Tessellated Field Synthesis (TFS): A Strain-Programmed Technology for Routing Heat, Stress, and Waves

Abstract

Tessellated Field Synthesis (TFS) is a proposed physical technology in which locked-in eigenstrain fields are used to program anisotropic transport and elastic properties in solid media. This framework enables routing of heat, mechanical stress, and acoustic energy through a medium without continuous power input or active force generation. Unlike conventional metamaterials, TFS employs rewritable, hysteretic microstructural tessellations to define stable and programmable effective transport tensors. This paper develops a rigorous mathematical foundation for TFS based on nonlinear elasticity, eigenstrain theory, homogenization, anisotropic diffusion, wave propagation, and PDE-constrained optimization. Practical design pathways, experimental realization strategies, and engineering applications are presented.

Contents

1 Introduction

Modern technologies manipulate energy flow primarily through geometry (e.g. waveguides), external fields, or continuous actuation. In contrast, Tessellated Field Synthesis (TFS) is based on controlling the *rules governing transport* by encoding spatially distributed, locked-in eigenstrain fields. These fields alter the anisotropy and magnitude of effective elasticity and transport tensors. Once written, the programmed state is stable in the absence of power.

This paper formalizes TFS as a new branch of *strain-programmable transport engineering*.

2 Domain and Field Definitions

Let the material occupy a bounded Lipschitz domain

$$\Omega \subset \mathbb{R}^3.$$

At each point $x \in \Omega$ we define:

- density $\rho(x)$
- displacement field $u(x,t)$
- infinitesimal strain tensor
$$\varepsilon(u) = \frac{1}{2}\left(\nabla u + \nabla u^\top\right)$$
- locked-in eigenstrain $\varepsilon^{\text{eig}}(x)$
- effective elasticity tensor $C_{\text{eff}}(x)$
- effective thermal conductivity tensor $\kappa_{\text{eff}}(x)$.

Total strain:

$$\varepsilon_{\text{total}} = \varepsilon(u) + \varepsilon^{\text{eig}}.$$

3 Constitutive Coupling to Eigenstrain

The defining principle of TFS is:

$$\boxed{\kappa_{\text{eff}} = \kappa_0 + \mathcal{H}_\kappa(\varepsilon^{\text{eig}})} \qquad \boxed{C_{\text{eff}} = C_0 + \mathcal{H}_C(\varepsilon^{\text{eig}})}$$

with nonlinear strain dependence.

A quadratic expansion gives:

$$\kappa_{\text{eff}} = \kappa_0 + A : \varepsilon^{\text{eig}} + \varepsilon^{\text{eig}} : B : \varepsilon^{\text{eig}}$$

$$C_{\text{eff}} = C_0 + D :: \varepsilon^{\text{eig}} + \varepsilon^{\text{eig}} :: E :: \varepsilon^{\text{eig}}.$$

4 Residual Stress

Residual stress:

$$\sigma^\star = C_{\text{eff}} : \varepsilon^{\text{eig}}.$$

Static equilibrium:

$$\nabla \cdot \sigma^\star = 0.$$

5 Governing Equations

5.1 Heat Transport

$$c(x)\partial_t T = \nabla \cdot (\kappa_{\text{eff}} \nabla T)$$

Heat rays follow geodesics of metric

$$g_{ij} = (\kappa_{\text{eff}}^{-1})_{ij}.$$

5.2 Elastic and Acoustic Waves

$$\rho \ddot{u} = \nabla \cdot (C_{\text{eff}} : \nabla^s u) .$$

5.3 Static Mechanics

$$\nabla \cdot \sigma = 0, \qquad \sigma = C_{\text{eff}} : \varepsilon(u).$$

Energy:

$$E = \frac{1}{2} \int_\Omega \varepsilon : C_{\text{eff}} : \varepsilon \, dx.$$

