Perfect Coherence Architecture:
A Formal Framework for Coherence-First Systems

OCTA Research (Conceptual Draft)

January 1, 2026

Abstract

This document introduces *Perfect Coherence Architecture* (PCA), a formal framework for *coherence-first systems*. Instead of optimizing isolated components, PCA treats the degree of global coherence—across geometry, dynamics, and intent—as the primary design variable. We define structural, dynamic, and intent coherence functionals on general state spaces, introduce a coherence index as an order parameter, and formulate attractor-driven dynamics that couple local optimization with a global coherence field. This provides a mathematically grounded way to describe and engineer phase transitions from locally governed behavior to globally unified, field-like modes in physical, computational, and socio-technical systems.

Contents

1 Introduction

Many complex systems exhibit a qualitative change in behavior when a sufficient degree of global alignment or *coherence* is reached. Examples include:

- phase-locked electronic states in extended molecular arrays,
- synchronized oscillations in neural populations,
- emergent routing backbones in mesh networks, and
- macro-scale order in socio-economic coordination.

Below a certain threshold, local behavior dominates; above it, the system behaves as a unified manifold rather than a collection of independent parts.

Perfect Coherence Architecture (PCA) is a paradigm in which coherence is elevated to a primary object of analysis and control. The central idea is that system design should prioritize the emergence and maintenance of coherent global modes, with traditional performance objectives treated as constraints or secondary criteria.

This document formalizes:

1. a general model of a system with geometry, dynamics, and intent;
2. coherence functionals along three axes: structural, dynamic, and intent coherence;
3. a coherence index as an order parameter and its role in phase transitions;
4. attractor-driven dynamics that couple local cost functions with a global coherence field; and
5. measurement functionals suitable for empirical estimation of coherence in real systems.

2 System Model

2.1 State Space and Geometry

Definition 2.1 (System)**.** A *system* in the sense of PCA is a tuple

$$\mathcal{S} = (M, G, \mu, \mathcal{X}, F, \Theta),$$

where:

- M is a smooth manifold (state space), typically $M \subseteq \mathbb{R}^d$ or a product thereof;
- $G = (V, E)$ is a graph encoding structural coupling, with $|V| = N$;
- μ is a reference measure on M (e.g., Lebesgue measure);
- $\mathcal{X} \subseteq M^N$ is the set of admissible global states $x = (x_1, \ldots, x_N)$, $x_i \in M$;
- $F : \mathcal{X} \times \Theta \to T\mathcal{X}$ is a vector field defining dynamics; and
- Θ is a parameter space for control, intent, and environment variables.

In many applications, x_i may represent:

- a node state in a network,
- a local field or order parameter,
- a dynamical variable (e.g., phase, voltage, probability vector), or
- a high-dimensional representation (e.g., an embedding or latent code).

2.2 Local and Global Observables

Definition 2.2 (Observables)**.** A *local observable* is a measurable function $o_i : M \to \mathbb{R}^k$ associated with node i. A *global observable* is a measurable function $O : \mathcal{X} \to \mathbb{R}^k$, typically constructed from $\{o_i\}_{i=1}^N$, for example:

$$O(x) = \frac{1}{N} \sum_{i=1}^{N} o_i(x_i).$$

Coherence will be defined in terms of the alignment and correlation of local observables across the system.

3 Coherence Axes

PCA decomposes coherence into three principal axes:

(a) structural coherence (geometry/topology),

(b) dynamic coherence (phase and temporal alignment), and

(c) intent coherence (alignment with goals or attractors).

Each axis is quantified by a functional taking values in $[0, 1]$, where larger values indicate greater coherence.

3.1 Structural Coherence

Let $G = (V, E)$ be a graph with $N = |V|$ nodes. Let L denote its graph Laplacian and let $x \in \mathcal{X} \subseteq M^N$.

Definition 3.1 (Structural Energy)**.** Define the *structural energy* of a global state x by

$$E_{\mathrm{str}}(x) = \frac{1}{2} \sum_{(i,j)\in E} w_{ij} \, \|x_i - x_j\|^2 = \frac{1}{2} x^\top (L \otimes I_d) x, \tag{1}$$

where $w_{ij} > 0$ are edge weights and I_d is the $d \times d$ identity matrix when $M \subseteq \mathbb{R}^d$.

The structural energy penalizes abrupt variations along edges, analogous to Dirichlet energy.

