Q3b, 8

ST2334 - Tutorial 2 Week 4

1. $P(A) = 0.7$

$P(B) = 0.4$

$P(A \cup B) = 0.8$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.7 + 0.4 - 0.8$

$= 0.3$

(b). $P((A \cap B)') = 1 - P(A \overset{\cup}{\cap} B)$

$A' \cap B' = (A \cup B)'$

$= 1 - 0.3 \; 0.8$

$= 0.7 \times 0.2$

2 (a). $n(\text{ways}) = {}_{30}C_5$

$= 142506$

(b). $P(\text{no minority}) = \dfrac{{}_{23}C_5}{{}_{30}C_5}$

$= 0.236$

(c). $P(\leq 1 \text{ minority}) = P(0 \text{ minorities}) + P(1 \text{ minority})$

$= \dfrac{{}_{23}C_5}{{}_{30}C_5} + \dfrac{{}_{7}C_1 \times {}_{23}C_4}{{}_{30}C_5}$

$= 0.671$

3 (a). $P(A) = \dfrac{{}_4C_1 \times {}_{13}C_5}{{}_{52}C_5}$

$= 0.00198$

? (b). $P(B) = \dfrac{{}_{52}C_1 \times ({}_4C_3)^4}{{}_{52}C_5}$

$= 0.00512$

choose suit

no. of straight flushes

(b) Number of straight hands with 1 as the smallest card is $({}_4C_1)^5 \times ({}_4C_0)^8 - 4 = 1020$. Similarly, the number of straight hands with 2 as the smallest card is ${}_4C_0 \times ({}_4C_1)^5 \times ({}_4C_0)^8 - 4 = 1020$ and so on. The smallest card can be any one from 1 to 10. Pr(a straight hand) = 10(1020)/2598960 = 10200/2598960 = 0.003925.

4. $A = \{\text{stops at first light}\}$

$B = \{\text{stops at second line}\}$

(a). $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.4 + 0.5 - 0.6$

$= 0.3$ ✓

(b). $P((A \cap B') \cup (A' \cap B)) = P(A \cup B) - 2P(A \cap B)$

$= 0.6 - 2(0.3)$

$= 0$ ✗ 0.3

(c). $P((A \cup B)') = 1 - P(A \cup B)$

$= 1 - 0.6$

$= 0.4$ ✓

(d). $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$

$= \dfrac{0.3}{0.4}$

$= 0.75$ ✓

$P(A)P(B) = 0.4 \times 0.5 = 0.2 \neq P(A \cap B) = 0.3$

$\therefore$ Not independent

5(a). $P(\text{no two consecutive same digits}) = \dfrac{{}_9C_1 \times ({}_8C_1)^8}{({}_9C_1)^9}$ $\qquad ({}_9C_1)^9$

$\qquad\qquad {}_9C_1 \times ({}_{10}C_1)^8$

$= 0.390$

(b). $P(0 \text{ appears three times}) = \dfrac{{}_8C_3 \times ({}_{\overset{9}{8}}C_1)^6}{({}_9C_1)^9}$ $\qquad {}_9C_1 \times ({}_{10}C_8)^8$

$= 0.0379$

6. $A = \{A \text{ enters}\}$

$B = \{B \text{ enters}\}$

$P(A|B) = \frac{1}{6} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{6}$

$P(A|B') = \frac{3}{4} \Rightarrow \frac{P(A \cap B')}{P(B')} = \frac{3}{4}$

$P(B) = \frac{1}{3} \Rightarrow P(B') = \frac{2}{3}$

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)P(B) + P(A|B')P(B') \\ &= \tfrac{1}{6}\left(\tfrac{1}{3}\right) + \tfrac{3}{4}\left(\tfrac{2}{3}\right) \\ &= \tfrac{5}{9} \end{aligned}$$

7. $A = \{ \text{from machine I} \}$

$B = \{ \text{from machine II} \}$

$C = \{ \text{nonconforming} \}$

$P(A \cap C) = 0.01$

$P(B \cap C) = 0.025$

$P(A) = P(B) = \frac{1}{2}$

(a). $P(C) = P(A \cap C) + P(B \cap C)$

$= 0.01 + 0.025$

$= 0.035$ ✓

(b). $P(B) = \frac{1}{2}$ ✓

(c). $P(B \cap C') = P(B) - P(B \cap C)$

$= \frac{1}{2} - 0.025$

$= 0.475$ ✓

$P(A) - P(A \cap C)$

(d). $P(A \cup C') = P(A) + P(C') - P(A \cap C')$

$= \frac{1}{2} + [1 - 0.35] - [\frac{1}{2} - 0.01]$

$= 0.66$ ✗ 0.975

(e). $P(C | A) = \dfrac{P(C \cap A)}{P(A)}$

$= \dfrac{0.01}{\frac{1}{2}}$

$= 0.02$ ✓

(f). $P(A | C) = \dfrac{P(A \cap C)}{P(C)}$

$= \dfrac{0.01}{0.035}$

$= \frac{2}{7}$ ✓

(g). $P(A | C) \neq P(C | A)$

Apparently the two events are independent.
How can we assume this though?

8. $A = \{\text{positive}\}$ Let $P = \{\text{pregnant}\}$,
$B = \{\text{pregnant}\}$ $T = \{\text{positive test}\}$
$P(B) = 0.75$ $Pr(P) = 0.75$, $Pr(T|P) = 0.99$, $Pr(T|P') = 0.02$
$P(B'|A) = 0.02$
$P(A) = 0.99$

Hence $\Pr(T) = \Pr(P)\Pr(T \mid P) + \Pr(P')\Pr(T \mid P^C) = 0.75(0.99) + 0.25(0.02) = 0.7475.$

(a). $P(B|A) = 1 - P(B'|A)$
$= 1 - 0.02$
$= 0.98$ ✗ 0.9933

(a) $\Pr(P \mid T) = \Pr(P \cap T)/\Pr(T) = 0.75(0.99)/0.7475 = 0.9933.$

(b). $$P(B'|A') = \frac{P(B' \cap A')}{P(A')}$$

$$= \frac{P((B \cup A)')}{1 - P(A)}$$

$$= \frac{1 - P(B \cup A)}{1 - P(A)}$$

$P(B|A)P(A)$

$$= \frac{1 - [P(B) + P(A) - P(B \cap A)]}{1 - P(A)}$$

$$= \frac{1 - [0.75 + 0.99 - 0.98(0.99)]}{1 - 0.99}$$

$= 23.02$ ✗ LOL
0.9703

(b) $\Pr(P^C \mid T^C) = \Pr(P^C \cap T^C)/\Pr(T^C) = (1 - 0.02)(0.25)/(1 - 0.7475) = 0.9703.$