

# CS2100 - L20 - Medium-Scale-Integration (MSI)

Week

## Components

11 + 12

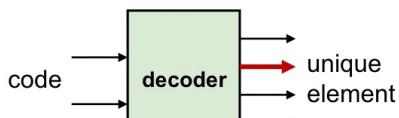
### 20.1 - MSI components

- Decoders
- Encoders
- Demultiplexers
- Multiplexers

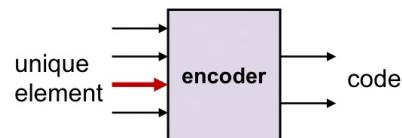
### 20.2 - Applications

## 20.1 - MSI components

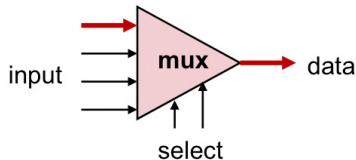
# Introduction to Four MSI Circuits



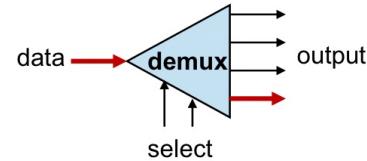
Decoder



Encoder



Multiplexer

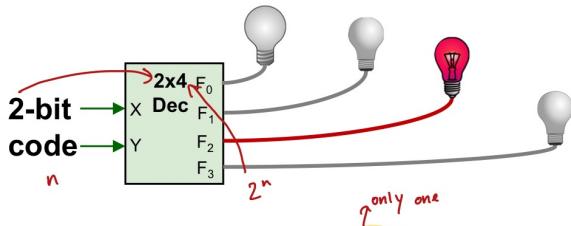


Demultiplexer

## Decoder : Overview

- A **N-bit** Binary values (codes) can represent up to  $2^N$  unique elements:
  - 2-bit = {00, 01, 10, 11}
- Decoder **takes the N-bit code** and **convert it into one of the  $2^N$  unique output**
  - $2^N$  is the upperbound as some of the N-bit code can be unused
- Known as ***n-to-m-line*** decoder, or simply ***n:m*** or ***n×m*** decoder ( $m \leq 2^n$ )

## 2 x 4 Decoder: Example



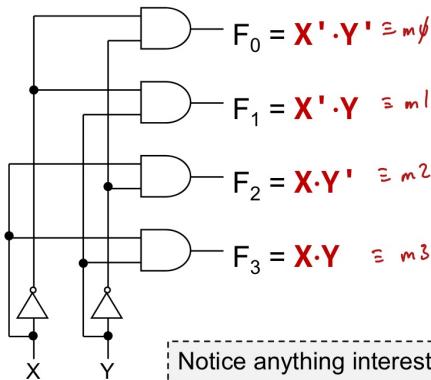
- Depending on the **2-bit code**, one of the light bulbs will be **switched on**
- Truth table:

X	Y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

[L20 - AY2021S1]

## Decoder: Internal Wiring

X	Y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



Notice anything interesting about the output expressions?

[L20 - AY2021S1]

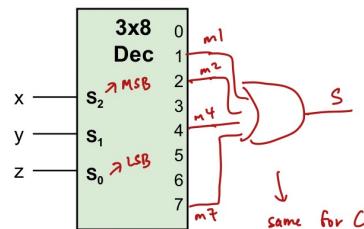
## Decoder: Implementing Boolean Function

### ■ Key Idea:

- Output of decoder corresponds to **minterms**

### ■ Example: Full adder

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$$S(x, y, z) = \Sigma m(1, 2, 4, 7)$$

$$C(x, y, z) = \Sigma m(3, 5, 6, 7)$$

[ L20 - AY2021S1 ]

## Decoder: Implementing Boolean Function

### ■ General Scheme:

- A Boolean function in sum-of-minterms form can be implemented by:
  - Decoder to generate the minterms with an **OR** gate to form the sum
- So, any combinational circuit with **n** inputs and **m** outputs can be implemented by:
  - **one [n:2<sup>n</sup> decoder] + m [OR gates]**
- Good for circuit with many outputs, and each function is expressed with just a few minterms

[ L20 - AY2021S1 ]

