

Q1 (d), (e), 8

Q2 (b), 6 (b)

ST2334 - Tutorial 3

Week 5

$$\begin{aligned} 1 \text{ (a). } \Pr(A \cap B \cap C) &= \Pr(A) \Pr(B|A) \Pr(C|A \cap B) \\ &= 0.75 \times 0.9 \times 0.8 \\ &= 0.54 \end{aligned}$$

$$\begin{aligned} \text{(b). } \Pr(B) &= \Pr(B \cap A) + \Pr(B \cap A') \\ &= \Pr(A) \Pr(B|A) + \Pr(A') \Pr(B|A') \\ &= 0.75 \times 0.9 + (1-0.75) \times 0.8 \\ &= 0.875 \end{aligned}$$

$$\begin{aligned} \text{(c). } \Pr(A|B) &= \frac{\Pr(A \cap B)}{\Pr(B)} \\ &= \frac{0.75 \times 0.9}{0.875} \\ &= 0.7714 \end{aligned}$$

(d).

2. Let $A = \{A \text{ is profitable}\}$, $B = \{B \text{ is profitable}\}$.

$$\begin{aligned} \text{(a). } Pr(A|B) &= \frac{Pr(A \cap B)}{Pr(B)} \\ &= \frac{0.05}{0.18} \\ &= 0.2778 \end{aligned}$$

$$\begin{aligned} \text{(b). } Pr(A|A \cup B) &= \frac{Pr(A \cap (A \cup B))}{Pr(A \cup B)} \\ &= \frac{Pr(\overset{A}{A \cap A} \cup (A \cap B))}{Pr(A) + Pr(B) - Pr(A \cap B)} \\ &= \frac{Pr(A \cap B)}{Pr(A) + Pr(B) - Pr(A \cap B)} \\ &= \frac{0.05}{0.18 + 0.18 - 0.05} \\ &= 0.1613 \end{aligned}$$

3. Let $A = \{\text{implemented TQM}\}$,
 $B = \{\text{increased in sales}\}$.

$$\Pr(A) = 0.3, \quad \Pr(B) = 0.6, \quad \Pr(A|B) = \frac{1}{3}$$

(a). $\Pr(A) = 0.3, \quad \Pr(B) = 0.6$ ✓

(b). $\Pr(A) \times \Pr(B) = 0.3 \times 0.6 = 0.18$

$$\Pr(A \cap B) = \Pr(B) \Pr(A|B)$$

$$= 0.6 \times \frac{1}{3}$$

$$= 0.2$$
 ✓

Since $\Pr(A) \Pr(B) \neq \Pr(A \cap B)$, A and B are dependent.

(c). New $\Pr(A|B) = 0.3$ ✓

New $\Pr(A \cap B) = 0.18$

Since $\Pr(A) \Pr(B) = \Pr(A \cap B)$, A and B are independent.

→ Bayes' Theorem

4. Let $R_i = \{ \text{component from } A_i \text{ needs rework} \}$,
 $S_i = \{ \text{component from } A_i \text{ selected} \}$.
 $\Pr(R_1|S_1) = 0.05$, $\Pr(R_2|S_2) = 0.08$, $\Pr(R_3|S_3) = 0.1$
 $\Pr(S_1) = 0.5$, $\Pr(S_2) = 0.3$, $\Pr(S_3) = 0.2$

$$(a). \Pr(\text{needs rework}) = 0.05 \times 0.5 + 0.08 \times 0.3 + 0.1 \times 0.2 \\ = 0.069$$

$$\Pr(S_1 | \text{needs rework}) = \frac{0.05 \times 0.5}{0.069} \\ = 0.3623$$

$$(b). \Pr(S_2 | \text{needs rework}) = \frac{0.08 \times 0.3}{0.069} \\ = 0.3478$$

$$(c). \Pr(S_3 | \text{needs rework}) = \frac{0.1 \times 0.2}{0.069} \\ = 0.2899$$

5(a). $\Pr(A_1) = \Pr(A_2) = \Pr(A_3) = \frac{2}{4} = 0.5$

$$\Pr(A_1 \cap A_2) = \Pr(A_1 \cap A_3) = \Pr(A_2 \cap A_3) = \frac{1}{4} = 0.25$$

$$\Pr(A_1)\Pr(A_2) = \Pr(A_1)\Pr(A_3) = \Pr(A_2)\Pr(A_3) = 0.5 \times 0.5 = 0.25$$

Since $\Pr(A_1)\Pr(A_2) = \Pr(A_1 \cap A_2)$,

$$\Pr(A_1)\Pr(A_3) = \Pr(A_1 \cap A_3),$$

$$\Pr(A_2)\Pr(A_3) = \Pr(A_2 \cap A_3),$$

A_1, A_2 and A_3 are pairwise independent.

(b). $\Pr(A_1 \cap A_2 \cap A_3) = \frac{1}{4} = 0.25 \neq \Pr(A_1)\Pr(A_2)\Pr(A_3)$

$$= 0.5^3$$

$$= 0.125$$

6. let $A = \{A \text{ works}\},$
 \vdots
 $D = \{D \text{ works}\}.$

$$(a). \Pr(B \cup C) = 0.7 + 0.8 - 0.7 \times 0.8 \\ = 0.94$$

$$\Pr(\text{system works}) = \Pr(A \cap (B \cup C) \cap D) \\ = 0.95 \times 0.94 \times 0.9 \\ = 0.8037$$

$$(b). \Pr(C \mid \text{system works}) = \frac{0.95 \times 0.7 \times 0.9}{0.8037} \\ = 0.7447$$

7. Let $A_i = \{ \text{next } i\text{th vehicle passes} \}$.

$$\begin{aligned} \text{(a). } \Pr(A_1 \cap A_2 \cap A_3) &= 0.6^3 \\ &= 0.216 \end{aligned}$$

$$\begin{aligned} \text{(b). } \Pr(\text{at least one fails}) &= 1 - 0.216 \\ &= 0.784 \end{aligned}$$

$$\begin{aligned} \text{(c). } \Pr(\text{exactly one pass}) &= 3C_1 \times 0.6 \times 0.4^2 \\ &= 0.288 \end{aligned}$$

$$\begin{aligned} \text{(d). } \Pr(\text{all pass} \mid \text{at least one pass}) &= \frac{0.216}{1 - (0.4)^3} \\ &= 0.2308 \end{aligned}$$

8. Let $A = \{\text{has key for the house}\},$

$B = \{\text{unlocked}\}.$

$$\Pr(A) = \frac{3}{8}, \quad \Pr(B) = 0.4$$