# Calculation of convection induced dynamic heat flow in walls

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## Introduction

The energy balance and thermal comfort of a building are depending on the surrounding air temperature. With changing air temperature during a day, the heat flows through walls change, they become dynamic. The dynamic heat flow through a wall can be calculated with the Crank-Nicolson method in combination with the Thomas algorithm as explained in Bagda, Dlabal, Öztürk: Calculation of Dynamic Heat Flow in Walls [1, 2].

In this work calculation of dynamic heat flow~~s~~ induced by changing air temperatures is explained using the Crank-Nicolson method in combination with the Biot number.

## Calculation of dynamic heat transfer between air and wall

The temperature of the surface of a wall is depending on the convective heat transfer from and to the surrounding air. The heat flow density $\boldsymbol{q}$ [W/m²] between the fluid medium air and the wall is proportional to the surface heat transfer coefficient $\boldsymbol{h}$ [W/(m²·K)] (equation 1).

$$q = h \cdot \left(T_{air} - T_{surface}\right) \qquad (1)$$

$\boldsymbol{q}$ : heat flow density [W/m²]
$\boldsymbol{h}$ : surface heat transfer coefficient [W/(m²·K)]
$\boldsymbol{T_{air}}$ : temperature of the surrounding air [K]
$\boldsymbol{T_{surface}}$ : temperature of the solid surface [K]

To calculate the dynamic heat flow with the Crank-Nicolson method the wall is sliced in [n] imaginary elements perpendicular to the heat flow. Each element i has a thickness $\boldsymbol{x_i}$ [m], a thermal conductivity $\boldsymbol{\lambda_i}$ [W/(m²·K)] and a heat capacity $\boldsymbol{c_{v,i}}$ [J/(m³·K)].

It is assumed that the heat flow from the center of the imaginary element i[1] to the surface of the wall is the same as the heat flow from the surface of the wall to the surrounding air (equation 2):

$$\frac{\lambda_1}{\frac{x_1}{2}} \cdot \left(T_1 - T_{surface}\right) = h \cdot \left(T_{surface} - T_{air}\right) \qquad (2)$$

The Biot number $\boldsymbol{Bi}$, named after Jean-Baptiste Biot (1774–1862), is a dimensionless quantity, expressing the ratio of the heat transfer from inside a body to the surface and from the surface to the surrounding air (equation 3):

$$Bi = \frac{h}{\lambda_1} \cdot \frac{x_1}{2} \tag{3}$$

***Bi*** : Biot number [-]
***h*** : surface heat transfer coefficient [W/(m$^2$·K)]
***λ*** : thermal conductivity [W/(m·K)]
***x*** : thickness of the imaginary element i[1] [m]

Equation (2) can be written with the Biot number from equation (3) as:

$$T_1 - T_{surface} = Bi \cdot (T_{surface} - T_{air}) \tag{4}$$

$$\Leftrightarrow\ T_{surface} = \frac{T_1 + Bi \cdot T_{air}}{1+Bi} \tag{5}$$

For the calculation of the dynamic heat flow through a wall with the Crank-Nicolson method, the temperature $T_1$ in the centre of the first element i[1] needs to be known. As $T_1$ and the surface temperature $T_{surface}$ are unknown in equation (2), an additional equation is needed to calculate $T_1$ from the air temperature $T_{air}$ with the Biot number of the system. Therefore a virtual dummy element i[D] is introduced between the element i[1] and the surrounding air (see figure 1).