## Decoders with Enable(2/2)

- The previous decoder has a **one-enable** control signal
- Most MSI decoders is **zero-enable** instead, usually denoted by  $E'$  or  $\bar{E}$ :
  - The decoder is enabled when the signal is **zero** (low)

E	X	Y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	X	X	0	0	0	0

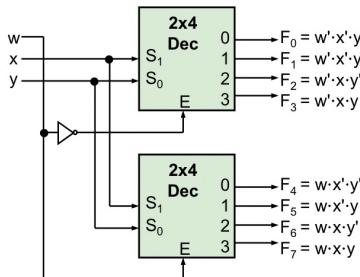
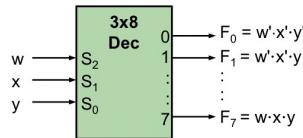
Decoder with **1-enable**

E'	X	Y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	1	0	0	0	1	0
0	1	1	0	0	0	1
1	X	X	0	0	0	0

Decoder with **0-enable**

## Larger Decoders (1/4)

- Larger decoders can be constructed from smaller decoders
- Example: A  $3 \times 8$  decoder can be built from:
  - Two  $2 \times 4$  decoders with one-enable AND
  - An inverter



[ L20 - AY2021S1 ]

## Larger Decoders (4/4)

- Decoder Variants:

### Enable:

- **Zero-enable:** Use '0' to activate decoder
- **One-enable:** Use '1' to activate decoder

### Output:

- **Normal (active-high):** Output is '1' when selected
- **Negated (active-low):** Output is '0' when selected

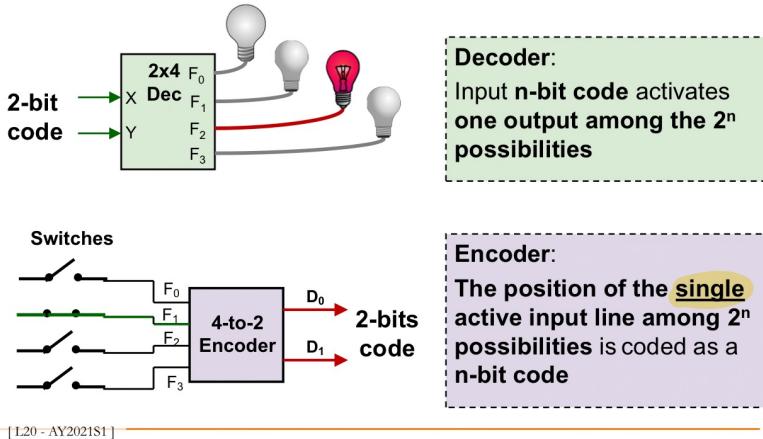
- Exercise: What modifications should be made to provide an ENABLE input for the  $3 \times 8$  decoder and the  $4 \times 16$  decoder created in the previous two slides?
- Exercise: How to construct a  $4 \times 16$  decoder using five  $2 \times 4$  decoders with enable?

[ L20 - AY2021S1 ]

## Encoders

### Encoders: Overview

- Encoding is the converse of decoding



### 4-to-2 Encoders: Implementation

- Truth table:

$F_0$	$F_1$	$F_2$	$F_3$	$D_1$	$D_0$
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1
0	0	0	0	X	X
0	0	1	1	X	X
0	1	0	1	X	X
0	1	1	0	X	X
0	1	1	1	X	X
1	0	0	1	X	X
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

- With K-map, we obtain:

$$D_0 = F_1 + F_3$$

$$D_1 = F_2 + F_3$$

- Circuit (Use OR gate):

also garbage  
∴ 00 used  
to represent  
 $F_0$  being on  
garbage  
input/  
output

## Priority Encoders (1/2)

- Suppose we assign **priority** to each input:
  - If two or more inputs are equal to 1, the input with the highest priority **takes precedence**
  - All inputs of 0 is considered **invalid**
- Example: **4-to-2 Priority Encoder**:

Inputs				Outputs		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	x	y	v
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

— [L20 - AY2021S1] —

## Priority Encoders (2/2)

- Be careful of the 'X's in the *input*
  - Don't interpret it as "either 1 or 0",
  - Instead it is actually "for **both** 1 and 0"
  - Essentially a "compressed" truth table

↓

for inputs:  
don't care  
for all

Inputs				Outputs		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	x	y	v
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

$$x = D_3 + D_2 \cdot D_3$$

$$= D_3 + D_2$$

$$y = D_3 + D_1 \cdot D_2 \cdot D_3$$

$$= D_3 + D_1 \cdot D_2$$

$$z = D_0 + D_1 + D_2 + D_3$$

$$= D_0 + D_1 + D_2 + D_3$$

- **Exercise:** Obtain the simplified expressions for x, y and v.

