boris

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Abstract

boris is a python module which calculates the dispersion characteristics of spin waves (SW) in ferromagnetic films based on the perturbation theory of Kalinikos and Slavin (K&S). Additionally, by calculating contours of the dispersion surface and via an inverse Fourier transform, boris calculates the emission pattern of a SW point source contacting a ferromagnetic medium.

1 Preliminaries

K&S [1, 2] have solved Maxwell's equation in the magnetostatic limit for a medium described by the linearised Landau-Lifshitz equation, subject to electromagnetic and "exchange" boundary conditions. Using perturbation theory, K&S have obtained an explicit dispersion relation for "spin-waves" in the medium. To wit,

$$\omega_n = \sqrt{(\omega_H + \alpha\omega_M k_n^2)(\omega_H + \alpha\omega_M k_n^2 + \omega_M F_{nn})} \tag{1}$$

where

$$F_{nn} = P_{nn} + \sin^2\theta \left(1 - P_{nn}\left(1 + \cos^2\phi\right) + \omega_M \frac{P_{nn}(1 - P_{nn})\sin^2\phi}{(\omega_H + \alpha\omega_M k_n^2)}\right) \tag{2}$$

and $k_n^2 = k_\zeta^2 + \kappa_n^2$. K&S defined $\omega_H = \mu_0|g|H_i$ and $\omega_M = \mu_0|g|M_0$, where μ_0 is the permeability of vacuum, $|g|$ is the gyromagnetic ratio, H_i is the magnitude of the internal field, and M_0 is the magnitude of the saturation magnetization. The constant α is the exchange constant.

We rely on the approximation of totally unpinnned surface spins, for which P_{nn} has the explicit expression

$$P_{nn} = \frac{k_\zeta^2}{k_n^2} - \frac{k_\zeta^4}{k_n^4} F_n \frac{1}{(1 + \delta_{0n})} \tag{3}$$

whereby

$$F_n = \frac{2}{k_\zeta L}[1 - (-1)^n e^{-k_\zeta L}]. \tag{4}$$

In this approximation, $\kappa_n = \frac{n\pi}{L}$ where L is the film thickness. Moreover, we write P_{nn} in the diagonal approximation, having taken $n = n'$. Lastly, we exclusively consider modes with a uniform profile across the film thickness, i.e. everywhere $n = 0$.

K&S utilize two coordinate systems. The first (ξ, η, ζ) system is oriented such that the ξ direction lies parallel to the film normal vector, if the film is considered as a plane with $L = 0$. The upper and lower surfaces of the film lie at $\xi = \frac{L}{2}$ and $\xi = \frac{-L}{2}$, respectively. Furthermore, the direction of spin-wave propagation is oriented along the ζ direction, i.e. $\vec{k} \parallel \hat{\zeta}$. The second (x, y, z) system is oriented such that the z axis lies parallel to the saturation magnetization $\vec{M}_0$ and the internal static magnetic field $\vec{H}_i$. The angle θ measures the rotation of the z axis relative to the ξ axis and takes values in the range $[0, \pi]$. The angle ϕ measures the rotation of the z axis relative to the ζ axis and takes values in the range $[0, 2\pi]$. If $\theta = \frac{\pi}{2}$, $\vec{M}_0$ lies "in-plane". If then $\phi = 0$, then $\hat{z} \parallel \hat{\zeta}$, i.e. $\vec{M}_0 \parallel \vec{k}$. If instead $\phi = \frac{\pi}{2}$, then $\hat{z} \parallel \hat{\eta}$, i.e. $\vec{M}_0 \perp \vec{k}$. Note that, by choosing the orientation of the axes, $\vec{k} = (0, 0, k_\zeta)$ in the (ξ, η, ζ) coordinate system.

We consider the physical quantities ω_H, ω_M, α, and L to be fixed. Then, for "in-plane" oriented magnetization, i.e. for fixed $\theta = \frac{\pi}{2}$, equation (1) defines a spin-wave dispersion surface

$$\omega_n = \omega_n \left(k_\parallel, k_\perp\right). \tag{5}$$

We have written $\left(k_\parallel, k_\perp\right)$ in place of (k_z, k_y) to denote the components of $\vec{k}$ relative to the external applied field.