Definition 3.2 (Structural Coherence)**.** Suppose E_{str} admits an a priori upper bound $E_{\max} \in (0, \infty)$ on $\mathcal{X}$. The *structural coherence* of x is

$$\mathrm{CS}(x) = 1 - \frac{E_{\mathrm{str}}(x)}{E_{\max}}, \quad \mathrm{CS}(x) \in [0, 1]. \tag{2}$$

In practice, $E_{\max}$ can be estimated from worst-case configurations, empirical extrema, or theoretical bounds.

Remark 3.3. High $\mathrm{CS}(x)$ corresponds to low curvature or tension across the network: states vary smoothly over the graph structure. In the continuum limit, this connects to the minimization of $\int \|\nabla\phi\|^2 \, d\mu$ for a field ϕ.

3.2 Dynamic Coherence

Dynamic coherence captures phase alignment, temporal correlation, or synchronization.

3.2.1 Oscillatory Case

Consider a phase description of the dynamics, where each node i is associated with a phase $\phi_i \in S^1$. Define the complex order parameter

$$R(x) = \frac{1}{N} \sum_{i=1}^{N} e^{\mathrm{i}\phi_i}. \tag{3}$$

Definition 3.4 (Dynamic Coherence: Oscillatory Case)**.** The *dynamic coherence* for an oscillatory system is

$$\mathrm{CD}(x) = |R(x)| = \left| \frac{1}{N} \sum_{i=1}^{N} e^{\mathrm{i}\phi_i} \right|, \quad \mathrm{CD}(x) \in [0, 1]. \tag{4}$$

$\mathrm{CD}(x)$ is maximal when all phases are equal and minimal when phases are uniformly spread.

3.2.2 General Case

For more general dynamics, let $v_i \in \mathbb{R}^d$ denote a local velocity or incremental state, and define the covariance structure:

$$C(x) = \frac{1}{N} \sum_{i=1}^{N} (v_i - \bar{v})(v_i - \bar{v})^\top, \quad \bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i.$$

Definition 3.5 (Dynamic Coherence: General Case)**.** Let $\lambda_{\max}(C)$ be the largest eigenvalue of $C(x)$, and let $0 < \Lambda_{\max} < \infty$ be a reference bound. Define

$$\mathrm{CD}(x) = 1 - \frac{\lambda_{\max}(C(x))}{\Lambda_{\max}}, \quad \mathrm{CD}(x) \in [0, 1]. \tag{5}$$

Here, low $\lambda_{\max}(C)$ implies small dispersion of local dynamics, corresponding to high alignment.

3.3 Intent Coherence

Intent coherence measures alignment of the system state with a target configuration, goal, or attractor field.

3.3.1 Cost and Attractor Potentials

Let $U : \mathcal{X} \to \mathbb{R}$ be a local cost (e.g., energy, loss, or negative utility), and let $\Phi : \mathcal{X} \to \mathbb{R}$ be an attractor potential encoding a global coherence field (e.g., a Perfect Attractor).

Definition 3.6 (Intent Field and Effective Potential)**.** For $\alpha \geq 0$, define the *intent field* as

$$V_\alpha(x) = U(x) - \alpha\,\Phi(x). \tag{6}$$

We interpret V_α as an effective potential that balances local optimization (U) and global coherence (Φ).

3.3.2 Gradient Alignment Measure

Define the (possibly generalized) gradient of V_α:

$$\nabla V_\alpha(x) = \nabla U(x) - \alpha \nabla \Phi(x),$$

where interpreted in a suitable Riemannian or metric sense if $\mathcal{X}$ is not Euclidean.

Definition 3.7 (Intent Coherence)**.** Let $C_{\mathrm{int}} > 0$ be a normalization constant. The *intent coherence* of state x at parameter α is

$$\mathrm{CI}_{\mathrm{int}}(x; \alpha) = 1 - \frac{\|\nabla V_\alpha(x)\|}{C_{\mathrm{int}}}, \quad \mathrm{CI}_{\mathrm{int}}(x; \alpha) \in [0, 1]. \tag{7}$$

High $\mathrm{CI}_{\mathrm{int}}$ corresponds to small gradient norm of the effective potential, signifying that the system is near a joint optimum of local and coherence objectives.

Remark 3.8. One may choose C_{int} as a bound on the gradient norm, an empirical maximum over a dataset, or a scale derived from physical parameters.

4 Coherence Index and Order Parameter

The three coherence components can be aggregated into a single *coherence index*.

