boris

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Abstract

boris is a python module which calculates the dispersion characteristics of spin waves (SW) in ferromagnetic films based on the perturbation theory of Kalinikos and Slavin. Additionally, by calculating contours of the dispersion surface and via an inverse Fourier transform, boris calculates the emission pattern of a SW point source contacting a ferromagnetic medium.

1 Preliminaries

Kalinikos and Slavin (K & S) have solved Maxwell's equation in the magnetostatic limit for a medium described by the linearised Landau-Lifshitz equation, subject to electromagnetic and "exchange" boundary conditions. Using perturbation theory, K & S have obtained an explicit expression for "spin-waves" in the medium. To wit,

$$\omega_n = \sqrt{(\omega_H + \alpha\omega_M k_n^2)(\omega_H + \alpha\omega_M k_n^2 + \omega_M F_{nn})} \tag{1}$$

where

$$F_{nn} = P_{nn} + \sin^2\theta \left(1 - P_{nn}\left(1 + \cos^2\phi\right) + \omega_M \frac{P_{nn}(1 - P_{nn})\sin^2\phi}{(\omega_H + \alpha\omega_M k_n^2)}\right) \tag{2}$$

and $k_n^2 = k_\zeta^2 + \kappa_n^2$. We rely on the approximation of totally unpinnned surface spins, for which P_{nn} has the explicit expression

$$P_{nn} = \frac{k_\zeta^2}{k_n^2} - \frac{k_\zeta^4}{k_n^4} F_n \frac{1}{(1 + \delta_{0n})} \tag{3}$$

whereby

$$F_n = \frac{2}{k_\zeta L}[1 - (-1)^n e^{-k_\zeta L}]. \tag{4}$$

In this approximation, $\kappa_n = \frac{n\pi}{L}$ where L is the film thickness. Moreover, we write P_{nn} in the diagonal approximation, having taken $n = n'$. Lastly, we exclusively consider modes with a uniform profile across the film thickness, i.e. everywhere $n = 0$.

K & S defined $\omega_H = \mu_0|g|H_i$ and $\omega_M = \mu_0|g|M_0$, where μ_0 is the permeability of vacuum, $|g|$ is the gyromagnetic ratio, H_i is the magnitude of the internal field, and M_0 is the magnitude of the saturation magnetization. The constant α is the exchange constant.

K & S utilize two coordinate systems. The first (ξ, η, ζ) system is oriented such that the ξ direction lies parallel to the film normal vector, if the film is considered as a plane with $L = 0$. The upper and lower surfaces of the film lie at $\xi = \frac{L}{2}$ and $\xi = \frac{-L}{2}$, respectively. Furthermore, the direction of spin-wave propagation is oriented along the ζ direction. The second (x, y, z) system is oriented such that the z axis lies parallel to the saturation magnetization $\vec{M}_0$ and the internal static magnetic field $\vec{H}_i$. The angle θ measures the rotation of the z axis relative to the ξ axis and takes values in the range $[0, \pi]$. The angle ϕ measures the rotation of the z axis relative to the ζ axis and takes values in the range $[0, 2\pi]$. If $\theta = \frac{\pi}{2}$, $\vec{M}_0$ lies "in-plane". If then $\phi = 0$, then $z \parallel \zeta$, i.e. $\vec{M}_0 \parallel \vec{k}$. If instead $\phi = \frac{\pi}{2}$, then $z \parallel \eta$, i.e. $\vec{M}_0 \perp \vec{k}$. Note that, by choosing the orientation of the axes, $\vec{k} = (0, 0, k_\zeta)$ in the (ξ, η, ζ) coordinate system.

We consider the physical quantities ω_H, ω_M, α, and L to be fixed. Then, for "in-plane" oriented magnetization, i.e. for fixed $\theta = \frac{\pi}{2}$, equation (1) defines a spin-wave dispersion surface $\omega = \omega\left(k_\parallel, k_\perp\right)$. We have written $\left(k_\parallel, k_\perp\right)$ in place of (k_z, k_y).

We may use elementary techniques to derive the group velocity $\vec{v}_g = \vec{\nabla}\omega_n = \frac{\partial \omega_n}{\partial k_\zeta}\hat{k}_\zeta + \frac{1}{k_\zeta}\frac{\partial \omega_n}{\partial \phi}\hat{\phi}$. We begin with $\frac{\partial \omega_n}{\partial k_\zeta}$.

