OCTA Hyperknot Algebra and Execution Semantics

A Rigorous Object-Based Topological Computation Framework
(with Worked Examples, Diagrams, and Operational Pseudocode)

OCTA Research Stack

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Contents

1 Foundational Postulate

Axiom 1.1 (OCTA Foundational Postulate)**.** Durable computation is realized as invariant-preserving transformations of structured objects under constrained field evolution.

Classical symbolic instruction execution and continuous numerical approximation are regarded as special-case encodings inside broader invariant dynamics.

2 Configuration Space and Hyperknot Objects

Let M^3 denote an observable three-manifold (typically a bounded subset of $\mathbb{R}^3$ with boundary conditions). Let X denote a configuration manifold (order-parameter space; internal phase manifold; or a composite bundle representing multi-field state).

Definition 2.1 (Configuration Field)**.** A configuration field is a continuous map

$$\Phi : M^3 \to X.$$

Definition 2.2 (Admissible Deformation)**.** An admissible deformation is a homotopy Φ_s ($s \in [0,1]$) such that Φ_s remains regular (no singular reconnection events) for all s.

Definition 2.3 (Hyperknot)**.** A Hyperknot is an equivalence class

$$H := [\Phi]$$

under admissible deformations of Φ.

Remark 2.1. The visible "knot" in M^3 is a shadow of the field topology in X. The Hyperknot object is the equivalence class in configuration space, not a geometric curve in M^3.

3 Invariant Stack and Identity

Definition 3.1 (Invariant Stack)**.** Each Hyperknot $H \in \mathcal{H}$ carries an invariant stack

$$\mathcal{Q}(H) = \begin{bmatrix} Q_{\text{top}}(H) \\ Q_{\text{phase}}(H) \\ Q_{\text{ori}}(H) \\ Q_{\text{bind}}(H) \end{bmatrix} \in \mathbb{Z} \times \mathbb{S} \times \mathcal{O} \times \mathcal{G}.$$

Interpretation of components:

- $Q_{\text{top}} \in \mathbb{Z}$: integer-valued topological charge (primary identity spine).
- $Q_{\text{phase}} \in \mathbb{S}$: internal winding/phase class (cyclic; may be multi-component).
- $Q_{\text{ori}} \in \mathcal{O}$: orientation/chirality/polarity label (finite set or group action).
- $Q_{\text{bind}} \in \mathcal{G}$: binding graph (hierarchical compositional structure).

Axiom 3.1 (Invariant Primacy)**.** Hyperknot identity is defined exclusively by $\mathcal{Q}(H)$, and two Hyperknots are identical if and only if their invariant stacks match.

Axiom 3.2 (Conservation Outside Events)**.** During admissible deformations (no events), $\mathcal{Q}(H)$ is conserved.

4 Projection and Shadow Observables (Shadow Math)

Definition 4.1 (Projection / Shadow Map)**.** A shadow observable is a projection

$$\pi : \Phi \mapsto \mathrm{Obs}(M^3),$$

where $\mathrm{Obs}(M^3)$ denotes the measurable field image in the observable space.

Theorem 4.1 (Projection Non-Invertibility and Invariant Persistence)**.** *In general, π is many-to-one (non-invertible). Distinct Hyperknots may share identical shadows; however, admissible evolution cannot change $\mathcal{Q}(H)$.*

Proof. Non-invertibility follows from the existence of distinct configuration classes mapping to the same measured observables (loss of internal degrees of freedom under projection). Conservation is an axiom of admissible deformation. □

5 Events: The Only Computation Steps

Definition 5.1 (Event)**.** An event is a localized singular reconnection of Φ supported in a compact region $U \subset M^3$ that changes the binding/geometry of the field while preserving allowable invariant constraints.

We distinguish primitive event types:

- **Fusion** ($\otimes$): two Hyperknots merge into a composite.
- **Fission** (Δ): one Hyperknot decomposes into a pair (or tuple).
- **Braid/Slide** (β): reorders constituents without changing Q_{top}.
- **Neutral pair creation/annihilation** (optional): allowed only if the net identity constraints remain satisfied.

Axiom 5.1 (Event Locality)**.** All events are local in M^3 (supported on compact regions), even though their invariant impact is global in configuration space.

6 Fusion and Fission Operators

6.1 Fusion

Definition 6.1 (Fusion Operator)**.** Fusion is a partial binary operator

$$\otimes : \mathcal{H} \times \mathcal{H} \rightharpoonup \mathcal{H}$$

defined when compatibility constraints are satisfied.