Definition 4.1 (Coherence Index)**.** Let $w_S, w_D, w_I \geq 0$ with $w_S + w_D + w_I = 1$. For a state $x \in \mathcal{X}$ and parameter α, define the *coherence index* by

$$\mathrm{CI}(x; \alpha) = w_S\ \mathrm{CS}(x) + w_D\ \mathrm{CD}(x) + w_I\ \mathrm{CI}_{\mathrm{int}}(x; \alpha), \tag{8}$$

with $\mathrm{CI}(x; \alpha) \in [0, 1]$.

Remark 4.2. CI acts as an *order parameter* for coherence-driven phase transitions: below a critical threshold, the system behaves as a collection of weakly coupled components; above it, coherent global modes dominate.

4.1 Phase Transition Regimes

We informally distinguish three regimes:

$$\begin{aligned} \text{(I) Subcritical:} \quad & 0 \le \mathrm{CI} < \mathrm{CI}_c, \\ \text{(II) Critical:} \quad & \mathrm{CI} \approx \mathrm{CI}_c, \\ \text{(III) Supercritical:} \quad & \mathrm{CI} > \mathrm{CI}_c, \end{aligned}$$

for some $\mathrm{CI}_c \in (0, 1)$.

Definition 4.3 (Coherence Phase)**.** A coherence phase is an equivalence class of system states with qualitatively similar macroscopic behavior, distinguished by the value of CI.

Rigorous characterization of CI_c will depend on the specific system class. Section 6 introduces a spectral criterion in a simplified setting.

5 Attractor-Driven Dynamics

5.1 General Form of Dynamics

Consider a continuous-time dynamical system on $\mathcal{X}$:

$$\dot{x}(t) = F(x(t); \theta), \quad x(0) = x_0, \tag{9}$$

for $\theta \in \Theta$.

Within PCA, we model F as a combination of:

- local cost descent (gradient flow of U),
- coherence field ascent/descent (gradient flow of Φ),
- coupling-induced alignment, and
- noise or exogenous forcing.

Definition 5.1 (PCA Dynamics)**.** A vector field $F : \mathcal{X} \to T\mathcal{X}$ is said to be of *PCA type* if it admits a decomposition

$$\dot{x} = -\nabla U(x) + \alpha \, \nabla \Phi(x) - \beta \, L_{\mathrm{dyn}} x + \sigma \, \eta(t), \tag{10}$$

where:

- U is a local cost functional,
- Φ is an attractor potential,
- $\alpha, \beta, \sigma \ge 0$ are scalar parameters,
- L_{dyn} is a (possibly state-dependent) Laplacian-like operator encoding dynamic coupling,
- $\eta(t)$ is a noise term (e.g., white noise, colored noise, or exogenous input).

The competition between $-\nabla U$ and $\alpha \nabla \Phi$ is central: U may favor localized or heterogeneous solutions, while Φ encourages coherent global structure.

5.2 Energy Dissipation and Coherence Growth

Under suitable conditions, Φ can be chosen so that trajectories tend to regions of higher coherence index.

Proposition 5.2 (Monotonicity of Effective Potential (Heuristic)). *Assume:*

(i) U and Φ are continuously differentiable and bounded below;

(ii) L_{dyn} is positive semidefinite;

(iii) $\sigma = 0$ (deterministic case).

Consider the PCA dynamics (10). Then along any trajectory $t \mapsto x(t)$,

$$\frac{\mathrm{d}}{\mathrm{d}t} V_\alpha(x(t)) = -\left\|\nabla U(x(t))\right\|^2 + \alpha \left\langle \nabla \Phi(x(t)), \nabla U(x(t)) \right\rangle - \alpha \left\langle \nabla \Phi(x(t)), L_{dyn} x(t) \right\rangle . \tag{11}$$

In particular, if the cross-terms can be bounded appropriately and α is chosen suitably, then V_α can be made non-increasing, guiding the system toward a joint optimum of U and Φ.

Remark 5.3. The detailed conditions under which V_α is a Lyapunov function depend on the geometry of $\mathcal{X}$, the structure of L_{dyn}, and the interaction between ∇U and $\nabla \Phi$. One can derive sufficient conditions in specific models (e.g., Kuramoto-type oscillators, reaction-diffusion systems, or coupled gradient flows).

6 Phase Transition via Spectral Criteria

In this section, we outline a spectral perspective on coherence transitions, focusing on a linearized model.