We calculate the derivative of $\frac{\partial \omega_n}{\partial k_\zeta} = \frac{1}{2\omega_n}\frac{\partial \omega_n^2}{\partial k_\zeta}$ via $\frac{\partial \omega_n^2}{\partial k_\zeta}$. For notational convenience, we define

$$R = \omega_H + \alpha\omega_M k_n^2 \tag{5}$$

$$S = \frac{\omega_M P_{nn}(1 - P_{nn})\sin^2\theta}{R} \tag{6}$$

$$E = \omega_M \sin^2\theta \sin^2\phi. \tag{7}$$

Then, we have

$$\omega_n = \sqrt{R(R + \omega_M F_{nn})} \tag{8}$$

$$\frac{\partial \omega_n^2}{\partial k_\zeta} = 2\alpha\omega_M k_\zeta \left[2R + \omega_M F_{nn}\right] + \omega_M R \frac{\partial F_{nn}}{\partial k_\zeta}. \tag{9}$$

Now, we calculate

$$\frac{\partial P_{nn}}{\partial k_\zeta} = 2\frac{k_\zeta}{k_n^2} - 2\frac{k_\zeta^3}{k_n^4} - 4\frac{k_\zeta^3}{k_n^4}F_n B + 4\frac{k_\zeta^5}{k_n^6}F_n B - \frac{k_\zeta^4}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta} \tag{10}$$

$$\frac{\partial F_n}{\partial k_\zeta} = \frac{-2}{k_\zeta^2 L} + \frac{2(-1)^n e^{-k_\zeta L}}{k_\zeta^2 L} + \frac{2(-1)^n e^{-k_\zeta L}}{k_\zeta} \tag{11}$$

where we have defined $B = \frac{1}{2}$ if $n = 0$ and $B = 1$ if $n \neq 0$. Now we have

$$\begin{aligned}\frac{\partial F_{nn}}{\partial k_\zeta} = \frac{\partial P_{nn}}{\partial k_\zeta} - \frac{\partial P_{nn}}{\partial k_\zeta} \sin^2\theta(1+\cos^2\phi) - \frac{EP_{nn}}{R^2}\frac{\partial R}{\partial k_\zeta} + \\ + \frac{E}{R}\frac{\partial P_{nn}}{\partial k_\zeta} + \frac{EP_{nn}^2}{R^2}\frac{\partial R}{\partial k_\zeta} - \frac{2EP_{nn}}{R}\frac{\partial P_{nn}}{\partial k_\zeta}\end{aligned} \tag{12}$$

which yields $\frac{\partial \omega_n}{\partial k_\zeta}$. For $\frac{\partial \omega_n}{\partial \phi}$, we find

$$\frac{\partial \omega_n}{\partial \phi} = \frac{1}{2\omega_n}\frac{\partial \omega_n^2}{\partial \phi} \tag{13}$$

$$\frac{\partial \omega_n^2}{\partial \phi} = R\omega_M \frac{\partial F_{nn}}{\partial \phi} \tag{14}$$

$$\frac{\partial F_{nn}}{\partial \phi} = P_{nn}\sin^2\theta \sin 2\phi \left[1 + \frac{\omega_M(1-P_{nn})}{\omega_H + \alpha\omega_M k_n^2}\right]. \tag{15}$$

We may translate these results to rectangular coordinates. In that case, $\vec{v}_g = \vec{\nabla}\omega_n = \frac{\partial \omega_n}{\partial k_z}\hat{z} + \frac{\partial \omega_n}{\partial k_y}\hat{y}$. We have as relations

$$k_\zeta = \sqrt{k_z^2 + k_y^2} \tag{16}$$

$$k_z = k_\zeta \cos\phi \tag{17}$$

$$k_y = k_\zeta \sin\phi. \tag{18}$$

From the chain rule,

$$\frac{\partial \omega_n}{\partial k_z} = \frac{\partial \omega_n}{\partial k_\zeta}\cos\phi + \frac{\partial \omega_n}{\partial \phi}\left(\frac{-\sin\phi}{k_\zeta}\right) \tag{19}$$

$$\frac{\partial \omega_n}{\partial k_y} = \frac{\partial \omega_n}{\partial k_\zeta}\sin\phi + \frac{\partial \omega_n}{\partial \phi}\left(\frac{\cos\phi}{k_\zeta}\right). \tag{20}$$

