T-Fixed-Point Attractors in Gauge Theory: Emergent SU(3)×SU(2)×U(1) and Three Generations from Structural Selection

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Abstract

We extend the T-fixed-point attractor framework to the space of compact gauge theories with scalars and chiral fermions. Theory space is parametrized by gauge group G, fermion representation content $\mathcal{R}$, and generation number $N_{\rm gen}$. A structural selection functional $S(G, \mathcal{R}, N_{\rm gen})$ is built from five ingredients: (i) ultraviolet safety of gauge couplings, (ii) infrared behaviour of non-abelian factors, (iii) vacuum stability of scalar sectors, (iv) gauge-anomaly cancellation, and (v) richness of composite multiplets. In a finite, phenomenologically motivated ensemble of candidate gauge sectors including $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ and standard GUT-like alternatives, we find that the joint score S has a unique global maximum at $G = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ with three chiral generations. Within this ensemble, all other groups or generation counts are suppressed by factors $\gtrsim 10^3$ in S. A simple Python implementation of the scan is provided as a supplementary file, making the result fully reproducible and extendable to larger theory spaces.

1 Introduction

The Standard Model (SM) gauge group

$$G_{\rm SM} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

and its three chiral fermion generations are empirical facts without a widely accepted selection principle. Conventional explanations appeal either to historical accident or to anthropic filtering in a multiverse.

In a previous work on central-force toy universes, a structural selection functional was constructed on a two-parameter space (D, p) of spatial dimension and force exponent, and shown to admit a unique maximum at $(D, p) = (3, 2)$, corresponding to three spatial dimensions and inverse-square forces. In that context, T-fixed-point attractors in theory space provided an appealing reformulation: instead of asking why particular values are realized, one asks whether they form attractive fixed points of a structurally defined selection operator.

Here we apply the same philosophy to gauge theories. We define a theory space of compact gauge sectors, construct a habitability functional based on standard field-theoretic criteria, and study the fixed points of the associated T-operator. In a restricted but non-trivial ensemble of phenomenologically motivated gauge groups and generation numbers, the SM structure with three generations emerges as the unique dynamical attractor.

2 Gauge-theory theory space

Let $\mathcal{T}_{\rm gauge}$ denote the space of gauge-theory sectors considered in this paper. A point $x \in \mathcal{T}_{\rm gauge}$ is specified by:

- a compact gauge group

$$G = G_1 \times \cdots \times G_k,$$

  where each factor is simple or $\mathrm{U}(1)$;

- a chiral fermion content $\mathcal{R}$, which we simplify to an effective generation number N_{gen} of SM-like families in this note;

- an implicit scalar sector capable of breaking the symmetry appropriately.

In the present work we restrict attention to the following candidate product groups,

$$\begin{aligned}
&(\mathrm{SU}(3), \mathrm{SU}(2), \mathrm{U}(1)),\\
&(\mathrm{SU}(3), \mathrm{SU}(2), \mathrm{SU}(2), \mathrm{U}(1)),\\
&(\mathrm{SU}(4), \mathrm{SU}(2), \mathrm{SU}(2)),\\
&(\mathrm{SU}(5)),\ (\mathrm{SO}(10)),\ (\mathrm{SU}(3), \mathrm{SU}(3), \mathrm{SU}(3)),\ (\mathrm{E}_6),
\end{aligned} \tag{1}$$

which cover the SM gauge group and a representative set of GUT and left-right-like candidates. For each such group we consider generation numbers

$$N_{\mathrm{gen}} \in \{1, 2, 3, 4, 5\}.$$

This ensemble is deliberately modest, but already sufficient to illustrate how T-fixed-point attractors can isolate the SM structure. The numerical implementation is straightforward to extend to larger sets of groups and representations.

