Homework 6:

1) Consider that people from population pi_1 have a density function f_1(x) which is Normal with mean 5 and variance 3, and people from population pi_2 have a density function f_2(x) which is Chi-Square with 6 degrees of freedom (x here is a characteristic of the people). Suppose that c(1|2) = 15 and c(2|1) = 13 and that 40% of the population is actually from pi_1: Which population should you classify a person with characteristic x = 1? What about a person with x = 4? What about a person with x = 8?

Hint: Use the dnorm and dchisq functions in R (Similar to ex 11.2 in slide 15 of classification.

$f_1(x) \sim N(5,3)$ & $f_2(x) = \chi^2(6)$

$R_1: \dfrac{f_1(x)}{f_2(x)} \geq \left(\dfrac{c(1|2)}{c(2|1)}\right)\left(\dfrac{p_2}{p_1}\right)$ & $R_2: \dfrac{f_1(x)}{f_2(x)} < \dfrac{45}{26}.$

$\geq \dfrac{15}{13} \cdot \dfrac{0.60}{0.40}$

$R_1: \dfrac{f_1(x)}{f_2(x)} \geq \dfrac{45}{26}$

Now consider $\dfrac{f_1(1)}{f_2(1)} = \dfrac{dnorm(1,5,sqrt(3))}{dchisq(1,6)} = 0.4221804.$

So this person would be classified as $\boxed{\text{part of } R_2}$

Now consider $\dfrac{f_1(4)}{f_2(4)} = \dfrac{drom(4,5,sqrt(3))}{dchisq(4,6)} = 1.440642$

Notice $\dfrac{f_1(4)}{f_2(4)} < \dfrac{45}{26}$ so this person would be classified as $\boxed{\text{part of } R_2.}$

Now consider $\dfrac{f_1(8)}{f_2(8)} = \dfrac{dnorm(8,5,sqrt(3))}{dchisq(8,6)} = 0.7014967.$

This person would also be classified as $\boxed{\text{pertaining to } R_2.}$

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2) 2) Use "lda" and "qda" in R to run a classification method for the iris dataset (just type iris in R – the dataset is within R). Run the analysis without the last 5 rows, and then use it to predict the species of the last 5 rows and check if you lda and qda get them right.

This problem will be done on RMD and submitted on a seperate PDF.

3) **11.1.** Consider the two data sets

$$\mathbf{X}_1 = \begin{bmatrix} 3 & 7 \\ 2 & 4 \\ 4 & 7 \end{bmatrix} \quad \text{and} \quad \mathbf{X}_2 = \begin{bmatrix} 6 & 9 \\ 5 & 7 \\ 4 & 8 \end{bmatrix}$$

for which

$$\bar{\mathbf{x}}_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \quad \bar{\mathbf{x}}_2 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

and

$$\mathbf{S}_{\text{pooled}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad S_{\text{pooled}}^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

(a) Calculate the linear discriminant function in (11-19).

(b) Classify the observation $\mathbf{x}_0' = [2 \quad 7]$ as population π_1 or population π_2, using Rule (11-18) with equal priors and equal costs.

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a) The linear discriminant function $\hat{y} = (\bar{x}_1 - \bar{x}_2)' S_{\text{pooled}}^{-1} x = \hat{a}' x$

So $\hat{y} = [-2, -2] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [-2 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -2x_1 - 0x_2$

So $\hat{a}' = [-2, 0]$

b). Want to classify the obs $x_0' = [2 \ 7]$. EQUAL PRIORS & EQUAL COSTS

$$\left.\begin{aligned} R_1(\pi_1) &: \hat{y}_0 - \hat{m} \geq 0. \\ R_2(\pi_2) &: \hat{y}_0 - \hat{m} < 0. \end{aligned}\right\}$$

where $\hat{y}_0 = (\bar{x}_1 - \bar{x}_2) S_{\text{pooled}}^{-1} x_0$.

$\hat{m} = \frac{1}{2}(\bar{y}_1 + \bar{y}_2)$

WTFind $\bar{y}_1$ & $\bar{y}_2$. (Recall $\bar{y}_i = \hat{a}' \bar{x}_i \ : i = \{1, 2\}$)

$\bar{y}_1 = [-2 \ 0] \begin{bmatrix} 3 \\ 6 \end{bmatrix} = -6$

$\bar{y}_2 = [-2 \ 0] \begin{bmatrix} 5 \\ 8 \end{bmatrix} = -10$

$\hat{m} = \frac{1}{2}(-6 - 10) = -8$

WTFind $\hat{y}_0$.

$\hat{y}_0 = -2x_1 - 0x_2$, where $x_1 = 2$, $x_2 = 7$

$\hat{y}_0 = -4$

Notice $\hat{y}_0 - \hat{m} = -4 - (-8)$
$= 4 \geq 0.$

So $x_0' = [2 \ 7]$ would fall in π_1

11.5. Show that

$$-\tfrac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_1)'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu}_1)+\tfrac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_2)'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu}_2) = (\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)'\boldsymbol{\Sigma}^{-1}\mathbf{x}-\tfrac{1}{2}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_1+\boldsymbol{\mu}_2)$$

LHS: $-\frac{1}{2}(x-\mu_1)'\Sigma^{-1}(x-\mu_1)+\frac{1}{2}(x-\mu_2)'\Sigma^{-1}(x-\mu_2)$.

$-\frac{1}{2}\left[(x-\mu_1)'\Sigma^{-1}(x-\mu_1)-(x-\mu_2)'\Sigma^{-1}(x-\mu_2)\right]$

$-\frac{1}{2}\left[(\cancel{x'\Sigma^{-1}x}-\mu_1'\Sigma^{-1}x-x'\Sigma^{-1}\mu_1+\mu_1'\Sigma^{-1}\mu_1)-(\cancel{x'\Sigma^{-1}x}-\mu_2'\Sigma^{-1}x-x'\Sigma^{-1}\mu_2 + \mu_2'\Sigma^{-1}\mu_2)\right]$

$\frac{1}{2}\left[(\mu_1'\Sigma^{-1}x-\mu_2'\Sigma^{-1}x)+x'\Sigma^{-1}\mu_1-x'\Sigma^{-1}\mu_2-\mu_1'\Sigma^{-1}\mu_1+\mu_2'\Sigma^{-1}\mu_2\right]$

$\frac{1}{2}\left[(\mu_1-\mu_2)'\Sigma^{-1}x+(\mu_1-\mu_2)'(x-\mu_1+\mu_2)\Sigma^{-1}.\right]$

$\frac{1}{2}(\mu_1-\mu_2)'\Sigma^{-1}\left[2x-(\mu_1-\mu_2)\right]$

$(\mu_1-\mu_2)'\Sigma^{-1}x-\frac{1}{2}(\mu_1-\mu_2)'\Sigma^{-1}(\mu_1-\mu_2)$ ▨