Shadow Math Part II:
Shadow Information Geometry, Shadow Laplacians, and Learning Dynamics

Abstract

Part II extends the foundations of Shadow Math by introducing:

- The Fisher–Rao geometry of shadows and identity fibers,
- Shadow Laplacians separating fiber and base diffusion,
- Strong Data Processing for irreducible shadow projections,
- Shadow Learning: how gradient descent in a higher layer appears in lower layers,
- Shadow Curvature: how curvature contracts or amplifies through projections,
- Shadow Flow Equations: shadow Ricci flow, shadow entropy flow,
- Fixed points of shadow dynamics and dimensional attractors.

This establishes the full calculus of how identity behaves across dimensions under geometry, stochastic processes, and learning.

Contents

1 Shadow Information Geometry

We now add a *statistical manifold structure* to link identity, entropy, and geometry.

1.1 Fisher metric under shadow projection

Let $\mathcal{P}(\mathcal{L}_k)$ denote the manifold of smooth probability densities on layer k.

Definition 1.1 (Fisher–Rao metric). *For $p \in \mathcal{P}(\mathcal{L}_k)$ and tangent vectors u, v with $\int u = \int v = 0$, define*

$$g_k(u, v) = \int_{\mathcal{L}_k} \frac{u(x)v(x)}{p(x)} dx.$$

Higher layers have analogous metrics g_{k+1}.

Definition 1.2 (Shadow pushforward of tangent vectors). *Given a projection $\pi_{k+1\to k}$ and a density p_{k+1} with shadow $p_k = \mathsf{Sh}_k(p_{k+1})$, define*

$$(u_{k+1})_{\mathsf{Sh}} := (\pi_{k+1\to k})_{\#} u_{k+1}.$$

Theorem 1.3 (Fisher contraction under shadow). *Let u_{k+1}, v_{k+1} be tangent vectors at p_{k+1}. Then*

$$g_k((u_{k+1})_{\mathsf{Sh}},\ (v_{k+1})_{\mathsf{Sh}}) \ \le\ g_{k+1}(u_{k+1}, v_{k+1}),$$

with equality iff the projection is information-lossless on the support.

Remark 1.4. Shadows always *decrease Fisher information.* Identity becomes smoother, less distinguishable, as dimensionality drops.

1.2 Shadow geodesics

If $\gamma_{k+1}(t)$ is a Fisher geodesic in the higher layer:

$$\ddot{\gamma}_{k+1}(t) = 0,$$

then its shadow path

$$\gamma_k(t) = \mathsf{Sh}_k(\gamma_{k+1}(t))$$

is generally *not* a geodesic.

Proposition 1.5. *Shadowing a geodesic introduces curvature proportional to fiber entropy.*

$$\ddot{\gamma}_k(t) = \nabla H_{\text{fiber}}(\gamma_{k+1}(t)) + \textit{projection curvature.}$$

Remark 1.6. Even linear motion in a higher space appears curved after losing identity information.

2 Shadow Laplacians and Diffusion

We now construct diffusion operators that separate:
- horizontal movement (visible identity change) - vertical movement (fiber identity change invisible from below)

2.1 Fiber vs base Laplacian

Let $\mathcal{L}_{k+1}$ be a smooth manifold and π a smooth submersion.

The tangent space splits as:

$$T_x\mathcal{L}_{k+1} = V_x \oplus H_x$$

where - $V_x = \ker d\pi$ is the *vertical* fiber space - H_x is the horizontal complement.

Definition 2.1 (Fiber Laplacian)**.**

$$\Delta_{\text{fiber}} f(x) := \mathrm{Tr}\big(\nabla^2 f|_{V_x}\big)$$

Definition 2.2 (Base Laplacian pullback)**.**

$$\Delta_{\text{base}} f(x) := \Delta(f \circ \pi)(x)$$

using the base layer Laplacian.

Definition 2.3 (Shadow Laplacian)**.**

$$\Delta_{\mathsf{Sh}} f(x) := \Delta_{\text{base}} f(x).$$

Remark 2.4. The shadow Laplacian ignores all fiber motion. Shadow diffusion is strictly the diffusion the lower layer can detect.

2.2 Decomposition theorem

Theorem 2.5 (Laplacian decomposition)**.** *For any smooth function f on $\mathcal{L}_k$,*

$$\Delta(f^{\uparrow}) = \Delta_{\mathsf{Sh}} f + \Delta_{\text{fiber}}(f^{\uparrow}).$$

Remark 2.6. Fiber diffusion is the "hidden turbulence" above the dimensional layer. To the lower layer, this vanishes entirely.

