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CS325 – HW7

1. **Only d and g can be inferred** from the below statements:

a. If Y is NP-complete then so is X.

No, we cannot infer that this statement is true because X can be in NP, but might not be NP-complete.

b. If X is NP-complete then so is Y.

No, we can also not necessarily infer this statement because Y might be in a class harder than NP.

c. If Y is NP-complete and X is NP then X is NP-complete.

No, this cannot be inferred because X could just be in NP.

d. If X is NP-complete and Y is NP then Y is NP-complete.

Yes, this can be inferred because X reduces to Y and if Y is also NP, then when if we are able to use a black box to solve Y, we can also use it to solve X as X is not in a harder class than Y.

e. X and Y can't both be NP-complete.

No, this cannot be inferred because both X and Y can be NP-complete.

f. If X is in P, then Y is in P.

No, we cannot infer that this is true because Y is at least as hard as X and it could be in NP.

g. If Y is in P, then X is in P.

Yes, this can be inferred because if Y is in P and X can be reduced to Y, then X also has to be in P.

2.

a. This statement does not follow because since SUBSET-SUM is NP-complete, then it follows that we can reduce it to another NP-complete problem. We know that COMPOSITE is in NP, but we do not know if it is NP-complete, therefore we do not know for sure that SUBSET-SUM can be reduced to COMPOSITE which is what this question states.

b. We cannot necessarily say that this statement is true. As stated above, we know that SUBSET-SUM is in NP-complete, but we do not know for sure that COMPOSITE is as well. If COMPOSITE were also NP-complete, then we would be able to definitively say that it could also be solved in polynomial time is SUBSET-SUM can, but we don't know that for certain.

c. Yes, we can infer that this is true because since SUBSET-SUM is in NP-complete, then we know that P=NP. Since COMPOSITE is in NP and every algorithm in NP would also be able to be solved in P time, then COMPOSITE would also be included.

d. No, this statement cannot be true because since P is a subset of the class NP, we can only show that NP-complete problems cannot be solved in polynomial time, not NP problems.

3.

a. This statement is true because we know that 3-SAT can be reduced to DIR-HAM-CYCLE to HAM-CYCLE to TSP which shows that 3-SAT is a subset of TSP, therefore this is true.

b. This statement is false because 2-SAT can be solved in polynomial time and 3-SAT is NP-complete, so it does not follow that P != NP because S-SAT is within P and since 3-SAT is in NP-complete, then even if they could be reduced to each other, P != NP would not hold true.

c. This is true because we know that if one NP-complete problem can be solved in polynomial time then we know that all NP-complete problems can be solved in polynomial time. Therefore, we can say that if p != NP, then no NP-complete problem can be solved in polynomial time.

4. A problem B is NP-complete if:

1. B $\in$ NP 2. X $\leq_P$ B for all X $\in$ NP

First we must show that HAM-PATH $\in$ NP. This is simple to do because we can easily see this is the case because we can verify in polynomial time that every vertex is in the graph is connected by an edge and that no vertex is repeated in the cycle.

Next, we can use HAM-CYCLE to show that HAM-CYCLE $\leq_P$ HAMPATH. We use HAM-CYCLE because it is NP-Complete and has a very similar structure to HAMPATH. If R = HAM-CYCLE and Q = HAMPATH, then we can prove that R(x) if any only if Q(x^1) = yes by the following:

If we have our graph Q of our HAMPATH, for each edge we {u, v} we can create a new graph Q^1 by deleting this edge and adding vertex x onto u and vertex y onto v. Then we can check to see whether Q^1 has a HAMPATH. If it does, then it must start at x to u and end with v to y or vice versa. Using this new graph, we will be able to see that the original HAMPATH has a HAM-CYCLE and therefore it can be reduced from HAM-CYCLE. And since we know that HAM-CYCLE is NP-complete, then we know that HAMPATH is also NP-complete.

5. This problem is somewhat similar to problem 4 above as LONG-PATH reduces to HAMPATH. We can first prove that LONG-PATH $\in$ NP because we can trivially determine in polynomial time that there is a simple path in G from u to v of length at least k.

Since we know that HAMPATH is NP-complete, all we need to do to prove LONG-PATH reduces to HAMPATH (and is therefore also NP-complete) is to create a new instance of G^1, k such that G^1=G and k = n – 1 where n is the number of vertices in G. Then we can see that there exists a simple path of length k in G^1 that is also a HAMPATH. Thus we can see now that HAMPATH $\leq_P$ LONG-PATH, and since we have already proved above that HAMPATH is NP-complete, then we can determine that LONG-PATH is also NP-complete.