Pattern Classification (EET 3035)

Extra Lecture: 2D-Gabor Filters

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https://github.com/erkundanec/PatternClassification

Outline

Introduction

- Gabor filters are bandpass filters which are used in image processing for
  - feature extraction,
  - texture analysis,
  - stereo disparity estimation, etc.
- The kernel mask of these filters is created by multiplying a Gaussian envelop function with a complex oscillation.

$$g_{\lambda,\theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\exp\left(i\left(2\pi\frac{x'}{\lambda}+\varphi\right)\right)$$

- It was shown by several researchers that the profile of simple-cell receptive fields in the mammalian visual cortex can by described by oriented two-dimensional Gabor functions.

Visual Cortex

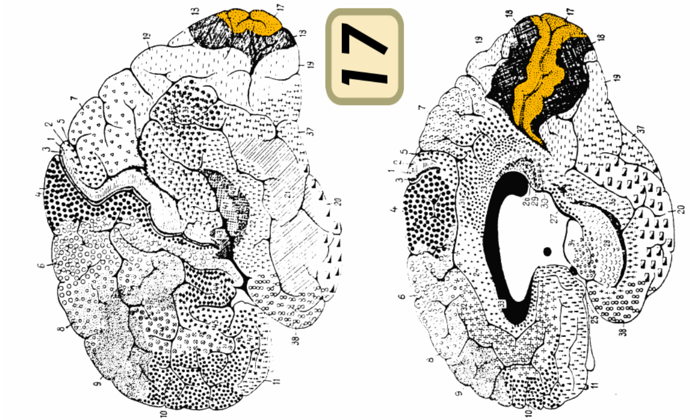
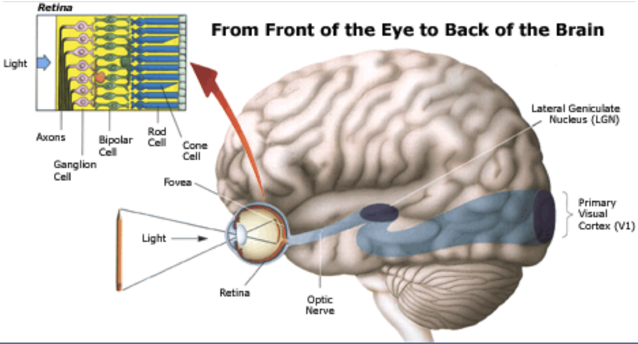


Figure: Broadmann area 17[1]

Visual System

- In the spatial domain, a 2D Gabor filter is a Gaussian kernel function modulated by a sinusoidal plane wave.
- Some authors claim that simple cells in the visual cortex of mammalian brains can be modelled by Gabor functions.
- Simple cells respond to bars and gratings of given orientation.



2D Gabor Functions

- Complex

$$g_{\lambda,\theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\exp\left(i\left(2\pi\frac{x'}{\lambda}+\varphi\right)\right)$$

- Real

$$g_{\lambda,\theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda}+\varphi\right)$$

- Imaginary

$$g_{\lambda,\theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\sin\left(2\pi\frac{x'}{\lambda}+\varphi\right)$$

where

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}$$

2D Gabor function parameters

$$g_{\lambda,\theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda}+\varphi\right)$$

where

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}$$

$\lambda \rightarrow$ wavelength of the sinusoidal factor,
$\theta \rightarrow$ orientation of the normal to the parallel stripes of a Gabor function,
$\varphi \rightarrow$ phase offset,
$\sigma \rightarrow$ standard deviation of the Gaussian envelope,
$\gamma \rightarrow$ spatial aspect ratio, specifies the ellipticity of the support of the Gabor function.

Wavelength (λ)

- Wavelength of the cosine factor of the Gabor filter kernel.
- Value is to be specified in number of pixels.
- Valid values are real numbers, $\lambda \geq 2$
- The value $\lambda = 2$ should not be used in combination with phase offset $\phi = -90$ or $\phi = 90$, because in these cases the Gabor function is sampled in its zero crossings.
- To avoid undesirable effect at the image borders, λ value should be smaller than one fifth of the input size.

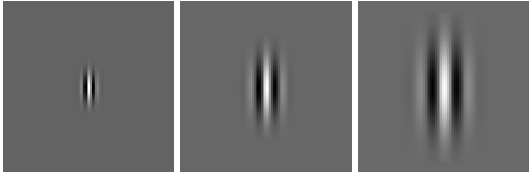


Figure: Image size is 100×100, $\lambda = 5, 10, 15$ from left to right, other parameters $\theta = 0$, $\varphi = 0$, $\gamma = 0.5$, $b = 1$

Orientations (θ)

- This parameter specifies the orientation of the normal to the parallel stripes of a Gabor function.
- Value is to be specified in degrees.
- Valid values are real numbers between $0 - 360$.

