Q2 (c).

Q5

ST2334 - Tutorial 6

Week 8

1 (a).

x	1	2	3
$f_X(x)$	0.10	0.35	0.55

(b).

y	1	2	3
$f_Y(y)$	0.20	0.50	0.30

(c).

$$\Pr(Y=3 \mid X=2) = f_{Y|X}(3|2) = \frac{f_{X,Y}(2,3)}{f_X(2)} = \frac{0.2}{0.35} = \frac{4}{7}$$

(d).

y	ecf. 1	2	3
$f_{Y\|X}(y\|\cancel{3}\ 2)$	$\frac{0.1}{0.55} = \frac{2}{11}$ $\frac{1}{7}$	$\frac{0.35}{0.55} = \frac{7}{11}$ $\frac{2}{7}$	$\frac{0.1}{0.55} = \frac{2}{11}$ $\frac{4}{7}$

$$\frac{f_{X,Y}(\cancel{3}\ 2,y)}{f_X(3)} = \frac{f_{X,Y}(\cancel{3}\ 2,y)}{0.55}$$

(e).

$f_{X,Y}(x,y)$		x = 1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.10	0.35
	3	0	0.2	0.1

$f_X(x) f_Y(y)$		x = 1	2	3
y	1	0.02	0.07	0.11
	2	0.05	0.175	0.275
	3	0.03	0.105	0.165

$\sim \left(f_{X,Y}(x,y) = f_X(x) f_Y(y)\right) \ \forall\, x,y \longrightarrow X$ and Y are dependent.

2(a).

$f_{X,Y}(x,y)$		x = 1	2	3
y	1	0	0	$\frac{{}^3C_3 \times {}^2C_1}{{}^8C_4} = \frac{1}{35}$
	2	0	$\frac{{}^3C_2 \times {}^2C_2}{{}^8C_4} = \frac{3}{70}$	0

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, & x = 0,1,2,3;\ y = 0,1,2;\ 1 \le x+y \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Subsequent sol$^{\underline{n}}$s use

(b).

$$Pr(X=1, Y=1) = \frac{\binom{3}{1}\binom{2}{1}\binom{3}{2}}{\binom{8}{4}}$$

$$= \frac{9}{35}$$

(c).

$$Pr(X+Y \le 2) = Pr(X+Y=1) + Pr(X+Y=2)$$

$$= Pr(X=0, Y=2) + Pr(X=1, Y=1) + Pr(X=2, Y=0)$$

$$+ Pr(X=0, Y=1) + Pr(X=1, Y=0)$$

$$= \frac{3}{70} + \frac{9}{35} + \frac{9}{70} + \frac{1}{35} + \frac{3}{70}$$

$$= \frac{22}{35} \quad 0.5$$

(d).

$$f_X(x) = \begin{cases} \dfrac{\binom{3}{x}\binom{5}{4-x}}{\binom{8}{4}} & x = 0,1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

(e).

$$f_{Y|X}(y|2) = \frac{f_{X,Y}(2,y)}{f_X(2)}$$

$$= \frac{\cancel{\binom{3}{2}}\binom{2}{y}\binom{3}{4-2-y}}{\cancel{\binom{8}{4}}\,1} \times \frac{\cancel{\binom{8}{4}}\,1}{\cancel{\binom{3}{2}}\binom{5}{4-2}}$$

$$= \begin{cases} \frac{1}{10}\binom{2}{y}\binom{3}{2-y} & y = 0,1,2; \\ 0 & \text{otherwise} \end{cases}$$

$$Pr(Y=0 \mid X=2) = \frac{1}{10}\binom{2}{0}\binom{3}{2-0}$$

$$= \frac{3}{10}$$

3 (a).

