Assignment 5

By

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Problem 1

(1) $\binom{n}{k}$ $\quad \frac{n!}{k!(n-k)!}$

$\alpha: \binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5\cdot 4}{2} = \frac{20}{2} = 10$

$\beta: \binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7\cdot 6\cdot 5}{6} = 7\cdot 5 = 35$

$\alpha\cdot\beta =$

$= 10\cdot 35 = 350$

With 2 feuding

$\underset{\alpha}{\binom{2}{2}}\underset{\beta}{\binom{7-2}{3-2}}$ = 5 fewer possible constitutions

$\binom{5}{2}\left(\binom{7}{3}-5\right) = 10\cdot(35-5) = 300$

Problem 2

(2)

a) possible rolls: with value larger than 8 (3 rolls)

(4,4,4) (4,4,4) (4,4,4) (4,4,3) (4,3,4) (3,4,4) (4,4,2)
(4,2,4) (2,4,4) (1,4,4) (4,1,4) (4,4,1) (3,3,3) (3,3,3) (3,3,3)
(2,3,4) (2,4,3) (3,2,4) (4,2,3) (4,3,2)

20 possible outcomes

Each roll has a $\frac{1}{4}$ to get the correct number.

3 rolls $\times \frac{1}{4} \rightarrow \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

answer:

$\frac{20}{64} / 4 = \frac{5}{16}$ ($\sim 31{,}3\%$) chans to get the the sum >8 with 3 rolls

b) $\binom{52}{2} = \frac{52!}{2!(52-2)!} = \frac{52 \cdot 51}{2} = \frac{2652}{2} = 1326$

$\binom{4}{2} = \frac{4!}{2!(4-2)} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2! \cdot 2} = \frac{4 \cdot 3 \cdot 1}{2} = \frac{12}{2} = 6$

$\frac{6}{1326} / 6 = \frac{1}{221}$

c) $p(b) = \frac{52-4}{52} \cdot \frac{51-4}{51} \cdot \frac{50-4}{50}$

$p(b) = \frac{4324}{5525}$

$p(a) = 1 - p(b) = \frac{1201}{5525}$

$p(a) = 0{,}22$

The change is $0{,}22 = 22\%$

Problem 3

B: women with breast cancer

A: women with positive mammogram

2% of of women have breast cancer

$P(B) = 0,02$

$P(\neg B) = 0,98$

90% of those women have a positive mammogram,
this gives $P(A|B) = 0,90$

8% don't have breast cancer, but positive mammogram

$$P(A|\neg B) = 0,08$$

Bayers Theorem

$$P(B|A) = \frac{P(D|A)\ P(H)}{P(A)} = \frac{P(A|B)\,P(B)}{P(A|B)\,P(A) + P(A|\neg B)\,P(\neg B)} =$$

$$= \frac{0,90 \cdot 0,02}{0,90 \cdot 0,02 + 0,08 \cdot 0,098} =$$

$$= 0,186721 \approx 18,67\%$$

Answer: The propability of women who has breast cancer give that they have a posetive mammogram

$$P(B|A) \approx 18,67\%$$

Problem 4

a) When $k=1$, there is only one neighbour to concider.

The propability P_1 is 0,15. $P = 0,15$

The propability for missclassification if $k=1$ are 0,15

Applying The binomial distribution law:

$$P(X=x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

$P(\alpha)$: k nearest neigbbour missclassification of x

$$p(\alpha) = \binom{1}{1} 0{,}15^1 \cdot 0{,}85^0 \Rightarrow$$

$$= 1 \cdot 0{,}15 \cdot 1 =$$

$$= \underline{0{,}15}$$

b)

$P(\alpha)$ = k Nearest Neighbour missclassification 2 of the 3 existing clusters neighbour to x

$P(\beta)$ = k Nearest Neighbour missclassification 3 of the 3 existing cluster neighbour to x

$\underbrace{P(\alpha \cup \beta)}$ = k Neares Neighbour missclassification of x

This gives $P(\alpha \cup \beta) = P(\alpha) + P(\beta)$

Apply Binomial distrubution law:

$$p(X=x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

$P(\alpha)$:

$$\binom{3}{2} 0,15^2 \cdot 0,85^{(3-2)} =$$

$$= \binom{3}{2} 0,15^2 \cdot 0,85^1 = 0,05737$$

$P(\beta)$:

$$\binom{3}{3} 0,15^3 \cdot 0,85^{(3-3)} =$$

$$= \binom{3}{3} 0,15^3 \cdot 0,85^0 = 0,00337$$

$$P(\alpha \cup \beta) = P(\alpha) + P(\beta) =$$

$$= \underline{\underline{0,06074}}$$