Q1 (d), (e), 8

Q2 (b), 6 (b)

ST2334 - Tutorial 3 Week 5

1 (a). $Pr(A \cap B \cap C) = Pr(A)\, Pr(B|A)\, Pr(C|A \cap B)$

$= 0.75 \times 0.9 \times 0.8$

$= 0.54$

(b). $Pr(B) = Pr(B \cap A) + Pr(B \cap A')$

$= Pr(A)\, Pr(B|A) + Pr(A')\, Pr(B|A')$

$= 0.75 \times 0.9 + (1-0.75) \times 0.8$

$= 0.875$

(c). $Pr(A|B) = \dfrac{Pr(A \cap B)}{Pr(B)}$

$= \dfrac{0.75 \times 0.9}{0.875}$

$= 0.7714$

$P(A')\, P(B|A')\, P(C|A' \cap B) = 0.25 \times 0.8 \times 0.7 = 0.14$

$P(A' \cap B)\, P(C|A' \cap B)$

(d). $P(B \cap C) = P((B \cap C) \cap A) + P((B \cap C) \cap A')$

$= 0.54 + 0.14$

Q1 (a).

$= 0.68$

(e). $P(A|B \cap C) = \dfrac{P(A \cap (B \cap C))}{P(B \cap C)}$

$= \dfrac{0.54}{0.68}$

$= 0.7941$

2. Let $A = \{A \text{ is profitable}\}$, $B = \{B \text{ is profitable}\}$.

(a). $$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$= \frac{0.05}{0.18}$$

$$= 0.2778$$

(b). $$Pr(A|A \cup B) = \frac{Pr(A \cap (A \cup B))}{Pr(A \cup B)}$$

$$= \frac{Pr((\overset{A}{A \cap A}) \cup (A \cap B))}{Pr(A) + Pr(B) - Pr(A \cap B)}$$ Careless!

$$= \frac{Pr(A \cap B) \; P(A)}{Pr(A) + Pr(B) - Pr(A \cap B)}$$

$$= \frac{\cancel{0.05} \; 0.18}{0.18 + 0.18 - 0.05}$$

$$= 0.1613 \quad 0.5806$$

3. Let $A = \{\text{implemented TQM}\}$,

$B = \{\text{increased in sales}\}$.

$Pr(A) = 0.3$, $Pr(B) = 0.6$, $Pr(A|B) = \frac{1}{3}$

(a). $Pr(A) = 0.3$, $Pr(B) = 0.6$

(b). $Pr(A) \times Pr(B) = 0.3 \times 0.6 = 0.18$

$Pr(A \cap B) = Pr(B)\, Pr(A|B)$

$= 0.6 \times \frac{1}{3}$

$= 0.2$

Since $Pr(A)\, Pr(B) \neq Pr(A \cap B)$, A and B are dependent.

(c). New $Pr(A|B) = 0.3$

New $Pr(A \cap B) = 0.18$

Since $Pr(A)\, Pr(B) = Pr(A \cap B)$, A and B are independent.

Bayes' Theorem

4. Let $R_i = \{$ component from A_i needs rework $\}$,

$S_i = \{$ component from A_i selected $\}$.

$Pr(R_1 | S_1) = 0.05$, $Pr(R_2 | S_2) = 0.08$, $Pr(R_3 | S_3) = 0.1$

$Pr(S_1) = 0.5$, $Pr(S_2) = 0.3$, $Pr(S_3) = 0.2$

(a). $$Pr(\text{needs rework}) = 0.05 \times 0.5 + 0.08 \times 0.3 + 0.1 \times 0.2 = 0.069$$

$$Pr(S_1 | \text{needs rework}) = \frac{0.05 \times 0.5}{0.069} = 0.3623$$

(b). $$Pr(S_2 | \text{needs rework}) = \frac{0.08 \times 0.3}{0.069} = 0.3478$$

(c). $$Pr(S_3 | \text{needs rework}) = \frac{0.1 \times 0.2}{0.069} = 0.2899$$

5(a). $Pr(A_1) = Pr(A_2) = Pr(A_3) = \frac{2}{4} = 0.5$

$Pr(A_1 \cap A_2) = Pr(A_1 \cap A_3) = Pr(A_2 \cap A_3) = \frac{1}{4} = 0.25$

$Pr(A_1)Pr(A_2) = Pr(A_1)Pr(A_3) = Pr(A_2)Pr(A_3) = 0.5 \times 0.5 = 0.25$

Since $Pr(A_1)Pr(A_2) = Pr(A_1 \cap A_2)$,

$Pr(A_1)Pr(A_3) = Pr(A_1 \cap A_3)$,

$Pr(A_2)Pr(A_3) = Pr(A_2 \cap A_3)$,

A_1, A_2 and A_3 are pairwise independent.

(b). $Pr(A_1 \cap A_2 \cap A_3) = \frac{1}{4} = 0.25 \neq Pr(A_1)Pr(A_2)Pr(A_3)$

$= 0.5^3$

$= 0.125$

6. Let $A = \{A \text{ works}\}$,

$\vdots$

$D = \{D \text{ works}\}$.

(a). $Pr(B \cup C) = 0.7 + 0.8 - 0.7 \times 0.8$

$= 0.94$

$Pr(\text{system works}) = Pr(A \cap (B \cup C) \cap D)$

$= 0.95 \times 0.94 \times 0.9$

$= 0.8037$

Forgot to include $P(C')$

(b). $Pr(C \mid \text{system works}) = \dfrac{\overset{P(A)}{0.95} \times \overset{P(B)}{0.7} \times \overset{P(D)}{0.9} \times (1-0.8)}{0.8037}$

$= 0.7447$ 0.1489

7. Let $A_i = \{$ next ith vehicle passes $\}$.

(a). $Pr(A_1 \cap A_2 \cap A_3) = 0.6^3$

$= 0.216$

(b). Pr (at least one fails) $= 1 - 0.216$

$= 0.784$

(c). Pr (exactly one pass) $= {}_3C_1 \times 0.6 \times 0.4^2$

$= 0.288$

(d). Pr (all pass | at least one pass) $= \dfrac{0.216}{1-(0.4)^3}$

$= 0.2308$

8. Let $A = \{$ ~~has key for the house~~ Agent gets into house $\}$,

$B = \{$ unlocked $\}$.

$C = \{$ Agent has correct key

$$Pr(C) = \frac{{}_7C_2}{{}_8C_3}, \quad Pr(B) = 0.4$$

why?

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)\,P(B) + \underbrace{P(A|B')}_{P(C)}\,P(B') \\ &= 1 \times 0.4 + \frac{{}_7C_2}{{}_8C_3} \times 0.6 \end{aligned}$$