Q2(a).
Q3(b).
Q11.

ST2334 – Tutorial 9

Week 11

1(a). $X \sim \exp(1)$

$E(X) = \frac{1}{1} = 1$

(b) $\sigma = \sqrt{\frac{1}{1^2}} = 1$

(c). $Pr(X \leq 4) = 0.98168$

$Pr(2 \leq X \leq 5) = 0.99326 - 0.86466$

$= 0.1286$

2(a). $X \sim \exp\left(\frac{1}{25000}\right)$

$Pr(X = 20000) =$

0.44493 $Pr(X \leq 30000) = 0.69881$

$Pr(20000 \leq X \leq 30000) = Pr(X \leq 30000) - Pr(X \leq 20000)$

$= 0.69881 - 0.55067$

$= 0.14814$

(b) $\sigma_X = 25000$

$Pr(X > 25000 + 2\sigma) = 0.04979$

3(a). $X \sim \exp\left(\frac{1}{2}\right)$

$V(X) = 2^2 = 4$

(b). Let Y be R.V. that a switch fails in the first year.

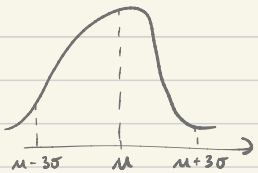
$Pr(X < 1) = 0.39347$

0.0335 $Y \sim B(100, 0.39347)$

$Pr(Y = 30) = 0.013030$

4

0.9973



5(a). $X \sim N(200, 15^2)$

$Pr(X > 224) = 1 - 0.945 = 0.055$

(b). $Pr(191 < X < 209) = 0.726 - 0.274$

$= 0.452$

(c). $Pr(X > 230) = 1 - 0.977 = 0.023$

Let Y be the no. of overflowed cups.

$Y \sim B(1000, 0.023)$

$E(Y) = 1000(0.023) = 23$

(d). $Pr(X < a) = 0.25$

$a = 189.883$

6(a). $X \sim N(24, 3.8^2)$

$Pr(X \geq 30) = 1 - 0.943$

$= 0.057$

(b). $Pr(X > 15) = 1 - 0.009$

$= 0.991$

(c). Let Y be the no. of trips > ½ h.

$Y \sim B(3, 0.057)$

$Pr(Y = 2) = 0.0091914$

7(a). Let X be the no. of heads.

$X \sim B(400, \frac{1}{2}) \overset{\text{approx.}}{\sim} N(200, 10^2)$

$Pr(185 \leq X \leq 210) \approx Pr(184.5 < X < 210.5)$

$= 0.853 - 0.061$

$= 0.792$

(b). $Pr(X = 205) \approx Pr(204.5 < X < 205.5)$

$= 0.709 - 0.674$

$= 0.035$

(c). $Pr(X < 176) + Pr(X > 229) \approx Pr(-\frac{1}{2} < X < 175.5) + Pr(229.5 < X < 400.5)$

$= (0.007 - 0.000) + (1.000 - 0.997)$

$= 0.01$

8 (a) $X \sim B(400, 0.1) \overset{\text{approx.}}{\sim} N(40, 6^2)$

$Pr(X < 32) \approx Pr(-\frac{1}{2} < X < 31.5)$

$= 0.078$

(b). $Pr(X > 49) \approx Pr(49.5 < X < 400.5)$

$= 1.000 - 0.943$

$= 0.057$

(c). $Pr(35 \leq X < 47) \approx Pr(34.5 < X < 46.5)$

$= 0.861 - 0.180$

$= 0.681$

2.179449^2

9(a). $X \sim B(100, 0.05) \overset{\text{approx.}}{\sim} N(5, 4.75)$

$Pr(X > 2) \approx Pr(2.5 < X < 100.5)$

$= 1.000 - 0.126 = 0.874$

(b). $Pr(X > 10) \approx Pr(10.5 < X < 100.5)$

$= 1.000 - 0.994 = 0.006$

10(a). $\mu_X = \sum x \, Pr(X = x) = 5.3$

$\sigma_X^2 = \sum (x - \mu_X)^2 Pr(X = x) = 0.81 = 0.9^2$

(b). $\mu_{\bar{X}} = \mu_X = 5.3$

$\sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{n} = \frac{0.81}{36} = 0.0225 = 0.15^2$

(c). $\bar{X} \overset{\text{approx}}{\sim} N(5.3, 0.15^2)$

$Pr(X < 5.5) = 0.909$

11(a). $\sigma_{\bar{X}} = \frac{100}{\sqrt{25}} = 20$

0.5 $\bar{X} \overset{\text{approx.}}{\sim} N(7960, 20^2)$

$Pr(\bar{X} > 7950) = 1 - 0.309$

$= 0.691$

(b).

0.3085

No