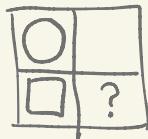


moone (5)

Goal :

$$1+2+3+\dots+n$$

moony-1:  
(?)



$$= \frac{n}{2} (n+1)$$

$$= \cancel{\frac{n^2}{2}} \cancel{n}$$

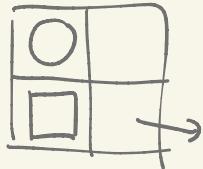
$$= O(n^2)$$

Wishful thinking

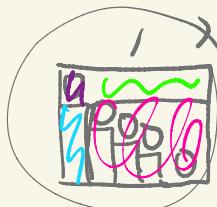
↳ base case

↳ inductive step:  
 $f(n-1) \rightarrow f(n)$

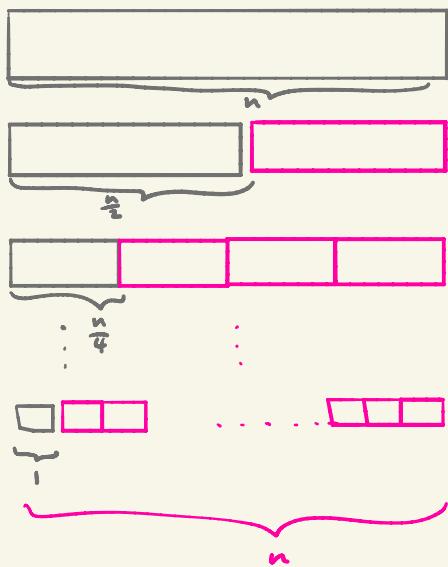
moony-2:  
(n)



moony-3:



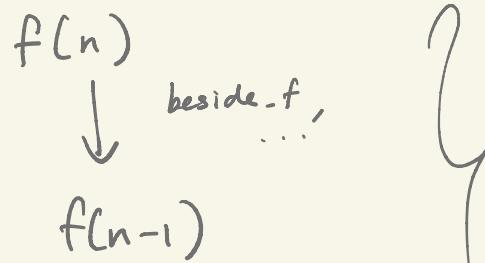
sf( $\frac{1}{n}$ , b.f( $\frac{1}{n}$ , 0, □),  
b.f( $\frac{1}{n}$ , □, moony-2(n-1)))



```

fn_ cone(n, rune) {
    fn_ cone_helper(n, rune, max) {
        ↳ n == 1
        ? rune
        : overlay_f(  $\frac{1}{n}$ ,  $1 - \frac{n-1}{max}$ 
                     scale(n, rune), max
                     cone_helper(n-1, rune));
    }
    ↳ cone_helper(n, rune, n);
}

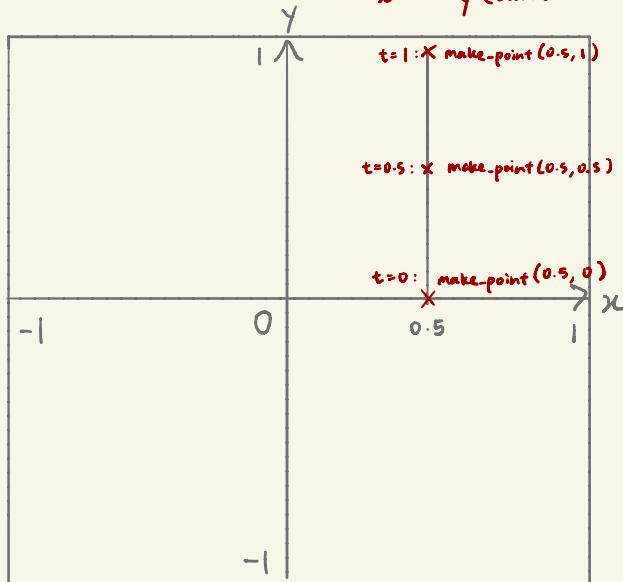
```



