

Math 440B HW 1

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Problem 8

- When $n = m = 20$, the sample size for both the control and treatment groups is below 30, thus we cannot apply the Central Limit Theorem to assume that the data is normally distributed. Therefore, the t-test would not be appropriate to use in this case.
- It would be appropriate to use the Mann-Whitney Test with $n = m = 20$. This is because Mann-Whitney does not require a normality assumption of the data, thus making it an effective test, especially for this small sample size.

Problem 12

Part 1: When comparing two samples, if the constructed confidence interval contains 0, then there is insufficient evidence to suggest that there is a difference between both samples.

This is similar to the sign test because the sign test assumes that the probability of getting either a positive or negative value is 50%. If $\eta = 0$, means that the differences of the samples has a median of zero. This median of zero implies that there are equal amounts of positive and negative values, which is similar to the sign test assumption of 50/50 positive and negative values.

Part 3: The data has 14 positive values and 10 negative values. Let $W = Y_i - X_i > 0$ for $i = \{1, \dots, 24\}$. Because we are using the sign test, $H_0 : p = 0.5$. Assuming the null hypothesis is true, then $W \sim \text{bin}(24, 0.5)$. Thus, I want to know if there is sufficient evidence to suggest the positive and negative values are evenly split.

WTFind $P(W \geq 14) = 1 - P(W \leq 13)$

```
1 - pbinom(13, size = 24, prob = 0.5)
```

```
## [1] 0.2706281
```

Above we can see this value is about 0.271. The book suggests the use of a two-sided tail test, thus $0.271 * 2 = 0.542$. This p-value is huge, thus not significant. There is insufficient evidence to suggest that the probability of getting a positive or negative difference is not 50%.

Problem 24

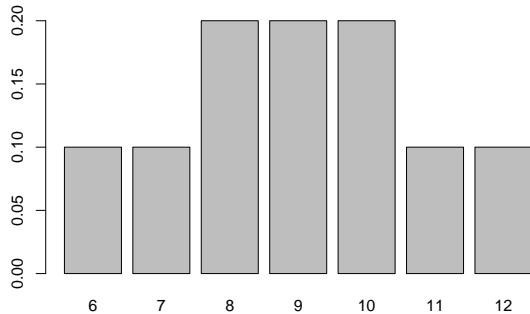
WTFind the null distribution of U_Y when $m = 3$ and $n = 2$. Recall n is the treatment group and m is the control group. This means that there are $\binom{m+n}{m} = \binom{5}{3} = 10$ different assignments for the control group.

The different types of ranks are as follows: $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 2, 5\}$, $\{1, 3, 4\}$, $\{2, 3, 4\}$, $\{1, 3, 5\}$, $\{2, 3, 4\}$, $\{1, 4, 5\}$, $\{2, 3, 5\}$, $\{2, 4, 5\}$, $\{3, 4, 5\}$.

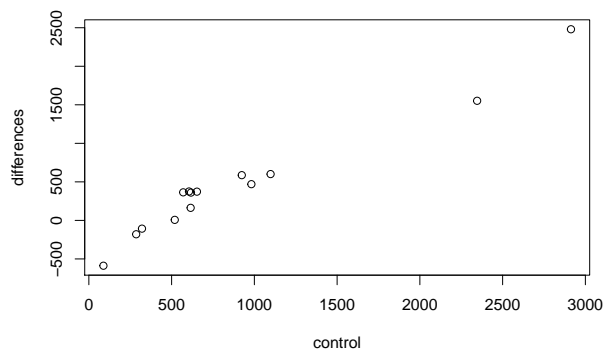
Their corresponding sum of ranks: 6, 7, 8, 8, 9, 9, 10, 10, 11, 12

This means that the probability of getting 6, 7, 11, or 12 as a sum of ranks each have a $\frac{1}{10}$ chance of occurring, while getting a 8, 9, or 10 have a $\frac{1}{5}$ chance of occurring.

The corresponding bar graph of the null distribution will appear as follows:



Problem 39



a. To summarize the above plot, as the size of the control rate increases, so does the size of the differences between the control and test groups.

```
##
## Paired t-test
##
## data: control and test
## t = 2.2776, df = 13, p-value = 0.0403
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## 23.73969 898.83174
## sample estimates:
## mean of the differences
## 461.2857

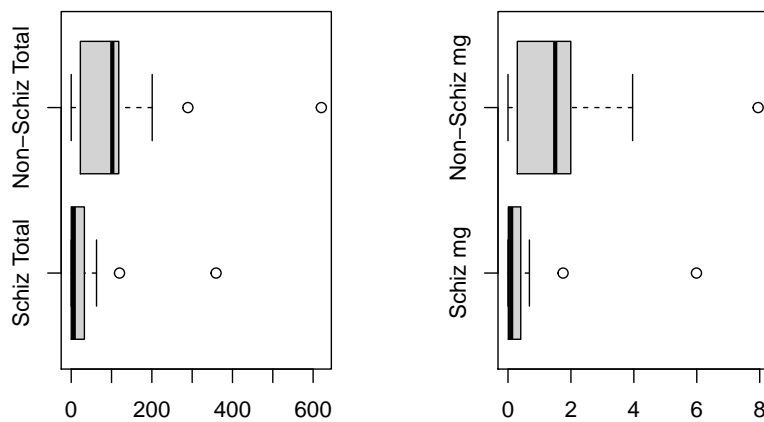
##
## Wilcoxon signed rank exact test
##
```

```
## data: control and test
## V = 88, p-value = 0.02454
## alternative hypothesis: true location shift is not equal to 0
```

d. The nonparametric method would be the most appropriate method to test the differences between test and control. This is because the sample size of both the test and control are too small to assume that normality is present.

