Q1(c), (d).
Q2
Q5(b).
Q6.
Q8.

ST2334 – Tutorial 11 Week 13

1(a). $X \sim N(800, 40)$

$H_0 : \mu = 800$

$H_1 : \mu \neq 800$

critical value $= z_{\alpha/2} = z_{0.025} = 1.960$

$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{788 - 800}{40/\sqrt{30}} = -1.643$

Since $-z_{\alpha/2} < z < z_{\alpha/2}$, there is no evidence.

(b). $\alpha = 0.05$

$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$788 - 1.960\left(\frac{40}{\sqrt{30}}\right) < \mu < 788 + 1.960\left(\frac{40}{\sqrt{30}}\right)$

$773.686 < \mu < 802.314$

Yes.

(c).

(d).

Inaccurate values.

2(a).

X	10.2	9.7	10.1	10.3	10.1	9.8	9.9	10.4	10.3	9.8
X^2	104.04	94.09	102.01	106.09	102.01	96.04	98.01	108.16	106.09	96.04

$E(X) \xrightarrow{} \bar{x}$ $= 10.06$, $E(X^2) = 101.258$, $V(X) \xrightarrow{} s^2$ $= E(X^2) - [E(X)]^2 = 0.0544$

$\alpha = 0.01$, $H_0: \mu = 10$, $H_1: \mu \neq 10$

crit. value $= t_{9;0.005} = 3.250$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{10.06 - 10}{\sqrt{0.0544}/\sqrt{10}} = 0.813$$

Since $|t| < t_{9;0.005}$, H_0 is not rejected

(b). $H_0: \sigma^2 = 0.03$, $H_1: \sigma^2 \neq 0.03$

$\chi^2_{n-1;1-\alpha/2} = 1.73$ $\chi^2_{n-1;\alpha/2} = 23.6$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{9(0.0544)}{0.03} = 16.32$$

Since $\chi^2_{n-1;1-\alpha/2} < \chi^2 < \chi^2_{n-1;\alpha/2}$, H_0 is not rejected.

(c).

3. $H_0 : \sigma^2 = 1.15$, $H_1 : \sigma^2 > 1.15$

$n = 25$, $S^2 = 2.03$, $\alpha = 0.05$

$\cancel{\chi^2_{n-1;\,1-\alpha/2} = 12.4}$ $\chi^2_{n-1;\,\cancel{\alpha/2}\,\alpha} = 39.4$ 36.415

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{24(2.03)}{1.15} = 42.365 > \chi^2_{n-1;\,\cancel{\alpha/2}\,\alpha}$$

$\therefore$ H_0 is rejected $\Rightarrow$ out of control

4 (a). $H_0 : \mu_A - \mu_B = 12$, $H_1 : \mu_A - \mu_B > 12$

$n_A = n_B = 50$, $\bar{x}_A = 86.7$, $\sigma_A = 6.28$,

$\alpha = 0.05$, $\bar{x}_B = 77.8$, $\sigma_B = 5.61$

crit. value $= z_{\cancel{\alpha/2}\,\alpha} = 1.960$ 1.645

$$z = \frac{\bar{x}_A - \bar{x}_B - 12}{\sqrt{\sigma_A^2/n_A + \sigma_B^2/n_B}} = \frac{86.7 - 77.8 - 12}{\sqrt{6.28^2/50 + 5.61^2/50}} = -2.603 < z_{\cancel{\alpha/2}\,\alpha}$$

$\therefore$ H_0 is not rejected.

(b). Type $\cancel{\text{I}}$ II error.

5 (a). $n_1 = 12, \quad \bar{x}_1 = 84, \quad S_1 = 4$

$n_2 = 18, \quad \bar{x}_2 = 77, \quad S_2 = 6$ $\Big\} \alpha = 0.01$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{197}{7}$$

0.2763

$$(\bar{x}_1 - \bar{x}_2) - t_{n_1+n_2-2;\,\alpha/2}\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) - t_{n_1+n_2-2;\,\alpha/2}\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

$$1.537 < \mu_1 - \mu_2 < 12.463$$

(b). $H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 > 0$

$\alpha = 0.05$

crit. value $= t_{n_1+n_2-2;\,\alpha} = 1.701$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{S_p^2/n_1 + S_p^2/n_2}} = 3.5406 > 1.701$$

$\therefore$ Enough evidence ?

6(a). Wa lazy

(a). $\bar{x}_d = 0.1417$

$S_d = 0.1975$

$(0.0162, 0.2672)$

(b). $t = 2.485$

$t_{11;0.05} = 1.796$

Reject H_0

7. $H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$, $H_1 : \frac{\sigma_1^2}{\sigma_2^2} > 1$, $\alpha = 0.05$

$n_1 = 11$, $S_1 = 6.1$

$n_2 = 14$, $S_2 = 5.3$

crit. value $= F_{n_1-1,\, n_2-2,\, \alpha} = 2.67$

$F = \frac{S_1^2}{S_2^2} = 1.325 < 2.67$

$\therefore$ Do not reject H_0.

8(a).

	$E(X^2)$	$E(X)$
1	9549.8	97.4
2	12882.857	110

$F = 0.0863$

$n_1 = 5$ $S_1^2 = 63.04$

$n_2 = 7$ $S_2^2 = 782.857$ Not accurate?

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$, $H_1 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$, $\alpha = 0.05$

$F_{n_1-1,\, n_2-2;\, 1-\alpha/2} = 0.11$ $F_{n_1-1,\, n_2-2;\, \alpha/2} = 6.23$

$F = \frac{S_1^2}{S_2^2} = 0.0805$

p-value $= \Pr(F < 0.0805) = 0.01$

Since $p < \alpha$, H_0 is rejected.

(b). CI : $(0.012921, 0.7406)$ Not accurate?

$(0.01385, 0.79375)$

(c).

$(0.1177, 0.8909)$

9. $E(W) = a_1\mu_1 + a_2\mu_2 + \ldots + a_n\mu_n$

$V(W) = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \ldots + a_n^2\sigma_n^2$