$\Pr(X=x, Y=y)$		$x = 0$	$x = 1$	$x = 2$
y	0	$\frac{4}{9}$	$2(\frac{1}{6})(\frac{4}{6}) = \frac{2}{9}$	$(\frac{1}{6})^2 = \frac{1}{36}$
	1	$2(\frac{1}{6})(\frac{4}{6}) = \frac{2}{9}$	$2(\frac{1}{6})^2 = \frac{1}{18}$	0
	2	$(\frac{1}{6})^2 = \frac{1}{36}$	0	0

(b). $\Pr(2X+Y < 3) = \Pr(X=0, Y=0) + \Pr(X=1, Y=0)$

$+ \Pr(X=0, Y=1) + \Pr(X=0, Y=2)$

$= \frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{36}$

$= \frac{11}{12}$

$(\frac{1}{6})^2$ each

(c). $\Pr(X=2, Y=2) = 0 \neq \Pr(X=2)\Pr(Y=2) = \frac{1}{1296}$

$\therefore$ X and Y are dependent

4(a). $\iint_{(x,y)\in R_{X,Y}} f_{X,Y}(x,y)\,dx\,dy = 1$

$$\int_3^5\int_3^5 k(x^2+y^2)\,dx\,dy = k\int_3^5 \left[\frac{x^3}{3} + xy^2\right]_{x=3}^{x=5} dy$$
$$= k\int_3^5 \frac{98}{3} + 2y^2 \, dy$$
$$= k\left[\frac{98}{3}y + \frac{2y^3}{3}\right]_{y=3}^{y=5}$$
$$= \frac{392}{3}k = 1$$
$$\therefore \quad k = \frac{3}{392}$$

(b). $$f_{X,Y}(x,y) = \begin{cases} \frac{3}{392}(x^2+y^2) & 3 \le x \le 5;\ 3 \le y \le 5; \\ 0 & \text{otherwise} \end{cases}$$

$$Pr(3 \le X \le 4 \text{ and } 4 \le Y < 5) = \int_4^5\int_3^4 \frac{3}{392}(x^2+y^2)\,dx\,dy$$
$$= \frac{3}{392}\int_4^5 \left[\frac{x^3}{3} + xy^2\right]_{x=3}^{x=4} dy$$
$$= \frac{3}{392}\int_4^5 \frac{37}{3} + y^2 \, dy$$
$$= \frac{3}{392}\left[\frac{37}{3}y + \frac{y^3}{3}\right]_{y=4}^{y=5}$$
$$= \frac{3}{392}\left[\frac{98}{3}\right]$$
$$= 0.25$$

(c). $$f_X(x) = \int_3^5 \frac{3}{392}(x^2+y^2)\,dy$$
$$= \frac{3}{392}\left[yx^2 + \frac{y^3}{3}\right]_{y=3}^{y=5}$$
$$= \frac{3}{392}\left(2x^2 + \frac{98}{3}\right)$$
$$= \begin{cases} \frac{3}{196}x^2 + \frac{1}{4} & 3 \le x \le 5; \\ 0 & \text{otherwise} \end{cases}$$

$$Pr(3.5 < X < 4) = \int_{3.5}^4 \frac{3}{196}x^2 + \frac{1}{4}\,dx$$
$$= \left[\frac{1}{196}x^3 + \frac{1}{4}x\right]_{3.5}^4$$
$$= \frac{365}{1568}$$
$$= 0.2328$$

5 (a). Creams: $f_x(x) = \int_0^1 24xy \; dy$ $\quad 12x(1-x)^2$?

$= \left[12xy^2 \right]_{y=0}^{y=1}$

$= \begin{cases} 12x & 0 \le x \le 1; \\ 0 & \text{otherwise} \end{cases}$

Toffees: $f_y(y) = \int_0^1 24xy \; dx$

$= \left[12x^2y \right]_{x=0}^{x=1}$ $\quad 12y(1-y)^2$

$= \begin{cases} 12y & 0 \le x \le 1; \\ 0 & \text{otherwise} \end{cases}$

(b). $f_{x,y}(x,y) = 24xy \neq f_x(x)\, f_y(y) = 144xy$

$\longrightarrow$ X and Y are independent

(c). ?