- ① Base case ( $n=1$ )
- ② Inductive step

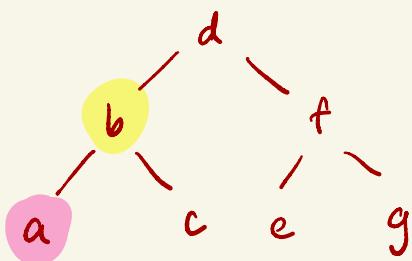
$\text{cone}(n)$  {  
 ↳  $n == 1$   
 ?    O  
 :               $\text{cone}(n-1)$

$t \Rightarrow \text{make-point}(0.5, t)$



$p(b, p(b, p(b, p(b, p(b, 0))))))$

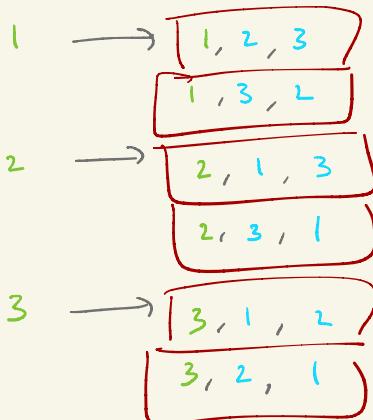
$$T(n) = n T$$



## permutations (list)

list (1, 2, 3):

- ① Wishful thinking. For each element,  
assume we have a solution  
for the list minus that element  
remove ( $x$ , list)

