SMATH-358 Homework-1, Due: Sept 30,Friday 11:59pm

Name: _____

1. Show that the general solution of the logistic differential equation

$$\begin{cases} \dfrac{dP}{dt} = rP(1 - P/K) \\ P(0) = P_0 \end{cases}$$

can be written by $P(t) = \dfrac{KP_0e^{rt}}{K + P_0(e^{rt} - 1)}$.

2. Consider the following coupled linear ODE system;

$$\begin{cases} \dfrac{dx_1}{dt} = 4x_1 + 2x_2 + 3e^t \\ \dfrac{dx_2}{dt} = 2x_1 + x_2 + e^t \end{cases}$$

Verify that the functions $x_1 = c_1 + 2c_2e^{5t} - \dfrac{e^t}{2}$ and $x_2 = -2c_1 + c_2e^{5t} - \dfrac{3e^t}{4}$ are the general solutions of this ODE system. Find a particular solution with the initial values $x_1(0) = 1, x_2(0) = 3$.

3. In this question, you will replicate some of the results in *Differences in predictions of ODE models of tumor growth: a cautionary example* by Murphy et.al.

   (a) First, click on the link and read the paper. The paper concerns different tumor growth models and easy to read.

   (b) In "Quantitative Example" section, they indicate that they used the data from Systemic treatment of xenografts with vaccinia virus GLV-1h68 reveals the immunologic facet of oncolytic therapy by Worschech et al by using WebPlotDigitizer. Click on paper link, go to Figure 1A and extract the data from GI-101A control group(red points) using WebPlotDigitizer. You should learn how to use this tool, it is super easy. Download your data as a "csv" file and name it "tumor.csv". Make sure that the first column is the day and second column is the tumor size in your csv file.

   (c) In Murphy et.al, replicate Figure-1 and Figure-2. Let's break down how we should do it;

      1. There are 7 different growth models are discussed in the paper, namely, *exponential, Mendelsohn, logistic, linear, surface, Gompertz, and Bertalanffy*. You will numerically solve each of them with matlab "ode45" command. To do so, we need a function to return these ODEs. Use the following template and complete the rest. Use the parameter values(a, b, c etc) given in Figure-1.

```
function dydt = GetODE(t,y,model)
    if model=='exponential'
        a = 0.0262;
        dydt = a*y;
    elseif model=='Mendelsohn'
        a = 0.286;
        b = 0.616;
        dydt = a*y^b;
    %complete rest of the code
    end
end
```

2. Just like we did in the class, define a step size dt, the last time point T and form "tspan" etc and plot the solutions for the each model on the same figure. Figure out what T should be to produce Figure-1.
3. Learn how to read a "csv" file and convert it to matrix. Read your "tumor.csv" you created and plot the first and second column just like in the paper. Notice that you should use **only the first 7 days** for this figure as indicated in the paper. Make sure all the plots you obtain in the previous item and these data points are on the same figure.
4. If you properly complete (1-3), you should be able to produce Figure-1.
5. Change T variable and and use the entire "tumor.csv" to produce Figure-2.

(d) Generate SSR and AIC_C values indicated in Figure-1. You can print them out with "fprintf" command.

(e) You should submit three files. First one is the pdf file for question #1 and #2, name it as [your first name].pdf. The second one is your Matlab code as ".m" file and format it as "[your first name].m". The last one is "tumor.csv" file. Make sure that when you run your matlab file, it correctly produces Figure-1 and item (d). I will not debug your code.

(f) You should need help to finish your homework, please visit me during my office hours. You have two weeks to finish and there is no deadline, cheers!