1.1 Group velocity

We derive the group velocity,

$$\vec{v}_g = \nabla_{\vec{k}} \omega_n(\vec{k}) = \frac{\partial \omega_n}{\partial k_\zeta} \hat{k}_\zeta + \frac{1}{k_\zeta} \frac{\partial \omega_n}{\partial \phi} \hat{\phi} \tag{6}$$

first in polar coordinates.

We calculate the derivative $\frac{\partial \omega_n}{\partial k_\zeta} = \frac{1}{2\omega_n} \frac{\partial \omega_n^2}{\partial k_\zeta}$ via $\frac{\partial \omega_n^2}{\partial k_\zeta}$. For notational convenience, we define

$$R = \omega_H + \alpha \omega_M k_n^2 \tag{7}$$

$$S = \frac{\omega_M P_{nn}(1 - P_{nn}) \sin^2 \theta}{R} \tag{8}$$

$$E = \omega_M \sin^2 \theta \sin^2 \phi. \tag{9}$$

Then, we have

$$\omega_n = \sqrt{R(R + \omega_M F_{nn})} \tag{10}$$

$$\frac{\partial \omega_n^2}{\partial k_\zeta} = 2\alpha \omega_M k_\zeta \left[2R + \omega_M F_{nn}\right] + \omega_M R \frac{\partial F_{nn}}{\partial k_\zeta}. \tag{11}$$

Now, we calculate

$$\frac{\partial P_{nn}}{\partial k_\zeta} = 2\frac{k_\zeta}{k_n^2} - 2\frac{k_\zeta^3}{k_n^4} - 4\frac{k_\zeta^3}{k_n^4}F_n B + 4\frac{k_\zeta^5}{k_n^6}F_n B - \frac{k_\zeta^4}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta} \tag{12}$$

$$\frac{\partial F_n}{\partial k_\zeta} = \frac{-2}{k_\zeta^2 L} + \frac{2(-1)^n e^{-k_\zeta L}}{k_\zeta^2 L} + \frac{2(-1)^n e^{-k_\zeta L}}{k_\zeta} \tag{13}$$

where we have defined $B = \frac{1}{2}$ if $n = 0$ and $B = 1$ if $n \neq 0$. Now we have

$$\begin{aligned}\frac{\partial F_{nn}}{\partial k_\zeta} = {} & \frac{\partial P_{nn}}{\partial k_\zeta} - \frac{\partial P_{nn}}{\partial k_\zeta}\sin^2\theta(1+\cos^2\phi) - \frac{EP_{nn}}{R^2}\frac{\partial R}{\partial k_\zeta} + \\ & + \frac{E}{R}\frac{\partial P_{nn}}{\partial k_\zeta} + \frac{EP_{nn}^2}{R^2}\frac{\partial R}{\partial k_\zeta} - \frac{2EP_{nn}}{R}\frac{\partial P_{nn}}{\partial k_\zeta}\end{aligned} \tag{14}$$

which yields $\frac{\partial \omega_n}{\partial k_\zeta}$. For $\frac{\partial \omega_n}{\partial \phi}$, we find

$$\frac{\partial \omega_n}{\partial \phi} = \frac{1}{2\omega_n}\frac{\partial \omega_n^2}{\partial \phi} \tag{15}$$

$$\frac{\partial \omega_n^2}{\partial \phi} = R\omega_M \frac{\partial F_{nn}}{\partial \phi} \tag{16}$$

$$\frac{\partial F_{nn}}{\partial \phi} = P_{nn}\sin^2\theta\sin 2\phi\left[1 + \frac{\omega_M(1-P_{nn})}{\omega_H + \alpha\omega_M k_n^2}\right]. \tag{17}$$

We may translate these results to rectangular coordinates. In that case, $\vec{v}_g = \nabla_{\vec{k}}\omega(\vec{k}) = \frac{\partial \omega_n}{\partial k_z}\hat{z} + \frac{\partial \omega_n}{\partial k_y}\hat{y}$. We have as relations

$$k_\zeta = \sqrt{k_z^2 + k_y^2} \tag{18}$$

$$k_z = k_\zeta \cos\phi \tag{19}$$

$$k_y = k_\zeta \sin\phi. \tag{20}$$

From the chain rule,

$$\frac{\partial \omega_n}{\partial k_z} = \frac{\partial \omega_n}{\partial k_\zeta}\cos\phi + \frac{\partial \omega_n}{\partial \phi}\left(\frac{-\sin\phi}{k_\zeta}\right) \tag{21}$$

$$\frac{\partial \omega_n}{\partial k_y} = \frac{\partial \omega_n}{\partial k_\zeta}\sin\phi + \frac{\partial \omega_n}{\partial \phi}\left(\frac{\cos\phi}{k_\zeta}\right). \tag{22}$$