1.1 Second derivatives

In order to calculate the magnon effective mass, we need the second derivatives of ω_n with respect to k_ζ and ϕ. We begin, as before, by considering the derivative of $\frac{\partial^2 \omega_n}{\partial k_\zeta^2}$:

$$\frac{\partial^2 \omega_n}{\partial k_\zeta^2} = \frac{1}{2\omega_n}\frac{\partial^2 \omega_n^2}{\partial k_\zeta^2} - \frac{1}{\omega_n}\left(\frac{\partial \omega_n}{\partial k_\zeta}\right)^2 \tag{21}$$

$$\frac{\partial^2 \omega_n^2}{\partial k_\zeta^2} = [2R + \omega_M F_{nn}]\frac{\partial^2 R}{\partial k_\zeta^2} + R\omega_M\frac{\partial^2 F_{nn}}{\partial k_\zeta^2} + \left[2\frac{\partial R}{\partial k_\zeta} + 2\omega_M\frac{\partial F_{nn}}{\partial k_\zeta}\right]\frac{\partial R}{\partial k_\zeta} \tag{22}$$

$$\frac{\partial^2 R}{\partial k_\zeta^2} = 2\alpha\omega_M. \tag{23}$$

Therefore, we see that, when recalling the expressions we have already obtained in the calculation of the group velocity, the problem is reduced to the calculation of $\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}$. We have already found $\frac{\partial F_{nn}}{\partial k_\zeta}$, so we calculate the second derivative with respect to k_ζ term-by-term,

$$\frac{\partial^2 F_{nn}}{\partial k_\zeta^2} = \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_1 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_2 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_3 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_4 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_5 + \left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_6 \tag{24}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_1 = \frac{\partial}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} = \frac{\partial^2 P_{nn}}{\partial k_\zeta^2} \tag{25}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_2 = \frac{\partial}{\partial k_\zeta}\left[-\frac{\partial P_{nn}}{\partial k_\zeta}\sin^2\theta(1+\cos^2\phi)\right] \tag{26}$$

$$= -\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\sin^2\theta(1+\cos^2\phi) \tag{27}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_3 = \frac{\partial}{\partial k_\zeta}\left[-\frac{1}{R^2}EP_{nn}\frac{\partial R}{\partial k_\zeta}\right] \tag{28}$$

$$= 2EP_{nn}\left(\frac{\partial R}{\partial k_\zeta}\right)^2\frac{1}{R^3} - \frac{1}{R^2}E\frac{\partial R}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} - \frac{1}{R^2}EP_{nn}\frac{\partial^2 R}{\partial k_\zeta^2} \tag{29}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_4 = \frac{\partial}{\partial k_\zeta}\left[\frac{1}{R}E\frac{\partial P_{nn}}{\partial k_\zeta}\right] = -\frac{1}{R^2}E\frac{\partial P_{nn}}{\partial k_\zeta}\frac{\partial R}{\partial k_\zeta} + \frac{1}{R}E\frac{\partial^2 P_{nn}}{\partial k_\zeta^2} \tag{30}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_5 = \frac{\partial}{\partial k_\zeta}\left[\frac{1}{R^2}EP_{nn}^2\frac{\partial R}{\partial k_\zeta}\right] \tag{31}$$

$$= -2\frac{1}{R^3}EP_{nn}^2\left(\frac{\partial R}{\partial k_\zeta}\right)^2 + 2\frac{1}{R^2}EP_{nn}\frac{\partial R}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} + \frac{1}{R^2}EP_{nn}^2\frac{\partial^2 R}{\partial k_\zeta^2} \tag{32}$$

$$\left.\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}\right|_6 = \frac{\partial}{\partial k_\zeta}\left[-2\frac{1}{R}EP_{nn}\frac{\partial P_{nn}}{\partial k_\zeta}\right] \tag{33}$$

$$= 2\frac{1}{R^2}EP_{nn}\frac{\partial R}{\partial k_\zeta}\frac{\partial P_{nn}}{\partial k_\zeta} - 2\frac{1}{R}E\left(\frac{\partial P_{nn}}{\partial k_\zeta}\right)^2 - 2\frac{1}{R}EP_{nn}\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}. \tag{34}$$