Axiom 6.1 (Fusion Charge Law)**.** If $H_3 = H_1 \otimes H_2$ is defined, then

$$Q_{\mathrm{top}}(H_3) = Q_{\mathrm{top}}(H_1) + Q_{\mathrm{top}}(H_2),$$

and $Q_{\mathrm{bind}}(H_3)$ records a binding edge between the constituents.

Definition 6.2 (Compatibility Predicate)**.** Fusion is permitted iff

$$\mathrm{Compat}(H_1, H_2) = \texttt{true},$$

where Compat enforces phase-window constraints on Q_{phase} and chirality/polarity constraints on Q_{ori}.

Remark 6.1. OCTA Hyperknot fusion is intentionally *not* universally defined: illegitimate compositions become metastable or forbidden.

6.2 Fission

Definition 6.3 (Controlled Fission)**.** Controlled fission is a multi-valued map

$$\Delta_{u(t)}(H) \subseteq \mathcal{H} \times \mathcal{H}.$$

Axiom 6.2 (Fission Charge Law)**.** If $(H_a, H_b) \in \Delta_{u(t)}(H)$, then

$$\mathcal{Q}(H) = \mathcal{Q}(H_a) + \mathcal{Q}(H_b),$$

interpreted componentwise (with Q_{bind} splitting by edge removal).

Remark 6.2. Intent $u(t)$ selects *which binding cut* is enacted, not the numeric values of the conserved charges.

7 Braiding and Non-Commutativity

Definition 7.1 (Braid Operator)**.** A braid operator is a morphism

$$\beta_{12} : H_1 \otimes H_2 \to H_2 \otimes H_1$$

that preserves Q_{top} and transforms Q_{phase} by an allowed rule.

Theorem 7.1 (Non-Commutativity)**.** *In general,*

$$H_1 \otimes H_2 \ncong H_2 \otimes H_1,$$

and commutativity holds only up to the existence of a braid morphism β.

Proof. If Q_{phase} and Q_{bind} encode ordering-dependent structure, then swapping constituents changes those components unless a braid morphism exists that accounts for the induced phase/binding transformation while preserving identity constraints. □

8 Stability, Metastability, and Failure Modes

Definition 8.1 (Stability Class)**.** A Hyperknot H has stability class $\sigma(H) \in \{\text{stable}, \text{metastable}, \text{unstable}\}$.

Definition 8.2 (Stable Hyperknot)**.** H is stable if no admissible event sequence reduces its invariants or violates constraints under small perturbations.

Definition 8.3 (Metastable Hyperknot)**.** H is metastable if Q_{top} is protected but secondary constraints (phase/orientation/binding) are frustrated, permitting relaxation events that preserve Q_{top}.

Definition 8.4 (Unstable / Forbidden)**.** H is unstable if it cannot persist under admissible evolution and will necessarily decay to another class (or is not physically realizable under the substrate constraints).

Remark 8.1. Failure in OCTA Hyperknot computation is not state-corruption; it is reconfiguration constrained by preserved identity.

9 Category-Theoretic Formulation

Definition 9.1 (Hyperknot Category **HK**)**.** Define a category **HK** with:

- objects: Hyperknots $H \in \mathcal{H}$,
- morphisms: admissible event sequences (including identity morphisms),
- monoidal product: fusion $\otimes$ (partial, implemented as a subcategory of compatible pairs),
- unit object: vacuum $\nVdash$ with $\mathcal{Q}(\nVdash) = 0$.

Axiom 9.1 (Monoidal Laws up to Equivalence)**.** Associativity and unit laws hold up to admissible equivalence:

$$(H_1 \otimes H_2) \otimes H_3 \cong H_1 \otimes (H_2 \otimes H_3), \qquad H \otimes \nVdash \cong H \cong \nVdash \otimes H.$$

Theorem 9.1 (Braided Monoidal Structure)**.** *Under the braid morphisms β,* **HK** *is a braided monoidal category (restricted to the compatible-pairs domain).*

Proof. By construction: monoidal product is fusion with coherence maps given by admissible equivalences; braid morphisms satisfy the required coherence (hexagon) relations as identities in the event-sequence quotient. □

10 EXECUTION SEMANTICS (Operational Model)

10.1 State Representation

A system state is a multiset of Hyperknots with an interaction graph.