6.1 Linearized Dynamics

Suppose that near a reference state $x^\star \in \mathcal{X}$, we can approximate the dynamics by

$$\dot{y} = -H_U y + \alpha H_\Phi y - \beta L_{\mathrm{dyn}} y, \quad y = x - x^\star, \tag{12}$$

where H_U and H_Φ are Hessians of U and Φ at $x^\star$.

Definition 6.1 (Effective Stability Operator). Define the effective operator

$$\mathcal{L}_\alpha = H_U - \alpha H_\Phi + \beta L_{\mathrm{dyn}}. \tag{13}$$

Stability of $x^\star$ is determined by the spectrum of $\mathcal{L}_\alpha$.

Proposition 6.2 (Spectral Coherence Transition (Heuristic)). *Let $\lambda_{\min}(\mathcal{L}_\alpha)$ denote the smallest eigenvalue of $\mathcal{L}_\alpha$. Assume that as α increases:*

(i) $\lambda_{\min}(\mathcal{L}_\alpha)$ crosses zero at some α_c,

(ii) the associated eigenvector corresponds to a coherent global mode (e.g., low-frequency graph Fourier mode).

Then α_c marks a coherence transition, *where the reference state $x^\star$ loses stability and a more coherent pattern emerges.*

Remark 6.3. In many systems, the onset of coherence is associated with the activation of low-frequency modes of the Laplacian (large-scale patterns) or with phase-locking in oscillator networks. The spectral gap of L_{dyn} and the structure of H_Φ control the critical point.

7 Measurement and Estimation of Coherence

To apply PCA to real systems, one must estimate coherence functionals from data.

7.1 Empirical Structural Coherence

Given snapshots $\{x^{(k)}\}_{k=1}^{K}$ from the system, we estimate $E_{\max}$ empirically, e.g.

$$\widehat{E}_{\max} = \max_{1\leq k\leq K} E_{\mathrm{str}}(x^{(k)}). \tag{14}$$

Then

$$\widehat{\mathrm{CS}}(x^{(k)}) = 1 - \frac{E_{\mathrm{str}}(x^{(k)})}{\widehat{E}_{\max}}. \tag{15}$$

7.2 Empirical Dynamic Coherence

For oscillatory systems, phases $\phi_i^{(k)}$ at sample time k can be extracted from observed signals (e.g., via Hilbert transform). Then

$$\widehat{\mathrm{CD}}(x^{(k)}) = \left| \frac{1}{N} \sum_{i=1}^{N} e^{\mathrm{i}\phi_i^{(k)}} \right|. \tag{16}$$

For general dynamics, one can estimate local velocities $v_i^{(k)}$ via finite differences and compute:

$$\widehat{C}^{(k)} = \frac{1}{N} \sum_{i=1}^{N} \left(v_i^{(k)} - \bar{v}^{(k)}\right) \left(v_i^{(k)} - \bar{v}^{(k)}\right)^{\top}, \tag{17}$$

then approximate $\Lambda_{\max}$ from the empirical maximum of $\lambda_{\max}(\widehat{C}^{(k)})$.

7.3 Empirical Intent Coherence

Assuming estimates $\widehat{\nabla U}$ and $\widehat{\nabla \Phi}$ are available (e.g., via differentiable models or surrogate modeling), we may define:

$$\widehat{\mathrm{CI}_{\mathrm{int}}}(x^{(k)};\alpha) = 1 - \frac{\left\| \widehat{\nabla U}(x^{(k)}) - \alpha \widehat{\nabla \Phi}(x^{(k)}) \right\|}{\widehat{C}_{\mathrm{int}}}, \tag{18}$$

where $\widehat{C}_{\mathrm{int}}$ is an empirical scale.

7.4 Empirical Coherence Index

Combining the components:

$$\widehat{\mathrm{CI}}(x^{(k)};\alpha) = w_S \widehat{\mathrm{CS}}(x^{(k)}) + w_D \widehat{\mathrm{CD}}(x^{(k)}) + w_I \widehat{\mathrm{CI}_{\mathrm{int}}}(x^{(k)};\alpha). \tag{19}$$

Time series of $\widehat{\mathrm{CI}}$ can reveal coherence transitions, metastable states, and the impact of control parameters.

8 Multiscale Coherence

Real systems often possess a hierarchy of scales. We sketch a multiscale generalization of PCA.