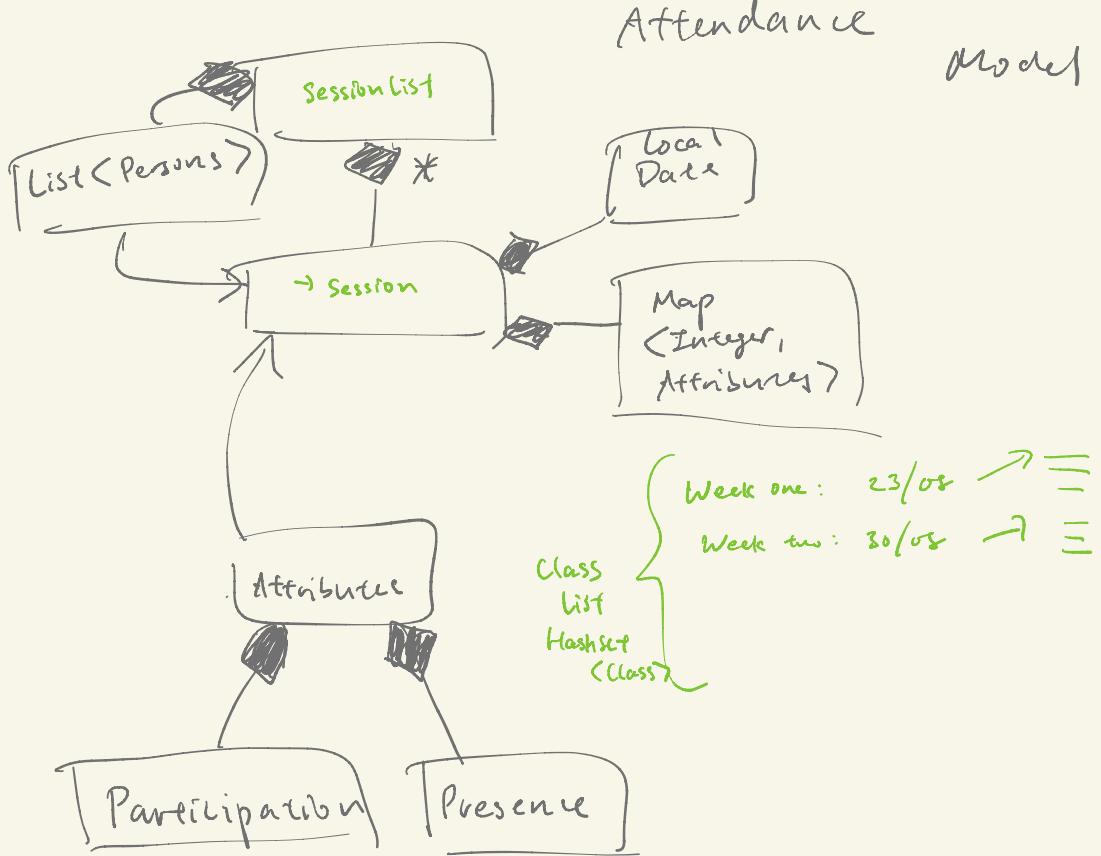


- ② Wishful thinking:  
permutations (remove( $x$ , list))

- ③ Map list in wishful thinking solution  
map ( $p \Rightarrow$  pair ( $x$ ,  $p$ ),  
permutations (remove ( $x$ , list)))

- ④ Map element in list  
map ( $x \Rightarrow$  map ( $p \Rightarrow$  pair ( $x$ ,  $p$ ),  
permutations (remove ( $x$ , list))),  
list)

- ⑤ Final step: combine solutions using accumulate (append, null, ...)

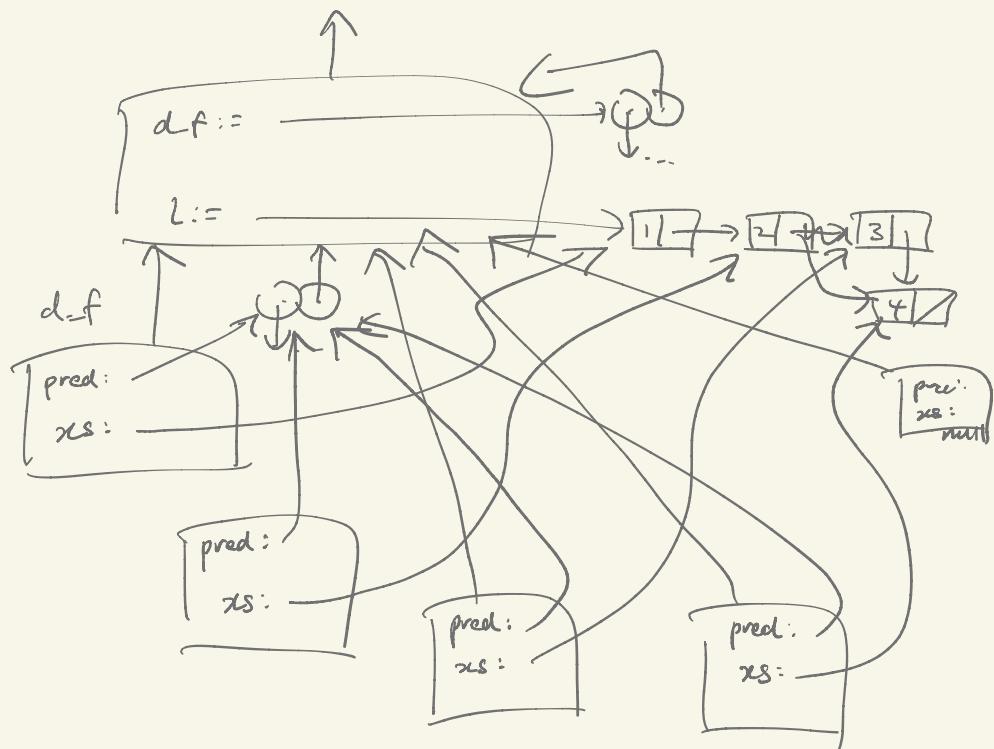
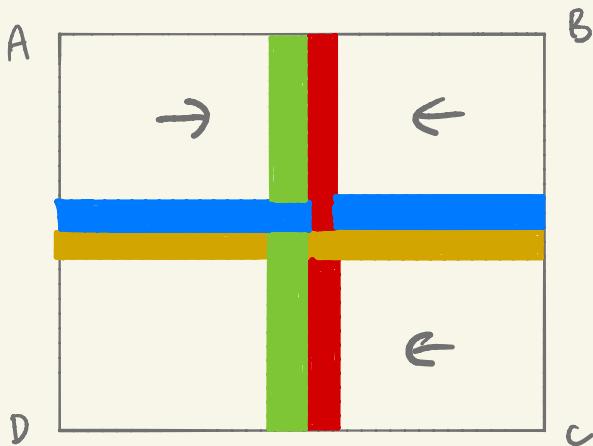


