Section 14.1

Problem 1: This problem asks to convert the given relationships into linear relationships.

a) $y = \frac{a}{b+cx}$. Notice, $(a,b,c)$ are constants

$y^{-1} = \frac{b+cx}{a} = \frac{b}{a} + \frac{c}{a}x. \Rightarrow$

Let $y^{-1} = y'$; $\frac{b}{a} = \beta_0$; $\frac{c}{a} = \beta_1$. Then $y' = \beta_0 + \beta_1 x$.

b) $y = ae^{-bx}$. Where $(a,b)$ are constants.

$\ln(y) = \ln(a) - bx$. Let $\ln(y) = y'$, $\ln(a) = \beta_0$, $-b = \beta_1$.

So $y' = \beta_0 + \beta_1 x$.

c) $y = ab^x$ $(a,b)$ are constants.

$\ln(y) = \ln(a) + x\ln(b)$.

$\ln(y) = y'$
$\ln(a) = \beta_0$
$\ln(b) = \beta_1$

$y' = \beta_0 + \beta_1 x$.

d) $y = \frac{x}{a+bx}$. $\Rightarrow y^{-1} = \frac{a+bx}{x} = \frac{a}{x} + b$.

$y^{-1} = ax^{-1} + b$. This looks pretty as it is.

e) $y = \frac{1}{1+e^{bx}}$. $\Rightarrow y^{-1} = 1 + e^{bx} \Rightarrow y^{-1} - 1 = e^{bx}$.

$\Rightarrow \ln(y^{-1} - 1) = bx$.

$\ln(y^{-1} - 1) = y'$
$b = \beta_1$

$y' = \beta_1 x$

5. Find the LSE of $\beta$ for fitting the line $y = \beta x$ to the points $(x_i, y_i)$, $i = \{1, 2, \dots, n\}$.

Recall $S = \sum_i (y_i - g(x_i))^2$ is what we need to estimate.

$$S = \sum (y_i - \beta x_i)^2$$

$$\frac{dS}{d\beta} = -2\sum (y_i - \beta x_i)x_i = 0$$

$$= \sum_i (y_i - \beta x_i)(x_i) = 0.$$

$$= \sum (x_i y_i) - \beta (x_i)^2 = 0. \quad \text{(divide both sides by } n\text{)}.$$

$$= \overline{xy} - \beta \overline{x^2} = 0$$

$$\boxed{\beta = \frac{\overline{xy}}{\overline{x^2}}}$$

Thus this is the LS estimate.