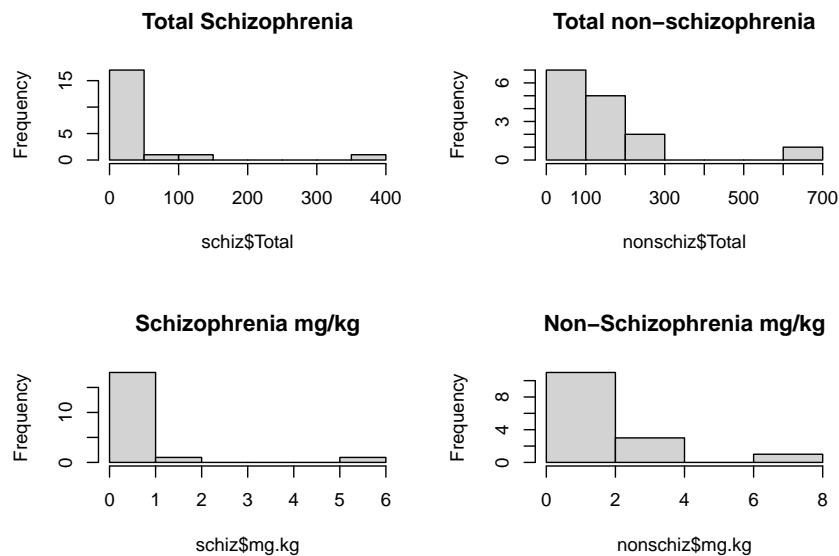
Running both tests shows the same result, that there is a significant difference between test and control.

Problem 44



```
##      Total      mg.kg
## Min.   : 0.00   Min.   :0.0000
## 1st Qu.: 0.10   1st Qu.:0.0100
## Median : 6.30   Median :0.1000
## Mean   : 35.78   Mean   :0.5295
## 3rd Qu.: 32.40   3rd Qu.:0.3975
## Max.   :359.30   Max.   :5.9900
```

```
##      Total      mg.kg
## Min.   : 0.0    Min.   :0.000
## 1st Qu.: 22.7    1st Qu.:0.295
## Median :101.9    Median :1.500
## Mean   :123.0    Mean   :1.779
## 3rd Qu.:117.9    3rd Qu.:1.995
## Max.   :620.4    Max.   :7.950
```



d. See the above plots. Each of the plots appeared skewed (mostly to the right).

```
t.test(schiz$Total,nonschiz$Total)
```

```
##
## Welch Two Sample t-test
##
## data: schiz$Total and nonschiz$Total
## t = -1.9403, df = 19.502, p-value = 0.06694
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -181.085618 6.698951
## sample estimates:
## mean of x mean of y
## 35.7800 122.9733
```

```
t.test(schiz$mg.kg,nonschiz$mg.kg)
```

```
##
## Welch Two Sample t-test
##
## data: schiz$mg.kg and nonschiz$mg.kg
## t = -2.0252, df = 22.503, p-value = 0.05487
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -2.5280682 0.0284015
## sample estimates:
## mean of x mean of y
## 0.529500 1.779333
```

e. The t-test was applied on both groups and on both their variables. The results are output above. However, performing a t-test is unreasonable since the data does not appear to be normally distributed, given the above plots. Therefore the output from such a test has dubious results.

```
wilcox.test(schiz$Total,nonschiz$Total)
```

```
## Warning in wilcox.test.default(schiz$Total, nonschiz$Total): cannot compute  
## exact p-value with ties
```

```
##  
## Wilcoxon rank sum test with continuity correction  
##  
## data: schiz$Total and nonschiz$Total  
## W = 76, p-value = 0.01404  
## alternative hypothesis: true location shift is not equal to 0
```

```
wilcox.test(schiz$mg.kg,nonschiz$mg.kg)
```

```
## Warning in wilcox.test.default(schiz$mg.kg, nonschiz$mg.kg): cannot compute  
## exact p-value with ties
```

```
##  
## Wilcoxon rank sum test with continuity correction  
##  
## data: schiz$mg.kg and nonschiz$mg.kg  
## W = 74, p-value = 0.01156  
## alternative hypothesis: true location shift is not equal to 0
```

f. The results from the Mann-Whitney test are completely different from the results of the t-test. This is expected as the data was not normally distributed to begin with. The results from the Mann-Whitney test show that there is sufficient evidence to suggest that both the schizophrenic and non-schizophrenic do have a systematic difference in terms of Vitamin C and waste production (mg/kg).

Additional Question

Two methods for testing for presence of a disease were compared. The traditional method was used on 8 subjects and the new method was used on 2 subjects. When the times to obtain results were compared for all 10 tests, the fastest and third fastest results were obtained for the subjects that received the new method. Determine the exact p-value for a one-sided test. Do the data provide statistically significant evidence at $\alpha = 5\%$ that the new method tends to be faster than the traditional one?

Response: Notice that there are $\binom{10}{2} = 45$ ways to have ranks placed in the new method. From what's given in the question, I know that the sum of the ranks for the new method is 4. This means that the p-value to test if the new method is effective comes from $P(r^* \leq 4)$ where r^* is the sum of the ranks. There is only one way to get $r^* = 4$ and one way to get $r^* = 3$, then this is a sum of $\frac{1}{45} + \frac{1}{45} = 0.0444\dots$ Thus this p-value is enough to suggest that the new method is faster than the traditional method.

However, there does seem to be something off about this whole statistical measure. It would be better if the sample size was larger overall.