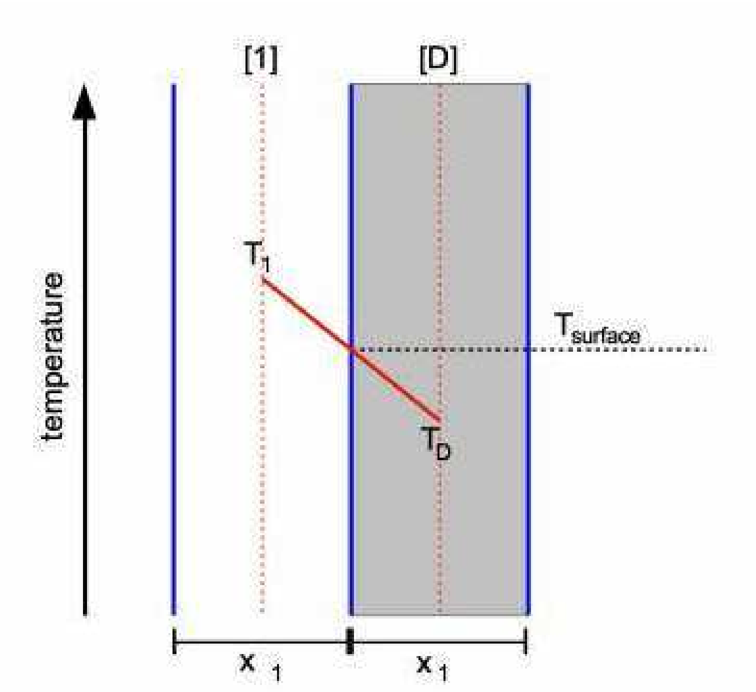


*Figure 1: Dummy element i[D] between element i[1] and air. $T_{surface}$ is the contact temperature of the two elements on their common surface.*

Subsequent assumptions are made:

1.) The dummy element i[D] has the same thickness $x_1$ and thermal conductivity $\lambda_1$ as its adjoining element i[1].

2.) The contact temperature $T_{surface}$ between element i[1] and its dummy element i[D] is the mean value of the temperatures $T_1$ in the element i[1] and the temperature $T_D$ of the dummy element i[D].

3.) The heat flow from the center of element i[1] to the surface is the same as the heat flow from the surface to the center of the dummy element i[D] (equation 6).

$$\frac{\lambda_1}{x_1/2} \cdot \left(T_1 - T_{surface}\right) = \frac{\lambda_1}{x_1/2} \cdot \left(T_{surface} - T_D\right) \tag{6}$$

$$\Leftrightarrow \quad T_D = 2 \cdot T_{surface} - T_1 \tag{7}$$

Replacing $T_{surface}$ in equation (7) with equation (5) leads to equation (8):

$$T_D = \frac{2 \cdot T_1}{1+Bi} + \frac{2 \cdot Bi \cdot T_{air}}{1+Bi} - T_1$$

$$\Leftrightarrow \quad T_D = \frac{(1-Bi) \cdot T_1 + 2 \cdot Bi \cdot T_{air}}{1+Bi} \tag{8}$$

Equation (8) states, that the temperature in the centre of the dummy element i[D] can be calculated from the known temperatures $T_1$ and $T_{air}$.

The principle above, as described for the “first” element i[1] of the wall in contact with air, is also valid for the “last” element i[n] on the other side of the wall.

To calculate the dynamic heat flow through a wall with the Crank-Nicolson method at changing surrounding air temperatures from time [j] to [j+1], the temperatures $T_{0,j+1}$ and $T_{n+1,j+1}$ of the dummy elements i[0] and i[n+1] are needed at time [j+1]. This are the boundary conditions for the heat flow through the wall, as explained in the former work [1, 2]. $T_{0,j+1}$ and $T_{n+1,j+1}$ are calculated with the Biot numbers of the system with the “new” $T_{air,j+1}$ at time [j+1] (see Code: BIOT_01.py version 2020_07_15 line 96 and 101).

$$T_{0,j+1} = \frac{(1-Bi) \cdot T_{1,j} + 2 \cdot Bi \cdot T_{air\_int,j+1}}{1+Bi} \quad and \quad T_{n+1,j+1} = \frac{(1-Bi) \cdot T_{n,j} + 2 \cdot Bi \cdot T_{air\_ext,j+1}}{1+Bi} \tag{9}$$

Note:
Python loops start at “0” and end at “n-1”. Therefore, there is a deviation between the indexes for n in this text and the code BIOT_01.py.

With the dummy Temperatures $T_{0,j+1}$ and $T_{n+1,j+1}$ at time [j+1] calculated with equation (9) the unknown temperatures $T_{1,j+1}$ to $T_{n,j+1}$ are calculated with the Crank Nicolson method and Thomas algorithm [1,2].