Figure: Image size is 100×100, $\theta = 0, 45, 90$ from left to right, other parameters $\lambda = 10$, $\varphi = 0$, $\gamma = 0.5$, $b = 1$

Phase offset (φ)

- The phase offset φ in the cosine factor of the Gabor function is specified in degree.
- Valid values are real number in between -180 and 180.
- The values 0 and 180 corresponds to center-symmetric 'center-on' and 'center-off' function, respectively. While -90 and 90 corresponds to anti-symmetric functions

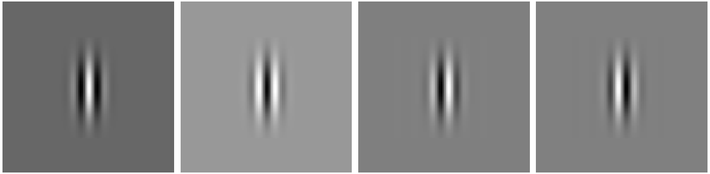


Figure: Image size is 100×100, $\phi = 0,\ 180,\ -90,\ and\ 90$ degrees from left to right, other parameters $\lambda = 10,\ \theta = 0,\ \gamma = 0.5,\ b = 1$

Aspect ratio (γ)

- This is spatial aspect ratio which specifies the ellipticity of the support of the Gabor function.
- For $\gamma = 1$, the support is circular.
- For $\gamma < 1$ the support is elongated in orientation of the parallel stripes of the function
- Normal value is $\gamma = 0.5$

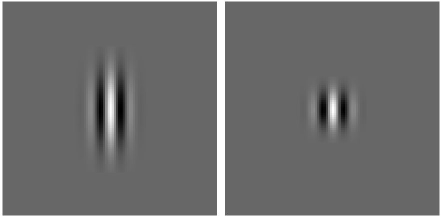


Figure: Image size is 100×100, $\gamma = 0.5\ and\ 1$ degrees from left to right, other parameters $\lambda = 10$, $\theta = 0$, $\varphi = 0$, $b = 1$

Bandwidth (b)

- The half-response spatial frequency bandwidth b (in octaves) of a Gabor filter is related to the ratio $\frac{\sigma}{\lambda}$.

$$b = \log_2 \frac{\frac{\sigma}{\lambda}\pi + \sqrt{\frac{\ln 2}{2}}}{\frac{\sigma}{\lambda}\pi - \sqrt{\frac{\ln 2}{2}}}, \quad \frac{\sigma}{\lambda} = \frac{1}{\pi}\sqrt{\frac{\ln 2}{2}} \cdot \frac{2^b + 1}{2^b - 1}$$

- The value of σ cannot be specified directly. It can only be changed through the bandwidth b.
- Must be a real positive number.
- The smaller the bandwidth, the larger σ,

Spatial frequency ($1/\lambda$)

- Preferred spatial frequency, $1/\lambda$, and size σ are not fully independent. Values are related with a relation
$$\sigma = a\lambda$$
- a varies in between 0.03 and 0.6 for most cells.
- In many experiments, $a = 0.56$ is used, i.e., $\sigma = 0.56\lambda$.

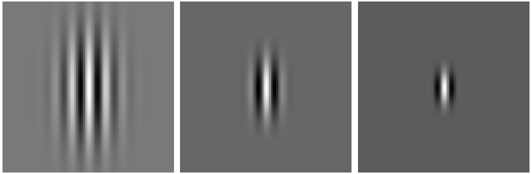
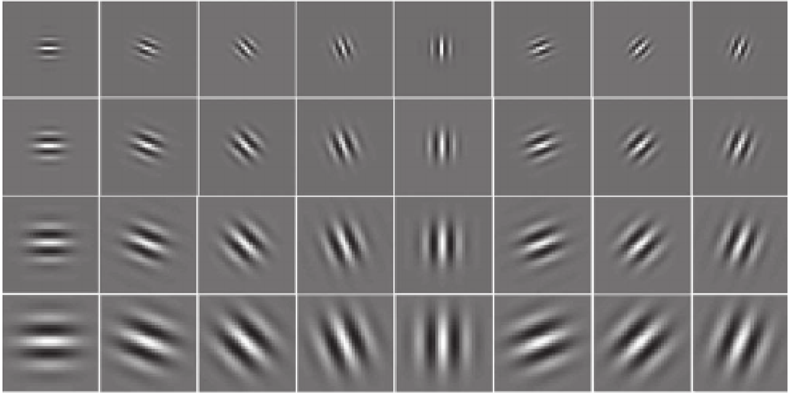
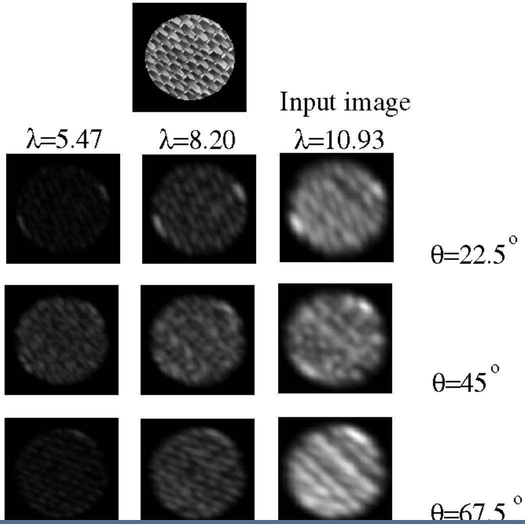