Inputs				Outputs		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	x	y	v
0	0	0	0	x	x	0
1	0	0	0	0	0	1
0	1	0	0	0	1	1
1	1	0	0	0	1	1
0	0	1	0	1	0	1
0	1	1	0	1	0	1
1	0	1	0	1	0	1
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

— [L20 - AY2021S1] —

## Demultiplexers

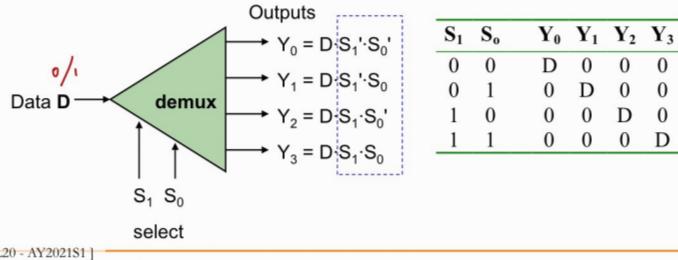
### Demultiplexers (1/2)

- Given:

- One input data line
- $N$  selection lines

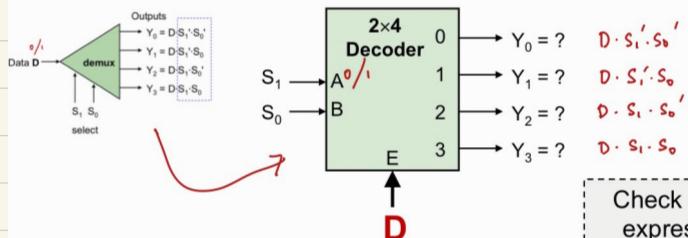
→ Demultiplexer directs the input data to the selected output line among  $2^N$  possibilities

- Example (1-to-4 Demultiplexer):



### Demultiplexers(2/2)

- It turns out that the demultiplexer circuit is actually identical to a decoder with enable:

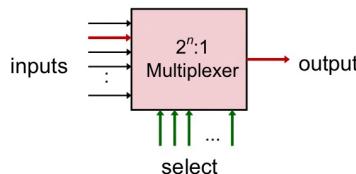


Check the output expression and compared with Demultiplexer's

## Multiplexers

### Multiplexers: Overview

- Given:
  - $2^n$  input lines
  - $n$  selection lines
  - One output line
- Using  $n$  selection lines, **multiplexer** steers one of  $2^n$  inputs to the output line
  - also known as a **data selector**

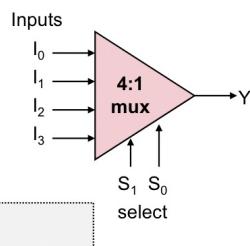
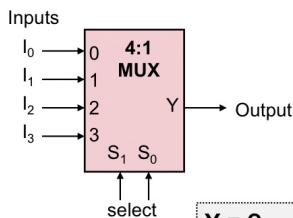


[L20 - AY2021S1]

### 4-to-1 Multiplexers: Truth Table

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	S <sub>1</sub>	S <sub>0</sub>	Y
d <sub>0</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	0	0	d <sub>0</sub>
d <sub>0</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	0	1	d <sub>1</sub>
d <sub>0</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	1	0	d <sub>2</sub>
d <sub>0</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	1	1	d <sub>3</sub>

S <sub>1</sub>	S <sub>0</sub>	Y
0	0	I <sub>0</sub>
0	1	I <sub>1</sub>
1	0	I <sub>2</sub>
1	1	I <sub>3</sub>

