CSCI 493.69 / CSCI 795.20
Computer Vision
Homework 1 [theory]
40 points (45 points for MA students)
Due Thursday February 13

Justification required in order receiving credit. The following problems will test your understanding of the pinhole projection model. Submit your answers in electronic form (pdf) via blackboard.

Questions

[A] (5 pts) Let **C** be a black circular disk in front of a white background. The circular disk is parallel to the image plane. This disk is projected on the image plane through a pinhole. What is the shape of the disk's projection on the image plane? *[Hint: A circular disk parallel to the image plane is described algebraically as all 3D points $[X,Y,Z]^T$ with $(X-X_0)^2+(Y-Y_0)^2=R^2$, $Z=Z_0$, where $[X_0,Y_0,Z_0]^T$ is the center of the disk and R is its radius.]*

[B] (5 pts) Let **S** be a black sphere in front of a white background in the scene. The sphere is projected on the image plane through a pinhole. What is the shape of the sphere's projection on the image plane? You can use a drawing in order to justify your answer.

[C] (5 pts) You are given a set of parallel lines in the scene. Compute mathematically their vanishing point on the image plane.
[Hint: Note that a 3D line is represented by the set of points $[x(t), y(t), z(t)]^T$,
where $x(t)=A_1+B_1*t$, $y(t)=A_2+B_2*t$, $z(t)=A_3+B_3*t$ (t ranges from negative infinity to infinity).
The 3D point $[A_1, A_2, A_3]^T$ is one point on the line, and the 3D vector
$[B_1, B_2, B_3]^T$ is the orientation of the line.]

[D] (10 pts) Imagine a 3D plane **P** in your 3D scene. An infinite number of lines can lie on that plane. Consider one set of parallel lines on this 3D plane. This set will produce one vanishing point when projected on the image plane. Consider now all possible sets of parallel line on the plane **P**. What is the locus of all vanishing points produced by all sets of parallel lines? *[Note that you do not have to right down a formula in order to solve this. You need to use a geometric argument.]*

[E] (10 pts) You are given a cube C. Its center is at point (20,20,20) on camera's coordinate frame. The size of the cube is 2 (length of one edge) and one of its face is oriented parallel to the image plane.

- If the effective focal length of the camera is f=10, give the coordinates of the projections of the cube's vertices (eight in total) on the image assuming perspective projection.
- Same problem, but assume weak-perspective projection.

[F] (5 pts) Show that in the pinhole camera model, three collinear points in 3-D space are imaged into three collinear points on the image plane.

(**MA students**) (5 pts) Describe how would you increase the magnification and then refocus using a two-lens system. Use the figure of a two-lens system presented in the second lecture. Be specific.