1.2 Magnon effective mass

Corpuscular picture, magnons. The magnon effective mass tensor has elements,

$$\left(\frac{1}{m}\right)_{\parallel,\perp} = \frac{1}{\hbar}\frac{\partial^2 \omega_n}{\partial k_\parallel \partial k_\perp}. \tag{23}$$

Note that, besides the factor $\frac{1}{\hbar}$, the magnon effective mass tensor is the Hessian matrix of $\omega_n(k_\parallel, k_\perp)$.

Once again, we proceed via $\frac{\partial^2 \omega_n}{\partial k_\zeta^2}$:

$$\frac{\partial^2 \omega_n}{\partial k_\zeta^2} = \frac{1}{2\omega_n}\frac{\partial^2 \omega_n^2}{\partial k_\zeta^2} - \frac{1}{\omega_n}\left(\frac{\partial \omega_n}{\partial k_\zeta}\right)^2 \tag{24}$$

$$\frac{\partial^2 \omega_n^2}{\partial k_\zeta^2} = [2R + \omega_M F_{nn}]\frac{\partial^2 R}{\partial k_\zeta^2} + R\omega_M \frac{\partial^2 F_{nn}}{\partial k_\zeta^2} + \left[2\frac{\partial R}{\partial k_\zeta} + 2\omega_M \frac{\partial F_{nn}}{\partial k_\zeta}\right]\frac{\partial R}{\partial k_\zeta} \tag{25}$$

$$\frac{\partial^2 R}{\partial k_\zeta^2} = 2\alpha\omega_M. \tag{26}$$

We lack only $\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}$. Having $\frac{\partial F_{nn}}{\partial k_\zeta}$, we proceed term-by-term,

$$\frac{\partial^2 F_{nn}}{\partial k_\zeta^2} = \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_1 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_2 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_3 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_4 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_5 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_6 \tag{27}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_1 = \frac{\partial}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} = \frac{\partial^2 P_{nn}}{\partial k_\zeta^2} \tag{28}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_2 = \frac{\partial}{\partial k_\zeta}\left[-\frac{\partial P_{nn}}{\partial k_\zeta}\sin^2\theta(1+\cos^2\phi)\right] \tag{29}$$

$$= -\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\sin^2\theta(1+\cos^2\phi) \tag{30}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_3 = \frac{\partial}{\partial k_\zeta}\left[-\frac{1}{R^2}EP_{nn}\frac{\partial R}{\partial k_\zeta}\right] \tag{31}$$

$$= 2EP_{nn}\left(\frac{\partial R}{\partial k_\zeta}\right)^2\frac{1}{R^3} - \frac{1}{R^2}E\frac{\partial R}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} - \frac{1}{R^2}EP_{nn}\frac{\partial^2 R}{\partial k_\zeta^2} \tag{32}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_4 = \frac{\partial}{\partial k_\zeta}\left[\frac{1}{R}E\frac{\partial P_{nn}}{\partial k_\zeta}\right] = -\frac{1}{R^2}E\frac{\partial P_{nn}}{\partial k_\zeta}\frac{\partial R}{\partial k_\zeta} + \frac{1}{R}E\frac{\partial^2 P_{nn}}{\partial k_\zeta^2} \tag{33}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_5 = \frac{\partial}{\partial k_\zeta}\left[\frac{1}{R^2}EP_{nn}^2\frac{\partial R}{\partial k_\zeta}\right] \tag{34}$$

$$= -2\frac{1}{R^3}EP_{nn}^2\left(\frac{\partial R}{\partial k_\zeta}\right)^2 + 2\frac{1}{R^2}EP_{nn}\frac{\partial R}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} + \frac{1}{R^2}EP_{nn}^2\frac{\partial^2 R}{\partial k_\zeta^2} \tag{35}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_6 = \frac{\partial}{\partial k_\zeta}\left[-2\frac{1}{R}EP_{nn}\frac{\partial P_{nn}}{\partial k_\zeta}\right] \tag{36}$$

$$= 2\frac{1}{R^2}EP_{nn}\frac{\partial R}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} - 2\frac{1}{R}E\left(\frac{\partial P_{nn}}{\partial k_\zeta}\right)^2 - 2\frac{1}{R}EP_{nn}\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}. \tag{37}$$

