Assignment 1 TIM 206, Optimization Theory and Applications

Instructor: John Musacchio

Due April 13, 2017

1. A food processing factory has one oven, one mixer, and one cooling rack. The factory has to decide how many units of bread, cake, and muffins to make. The factory runs for 300 hours a month. The oven can accommodate 10 units at a time, the mixer 5 units at a a time, and the cooling rack 20 units. Thus the capacity of the oven, mixer, and cooling rack is 3000, 1500, and 6000 unit-hours over the month of interest respectively. Bread takes 2 hour of mixing, 1 hours of oven time, and 3 hours of cooling. Cake takes 0.5 hour of mixing, 1.5 hours of oven time, and 4 hours of cooling. Muffins take 1 hour of mixing, 2 hours of oven time, and 5 hours of cooling. The profit for bread is $10 a unit, for cake is $12 a unit, and muffins $30 a unit. Assume that the units of cake, muffins, and bread are infinitely divisible, and ignore the detailed sequencing of the steps to make each product – just focus on the capacity constraints of each of the three machines.

   (a) Formulate the profit maximization of the factory as a linear program.

   (b) Use Matlab function linprog to solve the problem. Your solution should specify the amount of each product to make, and also the resulting profit.

   On using Matlab: Type "help linprog" at the Matlab command prompt to learn how to use the function. You will express your constraints in the form $Ax \leq b$. For example to make a matrix

   $$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix},$$

   type "$A = [1, 2; 2, 1]$". The "," separates columns, and the ";" separates rows. To make a vector

   $$b = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

   type $b = [1; 2]$. Or you can type $b = [1, 2]'$ where the " ' " signifies that you are transposing a row vector to make a column vector. If your copy of Matlab does not have the optimization toolbox, (with linprog) you can log into an instructional machine to try this. For any argument of the linprog function that you want to not specify, simply use the expression "[ ]". For example to solve

   $$\min f^T x$$

   $$\text{subject to } Ax \leq b, x \geq 0$$

   where x is a 2 dimensional column vector, use the command "linprog(f,A,b,[],[], [0;0],[Inf,Inf]) ".

2. An online ad-network can place ads of types I, II, and III. There are three types of users (A,B,C), which the network can determine by their browsing behavior. Type A users can be shown ads of type II, and III. Type B users can be shown ads of types I and II. Type C users can be shown ads of types I and III. There will be 7 million, 8 million, and 12 million visits of user types A, B, and C respectively over the next week. At most, 1 ad is allowed to be shown per visit. The ad network gets paid 5 cents per showing (impression) of ad type I, and 10 and 7 cents for ad types II and III respectively. The ad-network is allowed to show ads at most 10, 15, and 5 million times for ad types I, II, III respectively. This is because the advertisers who have signed a contract with the ad-network have agreed to limited advertising budgets. The data is summarized in the table below.

		Ad Type			
		I	II	III	number
	A		✓	✓	7 million
User Type	B	✓	✓		8 million
	C	✓		✓	12 million
max ads		10 million	15 million	5 million	
value		5 cents	10 cents	7 cents	

   (a) Formulate the problem as an LP to find the number of ads to show each user type to maximize revenue. Your constraints may be left in the form $Ax \leq b, x \geq 0$. Keep your units units in millions and cents to avoid carrying around unnecessary zeros and decimals.

   (b) Solve the LP in Matlab, and report your solution.