One-Way ANOVA Homework

1. One of the symptoms of depression is a lack of enthusiasm for an activity that would normally be enjoyable. A medication was developed that was designed to address this problem. Fifteen volunteers were randomly divided into three groups of five each. One group received a low dose of the medicine, one group received a high dose, and one group received a placebo. The experiment was conducted as a double-blind study. Here are the results, where the response variable y is a clinical measure of enthusiasm:

Placebo	Low Dose	High Dose
2	5	7
3	2	4
1	4	5
1	2	3
4	3	6

(a) Why was it important to use a double-blind study?

This is important to prevent researches from unintentionally tipping off participants or biasing their results

P: Placebo
L: Low dose
H: High dose

(b) Make a dotplot of the data on the number line below, using different colors for each of the three treatments.

0	1	2	3	4	5	6	7	8
			H	H				
	P	L	L	L	H			
	P	P	P	P	L	H	H	

How strongly does the plot suggest that the medication has an effect?

There appears to be good evidence that the medication is effective at higher doses.

(c) Without the use of statistical software, perform a one-way analysis of variance. Show the group means and the grand mean, construct the ANOVA table, and test the hypothesis that the medication has no effect (also write the null hypothesis in parameter notation).

	P	L	H	
	2	5	7	
	3	2	4	
	1	4	5	
	1	2	3	
	4	3	6	
Means	2.2	3.2	5	3.467

Grand Mean

ANOVA

Source	df	Sum of Squares	Mean Square	F
Groups	2	20.31	10.155	5.163
Error	12	23.6	1.967	
Total	14	43.91		

$$SS_B = \sum_{i=1}^{3} J_i(\bar{Y}_{i.} - \bar{Y}_{..})^2 = 5\left[(2.2-3.467)^2 + (3.2-3.467)^2 + (5-3.467)^2\right]$$

$$= 5(1.605 + 0.0712 + 2.350) \boxed{\approx 20.131}$$

$$SS_W = \sum_i \sum_j (Y_{ij} - \bar{Y}_{i.})^2 = (2-2.2)^2 + (3-2.2)^2 + \ldots + (6-5)^2$$

$$= 0.64 + 0.04 + 1.44 + 1.44 + 3.24 + \ldots$$

$$SS_w = 0.04 + 0.64 + 1.44 + 1.44 + 3.24$$
$$+ 3.24 + 1.44 + 0.64 + 1.44 + 0.04$$
$$+ 4 + 1 + 0 + 4 + 1$$
$$= 23.6$$

$H_0: \alpha_P = \alpha_L = \alpha_H = 0.$

Because $5.16 \gg 1$, we can say that there is good evidence that the medication is effective.

(w/ technology p-value is 0.024).

$k = \binom{I}{2} = \binom{3}{2} = 3$

(d) Find the margin of error of a Bonferroni 95% confidence interval to compare any two groups means, and use it to determine if any of the group means can be considered significantly different.

Recall the moe for Bonferroni is given by $t_{I(J-1), \frac{\alpha}{2k}} \sqrt{\frac{2}{J}} \cdot s_p$.

$t_{I(J-1), \frac{\alpha}{2k}} = 2.7794$

$\sqrt{\frac{2}{J}} = 0.6323$

$s_p = \sqrt{1.967} = 1.4025$

$\Rightarrow$ moe ≈ 2.465

Groups 1 & 2

$2.2 - 3.2 \pm 2.465 \Rightarrow (-3.47, 1.4\ldots$

Groups 2 & 3

$3.2 - 5 \pm 2.465 \Rightarrow (-4.27, 0.66\ldots$

So groups 1 & 3 are significantly different.

Groups 1 & 3 : $2.2 - 5 \pm 2.465 \Rightarrow (-5.265, -0.335)$.

(e) What is the alternative for the F-test here? Also, can you think of another design that would be more appropriate for this study, and how would the ANOVA for that design differ?

Kruskal – Wallis Test.

Another design would be Randomized Complete Block Designs

2. For the worms data in Problem 21, pp. 508-509, the summary statistics are given below:

	Group 1	Group 2	Group 3	Group 4
n	5	5	5	5
Mean	290.40	323.20	274.80	371.20
S.D.	56.99	67.05	67.98	60.20

$\bar{Y}_{..} = 314.9$

Perform a one-way analysis of variance, without referring to the original data.

$SS_W = 4(56.99^2 + 67.05^2 + 67.98^2 + 60.20^2)$ → 63,955.5

$SS_B = 5((290.40 - 314.9)^2 + \ldots + (371.20 - 31\ldots$

$= 5(600.25 + 68.89 + 1608.01 + 3,169.7\ldots$

$= 5(5466.84) \approx 27,234.20$

ANOVA

Source	df	Sum of Squares	Mean Square	F
Groups	3	27,344.20	9,114.73	2.28
Error	16	63,955.53	3,997.22	
Totals	19	91,289.73		

$1 - pf(2.28, 3, 16) \approx 0.1185$