8.1 Scale Hierarchy

Let $\{\mathcal{X}_\ell\}_{\ell=0}^L$ be a sequence of state spaces at different scales, with $\mathcal{X}_0$ microscopic and $\mathcal{X}_L$ macroscopic. Assume there exist coarse-graining maps

$$\Pi_{\ell+1,\ell} : \mathcal{X}_\ell \to \mathcal{X}_{\ell+1}, \quad \ell = 0, \dots, L-1.$$

Definition 8.1 (Scale-ℓ Coherence)**.** At scale ℓ, define coherence components

$$\mathrm{CS}_\ell, \quad \mathrm{CD}_\ell, \quad \mathrm{CI}_{\mathrm{int}\ \ell},$$

and a coherence index

$$\mathrm{CI}_\ell(x_\ell; \alpha_\ell) = w_{S,\ell}\, \mathrm{CS}_\ell(x_\ell) + w_{D,\ell}\, \mathrm{CD}_\ell(x_\ell) + w_{I,\ell}\, \mathrm{CI}_{\mathrm{int}\ \ell}(x_\ell; \alpha_\ell).$$

8.2 Multiscale Coherence Functional

Definition 8.2 (Global Multiscale Coherence)**.** Define the *global multiscale coherence functional* as

$$\mathrm{CI}_{\mathrm{global}} = \sum_{\ell=0}^{L} \gamma_\ell\, \mathrm{CI}_\ell(x_\ell; \alpha_\ell), \tag{20}$$

with weights $\gamma_\ell \geq 0$, $\sum_\ell \gamma_\ell = 1$, and $x_\ell = \Pi_{\ell,\ell-1} \circ \cdots \circ \Pi_{1,0}(x_0)$.

Remark 8.3. Multiscale coherence captures the extent to which local coherence propagates upward, and global coherence feeds back into local behavior via attractor fields defined at higher scales.

9 Example Templates

We briefly sketch how PCA specializes to several domains.

9.1 Oscillator Networks

Let each node i have phase ϕ_i and intrinsic frequency ω_i. Consider Kuramoto-type dynamics with an attractor term:

$$\dot{\phi}_i = \omega_i + \frac{K}{N} \sum_{j=1}^{N} a_{ij} \sin(\phi_j - \phi_i) + \alpha \frac{\partial}{\partial \phi_i} \Phi(\phi), \tag{21}$$

where $A = (a_{ij})$ is the adjacency matrix and $\Phi(\phi)$ is an appropriate potential (e.g., favoring a particular phase configuration or pattern).

Then:

- CS is defined via the smoothness of ϕ over the graph,
- CD is the standard Kuramoto order parameter,
- $\mathrm{CI}_{\mathrm{int}}$ measures alignment with Φ.

9.2 Field-Coherent Materials

Let $u : \Omega \subset \mathbb{R}^d \to \mathbb{R}^m$ denote a field (e.g., electronic or order parameter field). Define:

$$E_{\text{str}}(u) = \int_{\Omega} \|\nabla u(x)\|^2 \, dx, \tag{22}$$

$$\Phi(u) = \int_{\Omega} \phi(u(x)) \, dx, \tag{23}$$

where ϕ is a potential favoring certain coherent configurations (e.g., planar alignment). Dynamics may follow a gradient flow of $U - \alpha\Phi$ under appropriate constraints.

9.3 Intent-Coherent Multi-Agent Systems

Let x_i denote agent states and θ represent global intent. Define:

$$U(x) = \sum_{i=1}^{N} u_i(x_i), \tag{24}$$

$$\Phi(x; \theta) = \sum_{i=1}^{N} \phi(x_i; \theta), \tag{25}$$

where ϕ encodes how well agent i's state aligns with the global intent. Then PCA dynamics can be implemented via local updates using gradient estimates, with CI used as a system-level metric of coordination.

10 Discussion and Outlook

Perfect Coherence Architecture provides a unified language for:

- defining and quantifying coherence along structural, dynamic, and intent axes;
- constructing attractor-driven dynamics that explicitly trade off local optimization and global field alignment; and
- analyzing coherence transitions via spectral and multiscale criteria.

Future work includes:

1. rigorous derivation of phase-transition thresholds in concrete model classes;
2. extension to stochastic and non-equilibrium systems with strong noise;
3. integration with learning frameworks where U and Φ are learned from data; and
4. application to specific architectures in neuromorphic computing, mesh networking, and materials design.