Hence we need $\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}$. Having $\frac{\partial P_{nn}}{\partial k_\zeta}$, we proceed term-by-term,

$$\frac{\partial^2 P_{nn}}{\partial k_\zeta^2} = \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_1 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_2 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_3 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_4 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_5 \tag{38}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_1 = \frac{\partial}{\partial k_\zeta}\left[2\frac{k_\zeta}{k_n^2}\right] = \frac{2}{k_n^2} - 4\frac{k_\zeta}{k_n^3}\frac{\partial k_n}{\partial k_\zeta} \tag{39}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_2 = \frac{\partial}{\partial k_\zeta}\left[-2\frac{k_\zeta^2}{k_n^3}\frac{\partial k_n}{\partial k_\zeta}\right] \tag{40}$$

$$= -4\frac{k_\zeta}{k_n^3}\frac{\partial k_n}{\partial k_\zeta} + 6\frac{k_\zeta^2}{k_n^4}\left(\frac{\partial k_n}{\partial k_\zeta}\right)^2 - 2\frac{k_\zeta^2}{k_n^3}\frac{\partial^2 k_n}{\partial k_\zeta^2} \tag{41}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_3 = \frac{\partial}{\partial k_\zeta}\left[-4\frac{k_\zeta^3}{k_n^4}F_n B\right] \tag{42}$$

$$= -12\frac{k_\zeta^2}{k_n^4}F_n B + 16\frac{k_\zeta^3}{k_n^5}F_n B\frac{\partial k_n}{\partial k_\zeta} - 4\frac{k_\zeta^3}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta} \tag{43}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_4 = \frac{\partial}{\partial k_\zeta}\left[4\frac{k_\zeta^4}{k_n^5}F_n B\frac{\partial k_n}{\partial k_\zeta}\right] \tag{44}$$

$$= 16\frac{k_\zeta^3}{k_n^5}F_n B\frac{\partial k_n}{\partial k_\zeta} - 20\frac{k_\zeta^4}{k_n^6}F_n B\left(\frac{\partial k_n}{\partial k_\zeta}\right)^2 + 4\frac{k_\zeta^4}{k_n^5}B\frac{\partial k_n}{\partial k_\zeta}\frac{\partial F_n}{\partial k_\zeta} + 4\frac{k_\zeta^4}{k_n^5}F_n B\frac{\partial^2 k_n}{\partial k_\zeta^2} \tag{45}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_5 = \frac{\partial}{\partial k_\zeta}\left[-\frac{k_\zeta^4}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta}\right] \tag{46}$$

$$= -4\frac{k_\zeta^3}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta} + 4\frac{k_\zeta^4}{k_n^5}B\frac{\partial F_n}{\partial k_\zeta}\frac{\partial k_n}{\partial k_\zeta} - \frac{k_\zeta^4}{k_n^4}B\frac{\partial^2 F_n}{\partial k_\zeta^2}. \tag{47}$$

To conclude, we calculate $\frac{\partial^2 F_n}{\partial k_\zeta^2}$,

$$\frac{\partial^2 F_n}{\partial k_\zeta^2} = \left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_1 + \left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_2 + \left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_3 \tag{48}$$

$$\left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_1 = \frac{\partial}{\partial k_\zeta}\left[-2\frac{1}{k_\zeta^2 L}\right] = 4\frac{1}{k_\zeta^3 L} \tag{49}$$

$$\left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_2 = \frac{\partial}{\partial k_\zeta}\left[2(-1)^n \frac{1}{k_\zeta^2 L} e^{-k_\zeta L}\right] \tag{50}$$

$$= -4(-1)^n \frac{1}{k_\zeta^3 L} e^{-k_\zeta L} - 2(-1)^n \frac{1}{k_\zeta^2} e^{-k_\zeta L} \tag{51}$$

$$\left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_3 = \frac{\partial}{\partial k_\zeta}\left[2(-1)^n \frac{1}{k_\zeta} e^{-k_\zeta L}\right] \tag{52}$$

$$= -2(-1)^n \frac{1}{k_\zeta^2} e^{-k_\zeta L} - 2L(-1)^n \frac{1}{k_\zeta} e^{-k_\zeta L}. \tag{53}$$