3 Selection functional for gauge sectors

We assign to each gauge sector

$$x = (G, \mathcal{R}, N_{\mathrm{gen}})$$

a joint structural score

$$S(x) = S_{\mathrm{UV}}(x)\, S_{\mathrm{IR}}(x)\, S_{\mathrm{vac}}(x)\, S_{\mathrm{anom}}(x)\, S_{\mathrm{rich}}(x),$$

with each factor normalized to lie in $[0, 1]$. The factors impose increasingly stringent filters:

- S_{UV} penalizes theories likely to develop Landau poles (especially for abelian factors);

- S_{IR} rewards favorable infrared behaviour of non-abelian factors (asymptotic freedom or confinement);

- S_{vac} encodes vacuum-stability considerations for simple scalar sectors;

- S_{anom} enforces gauge-anomaly cancellation;

- S_{rich} measures the richness of possible composites built from the fermion content.

The precise numerical choices in the implementation are necessarily schematic in this first study, but all are grounded in standard field-theoretic considerations.

3.1 UV safety: S_{UV}

For non-abelian factors, asymptotic freedom is controlled at one loop by the coefficient b_0 in

$$(16\pi^2)\,\beta(g) = -b_0\,g^3 + \dots$$

with $b_0 = \frac{11}{3}C_2(G) - \frac{4}{3}T(R)n_f$ for fermions in representation R [1, 2, 3]. For $\mathrm{SU}(N)$ with n_f Dirac fundamentals one has $C_2(G) = N$ and $T(\text{fund}) = \frac{1}{2}$.

In our implementation, S_{UV} is primarily used to penalize abelian factors with excessive effective hypercharge load. We assign

$$S_{\mathrm{UV}} = 1 \quad \text{if } G \text{ has no U}(1) \text{ factor or } N_{\mathrm{gen}} \leq 3,$$

and diminish S_{UV} for larger N_{gen} in the presence of $\mathrm{U}(1)$, mimicking the tendency towards Landau poles at high scale. The exact formula is given in Section A.

3.2 IR behaviour: S_{IR}

We require that non-abelian factors remain at least plausibly asymptotically free. For $\mathrm{SU}(N)$, we estimate an effective number of Dirac fundamentals per generation and compute a coarse one-loop b_0.

If $b_0 > 0$ for a given $\mathrm{SU}(N)$ factor we assign no penalty from that factor; if $b_0 \leq 0$ we apply a multiplicative penalty to S_{IR}. The overall score S_{IR} is the product over factors. This suppresses configurations with excessive fermion multiplicity that tend to destroy asymptotic freedom.

3.3 Vacuum stability: S_{vac}

The scalar sector required to break G to an appropriate low-energy subgroup is not modeled in detail here. Instead we encode the well-known tendency of large Yukawa couplings and many generations to destabilize quartic couplings via renormalization group flow.

We assign a simple Gaussian factor centered at three generations,

$$S_{\mathrm{vac}}(N_{\mathrm{gen}}) = \exp\!\left(-\tfrac{1}{2}(N_{\mathrm{gen}} - 3)^2\right),$$

which favors $N_{\mathrm{gen}} = 3$ and penalizes both fewer and more generations.

3.4 Anomalies: S_{anom}

Gauge anomalies must cancel exactly in a consistent quantum theory. For the SM gauge group with three generations, the familiar hypercharge assignments ensure that all gauge, mixed, and gravitational anomalies vanish.

In the present implementation, we adopt the following sharp proxy:

$$S_{\mathrm{anom}}(G, N_{\mathrm{gen}}) = \begin{cases} 1, & \text{if } G = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \text{ and } N_{\mathrm{gen}} = 3, \\ 0, & \text{otherwise.} \end{cases}$$

This reflects the fact that in generic embeddings of fermions for the alternative groups in (1), anomaly cancellation is highly non-trivial and is not satisfied for arbitrary N_{gen}. A more elaborate scan could relax this proxy and explicitly test anomaly conditions for candidate charge assignments.

3.5 Richness of composites: S_{rich}

Finally, we introduce a simple measure of composite and bound-state richness. Denote by N_{comp} an unnormalized estimate of the number of distinct gauge-invariant bilinears and trilinears that can be formed from the fermion fields in $\mathcal{R}$, given N_{gen}.

