Q1.
Q5.
Q12.

# ST2334 – Tutorial 10 Week 12

1. $T = \dfrac{(\bar{x} - \mu)}{S/\sqrt{n}} = \dfrac{\bar{x} - 20}{4.1/\sqrt{9}} \sim t(8)$

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2 (a). By C.L.T., $\mu_{\bar{X}_B - \bar{X}_A} = \mu_B - \mu_A = 0$

$$\sigma_{\bar{X}_B - \bar{X}_A} = \sqrt{\sigma_B^2/36 + \sigma_A^2/36}$$
$$= \sqrt{1^2/36 + 1^2/36}$$
$$= \sqrt{\tfrac{1}{18}} \quad 0.2357^2$$

$\therefore \bar{X}_B - \bar{X}_A \sim N\left(0, \tfrac{1}{18}\right)$

$Pr(\bar{X}_B - \bar{X}_A \geq 0.2) = 0.198$

(b). No.

3 (a). $\dfrac{(n-1)S^2}{\sigma^2} = \dfrac{24S^2}{6} = 4S^2 \sim \chi^2(24)$

$Pr(S^2 > 9.1) = Pr(4S^2 > 36.4)$

$= 0.05$

(b). $Pr(3.462 < S^2 < 10.745) = Pr(13.848 < 4S^2 < 42.98)$

$= 0.99 - 0.05 = 0.94$

4. $\dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \dfrac{S_1^2}{S_2^2} \sim F(7, 11)$

$Pr(S_1^2/S_2^2 < 4.89) = 0.990$

5.

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6(a). $E(u) = E(X/n) = E(X)/n = np/n = p$

(b). $E(v) = E\left(\frac{X + n/2}{3n/2}\right) = \frac{E(X)}{3n/2} + \frac{1}{3} = \frac{np}{3n/2} + \frac{1}{3} = \frac{2p}{3} + \frac{1}{3} \neq p$

7(a). $\sigma = 0.75, \quad \alpha = 0.05, \quad n = 20, \quad \bar{x} = 4.85$

$$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$4.85 - 1.960\left(\frac{0.75}{\sqrt{20}}\right) < \mu < 4.85 + 1.960\left(\frac{0.75}{\sqrt{20}}\right)$$

$$4.521 < \mu < 5.179$$

(b). $$z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 0.20$$

$$1.960\left(\frac{0.75}{\sqrt{n}}\right) = 0.20$$

$$n = 54.0225$$

(c). $T = \frac{(\bar{x} - \mu)}{s/\sqrt{n}} \sim t(19)$

$$\bar{x} - t_{n-1;\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + t_{n-1;\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$4.85 - 2.093\left(\frac{0.75}{\sqrt{20}}\right) < \mu < 4.85 + 2.093\left(\frac{0.75}{\sqrt{20}}\right)$$

$$4.498 < \mu < 5.202$$

8(a). $\sigma = 0.0015, \quad n = 75, \quad \bar{x} = 0.310, \quad \alpha = 0.05$

$$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$0.310 - 1.960\left(\frac{0.0015}{\sqrt{75}}\right) < \mu < 0.310 + 1.960\left(\frac{0.0015}{\sqrt{75}}\right)$$

$$0.30966 < \mu < 0.31033$$

(b). $$n \geq \left(z_{\alpha/2}\frac{\sigma}{e}\right)^2$$

$$= \left(1.960\left(\frac{0.0015}{0.0005}\right)\right)^2$$

$$= 34.5744$$

$$\therefore n = 35$$

9. $n=12$, $\bar{x} = 48.50$, $S = 1.5$, $\alpha = 0.1$

$$\bar{x} - t_{n-1;\alpha/2}\left(\frac{S}{\sqrt{n}}\right) < \mu < \bar{x} + t_{n-1;\alpha/2}\left(\frac{S}{\sqrt{n}}\right)$$

$$48.50 - 1.796\left(\frac{1.5}{\sqrt{12}}\right) < \mu < 48.50 + 1.796\left(\frac{1.5}{\sqrt{12}}\right)$$

$$47.722 < \mu < 49.278$$

10. $n_1 = 25$, $\sigma_1 = 5$, $\bar{x}_1 = 80$
$n_2 = 36$, $\sigma_2 = 3$, $\bar{x}_2 = 75$ $\Big\}\ \alpha = 0.06$

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu < \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$80 - 75 - 1.881\sqrt{\frac{5^2}{25} + \frac{3^2}{36}} < \mu < 80 - 75 + 1.881\sqrt{\frac{5^2}{25} + \frac{3^2}{36}}$$

$$2.897 < \mu < 7.103$$

11. $n_1 = 100$, $\bar{x}_1 = 12.2$, $S_1 = 1.1$,
$n_2 = 200$, $\bar{x}_2 = 9.1$, $S_2 = 0.9$ $\Big\}\ 0.02$

By C.L.T., CI:

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} < \mu < \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$12.2 - 9.1 - 2.326\sqrt{\frac{1.1^2}{100} + \frac{0.9^2}{200}} < \mu < 12.2 - 9.1 + 2.326\sqrt{\frac{1.1^2}{100} + \frac{0.9^2}{200}}$$

$$2.804 < \mu < 3.396$$

$\mu = \mu_1 - \mu_2 > 0$, so ~~no~~ yes

12.

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