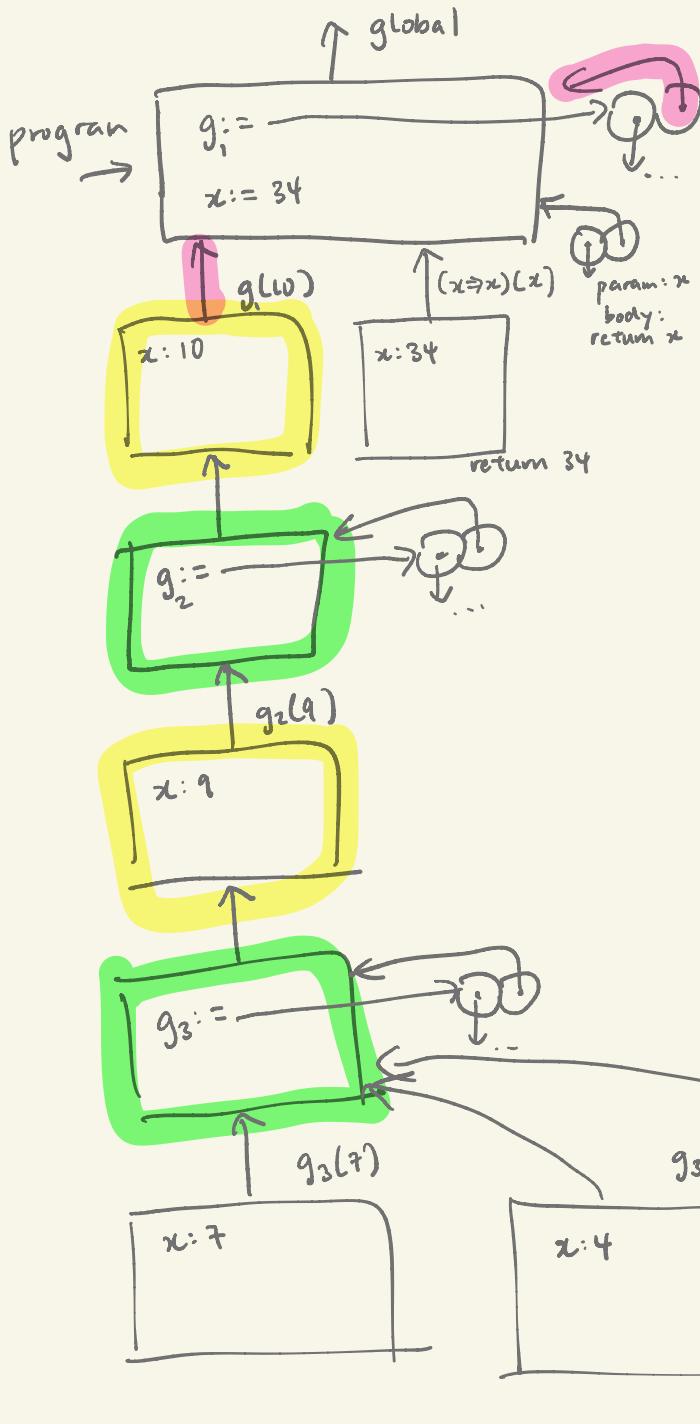
$\text{chain(twice, 3)}$   
 $\rightarrow \text{twice(twice)(twice)}$   
 $\rightarrow \text{four-times(twice)}$   
 $\rightarrow \text{four-times(fff)}$   
 $\rightarrow \text{three-times(ffff)}$   
 $\rightarrow \text{two-times(fff...f)}$   
 $\rightarrow \text{ff...f}_{16} = 2^{2^2} = 32$