6 Tessellated State Field

Partition:

$$\Omega = \bigcup_{i=1}^{N} \Omega_i,$$

and assign discrete states

$$\zeta(x) \in \{1, \ldots, M\}.$$

Free energy:

$$\mathcal{F} = \int_\Omega \left(V(\zeta) + \frac{\beta}{2} |\nabla \zeta|^2 \right) dx.$$

Dynamics:

$$\partial_t \zeta = -\Gamma \frac{\delta \mathcal{F}}{\delta \zeta}.$$

7 Writer Field Dynamics

Let $\phi(x, t)$ denote the writer field and define switching dynamics:

$$\partial_t \zeta = \alpha(\phi) \left(\zeta_{\text{target}}(\phi) - \zeta \right) \quad \text{if } |\phi| > \phi_{\text{thresh}}.$$

8 Homogenization

Let $y = x/\epsilon$.

Cell problem:

$$\nabla_y \cdot (C(y,\zeta) : \nabla_y w_{mn}) = 0.$$

Homogenized tensor:

$$C_{\text{eff}}^{ijkl} = \int_Y C^{pqrs} \left(\delta_{ip}\delta_{jq} + \partial_{y_p} w_{ij}^q\right) \left(\delta_{kr}\delta_{ls} + \partial_{y_r} w_{kl}^s\right) dy.$$

9 Stability Conditions

1. Compatibility:

$$\nabla \times \nabla \times \varepsilon^{\text{eig}} = 0$$

2. Equilibrium:

$$\nabla \cdot \sigma^\star = 0$$

3. Thermal stability:

$$\Delta V_{\text{well}} \gg k_B T$$

4. Negligible creep:

$$\dot{\varepsilon}^{\text{plastic}} \approx 0.$$

10 Inverse Design Problem

Subject to:

$$\mathcal{P}(u;\zeta) = 0,$$

define:

$$J(\zeta) = \int_\Omega L(x, u)\, dx + R(\zeta).$$

Solve:

$$\min_{\zeta \in \mathcal{A}} J(\zeta).$$

11 Case Study: Programmable Silence

Protected region $D \subset \Omega$.

Energy density:

$$E_a = \frac{1}{2} \left(\rho |\dot{u}|^2 + \varepsilon : C_{\text{eff}} : \varepsilon\right).$$

Objective:

$$J(\zeta) = \int_D E_a\, dx + \lambda \int_\Omega |\nabla \zeta|^2 dx.$$

12 Numerical Simulation Framework

1. Discretize Ω.
2. Assign ζ_i.
3. Map to $C_{\text{eff}}, \kappa_{\text{eff}}$.
4. Solve PDEs via FEM/FV.
5. Evaluate $J(\zeta)$.
6. Update ζ via:
    - simulated annealing
    - topology optimization
    - integer programming

7. Iterate.

13 Experimental Realization

Candidate materials include:

- ferroelastic ceramics
- shape-memory alloys
- graphene-reinforced composites
- amorphous strain-locking glasses.

Writer fields may include:

- phased ultrasonics
- localized EM heating
- stress-imprinting dies.

14 Measurement and Verification

- laser vibrometry
- scanning thermal microscopy
- acoustic spectroscopy
- XRD residual strain mapping.

15 Applications

- programmable acoustic cloaking
- heat routing in microelectronics
- impact-shunting protective systems
- friction-suppressed cutting tools
- vibration absorption architectures
- secure substrate-based communication.

16 Ethics and Risk

Potential misuse includes cloaking and weaponization. Governance mechanisms are recommended.

17 Roadmap

1. verify strain–transport coupling
2. demonstrate stable tessellations
3. implement inverse-design compiler
4. build first functional devices.

18 Open Problems

- optimal microstructure libraries
- stability bounds

- scaling limits
- safety frameworks.

19 Conclusion

This paper establishes a rigorous foundation for Tessellated Field Synthesis and identifies a practical path toward its engineering realization.

References

(To be populated.)