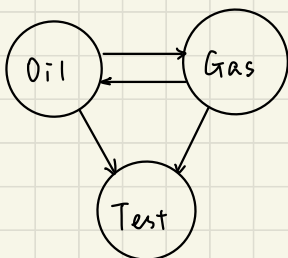


Q1. a)



$$Pr(Oil | Gas) = Pr(Oil | Gas, Test)$$

$$Pr(Gas | Oil) = Pr(Gas | Oil, Test)$$

$$Pr(Test | Oil, Gas)$$

$$b) Pr(Oil | Test=1) = \frac{Pr(Oil, Test)}{Pr(Test=1)} = \frac{Pr(Oil, Test=1)}{\sum_{oil} \sum_{gas} Pr(Test=1 | oil, gas) \cdot Pr(oil, gas)}$$

$$Pr(Oil, Test=1) = Pr(Test=1 | Oil) Pr(Oil) = 0.9 \times 0.5 = 0.45$$

$$\sum_{oil} \sum_{gas} Pr(Test=1 | oil, gas)$$

$$= Pr(Test=1 | oil=1, Gas=0) \cdot Pr(Oil) + Pr(Test=1 | Oil=0, Gas=1) \cdot Pr(Gas)$$

$$+ Pr(Test=1 | Oil=0, Gas=0) \cdot Pr(Oil=0, Gas=0)$$

$$= 0.5 \times 0.9 + 0.2 \times 0.3 + 0.3 \times 0.1$$

$$= 0.45 + 0.06 + 0.03$$

$$= 0.54$$

$$Pr(Oil | Test=1) = \frac{0.45}{0.54} = 0.833$$

Q2. a) $\Pr(A, B, C, D, E, F, G, H)$

$$= \Pr(A) \cdot \Pr(B) \cdot \Pr(C|A) \cdot \Pr(D|A, B) \cdot \Pr(E|B) \cdot \Pr(F|C, D) \cdot \Pr(G|F) \cdot \Pr(H|E, F)$$

c) $\Pr(a, \neg b, c, d, \neg e, f, \neg g, h)$

$$= \Pr(A) \cdot \Pr(\neg B) \cdot \Pr(C|A) \cdot \Pr(D|A, \neg B) \cdot \Pr(\neg E|\neg B) \cdot \Pr(F|C, D) \cdot \Pr(\neg G|F) \cdot \Pr(H|\neg E, F)$$

$$= 0.2 \times 0.3 \times \Pr(C|A) \times 0.6 \times 0.1 \times \Pr(F|C, D) \cdot \Pr(G|F) \cdot \Pr(H|\neg E, F)$$

$$= 0.0036 \times \Pr(C|A) \cdot \Pr(F|C, D) \cdot \Pr(G|F) \cdot \Pr(H|\neg E, F)$$

d) $\Pr(\neg a, b) = \Pr(A=0) \cdot \Pr(B=1) = 0.8 \times 0.7 = 0.56$

$$\Pr(\neg e|a) = \Pr(E=0) = \sum_b \Pr(E=0|b) \Pr(b)$$

$$= \Pr(E=0|B=0) \Pr(B=0) + \Pr(E=0|B=1) \Pr(B=1)$$

$$= 0.1 \times 0.3 + 0.9 \times 0.7$$

$$= 0.03 + 0.63$$

$$= 0.66$$

e) A is conditionally independent of all nodes

B is conditionally independent of all nodes

C is conditionally independent of all other nodes given A

D

...

A, B

E

...

B

F

...

C, D

G

...

F

H

...

E, F

f) A, B, C, F

g)	A	B	D	E	$\Pr(D AB) \cdot \Pr(E B)$
	0	0	0	0	$0.2 \times 0.1 = 0.02$
	0	0	0	1	$0.2 \times 0.9 = 0.18$
	0	0	1	0	$0.8 \times 0.1 = 0.08$
	0	0	1	1	$0.8 \times 0.9 = 0.72$
	0	1	0	0	$0.9 \times 0.9 = 0.81$
	0	1	0	1	$0.9 \times 0.1 = 0.09$
	0	1	1	0	$0.1 \times 0.9 = 0.09$
	0	1	1	1	$0.1 \times 0.1 = 0.01$
	1	0	0	0	$0.4 \times 0.1 = 0.04$
	1	0	0	1	$0.4 \times 0.9 = 0.36$
	1	0	1	0	$0.6 \times 0.1 = 0.06$
	1	0	1	1	$0.6 \times 0.9 = 0.54$
	1	1	0	0	$0.5 \times 0.9 = 0.45$
	1	1	0	1	$0.5 \times 0.1 = 0.05$
	1	1	1	0	$0.5 \times 0.9 = 0.45$
	1	1	1	1	$0.5 \times 0.1 = 0.05$