ST2334 - Chapter 1 - Basic Concepts of Probability

Part 1

1.1 - Sample Space and Sample Points

1.2 - Operations with Events

1.3 - Counting Methods

1.4 - Relative frequency and definition of probability

1.1 - Sample Space and Sample Points

1.1.1 - Sample Space

1.1.2 - Sample Points

1.1.3 - Events

1.1.4 - Simple and Compound Events

$S = \{1, 2, 3, 4, 5, 6\}$

sample space

sample points

e.g. compound: odd no. = $\{1, 3, 5\}$

simple: obtain a "six" = $\{6\}$

Sample space: collection of all possible outcomes

Sample point: outcome / element of sample space

Event: subset of a sample space

Compound event: > one outcome

Simple event: exactly one outcome

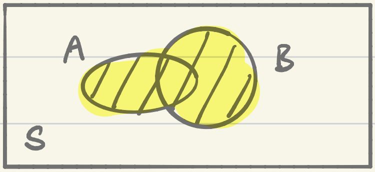
Sure event = sample space

Null event = empty set (ϕ)

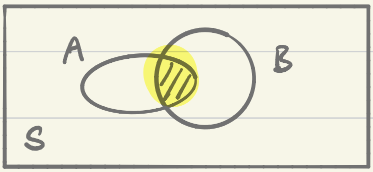
1.2 – Operations with Events

1.2.1 – Union and Intersection Events

Union : $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

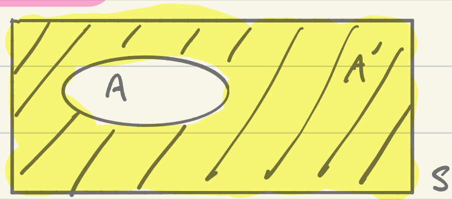


Intersection: $A \cap B = \{ x : x \in A \text{ and } x \in B \}$



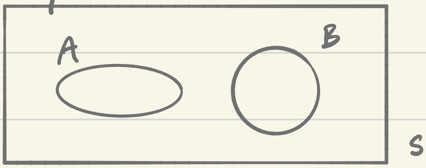
1.2.2 – Complement Event

Complement : A' or $A^c = \{ x : x \in S \text{ and } x \notin A \}$



1.2.3 - Mutually Exclusive Events

Mutually exclusive : $A \cap B = \phi$



1.2.4 - Union of n events

Union: $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x : x \in A_1 \text{ or} \dots \text{ or } x \in A_n\}$

1.2.5 - Intersection of n events

Intersection: $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x : x \in A_1 \text{ and} \dots \text{ and } x \in A_n\}$

1.2.6 - Some Basic Properties of Operations of Events

1. $A \cap A' = \phi$
2. $A \cap \phi = \phi$
3. $A \cup A' = S$
4. $(A')' = A$
5. $(A \cap B)' = A' \cup B'$
6. $(A \cup B)' = A' \cap B'$
7. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
8. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
9. $A \cup B = A \cup (B \cap A')$
10. $A = (A \cap B) \cup (A \cap B')$

1.2.7 - De Morgan's Law

1. $(A_1 \cup A_2 \cup \ldots \cup A_n)' = A_1' \cap A_2' \cap \ldots \cap A_n'$
2. $(A_1 \cap A_2 \cap \ldots \cap A_n)' = A_1' \cup A_2' \cup \ldots \cup A_n'$

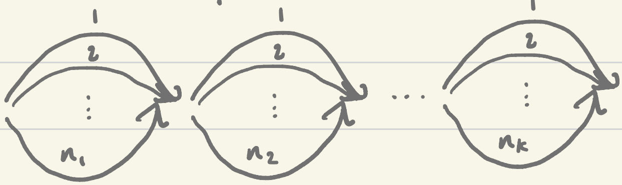
1.2.8 - Contained ($\subset$)

- $A \subset B$
- If $A \subset B$ and $B \subset A$, then $A = B$

1.3 – Counting Methods

1.3.1 – Multiplication Principle

Generalized Multiplication Rule : $n_1 n_2 \dots n_k$



1.3.2 – Addition Principle

Generalized Addition Principle : $n_1 + n_2 + \dots + n_k$

(assuming no two procedures may be performed together)

1.3.3 – Permutation

n distinct objects : $n!$

r boxes

1.3.3.1 – Permutations of n distinct objects taken r at a time

$$_nP_r = n(n-1)(n-2)\dots(n-(r-1)) = \frac{n!}{(n-r)!}$$

1.3.3.2 – Permutations of n distinct objects arranged in a circle

$(n-1)!$

1.3.3.3 - Permutations when not all n objects are distinct

$n_1 + n_2 + \dots + n_k = n$

$${}_nP_{n_1, n_2, \dots, n_k} = \frac{n!}{n_1!\, n_2! \dots n_k!}$$

1.3.4 — Combination

- $\binom{n}{r}$ or ${}_nC_r$ or C_r^n
- n distinct objects taken r at a time : $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$
- ${}_nC_r\, r! = {}_nP_r$

$\Rightarrow {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!} \frac{1}{r!} = \frac{n!}{r!\,(n-r)!}$

- Binomial coefficient = $\binom{n}{r}$

① $\binom{n}{r} = \binom{n}{n-r}$ for $r = 0, 1, \dots, n$

② $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ for $1 \le r \le n$

③ $\binom{n}{r} = 0$ for $r < 0$ or $r > n$

1.4 - Relative frequency and definition of probability

1.4.1 - Introduction

1.4.2 - Relative Frequency

- Relative frequency of event A in n repetitions of E : $f_A = \frac{n_A}{A}$
- Properties of f_A :

① $0 \leq f_A \leq 1$

② $f_A = 1$ iff A occurs every time among the n repetitions

③ $f_A = 0$ iff A never occurs among the n repetitions

④ If A and B are mutually exclusive, then $f_{A \cup B} = f_A + f_B$

⑤ f_A "stabilizes" near some definite numerical value as the experiment is repeated more and more times

1.4.3 - Axioms of Probability

Axiom ① : $0 \leq Pr(A) \leq 1$

Axiom ② : $Pr(S) = 1$

Axiom ③ : If $A_1, A_2, \ldots$ are mutually exclusive (disjoint) events, then

$$Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} Pr(A_i)$$

$\Rightarrow$ In particular, $Pr(A \cup B) = Pr(A) + Pr(B)$