Q3b, 8

ST2334 - Tutorial 2 Week 4

1. $P(A) = 0.7$
$P(B) = 0.4$
$P(A \cup B) = 0.8$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.7 + 0.4 - 0.8$
$= 0.3$

(b). $P((A \cap B)') = 1 - P(A \cap B)$ $\cup$
$A' \cap B'$ $= 1 - 0.3$ 0.8
$= (A \cup B)'$ $= 0.7$ $\times$ 0.2

2 (a). $n(\text{ways}) = {}^{30}C_5$
$= 142506$

(b). $P(\text{no minority}) = \frac{{}^{23}C_5}{{}^{30}C_5}$

$= 0.236$

(c). $P(\leq 1 \text{ minority}) = P(0 \text{ minorities}) + P(1 \text{ minority})$
$= \frac{{}^{23}C_5}{{}^{30}C_5} + \frac{{}^{7}C_1 \times {}^{23}C_4}{{}^{30}C_5}$

$= 0.671$

3 (a). $P(A) = \dfrac{{}_4C_1 \times {}_{13}C_5}{{}_{52}C_5}$

$= 0.00198$

? (b). $P(B) = \dfrac{{}_{52}C_1 \times ({}_4C_3)^4}{{}_{52}C_5}$

$= 0.00512$

4. $A = \{\text{stops at first light}\}$
$B = \{\text{stops at second line}\}$

(a). $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.4 + 0.5 - 0.6$
$= 0.3$ ✓

(b). $P((A \cap B') \cup (A' \cap B)) = P(A \cup B) - 2P(A \cap B)$
$= 0.6 - 2(0.3)$
$= 0$ ✗ 0.3

(c). $P((A \cup B)') = 1 - P(A \cup B)$
$= 1 - 0.6$
$= 0.4$ ✓

(d). $P(B|A) = \frac{P(B \cap A)}{P(A)}$
$= \frac{0.3}{0.4}$
$= 0.75$ ✓

$P(A)P(B) = 0.4 \times 0.5 = 0.2 \neq P(A \cap B) = 0.3$
$\therefore$ Not independent

5(a). $P(\text{no two consecutive same digits}) = \dfrac{{}_9C_1 \times ({}_8C_1)^8}{({}_9C_1)^9}$ $\quad ({}_9C_1)^9 \quad {}_9C_1 \times ({}_{10}C_1)^8$

$= 0.390$

(b). $P(0 \text{ appears three times}) = \dfrac{{}_8C_3 \times (\boxed{8}C_1)^{6}}{({}_9C_1)^9}$ $\quad 9 \quad {}_9C_1 \times ({}_{10}C_8)^8$

$= 0.0379$

6. $A = \{A \text{ enters}\}$

$B = \{B \text{ enters}\}$

$P(A|B) = \frac{1}{6} \Rightarrow \dfrac{P(A \cap B)}{P(B)} = \frac{1}{6}$

$P(A|B') = \frac{3}{4} \Rightarrow \dfrac{P(A \cap B')}{P(B')} = \frac{3}{4}$

$P(B) = \frac{1}{3} \Rightarrow P(B') = \frac{2}{3}$

$P(A) = P(A \cap B) + P(A \cap B')$

$= P(A|B)P(B) + P(A|B')P(B')$

$= \frac{1}{6}\left(\frac{1}{3}\right) + \frac{3}{4}\left(\frac{2}{3}\right)$

$= \frac{5}{9}$

7. $A = \{\text{from machine I}\}$

$B = \{\text{from machine II}\}$

$C = \{\text{nonconforming}\}$

$P(A \cap C) = 0.01$

$P(B \cap C) = 0.025$

$P(A) = P(B) = \frac{1}{2}$

(a). $P(C) = P(A \cap C) + P(B \cap C)$

$= 0.01 + 0.025$

$= 0.035$ ✓

(b). $P(B) = \frac{1}{2}$ ✓

(c). $P(B \cap C') = P(B) - P(B \cap C)$

$= \frac{1}{2} - 0.025$

$= 0.475$ ✓

(d). $P(A \cup C') = P(A) + P(C') - \overbrace{P(A \cap C')}^{P(A) - P(A \cap C)}$

$= \frac{1}{2} + [1 - 0.35] - [\frac{1}{2} - 0.01]$

$= 0.66$ ✗ 0.975

(e). $P(C|A) = \frac{P(C \cap A)}{P(A)}$

$= \frac{0.01}{\frac{1}{2}}$

$= 0.02$ ✓

(f). $P(A|C) = \frac{P(A \cap C)}{P(C)}$

$= \frac{0.01}{0.035}$

$= \frac{2}{7}$ ✓

(g). $P(A|C) \neq P(C|A)$

8. $A = \{\text{positive}\}$

$B = \{\text{pregnant}\}$

$P(B) = 0.75$

$P(B' \mid A) = 0.02$

$P(A) = 0.99$

(a). $P(B \mid A) = 1 - P(B' \mid A)$

$= 1 - 0.02$

$= 0.98$ ✗ 0.9933

(b). $$P(B' \mid A') = \frac{P(B' \cap A')}{P(A')}$$

$$= \frac{P((B \cup A)')}{1 - P(A)}$$

$$= \frac{1 - P(B \cup A)}{1 - P(A)}$$

$P(B \mid A)P(A)$

$$= \frac{1 - [P(B) + P(A) - P(B \cap A)]}{1 - P(A)}$$

$$= \frac{1 - [0.75 + 0.99 - 0.98(0.99)]}{1 - 0.99}$$

$= 23.02$ ✗ LOL

0.9703