T-Fixed-Point Attractors and Central-Force Toy Universes: A Framework for Dynamical Selection in Theory Space

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Abstract

We develop a framework for structural selection of physical law in theory space based on T-fixed-point attractors. Given a manifold of candidate theories and a habitability functional S, we define a selection operator T by gradient ascent on S. Attractive fixed points of T are structurally preferred theories. As a companion to earlier work on central-force toy universes, we show how that calculation can be recast in this language: a two-parameter family of universes labelled by spatial dimension D and force exponent p in a $1/r^p$ law admits a unique T-fixed-point attractor at $D = 3$ and $p = 2$. Within this model class, three spatial dimensions and inverse-square forces emerge as the unique dynamical fixed point of blind structural selection, without anthropic input.

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Part I: Companion Note — Central-Force Universes as a T-Attractor

In a separate paper [5], a detailed numerical and analytical study of central-force toy universes was carried out. This section summarizes that result and rephrases it in the language of T-fixed-point attractors developed in Part II of the present note.

1 Central-force theory space

The theory space considered in [5] consists of universes labelled by two parameters:

- the number of spatial dimensions $D \in \{2, 3, 4, 5\}$,
- the exponent $p > 1$ in an attractive long-range force of the form

$$F(r) = -\frac{1}{r^p}. \tag{1}$$

This defines a discrete set of "slices" (one for each integer D) in the (D, p)-plane, and a continuous range of admissible force exponents p (typically taken in an interval such as $[1.0, 4.0]$).

Two structural scores are attached to each pair (D, p):

- $S_{\text{grav}}(D, p)$, a classical score depending on the longevity and hierarchical structure of small N-body systems evolving under the $1/r^p$ force in D dimensions;
- $S_{\text{atom}}(D, p)$, a quantum score depending on the number and regularity of bound states for the radial Schrödinger equation with an attractive potential proportional to $-1/r^{p-1}$ in dimension D.

These scores are normalized and combined into a joint habitability functional

$$S(D, p) = S_{\text{grav}}(D, p)\, S_{\text{atom}}(D, p), \tag{2}$$

which is then evaluated on a grid of universes. The central question is whether this simple structural criterion singles out any particular combination (D, p).

2 Summary of the central-force result

The main findings of [5] can be summarized as follows.

- Analytic arguments show that the Kepler problem in three dimensions with an inverse-square force is special: it admits closed bounded orbits for arbitrary eccentricities [1], and the associated Coulomb problem in quantum mechanics supports the familiar hydrogenic Rydberg series only in three dimensions [2, 3].
- Numerical evaluation of S_{grav} over small N-body systems indicates that three dimensions with exponent p near two support long-lived, non-trivially structured configurations, while other combinations tend to suffer either rapid collapse or rapid dispersion.
- Numerical solution of the radial Schrödinger equation with potential proportional to $-1/r^{p-1}$ shows that bound-state richness and ladder regularity are maximized near three dimensions and the Coulomb exponent $p = 2$.
- When S_{grav} and S_{atom} are combined into the joint functional $S(D, p)$ and normalized so that $S(3, 2) = 1$, the point $(D, p) = (3, 2)$ is found to be the unique global maximum within the scanned domain, with substantial margins over all competitors in other dimensions.

A representative subset of the numerical values from [5] is given in Table 1.

The detailed study in [5] includes additional exponents and dimensions as well as continuous extensions in D and p. The overall conclusion is that the joint score has a single sharply isolated peak at three dimensions and inverse-square forces.

D	p	S_{grav}	S_{atom}	$S(D, p)$
3	2.000	1.000	1.000	1.000
3	2.025	0.991	0.995	0.980
3	1.975	0.987	0.992	0.970
2	2.000	0.412	0.683	0.281
4	2.000	0.378	0.591	0.223
5	2.000	0.105	0.443	0.047

Table 1: Representative joint scores $S(D, p)$ from the central-force ensemble, normalized so that $S(3, 2) = 1$. The three-dimensional Coulomb case dominates the alternatives by large factors.

