! For $f_x(x)$ or $F_x(x)$, use the format :

$$F_x(x) = \begin{cases} 0, & x \leq \dots ; \\ \dots, & 0 < x < \dots ; \\ 1, & x \geq \dots . \end{cases}$$

ST2334 - Tutorial 4

Week 6

1

cents

x	3	7	11
$f_x(x)$	$\frac{\binom{4}{3}\times\binom{2}{0}}{\binom{6}{3}} = 0.2$	$\frac{\binom{4}{2}\times\binom{2}{1}}{\binom{6}{3}} = 0.6$	$\frac{\binom{4}{1}\times\binom{2}{2}}{\binom{6}{3}} = 0.2$

2 (a). 3H 0T : 3

2H 1T : 1

1H 2T : -1

0H 3T : -3

(b).

x	3	1	-1	-3
$f_W(x)$	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$	$\left(\frac{1}{2}\right)^3 \times \binom{3}{2} = \frac{3}{8}$	$\left(\frac{1}{2}\right)^3 \times \binom{3}{1} = \frac{3}{8}$	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

(c). Let $a = Pr(T)$.

$Pr(H) = 2Pr(T) = 2a$

$Pr(H) + Pr(T) = 1 \Rightarrow 3a = 1 \Rightarrow a = \frac{1}{3}$

$\therefore Pr(H) = \frac{2}{3}, \; Pr(T) = \frac{1}{3}$

x	3	1	-1	-3
$f_W(x)$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$	$\left(\frac{2}{3}\right)^2 \times \frac{1}{3} \times \binom{3}{2} = \frac{4}{9}$	$\frac{2}{3} \times \left(\frac{1}{3}\right)^2 \times \binom{3}{1} = \frac{2}{9}$	$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

3. $\sum_{i=\cancel{1}\,0}^{\infty} f_X(x_i) = 1$

$\sum_{i=\cancel{1}\,0}^{3} f_X(x_i) + \cancelto{0}{\sum_{i=4}^{\infty} f_X(x_i)} = 1$

$c(0^2+4) + c(1^2+4) + c(2^2+4) + c(3^2+4) = 1$

$30\,\cancel{26}\,c = 1$

$c = \frac{1}{26}$ ✗ $\frac{1}{30}$

4(a). $\sum_{y=1}^{\infty} f_Y(y) = 1$

$\sum_{y=1}^{5} f_Y(y) + \cancelto{0}{\sum_{y=4}^{\infty} f_Y(y)} = 1$

$\sum_{y=1}^{5} ky = 1$

$15k = 1$

$k = \frac{1}{15}$

(b). $Pr(Y = c) = \frac{c}{15}$

$Pr(Y \leq 3) = Pr(Y=1) + Pr(Y=2) + Pr(Y=3)$

$= \frac{6}{15}$

$= \frac{2}{5}$

(c). $Pr(2 \leq Y \leq 4) = Pr(Y=2) + Pr(Y=3) + Pr(Y=4)$

$= \frac{9}{15}$

$= \frac{3}{5}$

(d). $F(y) = \sum_{t \leq y} f_Y(t) = \sum_{t \leq y} Pr(Y = t)$

$$F_Y(y) = \begin{cases} 0, & y < 1; \\ \frac{1}{15}, & 1 \leq y < 2; \\ \frac{3}{15}, & 2 \leq y < 3; \\ \frac{6}{15}, & 3 \leq y < 4; \\ \frac{10}{15}, & 4 \leq y < 5; \\ 1, & 5 \leq y. \end{cases}$$

5(a).

x	1	3	4	6	12
$f_X(x)$	0.3	0.1	0.05	0.15	0.4

(b). $Pr(3 \leq X \leq 6) = 0.1 + 0.05 + 0.15$

$= 0.3$

$Pr(4 \leq X) = 0.05 + 0.15 + 0.4$

$= 0.6$

6(a). $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

$\cancel{\int_{-\infty}^{0} 0\, dx}^{\,0} + \int_0^1 k\sqrt{x}\, dx + \cancel{\int_1^{\infty} 0\, dx}^{\,0} = 1$

$\left[\frac{2}{3} k x^{\frac{3}{2}}\right]_0^1 = 1$

$k = \frac{3}{2}$

Assume $0 < x < 1$

(b). $F_X(x) = \int_{-\infty}^{x} f(t)\, dt$

$= \int_0^x \frac{3}{2}\sqrt{t}\, dt$

$= \left[t^{\frac{3}{2}}\right]_0^x$

$= x^{\frac{3}{2}}$

Incomplete!

$$F_X(x) = \begin{cases} 0, & x \leq 0; \\ x^{\frac{3}{2}}, & 0 < x < 1; \\ 1, & x \geq 1. \end{cases}$$

$Pr(0.3 < X < 0.6) = Pr(0.3 < X \leq 0.6)$

$= F_X(0.6) - F_X(0.3)$

$= 0.3004$

7(a). $\int \frac{3}{4}(1-x^2)\,dx = \frac{3}{4}x - \frac{1}{4}x^3 + C$

$Pr(-\frac{1}{2} < X < \frac{1}{2}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{3}{4}(1-x^2)\,dx$

$= \left[\frac{3}{4}x - \frac{1}{4}x^3\right]_{-\frac{1}{2}}^{\frac{1}{2}}$

$= \frac{11}{16}$

(b). $Pr(X < -\frac{1}{4} \text{ or } X > \frac{1}{4}) = Pr(X < -\frac{1}{4}) + Pr(X > \frac{1}{4})$

$= \int_{-1}^{-\frac{1}{4}} \frac{3}{4}(1-x^2)\,dx + \int_{\frac{1}{4}}^{1} \frac{3}{4}(1-x^2)\,dx$

$= \left[\frac{3}{4}x - \frac{1}{4}x^3\right]_{-1}^{-\frac{1}{4}} + \left[\frac{3}{4}x - \frac{1}{4}x^3\right]_{\frac{1}{4}}^{1}$

$= \frac{81}{256} + \frac{81}{256}$

$= \frac{81}{128}$

(c). $F_X(x) = \int_{-1}^{x} \frac{3}{4}(1-t^2)\,dt$

$= \left[\frac{3}{4}t - \frac{1}{4}t^3\right]_{-1}^{x}$

$= \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2}$

Incomplete!

$$F_X(x) = \begin{cases} 0, & x < -1; \\ \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2}, & -1 \le x \le 1; \\ 1, & x > 1. \end{cases}$$

8(a). $Pr(X < 12) = F_X(12)$

$= 1 - e^{-8(\frac{12}{60})}$

$= 0.7981$

(b). $f_X(x) = \frac{d\,F_X(x)}{dx}$

$= 8e^{-8x}$

Incomplete!

$$f_X(x) = \begin{cases} 8e^{-8x}, & x \geq 0; \\ 0, & x < 0. \end{cases}$$