Figure: Image size is 100×100, $b = 0.5,\ 1$ and 2 from left to right, respectively. Other parameters $\lambda = 10$, $\theta = 0$, $\varphi = 0$, $\gamma = 0.5$

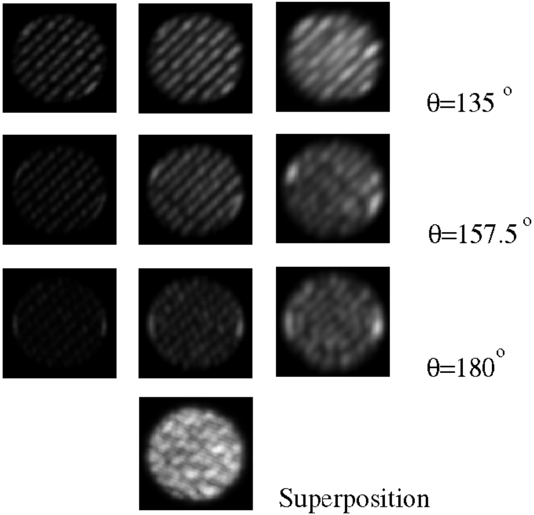
Set of Gabor filters



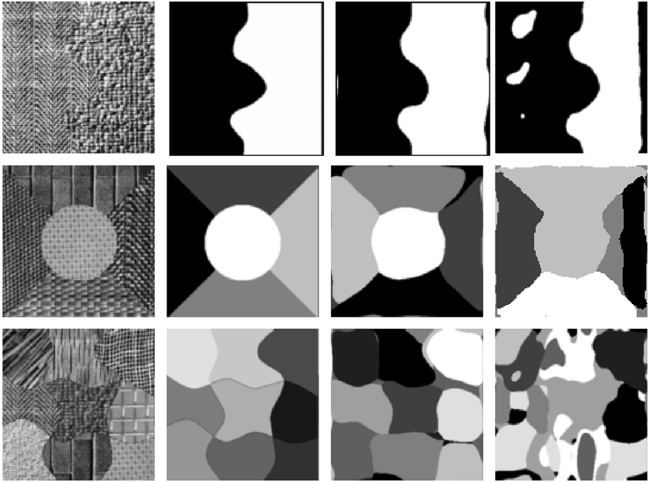
Application - Texture Segmentation



Application - Texture Segmentation



Application - Texture Segmentation



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[1] John G Daugman. "Uncertainty relation for resolution in space, spatial frequency, and orientation optimized by two-dimensional visual cortical filters". In: *JOSA A* 2.7 (1985), pp. 1160–1169.

[2] Cosmin Grigorescu, Nicolai Petkov, and Michel A Westenberg. "Contour detection based on nonclassical receptive field inhibition". In: *IEEE Transactions on image processing* 12.7 (2003), pp. 729–739.

[3] Simona E Grigorescu, Nicolai Petkov, and Peter Kruizinga. "Comparison of texture features based on Gabor filters". In: *IEEE Transactions on Image processing* 11.10 (2002), pp. 1160–1167.

[4] Anil K Jain and Farshid Farrokhnia. "Unsupervised texture segmentation using Gabor filters". In: *Pattern recognition* 24.12 (1991), pp. 1167–1186.

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[8] Nikolay Petkov. "Biologically motivated computationally intensive approaches to image pattern recognition". In: *Future Generation Computer Systems* 11.4-5 (1995), pp. 451–465.

References

[9] *Visual Cortex*. Wikipedia, the free encyclopedia, https://en.wikipedia.org/wiki/Visual_cortex.

Thank you!