**Steady state**

Changing air temperatures cause temperature changes in the wall. The course of temperatures in the elements is the temperature profile of the wall. When the temperature profile does not change any more in time the steady state situation is reached. In this case the heat flows between all elements are the same (see figure 2). The time to reach steady state is depending on the heat capacity and thermal conductivity of the materials of the wall. The steady state is characterized by a constant temperature profile and a static heat flow $q$ which can be calculated with the static heat flow equation (10).

$$q = U \cdot \left(T_{air\_ext} - T_{air\_int}\right) \tag{10}$$

$q$ : Heat flow density [W/(m²)]
$U$ : thermal transmittance [W/(m²·K)]
$T_{air}$ : temperature of surrounding air, intern and extern [°C]

The thermal transmittance of a wall is expressed as it's U-value. The U-value is the rate of heat transfer through a wall from interior air to exterior air. It is reciprocal to the thermal resistance $R$ of the wall according to equation (11):

$$U = \frac{1}{R} \tag{11}$$

$R$ : thermal resistance of the wall [m²·K/W]

The thermal resistance $R$ of a wall is calculated from the thermal resistances of the materials and the surface heat transfer coefficients of interior and exterior air according to equation (12).

$$R = \frac{1}{h_{int}} + \frac{x_1}{\lambda_1} + \frac{x_2}{\lambda_2} + \frac{1}{h_{ext}} \tag{12}$$

$h$ : surface heat transfer coefficient [W/(m²·K)]
$x$ : thickness of the material [m]
$\lambda$ : thermal conductivity [W/(m·K)]

The heat flow density calculated with the dynamic Crank-Nicolson method at steady state situation must be the same as calculated with the static heat flow equation (10).

## Dynamic heat flow at abrupt change of air temperature

### Example 1:  BIOT_01.py

(Version 2020_07_15)

In this example the heat flow through a wall consisting of 0.20 m cellular concrete and 0.05 m expanded polystyrene is calculated when the air temperature is decreased abruptly from 20°C to 0°C on one side. Following ISO 6946, internal surface heat transfer coefficient $h_{int}$ = 7.69 W/(m$^2$·K) and external surface heat transfer coefficient $h_{ext}$ = 25 W/(m$^2$·K) are used. According to our own measurements this corresponds to an internal air speed of 1 m/s and an external air speed of 2 m/s to 3 m/s. Figure 1 shows the development of the temperature profile after the abrupt change of the external air temperature from 20°C to 0°C.

NOTE:
The heat transfer coefficients depends on the air movement (air speed), the temperature difference between $T_{air}$ and $T_{surface}$, and also on the height of the wall due to the chimney effect.

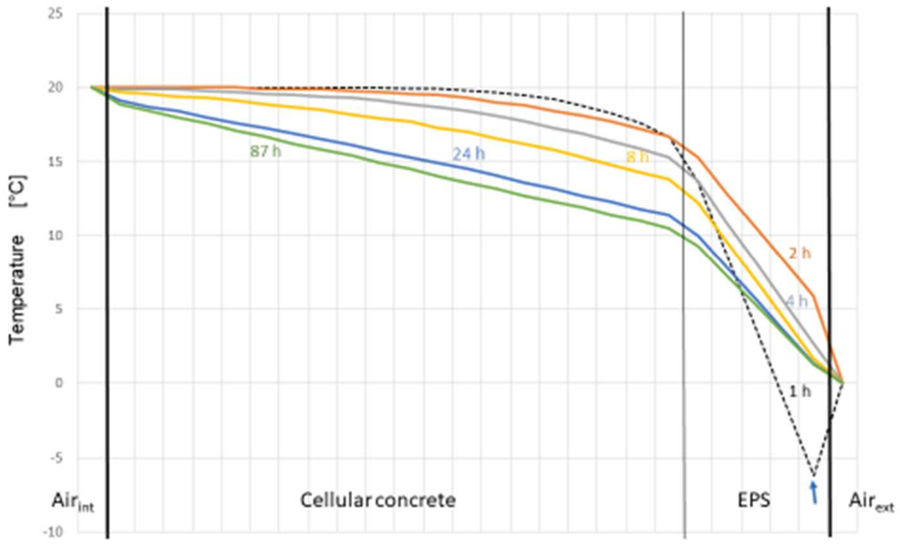


*Figure 1. Temperature profile of the wall at after 1h, 2h, 4h, 8h, 24 h and 87 h*

NOTE:
In figure 1 the discontinuity in the temperature profile after 1 hour (marked with an arrow) is physically not possible. The reason for this discontinuity is the temperature change of 20°C in the first minute of the calculation. This temperature step is too large for the mathematical stability of the system. After 2 hours the temperature profile is physically logic and the system is mathematically stable. To ensure a correct calculation of heat flows with the Crank-Nicolson method it is important to control the temperature profile and to use results after the system is settled and is mathematically stable, as shown above.

