If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges:

If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$ ← contrapositive

$\lim_{n\to\infty} S_n = L$ ↑ a real #

$\lim_{n\to\infty} S_{n-1} = L$ ↑ still technically going to ∞

$$S_n - S_{n-1} = a_n$$

$$\lim_{n\to\infty} \left(S_n - S_{n-1} \right) = \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} = \lim_{n\to\infty} a_n$$

$$L - L = \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} a_n = 0$$