3 Rephrasing as a T-Fixed-Point Attractor

In the language of the present note, we regard the set of central-force universes as a theory space

$$\mathcal{T} = \{2, 3, 4, 5\} \times [1.0, 4.0]$$

with coordinates (D, p) and selection functional $S(D, p)$ defined above. For concreteness, we may equip $\mathcal{T}$ with the Euclidean metric in these coordinates and consider a continuous extension of S to real D and p in some neighbourhood of the physically interesting region.

The gradient of S with respect to this metric defines a selection operator

$$T(D, p) = (D, p) + \eta \nabla S(D, p),$$

for a small step size $\eta > 0$. Fixed points of T are critical points of S, and attractive fixed points correspond to local maxima where the Hessian is negative-definite.

The numerical analysis in [5] indicates that:

- $(D, p) = (3, 2)$ is a global maximum of S within the scanned domain;
- in a continuous extension, the gradient of S vanishes near $(3, 2)$ and the Hessian has negative eigenvalues in both directions;
- gradient ascent from a wide range of initial points in the domain converges to $(3, 2)$.

Taken together, these properties mean that, within the central-force theory space and for the given selection functional, three spatial dimensions and an inverse-square force law form an attractive T-fixed-point attractor. This rephrasing emphasizes that the result is not simply the location of a high score, but the destination of a dynamics on theory space.

Part II: T-Fixed-Point Attractors in Theory Space

We now turn to the general formalism of T-fixed-point attractors, of which the central-force example is one instance.

4 Theory space and selection functionals

Let $\mathcal{T}$ be a smooth finite-dimensional manifold representing a space of theories or model universes. Points $x \in \mathcal{T}$ may encode, for example, values of couplings, choices of dimension, or shape data for potentials.

Definition 1 (Selection functional)**.** *A selection functional on $\mathcal{T}$ is a smooth map*

$$S : \mathcal{T} \to \mathbb{R}$$

that assigns to each theory x a real-valued score representing some notion of structural quality or habitability.

The choice of S is problem-dependent. Typical components include:

- dynamical stability (e.g. long-lived non-trivial solutions),
- spectral richness (e.g. number and spacing of bound states),
- robustness of structures under small perturbations,
- information-theoretic measures such as entropy or complexity.

When multiple criteria are present, they can be combined in various ways. A simple and often useful choice is a product,

$$S(x) = \prod_{i=1}^{k} S_i(x),$$

where each $S_i(x) \geq 0$ represents an independent structural axis; low values in any axis suppress the overall score.

5 Selection operator and T-fixed points

To convert S into a dynamical principle on $\mathcal{T}$, we introduce a gradient-based update rule.

Equip $\mathcal{T}$ with a Riemannian metric g. Denote by $\nabla_g S$ the gradient of S with respect to g, and by $\exp_x$ the Riemannian exponential map at x.

Definition 2 (Selection operator)**.** *Given a step size $\eta > 0$, the selection operator associated with S is the map*

$$T : \mathcal{T} \to \mathcal{T}, \qquad T(x) = \exp_x\big(\eta \nabla_g S(x)\big).$$

In local coordinates where the metric is approximately Euclidean, this becomes the familiar gradient ascent step

$$T(x) \approx x + \eta \nabla S(x),$$

valid for small enough η.

Definition 3 (T-fixed point)**.** *A point $x^\star \in \mathcal{T}$ is a T-fixed point if it satisfies*

$$T(x^\star) = x^\star.$$

In the gradient construction above, this is equivalent to $\nabla_g S(x^\star) = 0$.

Definition 4 (Attractive T-fixed point)**.** *A T-fixed point $x^\star$ is attractive if there exists a neighbourhood U of $x^\star$ such that for all $x \in U$, the iterates $T^n(x)$ converge to $x^\star$ as $n \to \infty$.*

Attractive T-fixed points serve as theory-space analogues of attractors: starting from generic initial theories in a basin of attraction U, repeated application of T drives the dynamics towards a structurally preferred law.

