CS2100 - L16 - Boolean Algebra

Week 9 + 10

16.1 - Basic boolean operations

16.2 - Basic laws

16.3 - Basic theorems

16.4 - Duality

16.5 - Boolean functions

- Complement of functions
- 2-variable boolean functions

16.1 - Basic boolean operations

Three Basic Boolean Operations

NOT	AND	OR
Negation	Conjunction	Disjunction
~A	A · B	A + B
A'	A ∧ B	A ∨ B
$\overline{A}$		

For CS2100

Be consistent with notation!

- Operator Precedence: **NOT > AND > OR**
- Example of ***boolean expressions***:
  - **A + ~B . C = A + ((~B) . C)**

16.2 - Basic laws

Boolean Algebra Laws

Identity laws	
$A + 0 = 0 + A = A$	$A \cdot 1 = 1 \cdot A = A$
Inverse/complement laws	
$A + A' = 1$	$A \cdot A' = 0$
Commutative laws	
$A \cdot B = B \cdot A$	$A + B = B + A$
Associative laws	
$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Distributive laws	
$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$

16.3 - Basic theorems

Idempotency	
$X + X = X$	$X \cdot X = X$
Zero and One elements	
$X + 1 = 1$	$X \cdot 0 = 0$
Involution	
$(X')' = X$	
Absorption	
$X + X \cdot Y = X$	$X \cdot (X + Y) = X$
Absorption (variant)	
$X + X' \cdot Y = X + Y$	$X \cdot (X' + Y) = X \cdot Y$

DeMorgan's	
$(X + Y)' = X' \cdot Y'$	$(X \cdot Y)' = X' + Y'$
*Can be generalized to more than two variables, e.g. $(A + B + \ldots + Z)' = A' \cdot B' \cdot \ldots \cdot Z'$	
Consensus	
$X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$	$(X+Y) \cdot (X'+Z) \cdot (Y+Z) = (X+Y) \cdot (X'+Z)$

16.4 - Duality

Duality

- If the AND/OR operators and identity elements 0/1 in a **Boolean equation** are interchanged, it remains valid.

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$

$$\Updownarrow$$

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

- Two-for-one: If you prove one theorem, its dual form is also true!
  - e.g.

$$(x + y + z)' = x' \cdot y' \cdot z' \quad \Updownarrow \quad (x \cdot y \cdot z)' = x' + y' + z'$$

$$x + 0 = x \quad \Updownarrow \quad x \cdot 1 = x$$

16.5 - Boolean functions

Boolean Functions (Logic Equations)

- Function of the form $f = B^k \rightarrow B$
  - **Input** (i.e. Domain): ***k*** number of Boolean variables
  - **Output** (i.e. Range): Boolean value

`F1(x,y,z) = x·y·z'`

`F2(x,y,z) = x + y'·z`

`F3(x,y,z) = x'·y'·z + x'·y·z + x·y'`

`F4(x,y,z) = x·y' + x'·z`

x	y	z	F1	F2	F3	F4
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

From the truth table, **F3 = F4**

Try to prove **F3 = F4** by using Boolean Algebra?

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Complement of Function

- **F'**, the ***complement function*** of **F**
  - Obtained by interchanging **1** with **0** in the function's output values

- E.g.: `F1(x,y,z) = x·y·z'`

- What is **F1'** ?

```
F1'
= (x·y·z')'
= x' + y' + (z')'     (DeMorgan's)
= x' + y' + z         (Involution)
```

x	y	z	F1	F1'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

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