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Math 445

Problem 1: Suppose we have $X_1, \cdots, X_n$ random sample from Poisson. Find $\hat{\lambda}_{MLE}$.

Recall $f(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$

So $L = \prod_{i=1}^{n} f(\lambda ; x_i) = \prod_{i=1}^{n} \dfrac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \dfrac{e^{-n\lambda}\lambda^{\Sigma x_i}}{\prod x_i!}$ gives the Likelihood Function

en $\ell = \ln(L) = -n\lambda + \Sigma x_i \ln(\lambda) - \ln(\prod x_i!)$

$$\frac{d\ell}{d\lambda} = \ell' = -n + \frac{\Sigma x_i}{\lambda} = 0.$$

$$= \boxed{\hat{\lambda}_{MLE} = \frac{\Sigma x_i}{n} = \bar{x}}$$

Problem 2: Suppose $X_1, \ldots, X_n$ form a random sample with distribution

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1}, & \text{for } 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the MLE of θ.

$$L = \prod_{i=1}^{n} f(\lambda ; x_i) = \prod_{i=1}^{n} \theta x_i^{\theta-1} = \theta^{n} \cdot \prod_{i=1}^{n} x_i^{\theta-1}$$

$$\ell = \ln(L) = n\ln(\theta) + (\theta-1)\ln\left(\prod_{i=1}^{n} x_i\right) = n\ln(\theta) + \theta\ln\left(\prod_{i=1}^{n} x_i\right) - \ln\left(\prod_{i=1}^{n} x_i\right)$$

$$\frac{\partial \ell}{\partial \theta} = \ell' = \frac{n}{\theta} + \ln\left(\prod x_i\right) = 0.$$

$$= \frac{n}{\theta} = -\ln\left(\prod x_i\right)$$

$$\boxed{\hat{\theta}_{MLE} = \frac{n}{\Sigma \ln(x_i)}}$$