On the Chua Circuit

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1 Introduction

The Chua Circuit is an electronic circuit named after Leon Chua, who suggested it in 1983. It was reported by Matsumoto in 1984 [3] that (a simplified version of) this system exhibited chaotic behaviour in the form of a chaotic attractor, although it is a simple autonomous circuit.

The circuit in question is shown in figure 1. It contains four passive elements – two capacitors C_1, C_2, one inductor L and a resistor G – and one non-linear resistor R. In the original circuit the non-linearity of R was given by a three-component piecewise linear function, dependent on the voltage over the resistor [3, 4, 1], giving the current through it. We will, however, instead study a resistor with a non-linearity that is given by the function

$$\phi(v) = \rho v^3 + \sigma v \tag{1}$$

as suggested in [2, p. 379]. The study will also follow the "exploration" given in this publication and expand on it.

The dynamics of the original circuit can be described by the following equations [4, eq. (1.1)]:

$$\begin{cases} C_1 \dfrac{dv_{C_1}}{dt} = G(v_{C_2} - v_{C_1}) - \phi(v_{C_1}) \\ C_2 \dfrac{dv_{C_2}}{dt} = G(v_{C_1} - v_{C_2}) + i_L \\ L \dfrac{di_L}{dt} = -v_{C_2} \end{cases} \tag{2}$$

This set of equations can be rescaled to transform it into:

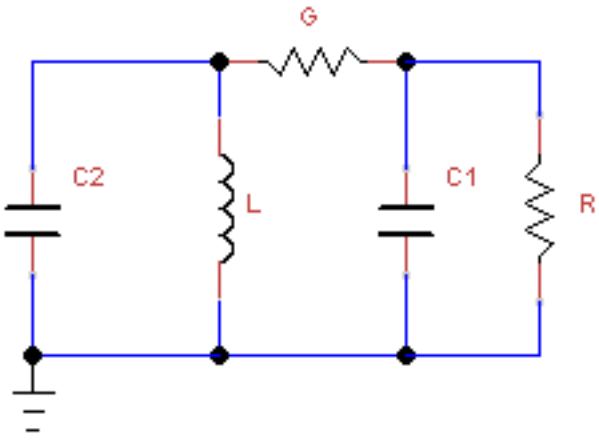

Figure 1: The Chua Circuit. The resistor R has a non-linear dependence on the voltage. After [3, 4].

$$\begin{cases} \dfrac{dx}{d\tau} = a(y - x - \phi(x)) \\ \dfrac{dy}{d\tau} = x - y + z \\ \dfrac{dz}{d\tau} = -by \end{cases} \tag{3}$$

We will now, in accordance with [2], modify this circuit by taking $\phi(x)$ as in eq. 1 and looking at the following system:

$$\begin{cases} x' = a(y - \phi(x)) \\ y' = x - y + z \\ z' = -bz \end{cases} \tag{4}$$

As suggested in [2], we will take $\rho = \frac{1}{16}, \sigma = -\frac{1}{6}$ for the parameters in eq. 1.

References

[1] L. O. Chua, M. Komuro, and T. Matsumoto. The double scroll family. *IEEE Transactions on Circuits and Systems*, 33(11):1072–1118, November 1986.

[2] M. W. Hirsh, S. Smale, and R. L. Devaney. *Differential Equations, Dynamical Systems, and an Introduction to Chaos*. Academic Press, 3rd edition, 2012. ISBN: 9780123820112.

[3] T. Matsumoto. A chaotic attractor from chua's circuit. *IEEE Transactions on Circuits and Systems*, 31(12):1055–1058, December 1984.

[4] T. Matsumoto, L. O. Chua, and M. Komuro. The double scroll. *IEEE Transactions on Circuits and Systems*, 32(8):797–818, August 1985.