3 Shadow Channels and Strong Data Processing

3.1 Shadow channel coefficient

For a Markov channel T and projection π:

Definition 3.1 (Shadow contraction coefficient)**.**

$$\eta_{\mathsf{Sh}}(T) := \sup_{p \neq q} \frac{D(\mathsf{Sh}_k(Tp) \parallel \mathsf{Sh}_k(Tq))}{D(\mathsf{Sh}_{k+1}(p) \parallel \mathsf{Sh}_{k+1}(q))}.$$

Theorem 3.2 (Strong Data Processing for shadows)**.** *For any shadow channel,*

$$\eta_{\mathsf{Sh}}(T) \leq \eta(T),$$

where $\eta(T)$ is the usual SDPI coefficient.

Remark 3.3. Shadowing amplifies contraction: differences in identity shrink faster after projection.

4 Shadow Learning Dynamics

We consider gradient descent in layer $k+1$:

$$\dot{\theta}_{k+1}(t) = -\nabla_{\theta_{k+1}} L(\theta_{k+1}(t)).$$

Let

$$\theta_k(t) = \pi(\theta_{k+1}(t))$$

be its shadow.

4.1 Shadow gradient

Definition 4.1 (Shadow gradient)**.**

$$\nabla_{\mathsf{Sh}} L(\theta_k) := (\nabla L(\theta_{k+1}))_{horizontal}.$$

Theorem 4.2 (Learning viewed through a shadow)**.**

$$\dot{\theta}_k(t) = -\nabla_{\mathsf{Sh}} L(\theta_k(t)).$$

Remark 4.3. The lower layer perceives only the horizontal component of the true gradient. Vertical identity adjustments are invisible.

4.2 Identity regularization

Definition 4.4 (Fiber regularizer)**.**

$$R_{id}(\theta_{k+1}) := \|\nabla L(\theta_{k+1})_{\text{vertical}}\|^2.$$

Remark 4.5. This measures how much the true gradient tries to change hidden identity that cannot be seen by the shadow layer.

Proposition 4.6. *Shadow gradient descent minimizes the composite functional:*

$$L_{\mathsf{Sh}}(\theta_k) = L(\theta_{k+1}) - \frac{1}{2} R_{id}(\theta_{k+1}),$$

projected to layer k.

5 Shadow Curvature and Flow

5.1 Shadow Ricci curvature

Let Ric_{k+1} be the higher layer Ricci tensor.

Definition 5.1 (Shadow Ricci tensor)**.**

$$\mathrm{Ric}_{\mathsf{Sh}} := (\pi_{k+1\to k})_* \mathrm{Ric}_{k+1}.$$

Remark 5.2. Shadow curvature encodes how the "felt geometry" changes after identity loss.

5.2 Shadow curvature contraction

Theorem 5.3. *If* $\mathrm{Ric}_{k+1} \geq \kappa g_{k+1}$ *for some* $\kappa > 0$*, then*

$$\mathrm{Ric}_{\mathsf{Sh}} \geq \kappa g_k.$$

6 Shadow Entropy Flow

For a heat flow p_t in layer $k+1$, the entropy evolves as:

$$\frac{d}{dt} H(p_t) = -I(p_t),$$

where I is Fisher information.

Theorem 6.1 (Shadow entropy flow)**.** *For* $q_t = \mathsf{Sh}_k(p_t)$*,*

$$\frac{d}{dt} H(q_t) = -I_{\mathsf{Sh}}(p_t)$$

with

$$I_{\mathsf{Sh}}(p_t) \leq I(p_t).$$

Remark 6.2. The shadow always sees slower entropy dissipation than the higher layer.

7 Shadow Attractors and Dimensional Fixed Points

We define attractors in layered identity spaces.

Definition 7.1 (Shadow attractor)**.** *A distribution* μ *in layer* k *is a* shadow attractor *if:*

$$\mathsf{Sh}_k(\mu_{k+1}(t)) \to \mu$$

for a wide class of higher-layer trajectories.

Theorem 7.2 (Dimensional fixed point)**.** *If a measure* $\mu^\star_{k+1}$ *is an invariant for a channel* T*, then*

$$\mu^\star_k = \mathsf{Sh}_k(\mu^\star_{k+1})$$

is an invariant for the shadow channel.

Remark 7.3. Dimensional fixed points project downward: the shadow of a steady state is a steady state.

8 Outlook for Part III

Part III will introduce:

- Shadow Yang–Mills identities,
- Shadow spectral theory,
- Shadow quasicrystal information layers,
- Shadow Qubit (Digi) geometry,
- Full Shadow Field Equations for AGI state evolution,
- Links to Kuramoto dynamics and P3P/OCTA identity layers.