

Q2(e).

Q5

Week 8

## ST2334 - Tutorial 6

(a).

$x$	1	2	3
$f_x(x)$	0.10	0.35	0.55



(b).

$y$	1	2	3
$f_y(y)$	0.20	0.50	0.30



$$(c). \Pr(y=3 | X=2) = f_{y|X}(3|2)$$

$$= \frac{f_{x,y}(2,3)}{f_x(2)}$$

$$= \frac{\frac{0.2}{0.55}}{\frac{0.35}{0.55}}$$

$$= \frac{4}{7}$$

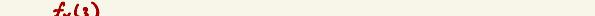
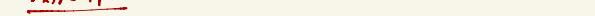


(d).

$y$	1	2	3
$f_{y x}(y 2)$	$\frac{0.1}{0.55} = \frac{2}{11} \cancel{\frac{1}{7}}$	$\frac{0.35}{0.55} = \frac{7}{11} \cancel{\frac{2}{7}}$	$\frac{0.1}{0.55} = \frac{2}{11} \cancel{\frac{4}{7}}$

$$\frac{f_{x,y}(2,y)}{f_x(2)}$$

$$= \frac{f_{x,y}(2,y)}{0.55}$$



(e).

$f_{X,Y}(x,y)$		$x$		
		1	2	3
$y$	1	0.05	0.05	0.1
	2	0.05	0.10	0.35
	3	0	0.2	0.1

		$x$		
		1	2	3
$y$	1	0.02	0.07	0.11
	2	0.05	0.175	0.275
	3	0.03	0.105	0.165

$$\sim (f_{x,y}(x,y) = f_x(x)f_y(y)) \quad \forall x, y \rightarrow x \text{ and } y \text{ are dependent.}$$



2(a).

$f_{X,Y}(x,y)$		x		
		1	2	3
y	1	0	0	$\frac{3C_2 \times 2C_1}{35}$
	2	0	$\frac{3C_2 \times 2C_2}{35}$	0

$$f_{X,Y}(x,y) = \begin{cases} \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, & x=0,1,2,3; \\ 0, & y=0,1,2; \\ 1 \leq x+y \leq 4 \\ \text{otherwise} \end{cases}$$

Subsequent solns use

(b).

$$\Pr(X=1, Y=1) = \frac{\binom{3}{1}\binom{2}{1}\binom{3}{2}}{\binom{8}{4}}$$

$$= \frac{9}{35}$$

(c).

$$\begin{aligned} \Pr(X+Y \leq 2) &= \Pr(X+Y=1) + \Pr(X+Y=2) \\ &= \Pr(X=0, Y=2) + \Pr(X=1, Y=1) + \Pr(X=2, Y=0) \\ &\quad + \Pr(X=0, Y=1) + \Pr(X=1, Y=0) \\ &= \frac{3}{70} + \frac{9}{35} + \frac{9}{70} + \frac{1}{35} + \frac{3}{70} \\ &= \frac{22}{35} \quad \times 0.5 \end{aligned}$$

(d).

$$f_X(x) = \begin{cases} \frac{\binom{3}{x}\binom{5}{4-x}}{\binom{8}{4}}, & x=0,1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} (e). \quad f_{Y|X}(y|x) &= \frac{f_{X,Y}(x,y)}{f_X(x)} \\ &= \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}} \times \frac{\binom{8}{4}_1}{\binom{3}{x}\binom{5}{4-x}} \\ &= \begin{cases} \frac{1}{10}\binom{2}{y}\binom{3}{2-y}, & y=0,1,2; \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{aligned} \Pr(Y=0 | X=2) &= \frac{1}{10}\binom{2}{0}\binom{3}{2-0} \\ &= \frac{3}{10} \end{aligned}$$

3(a).