6 Local properties of T-fixed points

We recall standard results from the theory of gradient flows and recast them in this setting.

Let $x^\star$ be a point where $\nabla_g S(x^\star) = 0$. The Hessian of S at $x^\star$, denoted $\mathrm{Hess}_g S(x^\star)$, is a symmetric bilinear form on the tangent space $T_{x^\star}\mathcal{T}$.

Proposition 1 (Local characterization of attractive T-fixed points)**.** *Let $x^\star$ be a non-degenerate critical point of S (that is, $\nabla_g S(x^\star) = 0$ and $\mathrm{Hess}_g S(x^\star)$ is non-singular). For sufficiently small step size $\eta > 0$, the following are equivalent:*

1. *$x^\star$ is a strict local maximum of S,*
2. *the Hessian $\mathrm{Hess}_g S(x^\star)$ is negative-definite,*
3. *$x^\star$ is an attractive T-fixed point of the selection operator T.*

Remark 1. *The choice of metric g affects the shape of the ascent dynamics but not the location of critical points. Different metrics may, however, change basins of attraction and convergence rates. In high-dimensional or anisotropic theory spaces, a natural metric might be derived from an information geometry or from kinetic terms in an action.*

7 Continuous and discrete dynamics

The discrete selection operator T can be viewed as a time-η map of a continuous flow.

Definition 5 (Gradient flow of the selection functional)**.** *The continuous-time gradient flow associated with S is the differential equation*

$$\frac{dx}{dt} = \nabla_g S(x).$$

Fixed points of this flow coincide with T-fixed points of the discrete operator, and local stability properties are determined by the same Hessian. For sufficiently small η, a single step of T is a first-order approximation to the flow over time η.

From a modelling perspective, either viewpoint may be appropriate:

- In contexts where theories are updated in discrete stages (for example, iterative model selection procedures), the discrete operator T is natural.
- In contexts where one thinks of theory space evolving continuously (for example, under a renormalization group flow), the differential equation may be more natural, and T can be interpreted as its time-η map.

8 Beyond the central-force example

The central-force application is a proof-of-principle: a low-dimensional theory space with a minimal selection functional can already exhibit a unique, sharply isolated T-attractor coinciding with the observed combination of dimension and force law.

The same framework can be applied to more elaborate theory spaces, for example:

- spaces of gauge theories characterized by gauge group, matter content, and coupling constants;
- families of scalar potentials characterized by masses and self-couplings;

- cosmological models characterized by curvature, cosmological constant, and dark sector parameters;
- effective field theories characterized by higher-dimensional operators with bounded coefficients.

In each case, the central tasks are:

1. to define a theory space $\mathcal{T}$ with a reasonable topology and metric;
2. to construct a structurally meaningful selection functional S;
3. to analyze the T-fixed points and their basins of attraction.

These steps can be carried out analytically in simple settings and numerically in more complex ones. The goal is not to claim that any particular S is unique or fundamental, but to demonstrate that structurally reasonable criteria can produce attractor behaviour in theory space.

9 Conclusion

We have presented a general framework for structural selection of physical law in theory space based on T-fixed-point attractors. The key elements are:

- a manifold of theories $\mathcal{T}$ equipped with a metric,
- a selection functional S encoding structural desirability,
- a gradient-based selection operator T whose attractive fixed points represent structurally preferred theories.

In the central-force example summarized in Part I, three spatial dimensions and an inverse-square interaction emerge as a unique T-attractor within a broad class of universes, with no anthropic assumptions. This suggests that at least some features of our observed laws may be understood as outcomes of dynamical selection in theory space.

Further work is needed to explore richer theory spaces and more sophisticated selection functionals. The present formalism is designed to be compatible with both analytical and numerical approaches, and to provide a language in which questions of the form "why these parameters?" can be reformulated as questions about attractors.

Acknowledgements

The numerical work in the companion central-force study [5] uses standard N-body and spectral codes and is designed to be fully reproducible from published scripts and data.

References

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