ST2334 - Tutorial 8

Q1(c).
Q2(d).
Q3(c). } Applying Chebyshev's
Q6(b).
Q11(c).

Week 10

1. Let X be the number of failures due to operator error.
$X \sim B(20, 0.3)$

(a). $Pr(X \geqslant 10) = 0.04796$

(b). $Pr(X \leqslant 4) = 0.2375$

(c). $Pr(X = 5) = 0.1789$

Assume $p = 0.3$
$Pr(X=5) = 0.1789$ not small
$X = 5$ is not a rare event
$\Rightarrow p = 0.3$ is a reasonable event

2. Let X be the number of blowouts.
$X \sim B(15, 0.25)$

(a). $Pr(X = 0) = \cancel{0.1336}$ 0.0134

Just read wrongly I think

(b). $Pr(X \geqslant 8) = 0.01730$

(c). $E(X) = 15(0.25)$
$= 3.75$

$1 \leq X \leq 7$

(d). Chebyshev's inequality: $Pr(|X - \mu| < k\sigma) \geqslant 1 - \frac{1}{k^2}$

$\sigma = \sqrt{15(0.25)(1-0.25)}$
$= \frac{3\sqrt{5}}{4}$

$$\Rightarrow Pr\left(|X - 3.75| < k \cdot \frac{3\sqrt{5}}{4}\right) \geqslant 1 - \frac{1}{k^2}$$

Given: $1 - \frac{1}{k^2} = \frac{3}{4}$

$-\frac{1}{k^2} = -\frac{1}{4}$

$k = 2$

3. Let X be the number of forms with an error.

$X \sim B(10\,000, 0.001)$

(a). $Pr(6 \leq X \leq 8) = Pr(X \leq 8) - Pr(X \leq 5)$

$= 0.3327 - 0.06699$

$= 0.2657$

(b). $\mu = 10\,000(0.001) = 10$

$\sigma^2 = 10\,000(0.001)(1-0.001) = 9.99$

$1 \leq X \leq 19$

(c). $Pr(|X-\mu| < 3\sigma) > 1 - \frac{1}{3^2} = \frac{8}{9}$

$\mu - 3\sigma < X < \mu + 3\sigma$

$\mu \pm 3\sigma = 10 \pm 3\sqrt{9.99}$

Then round off

4. Let X be the number of people interviewed.

$X \sim NB(5, 0.3)$

$Pr(X = 10) = 0.05146$

5. Let X be the number of children the couple has.

$X \sim NB(2, 0.5)$

(a). $Pr(X = 7) = 0.04688$

(b). $E(X) = \frac{2}{0.5}$

$= 4$

6. Let X be the number of rounds.

$Pr(\text{success}) = Pr(\text{at least one different}) = 1 - Pr(\text{all same})$

$= 1 - 2\left(\frac{1}{2}\right)^3 = \frac{3}{4}$

$X \sim NB\left(1, \frac{3}{4}\right)$

(a). $Pr(X < 4) = Pr(X=1) + Pr(X=2) + Pr(X=3)$

$= 0.75 + 0.1875 + 0.046875$

$= \frac{63}{64}$

$1-\left(\frac{1}{4}\right)^x$ (b). General formula:

$Pr(X = x) = (1-p)^{x-1} \cdot p$

$Pr(X \leq x) = \sum_{n=1}^{x} \left(\frac{1}{4}\right)^{n-1} \cdot \frac{3}{4}$

$= \frac{3}{4} \sum_{n=1}^{x} \left(\frac{1}{4}\right)^{n-1}$ GP

$= \frac{3}{4}\left(\frac{1-\left(\frac{1}{4}\right)^x}{1-\frac{1}{4}}\right)$

$= 1 - \left(\frac{1}{4}\right)^x$

7. Let X be the number of errors per page.

$X \sim Po(2)$

(a). $\sigma^2 = 2$

(b). $Pr(X \geq 4) = 0.1429$

$Pr(X=0) = 0.1353$

8. Let X be the number of emergencies per hour.

$X \sim Po(5)$

(a). $Pr(X=0) = 0.00674$

(b). $Pr(X > 10) = 0.01370$

(c). Let Y be the number of emergencies in a 3-hour shift.

$Y \sim Po(15)$

$Pr(Y > 20) = 0.08297$

9. Let X be the number of cars with the defect.

$X \sim B(10000, 0.0005)$ → Can use Poisson to Binom. approx.

$X \sim Po(\lambda = np)$

(a). $E(X) = 10000(0.0005) = 5$

$\sigma = \sqrt{10000(0.0005)(1-0.0005)} = 2.236$

(b). $Pr(X \geq 10) = 0.03179$

(c). $Pr(X=0) = 0.006730$

10 (a). $f_X(x) = \begin{cases} \frac{1}{4} & 0 \le x \le 4; \\ 0 & \text{otherwise.} \end{cases}$

(b). $Pr(X \geq 3) = (4-3)(\frac{1}{4})$
$= \frac{1}{4}$

(c). $E(X) = \frac{0+4}{2} = 2$
$V(X) = \frac{1}{12}(4-0)^2 = \frac{4}{3}$

11. Let X be the length to serve a person.
$f_X(x) = \begin{cases} \frac{1}{4}e^{-\frac{1}{4}x} & x > 0; \\ 0 & \text{otherwise} \end{cases}$

i.e. $X \sim \exp(\frac{1}{4})$.

(a). $Pr(X > 3) = 0.4724$

(b). $Pr(X < 3) = 0.5276$

0.3968 (c). Let Y = no. of days being served in < 3 min.
$Y \sim B(6, 0.5276)$
$Pr(Y \geq 4) = 0.3968$