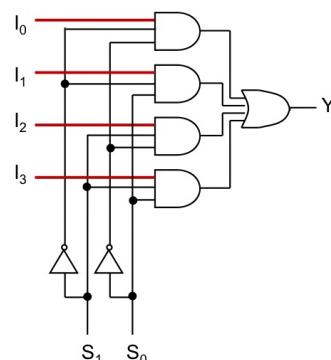
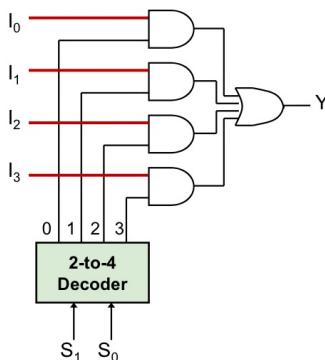


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## Multiplexers: Construction with Decoder

- Multiplexer's Output =

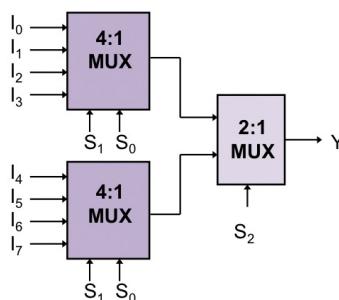
"sum of the (product of **data lines** and **selection lines**)"



## Larger Multiplexers: Overview

- Larger multiplexers can be constructed from smaller ones
- An **8-to-1 multiplexer** example:
  - Note the placement of selector lines

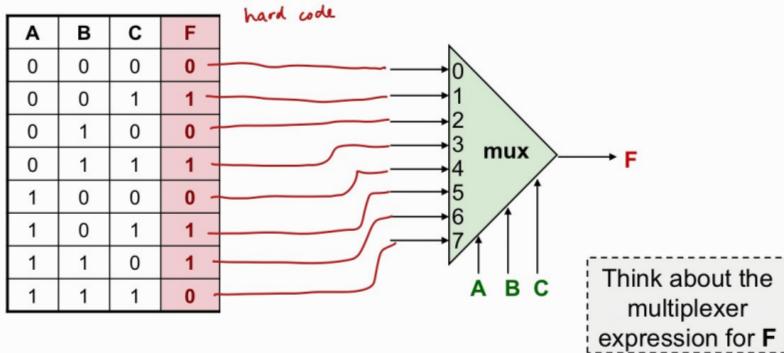
$S_2$	$S_1$	$S_0$	$Y$
0	0	0	$I_0$
0	0	1	$I_1$
0	1	0	$I_2$
0	1	1	$I_3$
1	0	0	$I_4$
1	0	1	$I_5$
1	1	0	$I_6$
1	1	1	$I_7$



[ L20 - AY2021S1 ]

## Implementing Functions: Overview

- Boolean functions can be implemented using multiplexers
- Try for  $F(A,B,C) = \sum m(1,3,5,6)$



## Implementing Functions: Generalization

- A  $2^n$ -to-1 multiplexer can implement a Boolean function of  $n$  input variables, as follows:

1. Express in sum-of-minterms form. Example:

$$\begin{aligned}F(A,B,C) &= A' \cdot B' \cdot C + A' \cdot B \cdot C + A \cdot B' \cdot C + A \cdot B \cdot C' \\&= \Sigma m(1,3,5,6)\end{aligned}$$

2. Connect  $n$  variables to the  $n$  selection lines

3. Put a '1' on a data line if it is a minterm of the function, or '0' otherwise

## Using Smaller Multiplexers: Procedure (1/4) 3

1. Express Boolean function in sum-of-minterms form.

Example:  $F(A,B,C) = \Sigma m(0,1,3,6)$

2. Reserve one variable for the input lines of multiplexer, and use the rest for selection lines

- We reserve the least significant bit (e.g. C) as input line in previous example

Example:

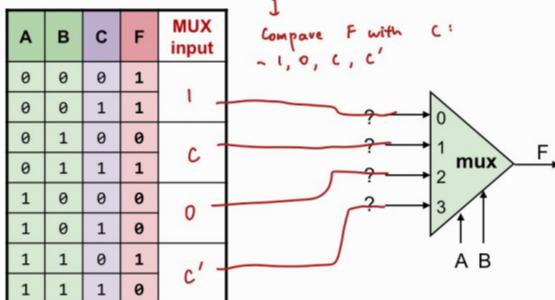
Use C for input lines; A and B for selection lines

[ L20 - AY2021S1 ]

## Using Smaller Multiplexers: Procedure (2/4) 3

3. Use truth table:

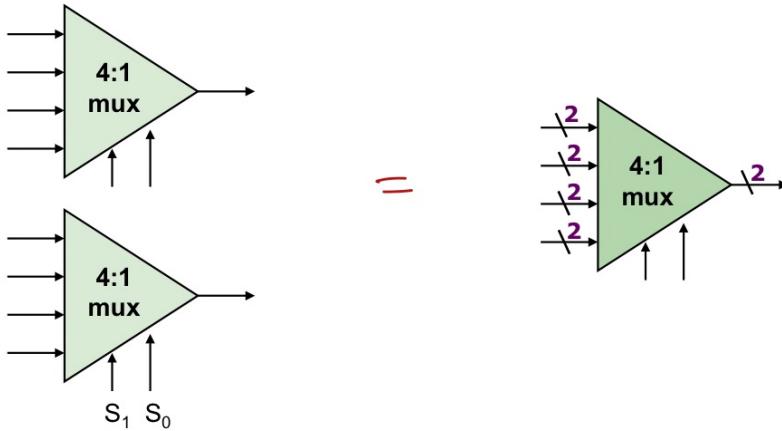
- Group inputs by **selection line values**
- Determine multiplexer inputs by **comparing input line and function output** for corresponding selection line value groups



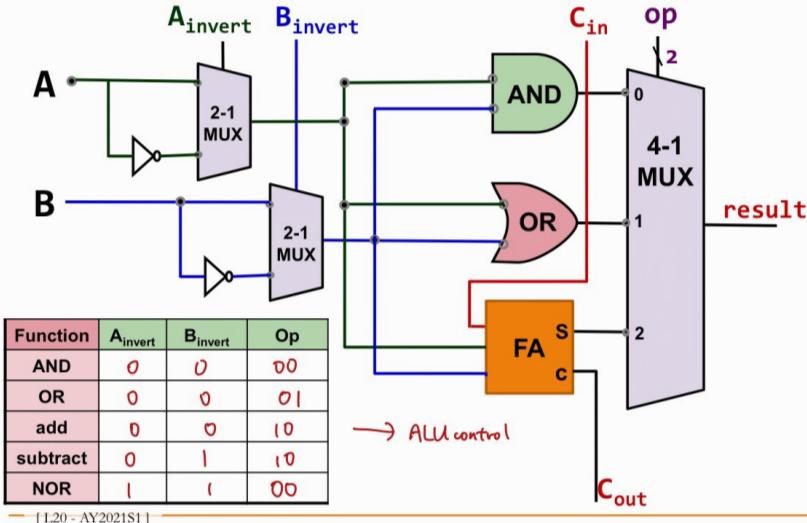
[ L20 - AY2021S1 ]

## Multiplexer: Selecting Multi-Bit Data?

- How to select between 4 input data **W, X, Y, Z**, each a 2-bit value, i.e.  $W_1 W_0$ ,  $X_1 X_0$ , etc?
  - The output **F<sub>1</sub>F<sub>0</sub>** is similarly a 2-bit value

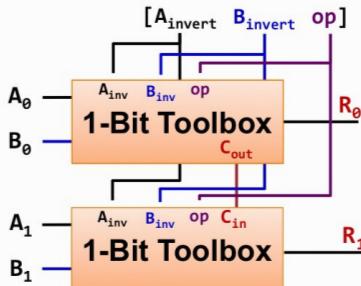


## V4.0: Summary



## "Swiss Army Knife" V4.0 – Multi-Bit?

- By connecting multiple 1-bit toolbox circuits, we can have a multi-purpose toolbox for multi bit inputs!
  - Below show a simplified 2-bit version:



### General Idea:

- Control Signals shared between all 1-bit toolbox
- C<sub>in</sub> connected to C<sub>out</sub> from previous bit position
- Each 1-bit toolbox at position x take a pair of input A<sub>x</sub>, B<sub>x</sub>