Definition 10.1 (System State)**.** A state is

$$S = (V, E, \{\mathcal{Q}(H_v)\}_{v \in V}),$$

where V indexes present Hyperknots, E encodes admissible interaction adjacency, and each node carries invariant stack $\mathcal{Q}$.

10.2 Event Scheduling

Definition 10.2 (Event Guard)**.** An event is enabled if all invariant guards and compatibility predicates are satisfied.

Definition 10.3 (Local Energy / Cost Functional (Substrate-Dependent))**.** Let $\mathcal{E}(S)$ be a substrate-dependent functional that ranks admissible moves without overriding invariants.

Axiom 10.1 (Invariant-First Dynamics)**.** Dynamics may minimize $\mathcal{E}$ subject to invariant constraints; invariants are never traded for energy improvement.

10.3 Operational Transition Rules

We define transitions $S \to S'$ via local rewrite rules:

- **Fusion:** choose nodes i, j with $\text{Compat}(H_i, H_j)$ true; replace them with k where $H_k = H_i \otimes H_j$ and update binding graph.
- **Fission:** choose node i and a cut selected by $u(t)$; replace H_i by $(H_a, H_b) \in \Delta_{u(t)}(H_i)$ and update binding graph.
- **Braid/Slide:** apply β to reorder constituents in a compatible composite.

10.4 Executable Pseudocode Semantics

Algorithm 1 OCTA Hyperknot Execution (Invariant-Guarded Event Dynamics)

Input: initial state S_0, intent/control schedule $u(t)$, step budget T
$S \leftarrow S_0$
for $t = 1$ to T **do**
  $\mathcal{M} \leftarrow \emptyset$ ▷ candidate moves
  Add all enabled fission moves to $\mathcal{M}$ using $\Delta_{u(t)}(H)$ and event guards
  Add all enabled fusion moves to $\mathcal{M}$ where $\text{Compat}(H_i, H_j)$ holds
  Add all enabled braid/slide moves to $\mathcal{M}$
  if $\mathcal{M} = \emptyset$ **then**
    break ▷ no admissible events: reached stable fixed point
  end if
  Choose $m^\star \in \mathcal{M}$ minimizing $\mathcal{E}$ (or by stochastic rule) ▷ substrate-dependent
  Apply $m^\star$ to obtain $S \leftarrow \text{Apply}(S, m^\star)$
  assert invariants conserved (componentwise) across the move
end for
Output: final state S

Theorem 10.1 (Termination to Local Fixed Points)**.** *Under finite state constraints and an energy functional $\mathcal{E}$ bounded below, the above dynamics reaches a local fixed point (no enabled moves) or cycles within an invariant-preserving equivalence class.*

Remark 10.1. OCTA computation is thus an event-driven relaxation process in a constrained rewrite system; there is no instruction pointer and no global clock requirement.

11 WORKED FORMAL EXAMPLES

This section provides explicit, checkable examples using a minimal algebraic model of $\mathcal{Q}$.

11.1 Example Model of the Invariant Stack

For worked examples, let

$$\mathcal{Q}(H) = (q, \theta, o, G),$$

where:

- $q \in \mathbb{Z}$ is topological charge,
- $\theta \in \mathbb{R}/2\pi\mathbb{Z}$ is phase,
- $o \in \{+, -\}$ is orientation,
- G is a rooted binding tree encoding composition.

Define componentwise addition:

$$(q_1, \theta_1, o_1, G_1) + (q_2, \theta_2, o_2, G_2) = (q_1 + q_2,\ \theta_1 + \theta_2 \pmod{2\pi},\ o_1 \cdot o_2,\ G_1 \cup G_2).$$

This is a *model* of conservation, not a claim about any specific substrate.

11.2 Example 1: Valid Fusion with Phase Window

Let H_1 and H_2 have:

$$\mathcal{Q}(H_1) = (1,\ 0.2,\ +,\ \bullet_1), \qquad \mathcal{Q}(H_2) = (2,\ 0.1,\ +,\ \bullet_2).$$

Define compatibility:

$$\mathrm{Compat}(H_1, H_2) \iff |\theta_1 - \theta_2| \le \delta \quad \text{and} \quad o_1 = o_2,$$

with $\delta = 0.2$.

Then $|0.2 - 0.1| = 0.1 \le 0.2$ and orientations match, hence fusion is enabled:

$$H_3 = H_1 \otimes H_2,$$

and

$$\mathcal{Q}(H_3) = \mathcal{Q}(H_1) + \mathcal{Q}(H_2) = (3,\ 0.3,\ +,\ (\bullet_1 \otimes \bullet_2)).$$

Example 11.1. This is a fully checkable case where fusion exists and conserves topological charge.