Note the derivative of k_n with respect to k_ζ,

$$\frac{\partial k_n}{\partial k_\zeta} = \frac{k_\zeta}{k_n} \tag{54}$$

$$\frac{\partial^2 k_n}{\partial k_\zeta^2} = \frac{1}{k_n} - \frac{k_\zeta^2}{k_n^3}. \tag{55}$$

Now, we calculate $\frac{\partial^2 \omega_n}{\partial \phi^2}$,

$$\frac{\partial^2 \omega_n}{\partial \phi^2} = \frac{1}{2\omega_n}\frac{\partial^2 \omega_n^2}{\partial \phi^2} - \frac{1}{\omega_n}\left(\frac{\partial \omega_n}{\partial \phi}\right)^2 \tag{56}$$

$$\frac{\partial^2 \omega_n^2}{\partial \phi^2} = R\omega_M \frac{\partial^2 F_{nn}}{\partial \phi^2} \tag{57}$$

$$\frac{\partial^2 F_{nn}}{\partial \phi^2} = 2\cos 2\phi \left(P_{nn}\sin^2\theta + \frac{\omega_M P_{nn}(1-P_{nn})\sin^2\theta}{R}\right). \tag{58}$$

Finally the mixed-derivatives,

$$\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} = \frac{\partial^2 \omega_n}{\partial \phi \partial k_\zeta} = \frac{\partial}{\partial k_\zeta}\frac{\partial \omega_n}{\partial \phi} = \frac{\partial}{\partial k_\zeta}\left[\frac{1}{2\omega_n}\frac{\partial \omega_n^2}{\partial \phi}\right] \tag{59}$$

$$= \frac{\partial}{\partial k_\zeta}\left[\frac{1}{2\omega_n} R P_{nn}\omega_M \sin 2\phi \sin^2\theta + \frac{1}{2\omega_n} R\omega_M^2 \sin 2\phi \frac{P_{nn}(1-P_{nn})\sin^2\theta}{R}\right] \tag{60}$$

$$\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} = \left.\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi}\right|_1 + \left.\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi}\right|_2 \tag{61}$$

$$\left.\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi}\right|_1 = \frac{-T}{2\omega_n^2} R P_{nn}\frac{\partial \omega_n}{\partial k_\zeta} + \frac{T}{2\omega_n} P_{nn}\frac{\partial R}{\partial k_\zeta} + \frac{T}{2\omega_n} R \frac{\partial P_{nn}}{\partial k_\zeta} \tag{62}$$

$$\left.\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi}\right|_2 = \frac{-U}{2\omega_n^2}\left[P_{nn} - P_{nn}^2\right]\frac{\partial \omega_n}{\partial k_\zeta} + \frac{U}{2\omega_n}\frac{\partial P_{nn}}{\partial k_\zeta}\left[1 - 2P_{nn}\right]. \tag{63}$$

The variables T and U have been defined for notational brevity:

$$T = \omega_M \sin 2\phi \sin^2\theta \tag{64}$$

$$U = \omega_M^2 \sin 2\phi \sin^2\theta. \tag{65}$$

To write down the Hessian matrix, we convert these results to Cartesian coordinates via

$$\frac{\partial^2 \omega_n}{\partial k_z^2} = \frac{\partial^2 \omega_n}{\partial k_z \partial k_\zeta}\frac{\partial k_\zeta}{\partial k_z} + \frac{\partial^2 \omega_n}{\partial k_z \partial \phi}\frac{\partial \phi}{\partial k_z} + \frac{\partial \omega_n}{\partial k_\zeta}\frac{\partial^2 k_\zeta}{\partial k_z^2} + \frac{\partial \omega_n}{\partial \phi}\frac{\partial^2 \phi}{\partial k_z^2} \tag{66}$$

