

Q26!

ST2334 - Tutorial 1

Week 3

$$1(a). S = \{3, 4, 5, 13, 14, 15, 23, 24, 25, 123, 124, 125, 213, 214, 215\}$$

$$(b). A = \{3, 4, 5\}$$

$$(c). B = \{5, 15, 25, 125, 215\}$$

$$(d). C = \{3, 4, 5, 23, 24, 25\}$$

$$(e). \text{No. } A \cap B = \{5\} \neq \phi$$

$$2. A = \{2, 4, 6, 8, 10\}, B = \{1, 3, 5, 7, 9\}, \\ C = \{2, 3, 4, 5\}, D = \{1, 6, 7\}$$

$$(a). A \cup C = \{2, 3, 4, 5, 6, 8, 10\}$$

$$(b). A \cap B = \phi$$

$$(c). C' = \{1, 6, 7, 8, 9, 10\}$$

$$(d). A \cap C \cap D' = \{2, 4\}$$

3. Digits: 0, 2, 4, 6, 8, 9

$$(a). \quad n(\text{three-digit numbers}) = 5 \times 5 \times 4 \\ = 100 \quad \checkmark$$

$$(b). \quad n(\text{odd numbers}) = 4 \times 4 \times 1 \\ = 16 \quad \checkmark$$

$$(c). \quad \begin{aligned} \text{Case 1: } 629 &: 1 \times 1 \times 1 = 1 \\ \text{Case 2: } > 62- &: 1 \times \cancel{4} \times 1 = 3 \quad \times 2 \\ \text{Case 3: } 8-9 &: 1 \times 4 \times 1 = 4 \\ \therefore \text{Total no.} &= 1 + \cancel{3} + 4 \\ &= 8 \quad \checkmark \end{aligned}$$

$$4(a). \quad n(\text{no restrictions}) = 8! \\ = 40320 \quad \checkmark$$

$$(b). \quad n(\text{couples together}) = 4! \times \cancel{2!} (2!)^4 \\ = 48 \quad \times \quad 384$$

$$(c). \quad n(\text{women left, men right}) = 4! \times 4! \\ = 576 \quad \checkmark$$

$$5(a). \quad n(\text{no restriction}) = \binom{7}{5} \\ = 21$$

$$(b). \quad n(\text{first two questions must be answered}) = \binom{5}{3} \\ = 10$$

$$(c). \quad \text{Case 1: exactly one of first two: } \binom{2}{1} \times \binom{5}{4} = 10$$

$$\text{Case 2: both: } 10 \quad (\text{from part (b)})$$

$$\therefore n(\text{at least one of first two must be answered}) = 10 + 10 \\ = 20$$

$$(d). \quad n(\text{exactly two from first three}) = \binom{3}{2} \times \binom{4}{3} \\ = 12$$

$$6(a). \quad n(\text{regardless}) = \frac{(13+8)!}{13! 8!}$$

13 W's, 8 R's \Rightarrow rearrange

$$? (b). \quad n(\text{pass Y}) = 16C_{10} \times 5C_3$$

$$n(\text{avoid Y}) = \text{total} - \uparrow$$

$$(c). \quad n(\text{ways}) = \underbrace{4C_2}_{A \rightarrow X} \times \left[\underbrace{17C_{11}}_{\substack{\text{total} \\ X \rightarrow B}} - \underbrace{12C_8}_{\substack{\text{pass Y} \\ X \rightarrow B}} \times 5C_3 \right]$$

$$7(a). n(\text{ways}) = 9 \times 21 \\ = 243$$

$$(b). n(\text{ways}) = 9 \times 27 \times 15 \\ = 3645$$

$$\therefore n(\text{years}) = 3645 \div 7 \div 52 \\ \approx 10$$

8. "w", "h", "i", "t", "e"

$$(a). n(\text{begins with consonant}) = {}^3P_1 \times 4! \\ = 72$$

$$(b). n(\text{ends with vowel}) = 4! \times {}^2P_1 \\ = 48$$

$$(c). n(\text{alt. vowels and consonants}) = 3! \times 2! \\ = 12$$

$$9. n(\text{ways}) = {}^9P_6 \times 6! \times 3! \\ = 362880$$

10. $n(\text{ways}) = \frac{9!}{3! 4! 2!}$

$= 1260$

11 (a). $A \cup B = A \rightarrow B \subset A$

(b). $A \cap B = A \rightarrow A \subset B$