11.3 Example 2: Failed Fusion and Metastable Constraint

Let

$$\mathcal{Q}(H_1) = (1,\ 0.0,\ +,\ \bullet_1), \qquad \mathcal{Q}(H_2) = (1,\ 1.2,\ +,\ \bullet_2),$$

with the same $\delta = 0.2$. Then $\mathrm{Compat}(H_1, H_2)$ fails. Two outcomes are possible:

- Fusion is undefined (forbidden).
- A metastable composite $H^\star$ forms but immediately admits a relaxation fission that restores separated objects while preserving q.

Example 11.2. This formalizes the OCTA principle: illegal compositions do not corrupt identity; they induce constrained reconfiguration.

11.4 Example 3: Controlled Fission by Binding-Cut Selection

Consider a composite Hyperknot H with binding tree

$$G = ((\bullet_a \otimes \bullet_b) \otimes \bullet_c).$$

Suppose

$$\mathcal{Q}(H) = (3,\ \theta,\ +,\ G).$$

A control schedule $u(t)$ selects a cut. Two admissible cuts:

- Cut the last edge: $((a \otimes b) \otimes c) \mapsto (a \otimes b,\ c)$
- Cut the first edge: $((a \otimes b) \otimes c) \mapsto (a,\ b \otimes c)$

Under the first cut,

$$(H_{ab}, H_c) \in \Delta_{u(t)}(H),$$

with

$$\mathcal{Q}(H_{ab}) = (2,\ \theta_{ab},\ +,\ (\bullet_a \otimes \bullet_b)), \quad \mathcal{Q}(H_c) = (1,\ \theta_c,\ +,\ \bullet_c),$$

and conservation yields

$$(3, \theta, +, G) = (2, \theta_{ab}, +, \cdot) + (1, \theta_c, +, \cdot)$$

componentwise, with phase conservation modulo 2π.

Example 11.3. Intent selects the *structural cut*. Charges follow conservation; intent does not "choose" the output charges.

11.5 Example 4: Braid-Induced Phase Shift (Order Matters)

Let H_1, H_2 be compatible. Define braid action:

$$\beta_{12} : H_1 \otimes H_2 \to H_2 \otimes H_1, \qquad \theta \mapsto \theta + \varphi,$$

for some allowed φ (substrate-dependent).

Then

$$\mathcal{Q}(H_1 \otimes H_2) = (q_1 + q_2, \theta_1 + \theta_2, +, G)$$

while

$$\mathcal{Q}(H_2 \otimes H_1) = (q_2 + q_1, \theta_2 + \theta_1 + \varphi, +, G').$$

Thus the order swap is not identity unless $\varphi \equiv 0 \pmod{2\pi}$ and binding graphs are equivalent under admissible rewrites.

Example 11.4. This provides a concrete mechanism for logic beyond integer arithmetic: braid-induced phase shifts.

12 DIAGRAM APPENDIX (TikZ Figures)

12.1 Figure A: Hyperknot as Field-Class with Shadow Projection

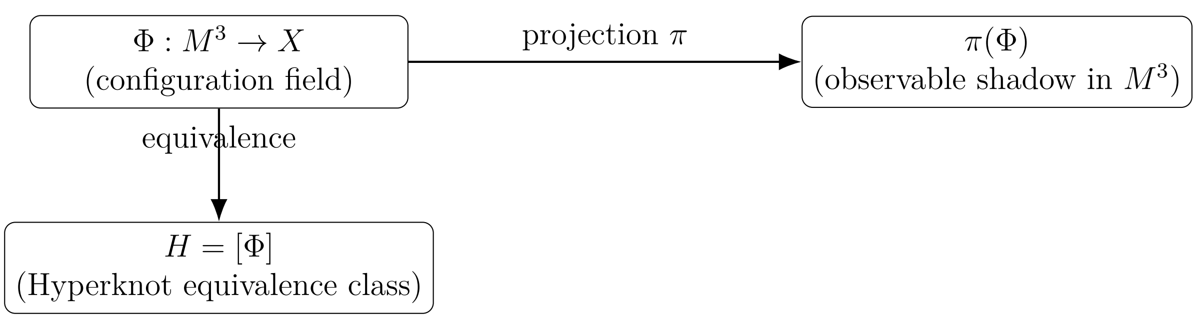

Figure 1: Hyperknot object is the field equivalence class; the visible knot is a shadow.