The target of the dynamic heat flow calculation is usually to assess the energy balance of an interior air volume (flat, room or zone) which is surrounded by walls, ceiling and floor. For this the resulting heat flows through the walls are important. Figure 2 shows the heat flow profile calculated with BIOT_01.py for different times.

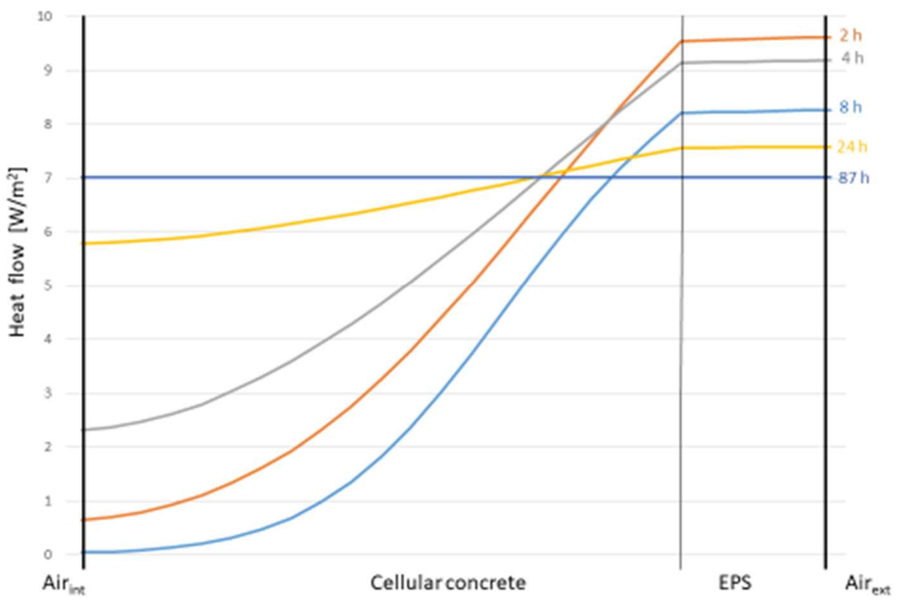


*Figure 2. Heat flow profile of the wall at 2h, 4h, 8h, 24 h and 87 h*

Over time the heat flow increases at the interior side and decreases on the exterior side. The steady state is reached after 87h at a heat flow density of $\boldsymbol{q}$ = 7.02 W/m$^2$.

The same heat flow density is calculated with the static heat equation (14). This shows that the heat flow density from dynamic calculation with the Crank-Nicolson method at steady state is the same as calculated with the static heat flow equation.

$$q = \frac{\Delta T}{\frac{1}{h_{int}} + \frac{x_1}{\lambda_1} + \frac{x_2}{\lambda_2} + \frac{1}{h_{ext}}} = \frac{20}{\frac{1}{7.69} + \frac{0,20}{0.160} + \frac{0.05}{0.035} + \frac{1}{25}} = 7.02 \quad \frac{W}{m^2} \tag{14}$$

**Dynamic heat flow at hourly changing air temperature**

**Example 2 BIOT_02.py**
(version 2020_07_15)

In this example the hourly change of $T_{air}$ on one side of the wall is simulated with a cosine step function with a period length of 24 hours and between its limit superior $T_{air\_max}$ and inferior $T_{air\_min}$ (equation 15).

$$T_{air\_hour} = \frac{T_{air\_max}+T_{air\_min}}{2} - \frac{T_{air\_max}-T_{air\_min}}{2} \cdot cos\left(\frac{2 \cdot \pi}{24} \cdot hour\right) \qquad (15)$$

A cosine function implies that the temperature at a time on a day is the same as the day before. This allows to analyse the heat flow in a wall during a day at periodic changes of air temperature.

In this example a median temperature of 20° and a temperature amplitude of 6°C is used. The resulting temperature profiles are given in figure 3.

