The Lattice of Computable Fields: Computons, Topological Logic, and Braided Field Computation

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Abstract

We develop a mathematically detailed framework in which computation is carried not by electrons or quantum amplitudes but by topologically protected structures arising at the intersections of propagating fields. We formalize the *Computon*: a particle-like, homology-defined, integer-valued excitation in an overlap domain of structured fields. We construct a computon algebra, embed it in a braided monoidal category, analyze robustness via homotopy and Mayer–Vietoris, and connect it to known topological excitations including skyrmions, solitons, magnon textures, photonic crystal defect modes, and quantum Hall edge states. We define the Lattice of Computable Fields (LCF), give scaling laws, and propose explicit device-level architectures. Multiple TikZ diagrams illustrate computon geometry, gates, and braiding. This establishes a physically plausible computational substrate grounded in topology and field theory.

Contents

1 Field Substrate and Overlap Geometry

Let $\Omega \subseteq \mathbb{R}^n$ be a bounded domain with piecewise smooth boundary. A *structured field* is a continuous map

$$F : \Omega \to \mathbb{R}^k$$

equipped with additional physical structure: phase, polarization, spin, chirality, or lattice embedding.

We consider a finite family of fields

$$\mathcal{F} = \{F_1, \ldots, F_m\}.$$

Define the support

$$(F_i) = \overline{\{x \in \Omega : \|F_i(x)\| > 0\}},$$

and the overlap domain

$$\mathcal{I}(\mathcal{F}) = \bigcap_{i=1}^{m} (F_i) \subseteq \Omega,$$

assumed to be compact with piecewise smooth boundary.

[Overlap Complex] We associate to $\mathcal{I}$ a CW-complex $K(\mathcal{I})$ obtained by triangulating $\mathcal{I}$. All homology groups will be taken on $K(\mathcal{I})$.

Thus, the topology of the overlap is captured by $H_k(K(\mathcal{I}); \mathbb{Z})$.

2 The Computon and Betti Vector Encoding

[Computon] Let $\mathcal{I} = \mathcal{I}(\mathcal{F})$. We say $\mathcal{I}$ contains a *Computon of order k* if

$$H_k(K(\mathcal{I}); \mathbb{Z}) \neq 0.$$

The *Betti numbers* of $\mathcal{I}$ are

$$\beta_k(\mathcal{I}) = \operatorname{rank} H_k(K(\mathcal{I}); \mathbb{Z}).$$

[Computon State Vector] The computon state vector of $\mathcal{I}$ is

$$\boldsymbol{\beta}(\mathcal{I}) = (\beta_0(\mathcal{I}), \beta_1(\mathcal{I}), \ldots, \beta_d(\mathcal{I})),$$

where $d \leq n$ is the maximal dimension of interest.

Logical state is encoded in $\boldsymbol{\beta}$. For example, in a 2D implementation, one can take

$$\text{logical bit} \quad b = \beta_1(\mathcal{I}) \bmod 2,$$

turning the presence of an odd number of loops into a 1, and even into 0.

3 Topological Stability via Homotopy

Let $\{\mathcal{I}_t\}_{t\in[0,1]}$ be a smooth one-parameter family of overlap regions, induced by smooth deformations of the fields $\{F_i^t\}$.

[Homotopy Stability] Suppose $\mathcal{I}_t$ varies such that:

(i) $\mathcal{I}_t$ remains a compact manifold with boundary (possibly with corners).

(ii) No topological events occur: no splitting, merging, or pinching that creates or destroys holes or connected components.

Then the homology groups $H_k(K(\mathcal{I}_t))$ are invariant in t.

Proof. These assumptions imply that the inclusion maps define a homotopy equivalence between $K(\mathcal{I}_0)$ and $K(\mathcal{I}_1)$. Homotopy-equivalent CW-complexes have isomorphic homology groups. $\square$

Thus computon existence is *topologically protected* against small geometric perturbations and noise.

4 Mayer–Vietoris and Logic from Overlaps

We now use the Mayer–Vietoris sequence to analyze how computons emerge from unions and intersections of subdomains.

Let $A, B \subseteq \Omega$ be two overlap candidates with $\mathcal{I} = A \cup B$ and $A \cap B \neq \varnothing$.

The Mayer–Vietoris long exact sequence in homology reads

$$\cdots \to H_k(A \cap B) \to H_k(A) \oplus H_k(B) \to H_k(A \cup B) \to H_{k-1}(A \cap B) \to \cdots$$

[Logical Creation via Intersection] Suppose $H_k(A) = 0$, $H_k(B) = 0$, but $H_k(A \cap B) \neq 0$. Then the map

$$H_k(A \cap B) \to H_k(A) \oplus H_k(B)$$

is trivial, and the boundary map into $H_{k-1}(A\cap B)$ is nontrivial, inducing a new class in $H_{k-1}(A\cup B)$. Thus, the union $A \cup B$ can carry a nontrivial toplogical class (triggering a computon) even when A and B do not.

