Information Loss in Black Holes

S.W.Hawking[*]
DAMTP, Center for Mathematical Sciences, university of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

The question of whether information is lost in black holes is investigated using Euclidean path integrals. The formation and evaporation of black holes is regarded as a scattering problem with all measurements being made at infinity. This seems to be well formulated only in asymptotically AdS spacetimes. The path integral over metrics with trivial topology is unitary and information preserving. On the other hand, the path integral over metrics with non-trivial topologies leads to correlation functions that decay to zero. Thus at late times only the unitary information preserving path integrals over trivial topologies will contribute. Elementary quantum gravity interactions do not lose information or quantum coherence.

PACS numbers: 04.70.Dy

I. INTRODUCTION

The black hole information paradox started in 1967 when Werner Israel showed that the Schwarzschild metric was the only static vacuum black hole solution [1]. This was then generalized to the no hair theorem, the only stationary rotating black hole solutions of the Einstein Maxwell equations are the Kerr Newman metrics [9]. The no hair theorem implied that all information about the collapsing body was lost from the outside region apart from three conserved quantities: the mass, the angular momentum, and the electric charge.

This loss of information wasn't a problem in the classical theory. A classical black hole would last for ever and the information could be thought of as preserved inside it, but just not very accessible. However, the situation changed when I discovered that quantum effects would cause a black hole to radiate at a steady rate [2]. At least in the approximation I was using the radiation from the black hole would be completely thermal and would carry no information[3]. So what would happen to all that information locked inside a black hole that evaporated away and disappeared completely? It seemed the only way the information could come out would be if the radiation was not exactly thermal but had subtle correlations. No one has found a mechanism to produce correlations but most physicists believe one must exist. If information were lost in black holes, pure quantum states would decay into mixed states and quantum gravity wouldn't be unitary.

I first raised the question of information loss in 75 and the argument continued for years without any resolution either way. Finally, it was claimed that the issue was settled in favor of conservation of information by ADS-CFT. ADS-CFT is a conjectured duality between string theory in anti de Sitter space and a conformal field theory on the boundary of anti de Sitter space at infinity [**?**]. Since the conformal field theory is manifestly unitary the argument is that string theory must be information preserving. Any information that falls in a black hole in anti de Sitter space must come out again. But it still wasn't clear how information could get out of a black hole. It is this question, I will address in this paper.

II. EUCLIDEAN QUANTUM GRAVITY

Black hole formation and evaporation can be thought of as a scattering process. One sends in particles and radiation from infinity and measures what comes back out to infinity. All measurements are made at infinity, where fields are weak and one never probes the strong field region in the middle. So one can't be sure a black hole forms, no matter how certain it might be in classical theory. I shall show that this possibility allows information to be preserved and to be returned to infinity.

I adopt the Euclidean approach [5], the only sane way to do quantum gravity nonperturbatively. One might think one should calculate the time evolution of the initial state by doing a path integral over all positive definite metrics that go between two surfaces that are a distance T apart at infinity. One would then Wick rotate the time interval T to the Lorentzian.

[*]Electronic address:

The trouble with this is that the quantum state for the gravitational field on an initial or final space-like surface is described by a wave function which is a functional of the geometries of space-like surfaces and the matter fields

$$\Psi[h_{ij}, \phi, t] \tag{1}$$

where h_{ij} is the three metric of the surface, ϕ stands for the matter fields and t is the time at infinity. However there is no gauge invariant way in which one can specify the time position of the surface in the interior. This means one can not give the initial wave function without already knowing the entire time evolution.

One can measure the weak gravitational fields on a time like tube around the system but not on the caps at top and bottom which go through the interior of the system where the fields may be strong. One way of getting rid of the difficulties of caps would be to join the final surface back to the initial surface and integrate over all spatial geometries of the join. If this was an identification under a Lorentzian time interval T at infinity, it would introduce closed time like curves. But if the interval at infinity is the Euclidean distance β the path integral gives the partition function for gravity at temperature $\Theta = \beta^{-1}$.

$$\begin{aligned} Z(\beta) &= \int DgD\phi e^{-I[g,\phi]} \\ &= \mathrm{Tr}(e^{-\beta H}) \end{aligned} \tag{2}$$

There is an infrared problem with this idea for asymptotically flat space. The partition function is infinite because the volume of space is infinite. This problem can be solved by adding a small negative cosmological constant Λ which makes the effective volume of the space the order of $\Lambda^{-3/2}$. It will not affect the evaporation of a small black hole but it will change infinity to anti-de Sitter space and make the thermal partition function finite.

It seems that asymptotically anti-de Sitter space is the only arena in which particle scattering in quantum gravity is well formulated. Particle scattering in asymptotically flat space would involve null infinity and Lorentzian metrics, but there are problems with non-zero mass fields, horizons and singularities. Because measurements can be made only at spatial infinity, one can never be sure if a black hole is present or not.

III. THE PATH INTEGRAL

The boundary at infinity has topology $S^1 \times S^2$. The path integral that gives the partition function is taken over metrics of all topologies that fit inside this boundary. The simplest topology is the trivial topology $S^1 \times D^3$ where D^3 is the three disk. The next simplest topology and the first non-trivial topology is $S^2 \times D^2$. This is the topology of the Schwarzschild anti-de Sitter metric. There are other possible topologies that fit inside the boundary but these two are the important cases, topologically trivial metrics and the black hole. The black hole is eternal: it can not become topologically trivial at late times.

The trivial topology can be foliated by a family of surfaces of constant time. The path integral over all metrics with trivial topology can be treated canonically by time slicing. The argument is the same as for the path integral for ordinary quantum fields in flat space. One divides the time interval T into time steps Δt. In each time step one makes a linear interpolation of the fields q_i and their conjugate momenta between their values on succesive time steps. This method applies equally well to topologically trivial quantum gravity and shows that the time evolution (including gravity) will be generated by a Hamiltonian. This will give a unitary mapping between quantum states on surfaces separated by a time interval T at infinity.

This argument can not be applied to the non-trivial black hole topologies. They can not be foliated by a family of surfaces of constant time because they don't have any spatial cross-sections that are a three cycle, modulo the boundary at infinity. Any global symmetry would lead to conserved global charges on such a three cycle. These would prevent correlation functions from decaying in topologically trivial metrics. Indeed, one can regard the unitary Hamiltonian evolution of a topologically trivial metric as a global conservation of information flowing through a three cycle under a global time translation. On the other hand, non-trivial black hole topologies won't have any conserved quantity that will prevent correlation functions from decaying. It is therefore very plausible that the path integral over a topologically non trivial metric gives correlation functions that decay to zero at late Lorentzian times. This is born out by explicit calculations. The correlation functions decay as more and more of the wave falls through the horizon into the black hole.