Figure 3 shows the temperature fluctuations of some selected elements in the wall over the course of the first two days. During the first 6 hours the system is mathematically instable, because of the abrupt change of $T_{air\_ext}$ from 20 °C to 14° C in the first minute. 24 hours later the system is the settled and mathematically stable. For this reason, it is recommended to start dynamic calculations with a pre-run and take results in account after the system is settled and mathematically stable.

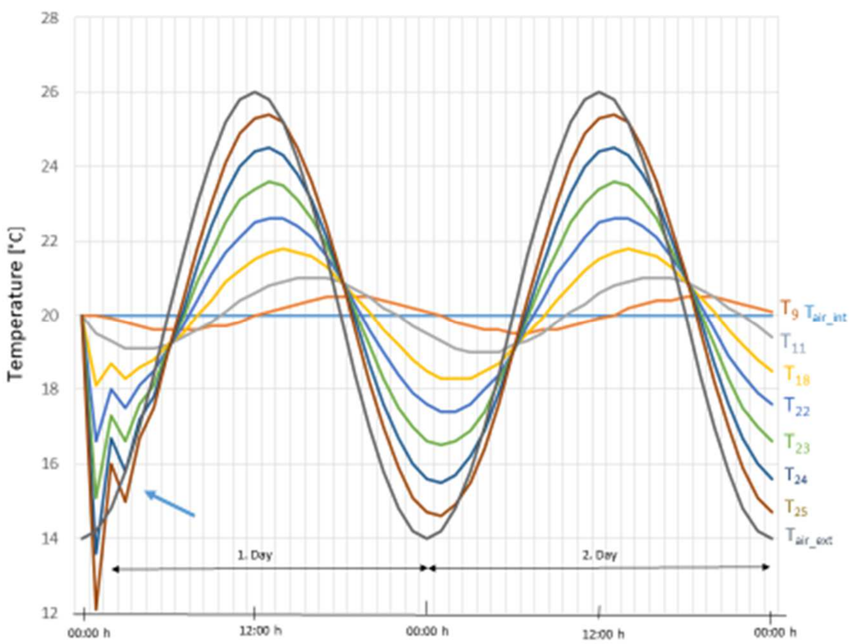


*Figure 3: Change of the air temperature with a cosine step function and resulting temperatures of selected elements in the wall. The arrow points to the mathematical instability at the beginning of calculation.*

Figure 3 illustrates the decrease of temperature amplitudes and the increase of temperature lag proceeding into the wall. The amplitude damping is important for the thermal comfort and will be dealt with in subsequent work. The amplitude lag is important for smart homes to save energy at heating and cooling at the “right” time when the temperature maximum or minimum reaches the inner side of the wall.

The hourly calculated temperature profiles of the wall are given in figure 4.

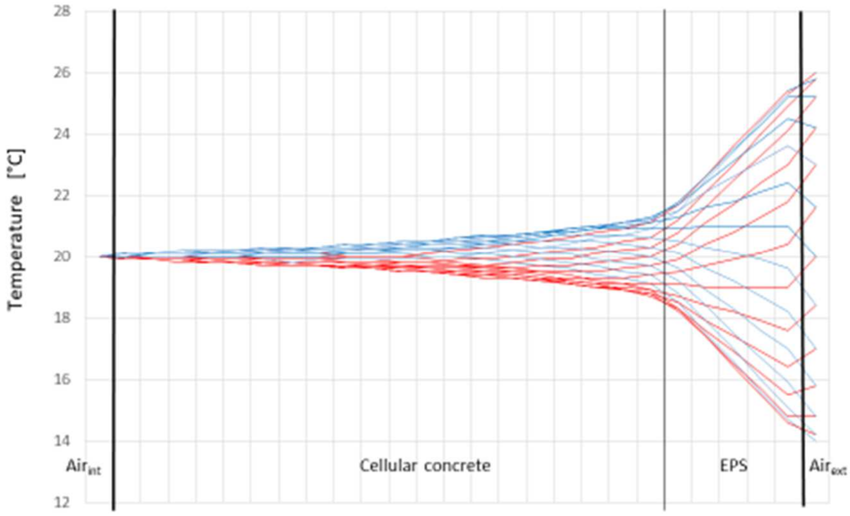


*Figure 4: Temperature profile of the wall at hourly changing temperatures. Red: increasing, blue: decreasing exterior temperatures.*

Figure 4 shows the decrease of temperature change from EPS to cellular concrete. The reason for this is the thermal conductivity of EPS (0.035 W/(m·K)), which is five times smaller than the thermal conductivity of cellular concrete (0.160 W/(m·K)). Therefore, the changes of heat flow through the wall at changing exterior temperatures are reduced in the EPS layer which is called “thermal insulation”.

