

**13.** In many pattern classification problems one has the option either to assign the pattern to one of  $c$  classes, or to *reject* it as being unrecognizable. If the cost for rejects is not too high, rejection may be a desirable action. Let

$$\lambda(\alpha_i|\omega_j) = \begin{cases} 0 & i = j \quad i, j = 1, \dots, c \\ \lambda_r & i = c + 1 \\ \lambda_s & \text{otherwise,} \end{cases}$$

where  $\lambda_r$  is the loss incurred for choosing the  $(c+1)$ th action, rejection, and  $\lambda_s$  is the loss incurred for making a substitution error. Show that the minimum risk is obtained if we decide  $\omega_i$  if  $P(\omega_i|\mathbf{x}) \geq P(\omega_j|\mathbf{x})$  for all  $j$  and if  $P(\omega_i|\mathbf{x}) \geq 1 - \lambda_r/\lambda_s$ , and reject otherwise. What happens if  $\lambda_r = 0$ ? What happens if  $\lambda_r > \lambda_s$ ?