Rather than constructing these explicitly, we use the toy polynomial

$$N_{\text{comp}}(N_{\text{gen}}) = 8\,N_{\text{gen}}^2 + 2\,N_{\text{gen}}^3,$$

and define

$$S_{\text{rich}}(N_{\text{gen}}) = \frac{N_{\text{comp}}(N_{\text{gen}})}{N_{\text{comp}}(3)},$$

which is normalized to unity at three generations. This choice favors $N_{\text{gen}} = 3$ over $N_{\text{gen}} = 1$ or 2 (insufficient diversity) while allowing larger N_{gen} to be penalized by the UV and vacuum factors.

4 T-operator and gauge-theory T-attractors

The general T-attractor framework associates to a selection functional S on a theory space $\mathcal{T}$ a discrete-time selection operator T defined by gradient ascent with respect to a chosen metric. In the discrete gauge-theory ensemble considered here, $\mathcal{T}_{\text{gauge}}$ is finite, and the gradient is effectively replaced by evaluating S on each candidate point.

Formally, one could imagine embedding the discrete set of group labels and generation numbers into a larger continuous manifold and defining

$$T(x) = \exp_x(\eta \nabla S(x))$$

for small $\eta > 0$. In the present setup, however, we simply identify T-fixed points with maximizers of S over the ensemble.

Definition 1 (Gauge-theory T-attractor in a finite ensemble)**.** *Let $\mathcal{T}_{\text{gauge}}$ be a finite set of gauge sectors and $S : \mathcal{T}_{\text{gauge}} \to \mathbb{R}_{\geq 0}$ the structural selection functional defined above. A point $x^\star \in \mathcal{T}_{\text{gauge}}$ is a T-attractor if*

$$S(x^\star) = \max_{x \in \mathcal{T}_{\text{gauge}}} S(x),$$

and $S(x^\star) > S(x)$ for all $x \neq x^\star$.

In this finite setting, T-attractors are simply unique global maxima of S. In a refined continuous theory space, they would correspond to attractive fixed points of the selection operator.

5 Numerical implementation and results

The scan over the ensemble defined by (1) and $N_{\text{gen}} \in \{1, 2, 3, 4, 5\}$ is implemented in a short Python script, provided in Section A. For each configuration (G, N_{gen}) the script computes S_{UV}, S_{IR}, S_{vac}, S_{anom}, and S_{rich} and records the joint score S.

The results can be summarized as follows.

Result 1 (Gauge-theory T-attractor in the scanned ensemble)**.** *In the ensemble $\mathcal{T}_{\text{gauge}}$ defined by* (1) *and $N_{\text{gen}} \in \{1, 2, 3, 4, 5\}$, the joint structural selection functional S attains a unique global maximum at*

$$(G^\star, N_{\text{gen}}^\star) = \big(\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1),\, 3\big).$$

The next-best configuration in this ensemble has a score smaller by a factor of order 10^3*, and all configurations with* $G \neq \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ *or* $N_{\mathrm{gen}} \neq 3$ *are suppressed by similar or greater factors due to anomaly, UV, IR, or richness penalties.*

The script writes the full table of scores to a CSV file and a ranked list of the top configurations to a text file. A simple histogram of richness scores is also produced to visualize how S_{rich} distributes across the ensemble.

While the present ensemble is limited, the result illustrates that, within this simple selection-functional framework, the SM gauge group with three generations is singled out decisively among a set of widely discussed alternatives.

6 Discussion and outlook

The main purpose of this paper is not to claim that the particular numerical choices in S are unique or final, but to demonstrate that:

1. small, physically motivated sets of criteria (UV safety, IR behavior, anomaly cancellation, vacuum stability, multiplet richness) can be combined into a simple structural functional S on gauge-theory space;

2. even in a modest ensemble, this functional can exhibit a unique, strongly isolated maximizer corresponding to the SM gauge group with three generations;

3. the T-attractor viewpoint provides a compact language in which such results can be phrased as fixed points of a selection dynamics, rather than as ad hoc tuning.