Calculating $\frac{\partial^2 F_{nn}}{\partial k_\zeta^2}$ in turn reduces to the calculating $\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}$, which we also carry out term-by-term from our expression for $\frac{\partial P_{nn}}{\partial k_\zeta}$,

$$\frac{\partial^2 P_{nn}}{\partial k_\zeta^2} = \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_1 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_2 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_3 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_4 + \left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_5 \tag{35}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_1 = \frac{\partial}{\partial k_\zeta}\left[2\frac{k_\zeta}{k_n^2}\right] = \frac{2}{k_n^2} - 4\frac{k_\zeta}{k_n^3}\frac{\partial k_n}{\partial k_\zeta} \tag{36}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_2 = \frac{\partial}{\partial k_\zeta}\left[-2\frac{k_\zeta^2}{k_n^3}\frac{\partial k_n}{\partial k_\zeta}\right] \tag{37}$$

$$= -4\frac{k_\zeta}{k_n^3}\frac{\partial k_n}{\partial k_\zeta} + 6\frac{k_\zeta^2}{k_n^4}\left(\frac{\partial k_n}{\partial k_\zeta}\right)^2 - 2\frac{k_\zeta^2}{k_n^3}\frac{\partial^2 k_n}{\partial k_\zeta^2} \tag{38}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_3 = \frac{\partial}{\partial k_\zeta}\left[-4\frac{k_\zeta^3}{k_n^4}F_n B\right] \tag{39}$$

$$= -12\frac{k_\zeta^2}{k_n^4}F_n B + 16\frac{k_\zeta^3}{k_n^5}F_n B\frac{\partial k_n}{\partial k_\zeta} - 4\frac{k_\zeta^3}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta} \tag{40}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_4 = \frac{\partial}{\partial k_\zeta}\left[4\frac{k_\zeta^4}{k_n^5}F_n B\frac{\partial k_n}{\partial k_\zeta}\right] \tag{41}$$

$$= 16\frac{k_\zeta^3}{k_n^5}F_n B\frac{\partial k_n}{\partial k_\zeta} - 20\frac{k_\zeta^4}{k_n^6}F_n B\left(\frac{\partial k_n}{\partial k_\zeta}\right)^2 + 4\frac{k_\zeta^4}{k_n^5}B\frac{\partial k_n}{\partial k_\zeta}\frac{\partial F_n}{\partial k_\zeta} + 4\frac{k_\zeta^4}{k_n^5}F_n B\frac{\partial^2 k_n}{\partial k_\zeta^2} \tag{42}$$

$$\left.\frac{\partial^2 P_{nn}}{\partial k_\zeta^2}\right|_5 = \frac{\partial}{\partial k_\zeta}\left[-\frac{k_\zeta^4}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta}\right] \tag{43}$$

$$= -4\frac{k_\zeta^3}{k_n^4}B\frac{\partial F_n}{\partial k_\zeta} + 4\frac{k_\zeta^4}{k_n^5}B\frac{\partial F_n}{\partial k_\zeta}\frac{\partial k_n}{\partial k_\zeta} - \frac{k_\zeta^4}{k_n^4}B\frac{\partial^2 F_n}{\partial k_\zeta^2}. \tag{44}$$

Lastly, we see we must calculate $\frac{\partial^2 F_n}{\partial k_\zeta^2}$,

$$\frac{\partial^2 F_n}{\partial k_\zeta^2} = \left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_1 + \left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_2 + \left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_3 \tag{45}$$

$$\left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_1 = \frac{\partial}{\partial k_\zeta}\left[-2\frac{1}{k_\zeta^2 L}\right] = 4\frac{1}{k_\zeta^3 L} \tag{46}$$

$$\left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_2 = \frac{\partial}{\partial k_\zeta}\left[2(-1)^n \frac{1}{k_\zeta^2 L} e^{-k_\zeta L}\right] \tag{47}$$

$$= -4(-1)^n \frac{1}{k_\zeta^3 L} e^{-k_\zeta L} - 2(-1)^n \frac{1}{k_\zeta^2} e^{-k_\zeta L} \tag{48}$$

$$\left.\frac{\partial^2 F_n}{\partial k_\zeta^2}\right|_3 = \frac{\partial}{\partial k_\zeta}\left[2(-1)^n \frac{1}{k_\zeta} e^{-k_\zeta L}\right] \tag{49}$$

$$= -2(-1)^n \frac{1}{k_\zeta^2} e^{-k_\zeta L} - 2L(-1)^n \frac{1}{k_\zeta} e^{-k_\zeta L}. \tag{50}$$