This illustrates a topological AND-like mechanism: logical structure emerges only in the presence of both inputs.

5 Category and Braided Monoidal Structure

We construct a category Comp:

- Objects: overlap complexes $K(\mathcal{I})$.
- Morphisms: continuous maps $f : K(\mathcal{I}) \to K(\mathcal{J})$ that respect allowed physical deformations (topology-preserving) or annihilations (collapse maps).

Composition is function composition; identities are identity maps.

[Monoidal Product] We define a monoidal product $\otimes$ on objects by disjoint union:

$$K(\mathcal{I}_1) \otimes K(\mathcal{I}_2) := K(\mathcal{I}_1 \sqcup \mathcal{I}_2).$$

On morphisms, $f \otimes g$ acts componentwise.

In two spatial dimensions, worldlines of moving computons can form braids. These define a braiding morphism

$$c_{X,Y} : X \otimes Y \to Y \otimes X$$

that encodes the exchange of two computons. This yields a braided monoidal category, reminiscent of anyon models.

6 Energetics and Topological Protection

Let $E[\mathcal{I}]$ be the energy of a computon-bearing domain. We assume the energy functional decomposes into

$$E[\mathcal{I}] = E_{\text{bulk}}[\mathcal{I}] + E_{\text{boundary}}[\partial\mathcal{I}]$$

with constraints that different topological sectors correspond to distinct local minima separated by energy barriers.

[Topological Gap] The topological gap ΔE is the minimal energy required to transition between distinct homology sectors:

$$\Delta E = \min_{H_k(\mathcal{I}_1) \ncong H_k(\mathcal{I}_2)} \big| E[\mathcal{I}_2] - E[\mathcal{I}_1] \big|.$$

A large ΔE supports thermal stability of logical state.

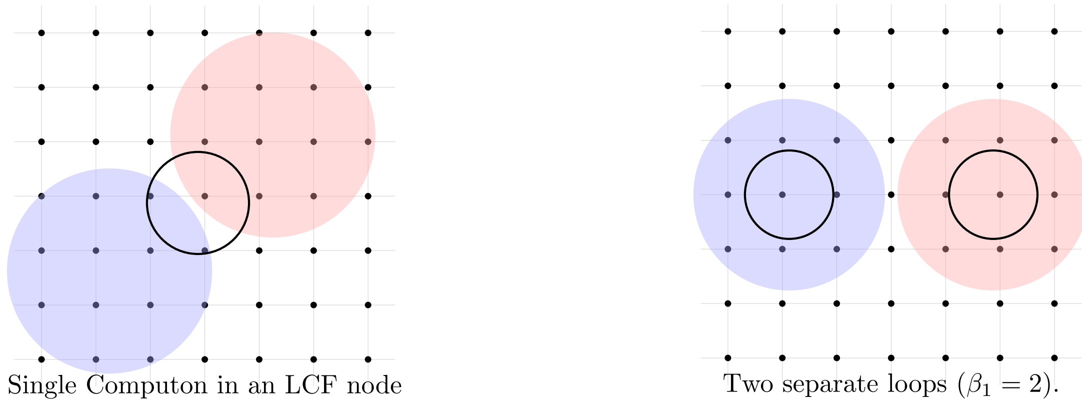
7 LCF Architecture and Computon Logic

An LCF processor consists of:

- a spatial lattice supporting wave propagation,
- field emitters (inputs),
- boundary-shaping actuators (logic routing),
- sensors that measure $\boldsymbol{\beta}(\mathcal{I})$ (state readout).

Logical gates are realized as controlled manipulations of field geometry.

7.1 TikZ: Simple Computon Lattice



(a) Single computon loop ($\beta_1 = 1$).

(b) Two computons ($\beta_1 = 2$).

Figure 1: Computon configurations in an LCF lattice; logical state can be encoded in β_1 mod 2.

8 Computon Logic Gates with TikZ

8.1 Topological NOT Gate

A NOT gate destroys a target homology class.

[Topological NOT] Let $\mathcal{I}$ be a domain with $\beta_1(\mathcal{I}) = 1$. A NOT operation is a controlled deformation Φ such that $\beta_1(\Phi(\mathcal{I})) = 0$.

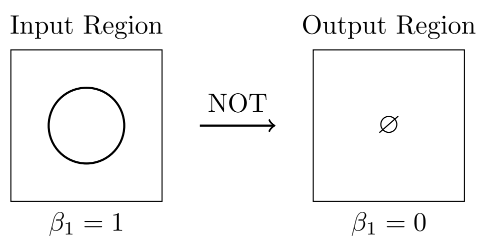


Figure 2: Schematic of a topological NOT gate implemented by loop collapse.