12.2 Figure B: Fusion and Fission as Graph Rewrites

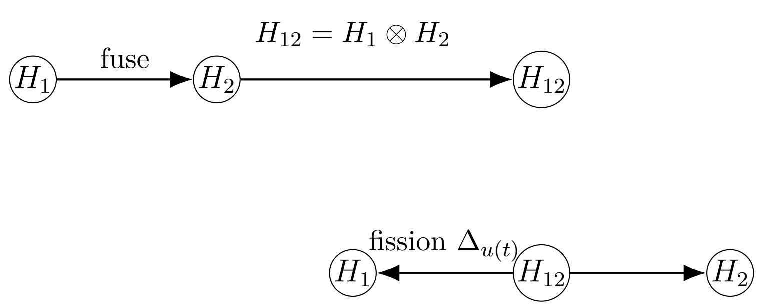

Figure 2: Fusion binds two Hyperknots into a composite; fission splits a composite under intent-selected cuts.

12.3 Figure C: Event-Driven Execution Loop

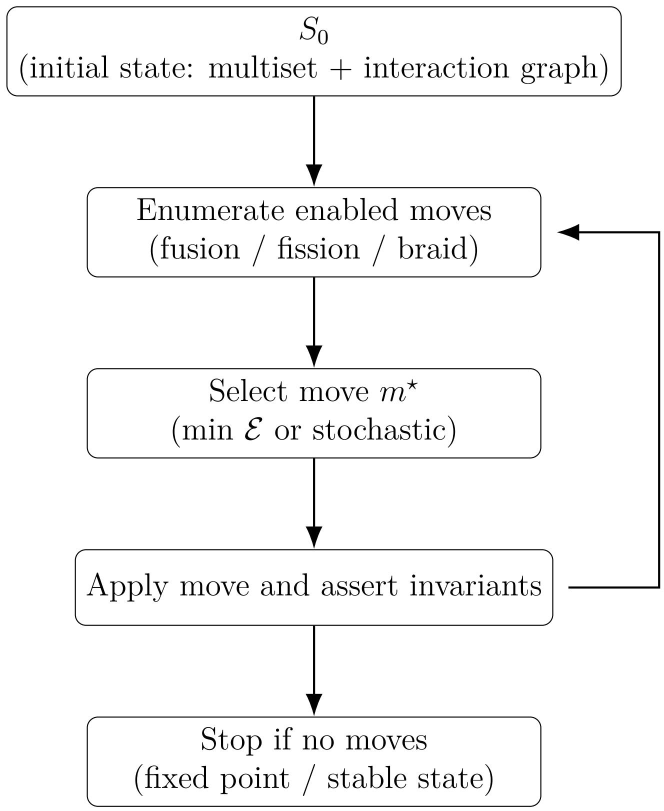

Figure 3: Operational semantics: event enumeration, guarded selection, invariant-preserving updates, termination at fixed points.

13 Scientific Rigor: Claims, Scope, and Testability

13.1 What Is Formally Claimed Here

This monograph claims:

- A closed algebraic and categorical specification of Hyperknot objects and event operators.
- A guarded operational semantics for computation by event-driven invariant dynamics.
- A set of falsifiable predictions: if a substrate cannot preserve invariants under local events, it cannot realize Hyperknot computation.

13.2 What Is Not Claimed

This monograph does not claim:

- That a particular physical substrate has been fully engineered.

- That Hyperknot computation solves arbitrary complexity classes without substrate constraints.
- That all invariants are realizable in all media.

13.3 Experimental / Engineering Validation Targets

Minimal validation:

1. Identify a substrate with a measurable Q_{top} invariant.
2. Demonstrate repeatable fusion/fission events with invariant conservation.
3. Demonstrate braid-induced secondary invariant modulation.
4. Build a pulse protocol $u(t)$ that selects fission channels.
5. Establish readout $\pi(\Phi)$ with stable classification by $\mathcal{Q}$.

14 Conclusion

The OCTA Hyperknot System defines computation as:

> *a constrained rewrite dynamics over invariant-bearing objects, executed via localized events and controlled by boundary selection rather than symbolic instruction.*

It is formally specified at:

- object level (Hyperknots)
- identity level (invariant stack)
- operator level (fusion/fission/braid)
- semantic level (operational execution)
- structural level (braided monoidal category)