Math 440B HW 3

Erick Castillo

2/12/2021

Problem 65

I begin with the given information:

- The sample size is $n = 20$ which has a distribution $N(\theta_0 = 10,\ \sigma_0^2 = 1)$.
- The posterior distribution is $N(\theta_{post} = 15,\ \sigma_{post}^2 = 0.1^2)$.

The question asks to find the parameters of the prior distribution given the above information. On page 291 there are two formulas given that can be used to solve this problem.

- $\xi_{post} = n\xi_0 + \xi_{prior}$
- $\theta_{post} = \frac{n\xi_0\bar{x} + \theta_{prior}\xi_{prior}}{n\xi_0 + \xi_{prior}}$

From here, I just need to plug in the values. Recall that $\xi = \frac{1}{\sigma^2}$.

Solving the first equation for ξ_{prior} grants $\xi_{prior} = \frac{1}{0.01^2} - 20(1) = 80$. Thus the standard deviation of the prior distribution is $\frac{1}{\sqrt{80}} \approx 0.1118....$

Now, solving for θ_{prior} in the second equation grants the following: $\theta_{prior} = \frac{\theta_{post}(n\xi_0 + \xi_{prior}) - n\bar{x}\xi_0}{\xi_{prior}}$.

Plugging in the given values on the right will result in $\theta_{prior} = 16.25$.

Problem 66

The problem begins by stating that a basketball player makes two shots with a basketball.

Part A:
This means the sample has a likelihood function with a binomial distribution:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Multiplying the above with the prior distribution of Unif$[0, 1]$ grants a distribution that is proportional to a beta distribution which looks like beta$(x + 1,\ n - x + 1)$. Here $x = n = 2$, thus the posterior distribution is beta$(3, 1)$.

Part B:
The probability of making the third shot is the posterior mean of the previously calculated distribution, making the probability $\frac{3}{4}$

Problem 31

Part A:
There are n^n ways to select samples with replacement given an original sample size of n.
Part B:
Given a sample size of 3, there will be 27 possible samples with replacement. They are:

$$(1,1,1); (1,1,3); (1,1,4); (1,3,1); (1,3,3); (1,3,4); (1,4,1); (1,4,3); (1,4,4)$$

$$(3,1,1); (3,1,3); (3,1,4); (3,3,1); (3,3,3); (3,3,4); (3,4,1); (3,4,3); (3,4,4)$$

$$(4,1,1); (4,1,3); (4,1,4); (4,3,1); (4,3,3); (4,3,4); (4,4,1); (4,4,3); (4,4,4)$$

Problem 32

By definition, bootstrapping begins with a sample of size n. Then, values from the sample are selected at random n times, in order to match the initial size of the sample. This process is repeated B times, and in each iteration, the MAD is calculated. From here, there will be a variety of values the MAD can take over the B iterations. A density plot can then be created to visualize the process. This density plot would be the sampling distribution of the MAD.