8.2 Topological AND Gate

An AND gate exists when a computon (nontrivial homology) appears only when both input fields are present.

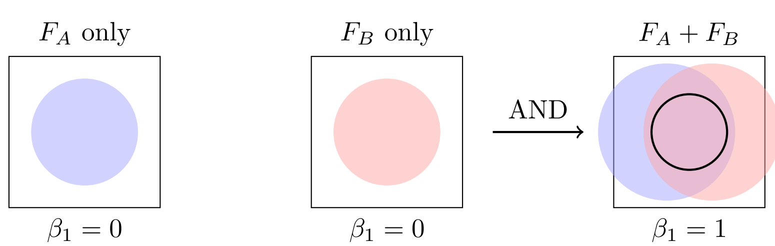
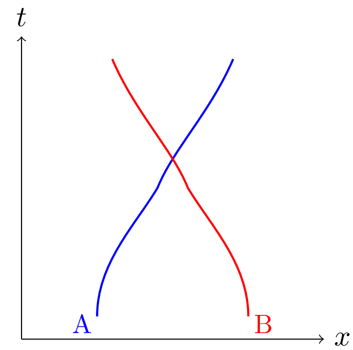


Figure 3: Topological AND gate: a loop computon appears only when both fields are present.

9 Computon Worldlines and Braiding

In a 2D LCF extended over time, we can represent computon worldlines in a (x, t) diagram. Braiding corresponds to exchanging two worldlines.



Braided worldlines

Figure 4: Braiding of two computon worldlines in an LCF. The topology of this braid can encode nontrivial logical operations.

In principle, different braid types can realize different unitary or classical transformations on computon states, analogous to anyonic computing but here grounded in homology of overlap domains rather than quantum phase alone.

10 LCF Configuration Space and Complexity

Let L be an LCF with N discrete nodes (e.g. lattice points, resonators, or cavities). We define a configuration of computons by placing up to M non-overlapping computon loops within the lattice.

[Configuration Space] The configuration space of M indistinguishable computons in L is

$$\mathcal{C}_M(L) = \frac{L^M - \Delta}{S_M},$$

where Δ excludes coincident placements and S_M is the symmetric group.

The cardinality of $\mathcal{C}_M(L)$ grows roughly like

$$|\mathcal{C}_M(L)| \sim \frac{N^M}{M!}$$

for $M \ll N$. Thus, the expressive capacity of the system is combinatorially large in the number of sites and computons.

If each configuration encodes a distinct logical state, the information capacity scales as

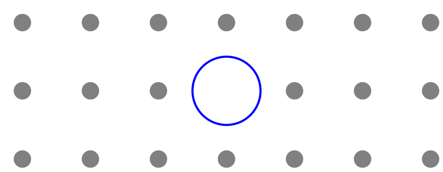
$$\log_2 |\mathcal{C}_M(L)| \approx M \log_2 N - \log_2 M!.$$

This suggests substantial combinatorial richness even for moderate N, M.

11 Candidate Physical Platforms and TikZ Layout

Platform	Field Type	Natural Topology
Polariton Lattices	Hybrid light–matter	Loops, bound modes
Magnonic Circuits	Spin waves	Skyrmion cores, spin textures
Photonic Crystals	EM fields	Defect cavities, ring modes
Acoustic Metamaterials	Pressure waves	Torus-like cavities
BEC Systems	Quantum phase	Quantized vortices

Table 1: Candidate physical implementations of LCF Computons.



Photonic crystal with defect cavity (loop mode as computon).

Figure 5: Simplified photonic crystal representation: removed central rod creates a defect; a loop mode in the defect acts as a computon ($\beta_1 = 1$).

12 Simulation Outline

A numerical LCF simulation pipeline:

1. Discretize Ω on a grid or mesh.
2. Solve field equations (e.g. wave, Helmholtz, or Schrödinger-type).
3. Threshold to obtain supports (F_i).
4. Compute $\mathcal{I} = \bigcap_i (F_i)$.
5. Construct a simplicial complex $K(\mathcal{I})$.
6. Compute homology or persistent homology to obtain $\boldsymbol{\beta}(\mathcal{I})$.

7. Apply logic rules by manipulating fields and boundaries.

13 Open Problems and Outlook

Key open directions:

- Constructive universality proofs for computon algebras.
- Quantitative noise thresholds and fault-tolerance bounds.
- Full braid group representations for computon worldlines.
- Realistic device design and fabrication constraints.

14 Conclusion

We have enriched the LCF framework with explicit homology, category theory, energetic considerations, braided worldlines, logical gate constructions, and multiple geometric diagrams. The result is a coherent, physically motivated model of computation whose primitive is a topological object, the Computon, rather than a transistor or qubit.

This opens a path to hardware in which information is literally *the shape of reality*.

Acknowledgements

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