Homework 1

Name: ___________ **NetID:_____________**

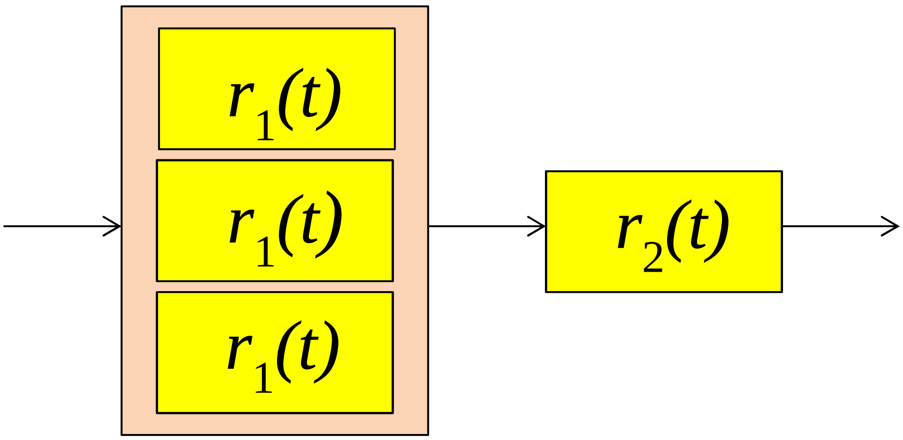
This homework is composed of three questions. Please work on the homework independently. Please print out this PDF file and fill-in answers within the spaces provided. The homework is due in hard copy on Sept 8th in class.

Q1: In March 2011, a massive Earthquake hit the coast of Japan, followed by a large tsunami. The Earthquake and the tsunami contributed different types of damage to the Fukushima nuclear reactor, setting off a series of events that ultimately led to core meltdown. Using Web resources at your disposal, plot a diagram showing the sequence of events that led to reactor failure. For an example of such a diagram, see lecture slides on the Three Mile Island reactor meltdown. **(5 points)**

Many references are available. For example, see the "Overview" section under:

https://en.wikipedia.org/wiki/Fukushima_Daiichi_nuclear_disaster

Q2: An avionics system consists of a navigation function that has triple-modular-redundancy, followed by a voter, as shown below. Each individual version of the navigation function has reliability r_1=0.95. The voter has reliability r_2=0.99. What is the reliability of the entire system? (Note: compute system reliability with accuracy of two digits after the decimal point.) Assume that, for the system to work properly, at least two of the three versions of the navigation function have to work, in addition to the voter. Please show your steps. **(3 points)**



$R(t) = (r_1(t)^3 + 3\, r_1(t)^2\, (1 - r(t)))\, r_2(t) = (0.95^3 + 3\,(0.95)^2(0.05))\, 0.99 = 0.98$

Q3: You are to rank three architectural choices for building a scientific exploration system to be deployed inside a volcano crater, where component repair and replacement are impossible. The system is to be composed of one or more components as will be described below. Assume that the reliability of a single component is given by the function, $r(t)=e^{-k\,(C/E)\,t}$ where C is component complexity and E is the budget allocated to writing the component. Let t be measured in units of years. Assume that the mean time to failure of a component of unit complexity given a unit budget is one year. (Hint: use this information to compute k.) For purposes of ranking the three architectures, an architecture is deemed "better" if retains a higher reliability by the end of the mission. The three architectures compared are:

1. An architecture composed of a single do-it-all component of complexity C=1.
2. A triple-modular-redundancy architecture composed of three components of redundant functionality, each of complexity C=1.
3. A system composed of one do-it-all component of complexity C=1 and another basic safety component of complexity C=0.1. The system remains safe as long as at least one of the two components works.

(a) Show your reliability computations for each of the above three architectures for the case of *short* missions that last 3 months. Assume that the total budget=1, and that it is split equally among the components of the architecture considered. (For example, if an architecture is composed of two components, then for each component, E=0.5). Rank the architectures from best to worst: **(3 points)**

Note: since mean time to failure is 1, when C=1 and E=1, then k=1. Let component reliability be denoted by $r(t)$ and system reliability be denoted by $R(t)$:

Case 1: $E = 1$, $R(t) = r(t) = e^{-(0.25)} = 0.779$

Case 2: $E = 0.333$, $r(t) = e^{-(0.25/0.333)} = 0.472$, $R(t) = (0.472)^3 + 3(0.472)^2 (1-0.472) = 0.4586$

Case 3: $E = 0.5$, $r_1(t) = e^{-(0.25/0.5)} = 0.607$, $r_2(t) = e^{-(0.1*0.25/0.5)} = 0.951$.

$R(t) = 1 - (1 - 0.607)(1 - 0.951) = 0.981$

Case 3 > Case 1 > Case 2

(b) Show your reliability computations for each of the three architectures for the case of *long* missions that last two years. Assume that the total budget=1, and that it is split equally among the components in the architecture considered (as before). Rank the architectures from best to worst: **(3 points)**

Case 1: $E = 1$, $R(\mathrm{t}) = r(t) = \mathrm{e}^{-2} = 0.1353$

Case 2: $E = 0.333$, $r(t) = \mathrm{e}^{-(2/0.333)} = 0.00248$, $R(t) = (0.00248)^3 + 3(0.00248)^2\,(1\text{-}0.00248) = 0.0000184$

Case 3: $E = 0.5$, $r_1(t) = \mathrm{e}^{-(2/0.5)} = 0.0183$, $r_2(t) = \mathrm{e}^{-(0.1*2/0.5)} = 0.6703$.
$R(t) = 1 - (1 - 0.0183)\,(1 - 0.6703) = 0.6764$

Case 3 > Case 1 > Case 2

(c) Repeat (a), except that this time assume a fixed budget E=1 *per component*: **(3 points)**

Case 1: $R(\mathrm{t}) = r(t) = \mathrm{e}^{-(0.25)} = 0.779$

Case 2: $R(t) = (0.779)^3 + 3(0.779)^2\,(1\text{-}0.779) = 0.875$

Case 3: $r_1(t) = \mathrm{e}^{-(0.25)} = 0.779$, $r_2(t) = \mathrm{e}^{-(0.1*0.25)} = 0.975$
$R(t) = 1 - (1 - 0.779)\,(1 - 0.975) = 0.995$

Case 3 > Case 2 > Case 1

(d) Repeat (b), except that this time assume a fixed budget E=1 *per component*: **(3 points)**

Case 1: $R(\mathrm{t}) = r(t) = \mathrm{e}^{-2} = 0.1353$

Case 2: $R(t) = (0.1353)^3 + 3(0.1353)^2\ (1\text{-}0.1353) = 0.05$

Case 3: $r_1(t) = \mathrm{e}^{-2} = 0.1353$, $r_2(t) = \mathrm{e}^{-0.2} = 0.8187$

$R(t) = 1 - (1 - 0.1353)\ (1 - 0.8187) = 0.843$

Case 3 > Case 1 > Case 2