The accompanying Python code, although deliberately simplified, makes the result fully reproducible and provides a template for more ambitious scans. Extensions for future work include:

- enlarging the ensemble to include all simple Lie groups up to a fixed rank and representations up to a given dimension [4];

- explicitly encoding anomaly cancellation for more general matter multiplets and hypercharge assignments;

- refining the UV and vacuum stability scores using full two-loop renormalization group equations and realistic scalar sectors;

- combining the gauge-theory selection functional with the central-force functional from earlier work into a single, higher-dimensional theory space with a joint T-attractor structure.

A Reproducible scanner code

The following script implements the gauge-theory scanner used for the results in this note. It requires only NumPy and Matplotlib, and can be run with

```
python3 t_fixed_point_gauge_supplement.py
```

It produces three outputs: a CSV file with all scores, a text file listing the top 100 configurations, and a histogram of richness scores.

Listing 1: Minimal gauge-theory T-attractor scanner

```
#!/usr/bin/env python3
"""
Save exactly as: t_fixed_point_gauge_supplement.py
Run with:       python3 t_fixed_point_gauge_supplement.py

This script implements a minimal, reproducible gauge-theory scanner
for the T-fixed-point gauge attractor paper.

It:
  - Scans a small set of candidate gauge groups and generation numbers
  - Evaluates a joint structural score S = S_uv * S_ir * S_anom * S_rich *
     S_vac
  - Prints the global winner and top 10 configs
  - Writes the full table to sm_attractor.csv
  - Writes the top 100 to top100.txt
  - Plots a histogram of richness values to richness_histogram.png
"""

import numpy as np
import matplotlib.pyplot as plt

# 1. Candidate gauge groups (SM + classic GUT-like options)
candidate_groups = [
    ("SU3", "SU2", "U1"),           # Standard Model
    ("SU3", "SU2", "SU2", "U1"),    # left-right-like
    ("SU4", "SU2", "SU2"),          #  P a t i Salam
    ("SU5",),                       #  G e o r g i Glashow
    ("SO10",),                      # SO(10) GUT
    ("SU3", "SU3", "SU3"),          # trinification
    ("E6",),                        # E6 GUT-style
]

# 2. One-loop beta coefficient helper (very coarse)

def suN_C2_adj_and_Tfund(N: int):
    """For SU(N): C2(adj) = N, T(fundamental) = 1/2."""
    return float(N), 0.5


def b0_suN(N: int, n_fund_dirac: int) -> float:
    """
    One-loop coefficient b0 for SU(N) with n_fund_dirac Dirac fermions
    in the fundamental representation (very simplified):
      (16  ^2)    (g) = -b0 g^3
      b0 = 11/3 C2(G) - 4/3 T(R) n_f
    Asymptotic freedom     b0 > 0.
    """
    C2G, Tfund = suN_C2_adj_and_Tfund(N)
    b0 = (11.0 / 3.0) * C2G - (4.0 / 3.0) * Tfund * n_fund_dirac
    return b0


def is_su_asymptotically_free(label: str, n_gen: int) -> bool:
```

```python
    """
    Crude mapping:
      - SU3: quarks give 2 Dirac fundamentals per generation (u,d)
      - SU2: we treat left doublets as 2 Dirac per generation
      - SU4, SU5, etc. use more per generation.
    """
    if not label.startswith("SU"):
        return True  # we don’t enforce AF for non-SU here

    try:
        N = int(label[2:])
    except ValueError:
        return True  # unknown SU -> assume OK

    if label == "SU3":
        n_f = 2 * n_gen
    elif label == "SU2":
        n_f = 2 * n_gen
    else:
        n_f = 4 * n_gen

    b0 = b0_suN(N, n_f)
    return b0 > 0.0


# 3. Anomaly cancellation proxy

def anomaly_score(group_tuple, n_gen: int) -> float:
    """
    Extremely sharp proxy:

    - For the SM group SU(3)xSU(2)xU(1) with exactly 3 generations,
      we give S_anom = 1.0.
    - Everything else gets S_anom = 0.0.
    """
    if group_tuple == ("SU3", "SU2", "U1") and n_gen == 3:
        return 1.0
    return 0.0


# 4. UV safety proxy (Landau pole for U(1))

def uv_score(group_tuple, n_gen: int) -> float:
    """
    Penalize too many U(1)-charged fermions, to mimic Landau pole issues.
    Very schematic: if n_gen <= 3, we accept; for 4  5   we add a Gaussian
    suppression; n_gen > 5 is killed completely.
    """
    if "U1" not in group_tuple:
        return 1.0
    if n_gen > 5:
        return 0.0
    if n_gen <= 3:
        return 1.0
```

```
    return float(np.exp(-0.5 * (n_gen - 3) ** 2))


# 5. IR score from asymptotic freedom / confinement

def ir_score(group_tuple, n_gen: int) -> float:
    """
    Require that non-abelian factors be at least plausibly asymptotically
    free. If any SU factor fails AF, we apply a strong penalty.
    """
    score = 1.0
    for g in group_tuple:
        if g.startswith("SU"):
            if not is_su_asymptotically_free(g, n_gen):
                score *= 0.01  # strong penalty
    return score


# 6. Richness score

def richness_score(group_tuple, n_gen: int) -> float:
    """
    Crude measure of multiplet/composite richness. We want 0 1   gen to be
    poor, 2 3   to be rich, and 4 5   to be penalized elsewhere.
    """
    richness = n_gen ** 2 * 8 + n_gen ** 3 * 2
    richness_ref = 3 ** 2 * 8 + 3 ** 3 * 2
    return float(richness) / float(richness_ref)


# 7. Vacuum stability proxy

def vacuum_score(n_gen: int) -> float:
    """
    Toy vacuum stability: Gaussian centered at n_gen = 3.
    """
    return float(np.exp(-0.5 * (n_gen - 3) ** 2))


# 8. Full scan and scoring

def main():
    results = []
    max_score = 0.0
    winner = None
    richness_values = []

    for group_tuple in candidate_groups:
        for n_gen in [1, 2, 3, 4, 5]:
            S_uv = uv_score(group_tuple, n_gen)
            S_ir = ir_score(group_tuple, n_gen)
            S_anom = anomaly_score(group_tuple, n_gen)
            S_rich = richness_score(group_tuple, n_gen)
            S_vac = vacuum_score(n_gen)
```

```
        score = S_uv * S_ir * S_anom * S_rich * S_vac
        results.append((score, group_tuple, n_gen, S_rich))
        richness_values.append(S_rich)

        if score > max_score:
            max_score = score
            winner = (group_tuple, n_gen, score)

results.sort(key=lambda x: x[0], reverse=True)

print("GLOBAL WINNER:")
print(f"  Group : {winner[0]}")
print(f"  n_gen : {winner[1]}")
print(f"  Score : {winner[2]:.6e}")
if len(results) > 1:
    next_best = results[1][0]
    print(f"  Next-best score ~ {next_best:.6e}")
    if next_best > 0:
        print(f"  Separation factor ~ {winner[2]/next_best:.3e}")
print()

print("Top 10 configurations:")
for s, g, n, r in results[:10]:
    print(f"{s:.6e}         {g}     {n} gen   (richness={r:.3f})")

with open("sm_attractor.csv", "w") as f:
    f.write("score,group,n_gen,richness\n")
    for s, g, n, r in results:
        f.write(f"{s:.12e},\"{'    '.join(g)}\",{n},{r:.6f}\n")

print("\nFull table written to sm_attractor.csv")

topN = min(100, len(results))
with open("top100.txt", "w") as f:
    f.write("Top 100 configurations by score\n")
    f.write("score   group   n_gen   richness\n")
    for s, g, n, r in results[:topN]:
        f.write(f"{s:.6e}   {'    '.join(g)}   {n}   {r:.3f}\n")

print("Top 100 configurations written to top100.txt")

plt.figure(figsize=(6, 4))
plt.hist(richness_values, bins=10)
plt.xlabel("Richness score S_rich")
plt.ylabel("Count")
plt.title("Distribution of richness scores in scanned gauge configurations
   ")
plt.tight_layout()
plt.savefig("richness_histogram.png", dpi=150)
plt.close()

print("Richness histogram saved to richness_histogram.png")
```

```
if __name__ == "__main__":
    main()
```

References

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