

FSA

① DFA: (Deterministic)

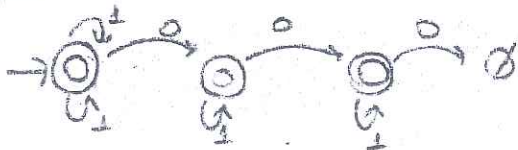
$\delta: Q \times \Sigma \rightarrow Q$ * Bir elemanı bir tere kullabiliriz.

② NFA: (Non-deterministic) $\delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow P(Q)$

Midterm

1) $L = \{w \mid w \text{ has at most 2 0's}\}$ $\Sigma = \{0,1\}$

a) DFA



$\epsilon^* 0 (1^* 0 (1^* 0))^* \epsilon$

b) Regular Expression

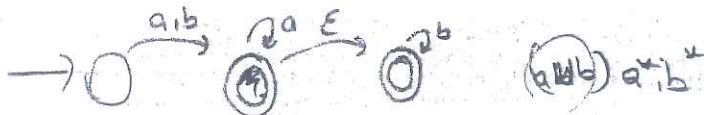
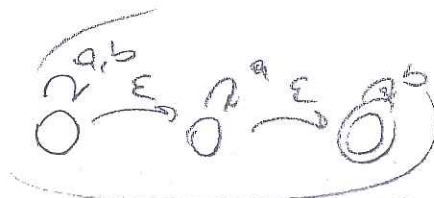
$(1^*) \cup (1^* 0) \cup (1^* 0 1^*) \cup (1^* 0 1^* 0) \cup (1^* 0 1^* 0)$

2) $R = \underbrace{(a \cup b)}_{R_1} \underbrace{a^*}_{R_2} \underbrace{b^*}_{R_3}$

$\Sigma = \{a, b\}$

$\Sigma^* = (a \cup b)^*$

NFA described by R



3) Let $B_n = \{a^k \mid k \text{ is a multiple of } n\}$

Show that for each $n \geq 1$, B_n is regular.

$B_3 = \{a^3, a^6, a^9, \dots\}$

$B_1 = \{a, aa, aaa, \dots\} = a^+$

Proof by Induction:

Base Step: B_1 : $\rightarrow q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \dots$

Inductive Step: Assume the assertion holds for B_m . Show that it holds for B_{m+1} .



$M_{B_n} = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, \dots, q_n\}$

$\Sigma = \{a\}$

$\delta(q_i, a) = \begin{cases} q_{i+1} & \text{if } i \neq n \\ q_0 & \text{if } i = n \end{cases}$

$F = \{q_n\}$

$B_n = \{a^n, a^{2n}, \dots\}$

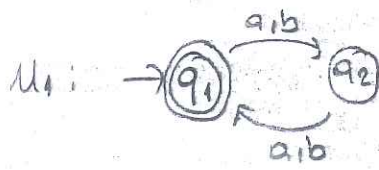
$F = \{q_n\}$

① Design the DFA recognizing the language

$$L_1 = \{w \mid w = 2n \text{ for } n \geq 0\}$$

$$L_2 = \{w \mid w \text{ has an odd \# of a's}\}$$

$$L = \{w \mid w \text{ has an even length or odd \# of a's}\} \text{ Consider } L = L_1 \cup L_2$$



① $Q = \{q_1, q_2\}$

② $\Sigma = \{a, b\}$

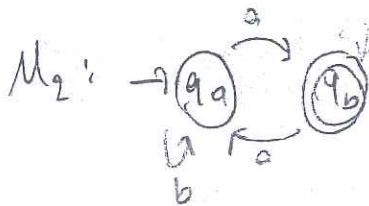
④ $q_0 = q_1$

⑤ $F = \{q_1\}$

$$M = (Q, \Sigma, \delta, q_0, F)$$

③

	a	b
q_1	q_2	q_1
q_2	q_1	q_2



① $Q = \{q_a, q_b\}$

② $\Sigma = \{a, b\}$

③

	a	b
q_a	q_b	q_a
q_b	q_a	q_b

$L_1 \rightarrow \text{regular}$ $L_2 \rightarrow \text{regular}$ $L_1 \cup L_2$ also regular

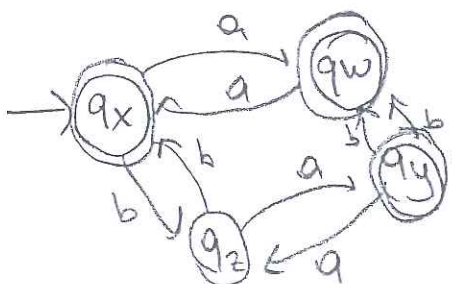
$$M = (Q, \Sigma, \delta, q_0, F)$$

① $Q = Q_1 \times Q_2 = \{(q_1, q_a), (q_1, q_b), (q_2, q_a), (q_2, q_b)\}$

② $\Sigma = \Sigma_1 = \Sigma_2$

③

	a	b
q_x	q_x, q_a	q_x, q_b
q_y	q_y, q_a	q_y, q_b
q_z	q_z, q_a	q_z, q_b
q_w	q_w, q_a	q_w, q_b



④ $q_0 = q_x$

⑤ $F = \{q_x, q_y, q_w\}$