

CS2100 - L16 - Boolean Algebra

Week

9 + 10

16.1 - Basic boolean operations

16.2 - Basic laws

16.3 - Basic theorems

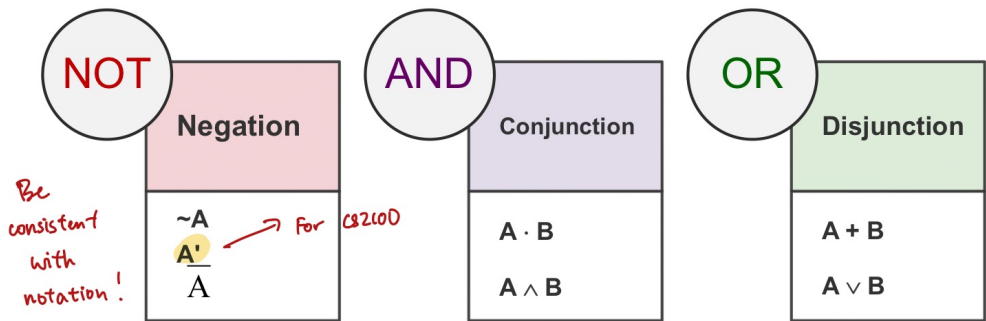
16.4 - Duality

16.5 - Boolean functions

- Complement of functions
- 2-variable boolean functions

16.1 - Basic boolean operations

Three Basic Boolean Operations



- Operator Precedence: **NOT > AND > OR**
- Example of *boolean expressions*:
 - $A + \sim B \cdot C = A + (\sim B) \cdot C$

Boolean Algebra Laws

Identity laws	
$A + 0 = 0 + A = A$	$A \cdot 1 = 1 \cdot A = A$
Inverse/complement laws	
$A + A' = 1$	$A \cdot A' = 0$
Commutative laws	
$A \cdot B = B \cdot A$	$A + B = B + A$
Associative laws	
$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Distributive laws	
$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$

16.3 - Basic theorems

Idempotency	
$X + X = X$	$X \cdot X = X$
Zero and One elements	
$X + 1 = 1$	$X \cdot 0 = 0$
Involution	
$(X')' = X$	
Absorption	
$X + X \cdot Y = X$	$X \cdot (X + Y) = X$
Absorption (variant)	
$X + X' \cdot Y = X + Y$	$X \cdot (X' + Y) = X \cdot Y$

DeMorgan's	
$(X + Y)' = X' \cdot Y'$	$(X \cdot Y)' = X' + Y'$
<p>*Can be generalized to more than two variables, e.g. $(A + B + \dots + Z)' = A' \cdot B' \cdot \dots \cdot Z'$</p>	
Consensus	
$X \cdot Y + X' \cdot Z + Y \cdot Z$ $= X \cdot Y + X' \cdot Z$	$(X+Y) \cdot (X'+Z) \cdot (Y+Z)$ $= (X+Y) \cdot (X'+Z)$

Duality

- If the AND/OR operators and identity elements 0/1 in a **Boolean equation** are interchanged, it remains valid.

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$



$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

- Two-for-one: If you prove one theorem, its dual form is also true!

□ e.g.

$$(x + y + z)' = x' \cdot y' \cdot z'$$



$$(x \cdot y \cdot z)' = x' + y' + z'$$

$$x + 0 = x$$



$$x \cdot 1 = x$$

16.5 - Boolean functions

Boolean Functions (Logic Equations)

- Function of the form $f = B^k \rightarrow B$
 - Input** (i.e. Domain): k number of Boolean variables
 - Output** (i.e. Range): Boolean value

$$F1(x,y,z) = x \cdot y \cdot z'$$

$$F2(x,y,z) = x + y' \cdot z$$

$$F3(x,y,z) = x' \cdot y' \cdot z + x' \cdot y \cdot z + x \cdot y'$$

$$F4(x,y,z) = x \cdot y' + x' \cdot z$$

x	y	z	F1	F2	F3	F4
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

From the truth table, $F3 = F4$

Try to prove $F3 = F4$ by using Boolean Algebra?

[L16 - AY2021S1]

Complement of Function

- F' , the **complement function** of F
 - Obtained by interchanging 1 with 0 in the function's output values

E.g.: $F1(x,y,z) = x \cdot y \cdot z'$

- What is $F1'$?

$$\begin{aligned} F1' &= (x \cdot y \cdot z')' \\ &= x' + y' + (z')' \quad (\text{DeMorgan's}) \\ &= x' + y' + z \quad (\text{Involution}) \end{aligned}$$

x	y	z	F1	F1'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

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