In the presence of multiple segments that can be seperated, the firm must solve the following two problems:
- What price to charge each segment.
- How to allocate limited capacity among the segments.

Pricing to Multiple Segments

- Consider a supplier that has identified *k* distinct customer segments that can be seperated. Assume that the demand curve for segments *i* is given by:

$$d_i = A_i - B_i \cdot p_i$$

- If the supplier has a cost *c* of production per unit and the goal of the supplier is o price so as to maximize its profits, the pricing problem can be formulated as follows:

$$\text{Max} \sum_{i=1}^{k} (p_i - c) \cdot (A_i - B_i \cdot p_i)$$

- The optimal price for each segment *i* is given by:

$$p_i = \frac{A_i}{2 \cdot B_i} + \frac{c}{2}$$

Example Problem:

• A contract manufacturer has identified two customer segments for its production capacity: one willing to place an order more than one week in advance and the other willing to pay a higher price as long as it can provide less than one week's notice for production.
• The customers that are unwilling to commit in advance are less price sensitive and have a demand curve d1 = 5,000 - 20p1. Customers willing to commit in advance are more price sensitive and have a demand curve of d2 = 5,000 - 40p2.
• Production cost is c = $10 per unit.
a) • What price should the contract manufacturer charge each segment if its goal is to maximize profits?
b) • If the contract manufacturer were to charge a single price over both segments, what should it be?
c) • How much increase in profits does differential pricing provide?
d) • If total production capacity is limited to 4,000 units, what should the contract manufacturer charge each segment?

Without a capacity constraint (a,b,c)

a) $p_i = \frac{A_i}{2 \cdot B_i} + \frac{c}{2}$ $\quad$ $p_1 = \frac{5000}{2 \cdot 20} + \frac{10}{2} = \$130$ $\quad$ $p_2 = \frac{5000}{2 \cdot 40} + \frac{10}{2} = \$67.5$

$d_1 = 5000 - 20 \cdot 130 = 2400$ $\quad$ $d_2 = 5000 - 40 \cdot (67.5) = 2300$

Total revenue = $130 \cdot 2400 + (67.5) \cdot 2300 = \$467250$

Total cost = $(2400 + 2300) \cdot 10 = 47000$

Profit = $467250 - 47000 = \$420250$

b) $(p-10)(5000-20p) + (p-10)(5000-40p) \rightarrow (\underset{C}{p-10})(\underset{A}{10000} - \underset{B}{60p})$

$p = \frac{10000}{2.60} + \frac{10}{2} = \$88.33$

$d_1 = 5000 - 20.(88.33) = 3233.4$ $\qquad d_2 = 5000 - 40.(88.33) = 1466.8$

Total Revenue $= (88.33).(3233.4 + 1466.8) = \$415169$

Total cost $= (3233.4 + 1466.8).10 = \$47002$

Profit $= 415169 - 47002 = \$368167$

c) $420250 - 368167 = 52083$

d) Max $(p_1 - 10).(5000 - 20p_1) + (p_2 - 10).(5000 - 40p_2)$

Constraint: $(5000 - 20p_1) + (5000 - 40p_2) \leq 4000$

$(5000 - 20p_1), (5000 - 40p_2) \geq 0$

Allocating Capacity to a Segment under Uncertainity

p_L : The price charged to the lower-price segment.

p_H : The price charged to the higher-price segment.

D_H : Mean demand for the higher-price segment.

σ_H : Standard deviation of demand for the higher-price segment.

C_H : The capacity reserved for the higher-price segment.

$R_H(C_H)$ = The expected marginal revenue from reserving more capacity for the higher-price segment

$R_H(C_H)$ = The probability that the demand for higher price segment is larger than $C_H \times p_H$

$R_L(C_L)$ = The expected marginal revenue from reserving more capacity for the lower-price segment

$R_L(C_L) = p_L$

- Prob (demand from higher-price segment > C_H) = p_L / p_H

- If *demand for the higher-price segment is normally distributed,* with a mean of D_H and a standard deviation of σ_H, we can obtain the quantity reserved for the higher price segment as

$z = \frac{x - \mu}{\sigma}$

- $C_H = F^{-1}(1 - p_L/p_H, D_H, \sigma_H) = NORMINV(1 - p_L/p_H, D_H, \sigma_H)$

Example Problem:

• ToFrom Trucking serves two segments of customers. One segment (A) is willing to pay \$3.50 per cubic foot but wants to commit to a shipment with only 24 hours notice. The other segment (B) is willing to pay only \$2.00 per cubic foot and is willing to commit to a shipment with up to one week notice.

• With two weeks to go, demand for segment A is forecast to be normally distributed, with a mean 3000 cubic feet and a standard deviation of 1000.

a) • How much of the available capacity should be reserved for segment A?

b) • How should ToFrom change its decision if segment A is willing to pay \$5 per cubic foot?