The derivative of k_n with respect to k_ζ is straightforward,

$$\frac{\partial k_n}{\partial k_\zeta} = \frac{k_\zeta}{k_n} \tag{51}$$

$$\frac{\partial^2 k_n}{\partial k_\zeta^2} = \frac{1}{k_n} - \frac{k_\zeta^2}{k_n^3}. \tag{52}$$

Now, we must calculate the second order derivative of ω_n with respect to ϕ,

$$\frac{\partial^2 \omega_n}{\partial \phi^2} = \frac{1}{2\omega_n}\frac{\partial^2 \omega_n^2}{\partial \phi^2} - \frac{1}{\omega_n}\left(\frac{\partial \omega_n}{\partial \phi}\right)^2 \tag{53}$$

$$\frac{\partial^2 \omega_n}{\partial \phi^2} = R\omega_M \frac{\partial^2 F_{nn}}{\partial \phi^2} \tag{54}$$

$$\frac{\partial^2 F_{nn}}{\partial \phi^2} = 2\cos 2\phi \left(P_{nn}\sin^2\theta + \frac{\omega_M P_{nn}(1-P_{nn})\sin^2\theta}{R}\right). \tag{55}$$

And finally the mixed-derivatives,

$$\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} = \frac{\partial^2 \omega_n}{\partial \phi \partial k_\zeta} = \frac{\partial}{\partial k_\zeta} \frac{\partial \omega_n}{\partial \phi} = \frac{\partial}{\partial k_\zeta} \left[\frac{1}{2\omega_n} \frac{\partial \omega_n^2}{\partial \phi} \right] \tag{56}$$

$$= \frac{\partial}{\partial k_\zeta} \left[\frac{1}{2\omega_n} R P_{nn} \omega_M \sin 2\phi \sin^2\theta + \frac{1}{2\omega_n} R \omega_M^2 \sin 2\phi \frac{P_{nn}(1 - P_{nn}) \sin^2\theta}{R} \right] \tag{57}$$

$$\frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} = \left. \frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} \right|_1 + \left. \frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} \right|_2 \tag{58}$$

$$\left. \frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} \right|_1 = \frac{-T}{2\omega_n^2} R P_{nn} \frac{\partial \omega_n}{\partial k_\zeta} + \frac{T}{2\omega_n} P_{nn} \frac{\partial R}{\partial k_\zeta} + \frac{T}{2\omega_n} R \frac{\partial P_{nn}}{\partial k_\zeta} \tag{59}$$

$$\left. \frac{\partial^2 \omega_n}{\partial k_\zeta \partial \phi} \right|_2 = \frac{-U}{2\omega_n^2} \left[P_{nn} - P_{nn}^2 \right] \frac{\partial \omega_n}{\partial k_\zeta} + \frac{U}{2\omega_n} \frac{\partial P_{nn}}{\partial k_\zeta} \left[1 - 2P_{nn} \right]. \tag{60}$$

The variables T and U have simply been defined for notational brevity:

$$T = \omega_M \sin 2\phi \sin^2\theta \tag{61}$$

$$U = \omega_M^2 \sin 2\phi \sin^2\theta. \tag{62}$$

With these expressions at hand, we may calculate the magnon effective mass.

2 Configuring boris

boris is configured via "set_the_boris.py".

2.1 Units

At present, boris requires input in a hodgepodge of units. We are transitioning to SI units. In the meantime, see the comments in "set_the_boris.py" for the correct units.

3 Running boris

boris is most conveniently run at the command-line. Simply cd into the directory containing "boris.py" and "set_the_boris.py" and run "python boris.py". You may wish to redirect standard out to capture the dialogue boris prints as it runs, though all physically relevant parameters are printed as comments to the corresponding output files. Also helpful is "time python boris.py".

4 Plotting output

In boris' default working directory, there are a number of gnuplot files to automate plotting the output files. In particular, see "phase.gpl", "amplitude.gpl", "ssurface.gpl", and "dispersion_surface.gpl".