$$\frac{\partial^2 \omega_n}{\partial k_y^2} = \frac{\partial^2 \omega_n}{\partial k_y \partial k_\zeta}\frac{\partial k_\zeta}{\partial k_y} + \frac{\partial^2 \omega_n}{\partial k_y \partial \phi}\frac{\partial \phi}{\partial k_y} + \frac{\partial \omega_n}{\partial k_\zeta}\frac{\partial^2 k_\zeta}{\partial k_y^2} + \frac{\partial \omega_n}{\partial \phi}\frac{\partial^2 \phi}{\partial k_y^2} \tag{67}$$

$$\frac{\partial^2 \omega_n}{\partial k_y \partial k_z} = \frac{\partial^2 \omega_n}{\partial k_z \partial k_\zeta}\frac{\partial k_\zeta}{\partial k_y} + \frac{\partial^2 \omega_n}{\partial k_z \partial \phi}\frac{\partial \phi}{\partial k_y} + \frac{\partial \omega_n}{\partial k_\zeta}\frac{\partial^2 k_\zeta}{\partial k_z \partial k_y} + \frac{\partial \omega_n}{\partial \phi}\frac{\partial^2 \phi}{\partial k_z \partial k_y} \tag{68}$$

$$\frac{\partial^2 \omega_n}{\partial k_y \partial k_z} = \frac{\partial^2 \omega_n}{\partial k_z \partial k_y}. \tag{69}$$

Hence we need

$$\frac{\partial^2 \omega_n}{\partial k_z \partial k_\zeta} = \frac{\partial k_\zeta}{\partial k_z}\frac{\partial^2 \omega_n}{\partial k_\zeta^2} + \frac{\partial \phi}{\partial k_z}\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} \tag{70}$$

$$\frac{\partial^2 \omega_n}{\partial k_z \partial \phi} = \frac{\partial k_\zeta}{\partial k_z}\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} + \frac{\partial \phi}{\partial k_z}\frac{\partial^2 \omega_n}{\partial \phi^2} \tag{71}$$

$$\frac{\partial k_\zeta}{\partial k_z} = \cos\phi \tag{72}$$

$$\frac{\partial k_\zeta}{\partial k_y} = \sin\phi \tag{73}$$

$$\frac{\partial^2 k_\zeta}{\partial k_z^2} = \frac{\sin^2\phi}{k_\zeta} \tag{74}$$

$$\frac{\partial^2 k_\zeta}{\partial k_y^2} = \frac{\cos^2\phi}{k_\zeta} \tag{75}$$

$$\frac{\partial^2 k_\zeta}{\partial k_y \partial k_z} = \frac{\partial^2 k_\zeta}{\partial k_z \partial k_y} = \frac{-\cos\phi\sin\phi}{k_\zeta} \tag{76}$$

$$\frac{\partial \phi}{\partial k_z} = \frac{-\sin\phi}{k_\zeta} \tag{77}$$

$$\frac{\partial \phi}{\partial k_y} = \frac{\cos\phi}{k_\zeta} \tag{78}$$

$$\frac{\partial^2 \phi}{\partial k_z^2} = \frac{\sin 2\phi}{k_\zeta^2} \tag{79}$$

$$\frac{\partial^2 \phi}{\partial k_y^2} = \frac{-\sin 2\phi}{k_\zeta^2} \tag{80}$$

$$\frac{\partial^2 \phi}{\partial k_y \partial k_z} = \frac{\partial^2 \phi}{\partial k_z \partial k_y} = \frac{1 - 2\cos^2\phi}{k_\zeta^2} \tag{81}$$

1.3 Density of modes/states

2 Steering magnetostatic waves

Wave sources. Control of real-space wave source distribution. Phase dispersion. Anisotropic dispersion (origin: (bi-)gyrotopy). Possibilities via control of spectrum of excitation. Directional emission for harmonic excitation. Validity of convolution as system operator—is magnetic medium under linearized L. L. equation an LTI system?

2.1 Point source emission pattern

References

[1] B. A. Kalinikos, Excitation of propagating spin waves in ferromagnetic films. *IEE Proc. H* **127**, 4 (1980).

[2] B. A. Kalinikos and A. N. Slavin, Theory of dipole-exchange spin-wave spectrum for ferromagnetic films with mixed exchange boundary conditions. *J. Phys. C* **19**, 7013 (1986).