{\beta_1(x-\beta_2)^2}{\beta_3^3} e^{\frac{-(x-\beta_2)^2}{2\beta_3^2}}$$



③ function <sup>dely =</sup> compute\_dely(i)  
• input ( $\vec{B}^P$ , xVec, yVec)  
• output dely

for i = 1 : length(yVec)  
dely(i) = yVec(i) - f(xVec(i);  $\vec{B}^P$ )  
% where  $f = B_1 e^{-\frac{(x - B_2)^2}{2B_3^2}}$   
end