In figure 5 the calculated heat flow densities on the interior and exterior side of the wall are plotted after the system was settled and stable.

Figure 5 shows that the daily heat flows on the interior and exterior side of the wall are fluctuating around the zero line. This is due to the fact that the outside temperature is symmetrically fluctuating around the median temperature of 20°C. Gains and losses of heat are cancelling each other out. To show this, daily heat flows are summarized at the end of each day in Biot_02.py. At the fourth day, after the system is settled and mathematically stable the sums of heat flows are 0.00 $Wh/m^2$.

NOTE:
Summarized over a period of time, heat flow density is of dimension $Wh/m^2$. This are 3600 $J/m^2$ in one hour, as 1 W is defined as 1 J/s.

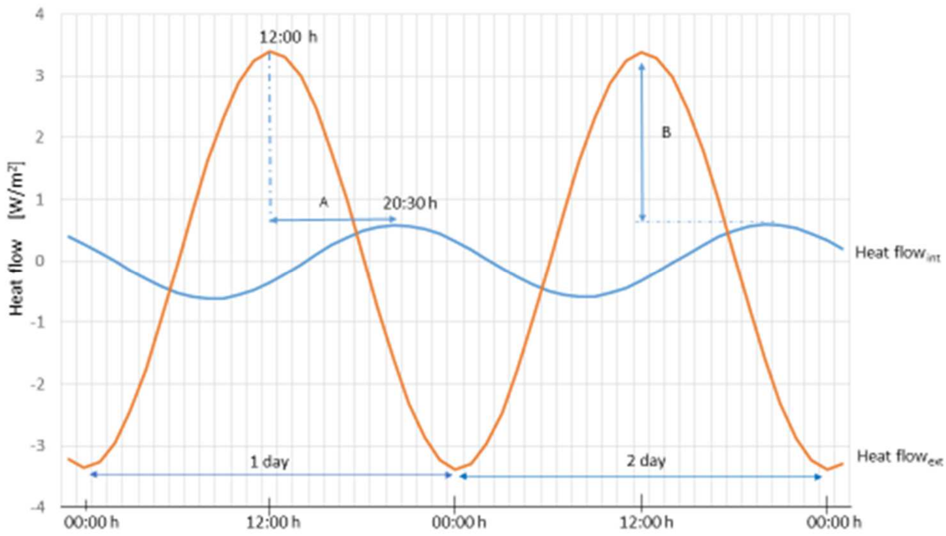


*Figure 5: Heat flows on the interior and exterior side of the wall*

The amplitude lag A is the phase difference between the heat exchange on the outer wall surface and the inner wall surface. The amplitude dampening B is the amplitude difference between the inside and outside heat exchange.

The lag between daily temperature and heat flow amplitude is depending on the heat capacity and the thermal conductivity of the materials of the wall and can be calculated with dynamic methods only.

The dampening of the temperature amplitude and heat flow amplitude is depending on the U-value of the wall and can be calculated with the static heat flow equation and with dynamic methods with the same results.

**Outlook**

The daily energy balance of a building solely caused by changing daily outside air temperature can be calculated with the U-Value of the wall. For this the daily median air temperature difference between interior and exterior air temperatures is sufficient enough.

A dynamic calculation of the heat flow is important, when energy gains and losses like sun irradiation, air exchange, living beings or heating and cooling equipment in the room are to be taken into account. This is the basis for the ”smart home” concept for high living comfort and energy saving by intelligent management of heating, cooling, air exchange and shading, as will be shown in the next publications.

**Literature**

1] http://www.u-cube.org/publikationen/pdf/Calc%20dyn%20heat%20flow.pdf
2] https://github.com/erkam-o/DynamicHeatFlow/blob/master/Calc_Dynamic%20Heat%20Flow_2020_07_05.pdf