$\Pr(X=x, Y=y)$		$x$		
		0	1	2
$y$	0	$\frac{4}{9}$	$2\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$	$\left(\frac{1}{3}\right)^2 = \frac{1}{36}$
	1	$2\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$	$2\left(\frac{1}{6}\right)^2 = \frac{1}{18}$	0
	2	$\left(\frac{1}{6}\right)^2 = \frac{1}{36}$	0	0

$$(b). \Pr(2X+Y < 3) = \Pr(X=0, Y=0) + \Pr(X=1, Y=0)$$

$$+ \Pr(X=0, Y=1) + \Pr(X=0, Y=2)$$

$$= \frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{36}$$

$$= \frac{11}{12}$$

↗  $(\frac{1}{6})^2$  each

$$(c). \Pr(X=2, Y=2) = 0 \neq \Pr(X=2) \Pr(Y=2) = \frac{1}{1296}$$

$\therefore X$  and  $Y$  are dependent

$$\begin{aligned}
 4(a). \quad & \iint_{(x,y) \in R_{x,y}} f_{x,y}(x,y) dx dy = \\
 & \int_3^5 \int_3^5 k(x^2+y^2) dx dy = k \int_3^5 \left[ \frac{x^3}{3} + xy^2 \right]_{x=3}^{x=5} dy \\
 & = k \int_3^5 \frac{98}{3} + 2y^2 dy \\
 & = k \left[ \frac{98}{3}y + \frac{2y^3}{3} \right]_{y=3}^{y=5} \\
 & = \frac{392}{3}k = 1 \\
 & \therefore k = \frac{3}{392}
 \end{aligned}$$

$$\begin{aligned}
 (b). \quad f_{x,y}(x,y) &= \begin{cases} \frac{3}{392}(x^2+y^2) & 3 \leq x \leq 5; 3 \leq y \leq 5; \\ 0 & \text{otherwise} \end{cases} \\
 \Pr(3 \leq X \leq 4 \text{ and } 4 \leq Y \leq 5) &= \int_4^5 \int_3^4 \frac{3}{392}(x^2+y^2) dx dy \\
 &= \frac{3}{392} \int_4^5 \left[ \frac{x^3}{3} + xy^2 \right]_{x=3}^{x=4} dy \\
 &= \frac{3}{392} \int_4^5 \frac{37}{3} + y^2 dy \\
 &= \frac{3}{392} \left[ \frac{37}{3}y + \frac{y^3}{3} \right]_{y=4}^{y=5} \\
 &= \frac{3}{392} \left[ \frac{98}{3} \right]
 \end{aligned}$$

$$\begin{aligned}
 (c). \quad f_x(x) &= \int_3^5 \frac{3}{392}(x^2+y^2) dy \\
 &= \frac{3}{392} \left[ yx^2 + \frac{y^3}{3} \right]_{y=3}^{y=5} \\
 &= \frac{3}{392} \left( 2x^2 + \frac{98}{3} \right) \\
 &= \begin{cases} \frac{3}{196}x^2 + \frac{1}{4} & 3 \leq x \leq 5; \\ 0 & \text{otherwise} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 \Pr(3.5 < X < 4) &= \int_{3.5}^4 \frac{3}{196}x^2 + \frac{1}{4} dx \\
 &= \left[ \frac{1}{196}x^3 + \frac{1}{4}x \right]_{3.5}^4 \\
 &= \frac{365}{1568} \\
 &= 0.2328
 \end{aligned}$$

5(a). Creams :  $f_x(x) = \int_0^1 24xy \, dy$   $\rightarrow 12x(1-x)^2 ?$   
 $= [12xy^2]_{y=0}^{y=1}$   
 $= \begin{cases} 12x & 0 \leq x \leq 1; \\ 0 & \text{otherwise} \end{cases}$

Toffees :  $f_y(y) = \int_0^1 24xy \, dx$   $\rightarrow 12y(1-y)^2$   
 $= [12x^2y]_{x=0}^{x=1}$   
 $= \begin{cases} 12y & 0 \leq y \leq 1; \\ 0 & \text{otherwise} \end{cases}$

(b).  $f_{x,y}(x,y) = 24xy \neq f_x(x)f_y(y) = 144xy$   
 $\rightarrow X \text{ and } Y \text{ are independent}$

(c). ?