$t(b, n-1) : \Theta(n^{-1}b)$   
 $t(b, n-2) : \Theta(n^{-2}b) < \Theta(n^{-1}b)$   
 $t(b, n-3) : \Theta(n^{-3}b) < \Theta(n^{-1}b)$   
 $\vdots$   
 $t(b, 1) : \Theta(1) < \Theta(n^{-1}b)$

$n$

- 1111 1111  
11 1111       $2^7 - 1$



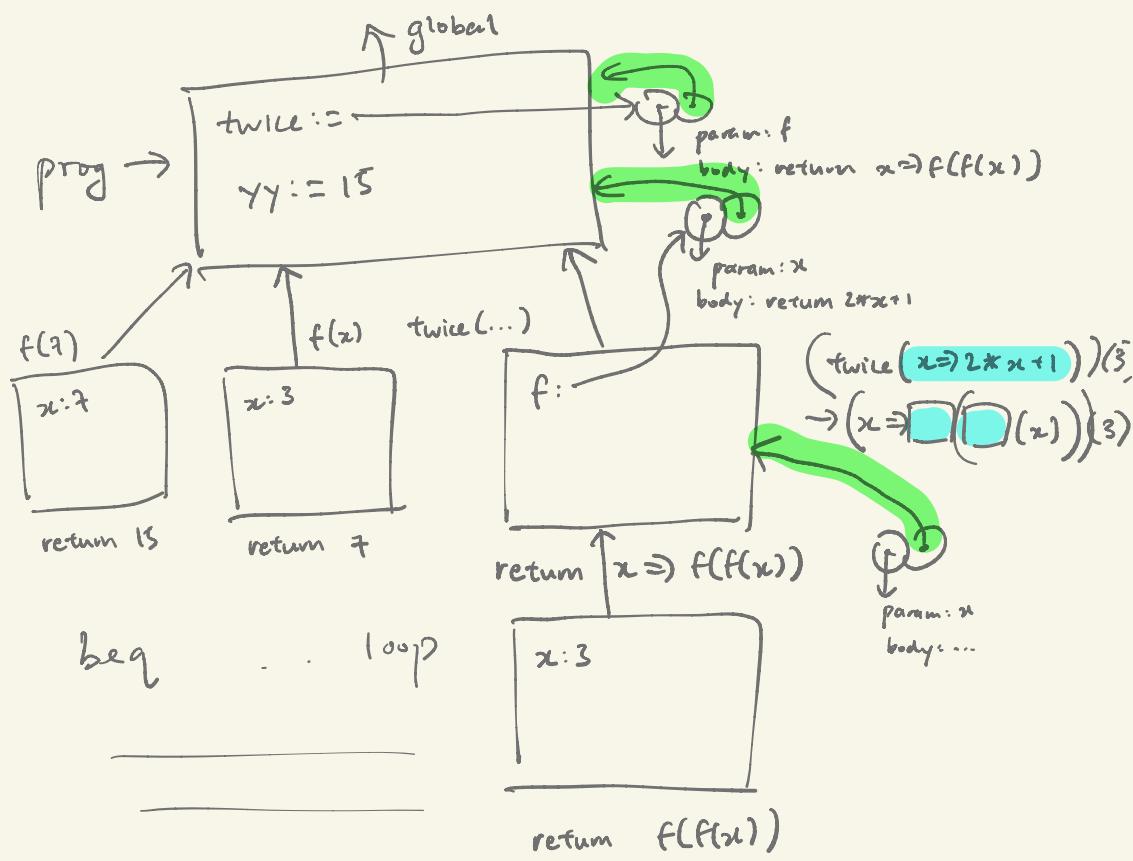


### Func. app.

- ↳ extend frame
- ↳ extend from frame in which fn obj. is created
- ↳ declare names
  - parameters
  - body (names)

### Func. obj.





beg . . . loop

---



---



---

loop:

