

$$\begin{aligned}
\mathbf{S}_T &= \sum_{i=1}^c \sum_{\mathbf{x} \in \mathcal{D}_i} (\mathbf{x} - \mathbf{m}_i + \mathbf{m}_i - \mathbf{m})(\mathbf{x} - \mathbf{m}_i + \mathbf{m}_i - \mathbf{m})^t \\
&= \sum_{i=1}^c \sum_{\mathbf{x} \in \mathcal{D}_i} (\mathbf{x} - \mathbf{m}_i)(\mathbf{x} - \mathbf{m}_i)^t + \sum_{i=1}^c \sum_{\mathbf{x} \in \mathcal{D}_i} (\mathbf{m}_i - \mathbf{m})(\mathbf{m}_i - \mathbf{m})^t \\
&= \mathbf{S}_W + \sum_{i=1}^c n_i (\mathbf{m}_i - \mathbf{m})(\mathbf{m}_i - \mathbf{m})^t.
\end{aligned}$$