$P_A = \$3.5 / cft \qquad D_A = 3000\ cft \qquad \sigma_A = 1000\ cft$

$P_B = \$2 / cft$

a) $C_A = F^{-1}\left(1 - \frac{P_A}{P_B}, D_A, \sigma_A\right) \rightarrow C_A = F^{-1}\left(\underbrace{1 - \frac{2}{3.5}}_{0.43}, 3000, 1000\right) \rightarrow -0.18 = \frac{C_A - 3000}{1000}$

$0.43 \rightarrow z = -0.18$

$\boxed{C_A = 2820\ cft}$

b) $P_A = \$5 / cft \qquad P_B = \$2 / cft \qquad D_A = 3000\ cft \qquad \sigma_A = 1000\ cft$

$C_A = F^{-1}\left(\underbrace{1 - \frac{2}{5}}_{0.6}, 3000, 1000\right) \rightarrow 0.253 = \frac{C_A - 3000}{1000} \qquad \boxed{C_A = 3253\ cft}$

$0.6 \rightarrow z = 0.253$

Overbooking

p : the price at which each unit of the asset is sold.

c : the cost of using or producing each unit of the asset.

b : the cost per unit at which a backup can be used.

C_w: $p - c$ = the marginal cost of having wasted capacity.

C_s : $b - p$ = the marginal cost of having a capacity shortage.

O^*: the optimal overbooking level.

s^* : the probability that cancellations will be less than or equal to $O^* = C_w / (C_w + C_s)$

If the distribution of cancellations is known in absolute terms to be normally distributed:

$O^* = F^{-1}(s^*, \mu_c, \sigma_c) = NORMINV(s^*, \mu_c, \sigma_c)$

If the cancellation distribution is known only as a function of the booking level:

$O^* = F^{-1}(s^*, \mu(L+O), \sigma(L+O)) = NORMINV(s^*, \mu(L+O), \sigma(L+O))$

Capacity **Overbooking**

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Example Problem:

- Consider an apparel supplier that is taking orders for dresses with a Christmas motif. The production capacity available from the supplier is 5,000 dresses, and it makes $10 for each dress sold. C_w
- The supplier is currently taking orders from retailers and must decide on how many orders to commit to at this time. If it has orders that exceed capacity, it has to arrange for backup capacity that results in a loss of $5 per dress. C_s

a) • Retailers have been known to cancel their orders near the winter season as they have better visibility into expected demand. How many orders should the supplier accept if cancellations are normally distributed, with a mean of 800 and a standard deviation of 400?

b) • How many orders should the supplier accept if cancellations are normally distributed, with a mean of 15 percent of the orders accepted and a coefficient of variation of 0.5?

a)

$C_w = \$10 / dress \qquad C_s = \$5 / dress$

$s^* = \frac{C_w}{C_w + C_s} = \frac{10}{10+5} = 0.667$

$O^* = F^{-1}(0.667, 800, 400) \rightarrow 0.43 = \frac{O^* - 800}{400} \rightarrow O^* = 973\ dress$

$z = 0.43$

The supplier should accept 5973 order

b) $CV = \frac{\sigma}{\mu} \rightarrow 0.5 = \frac{\sigma}{\mu}$

Coefficient of variation

$\sigma = (0.5) . \mu$

$O^* = F^{-1}\left(\overbrace{0.667}^{z=0.43}, \overbrace{(0.15).(5000 + O^*)}^{\mu}, \overbrace{(0.075).(5000 + O^*)}^{\sigma}\right)$

$0.43 = \frac{O^* - (0.15).(5000 + O^*)}{(0.075).(5000 + O^*)} \rightarrow \boxed{O^* = 1115\ dress}$

Total 6115 order

Pricing and Revenue Management for Bulk and Spot Contracts

c_B : the bulk price for the asset.

c_S : the spot market price for the asset.

Q^* : the optimal amount of the asset to be purchased in bulk.

p^* : the probability that demand for the asset does not exceed Q^*.

- The marginal cost of purchasing another unit in bulk: c_B
- The expected marginal cost of not purchasing another unit in bulk and then purchasing it in the spot market: $(1-p^*)c_S$.

- If the optimal amount of the asset is purchased in bulk, the marginal cost of the bulk purchase should equal the expected marginal cost of the spot market purchase; that is, $c_B = (1 - p^*)c_S$.
- Thus, the optimal value p^* is obtained as $p^*=(c_S - c_B)/c_S$
- *If demand is normally distributed,* with a mean of μ and a standard deviation of σ, the optimal amount, Q^*, of the asset purchased in bulk is obtained as

$$Q^* = F^{-1}(p^*, \mu, \sigma) = NORMINV(p^*, \mu, \sigma)$$

Example Problem:

- A manufacturer sources several components from China and has monthly transportation needs that are normally distributed, with a mean of μ = 10 million units and a standard deviation of σ= 4 million units. The manufacturer must decide on the portfolio of transportation contracts to carry. A long-term bulk contract costs $10,000 per month for a million units. Transportation capacity is also available in the spot market at an average price of $12,500 per million units. For how much transportation capacity should the manufacturer sign a long-term bulk contract?

$C_B = \$10000$ / mil units

$C_S = \$12500$ / mil units

$$p^* = \frac{C_S - C_B}{C_S} \rightarrow \frac{12500 - 10000}{12500} = 0.2$$

$$Q^* = F^{-1}(p^*, \mu, \sigma) \rightarrow F^{-1}(0.2, 10, 4) \rightarrow -0.84 = \frac{Q^* - 10}{4} \rightarrow \boxed{Q^* = 6.64 \text{ mil. unit}}$$

$z = -0.84$