Introduction to Digital Image Processing

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Image Digitization

Why Digitization?

Theory of Real Number:

- Between any two given points there are infinite number of points
- An image should be represented by infinite number of points.
- Each such image point may contain one of the infinitely many possible intensity/color values needing infinite number of bits
- Obviously such a representation is not possible in any digital computer Why Digitization

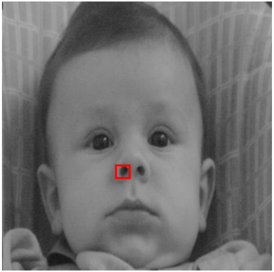
What is desired?

- An image to be represented in the form of a finite 2-D matrix

$$\begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,2) & \cdots & f(1,N-1) \\ f(2,0) & f(2,1) & f(2,2) & \cdots & f(2,N-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f(M-1,0) & f(M-1,1) & f(M-1,2) & \cdots & f(M-1,N-1) \end{bmatrix}$$

- Each of the matrix elements should assume one of finite discrete values.

Image as a Matrix of Numbers



Pixel values in highlighted region

99	71	61	51	49	40	35	53	86	99
93	74	53	56	48	46	48	72	85	102
101	69	57	53	54	52	64	82	88	101
107	82	64	63	59	60	81	90	93	100
114	93	76	69	72	85	94	99	95	99
117	108	94	92	97	101	100	108	105	99
116	114	109	106	105	108	108	102	107	110
115	113	109	114	111	111	113	108	111	115
110	113	111	109	106	108	110	115	120	122
103	107	106	108	109	114	120	124	124	132

A set of number in 2D grid

Camera → Digitizer →

What is digitization?

Sampling

- Image representation by 2-D finite matrix

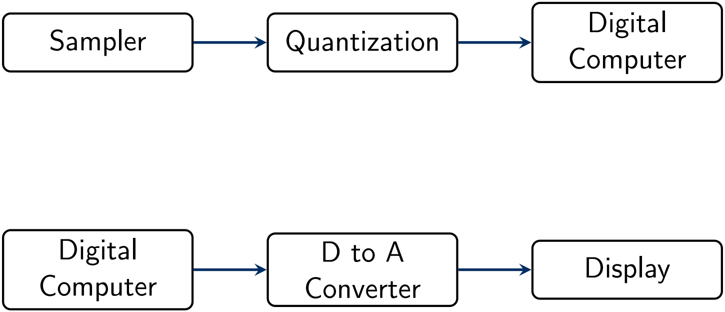
Quantization:

- Each Matrix element represented by one of the finite set of discrete values.

Sampling, Quantization, and Display

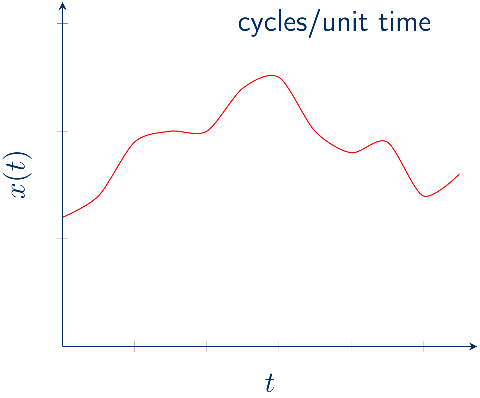
- Computer processing of images require that images be available in digital form.
- Digitization includes
  (a) Sampling,
  (b) Quantization.
- To display images it is first converted to analog signal which is scanned onto a display.

Sampling, Quantization, and Display



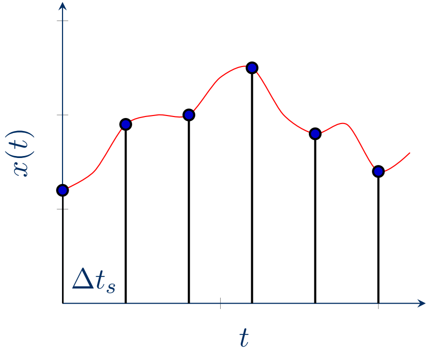
Sampling

- 1-D Sampling



Sampling

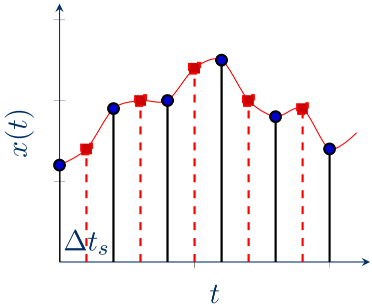
- 1-D Sampling



Sampling frequency $f_s = \frac{1}{\Delta t_s}$

Sampling

- 1-D Sampling



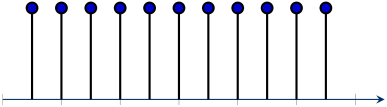
- Sampling frequency

$$f_s' = \frac{1}{\Delta t_s'} = \frac{2}{\Delta t_s} = 2f_s$$

where $\Delta t_s' = \frac{\Delta t_s}{2}$

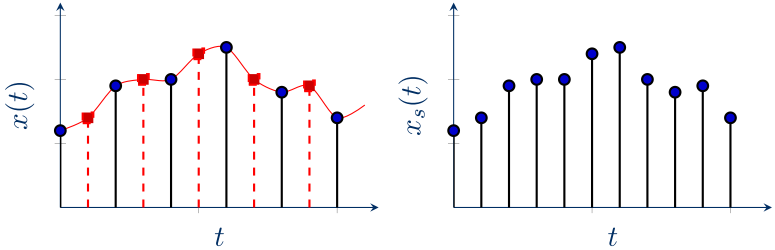
Sampling Theorem

- Sampling Function: 1-D array of Dirac delta function situated at regular spacing of Δt



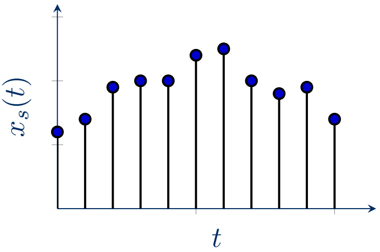
$$\text{comb}(t; \Delta t) = \sum_{m=-\infty}^{\infty} \delta(t - m\Delta t)$$

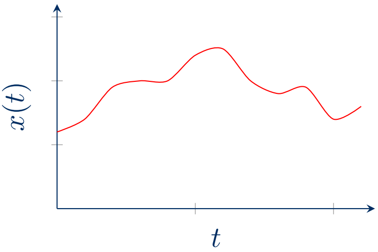
Sampling Theory



$$
\begin{aligned}
X_s(t) &= X(t).\mathrm{comb}(t, \Delta t) \\
&= \sum_{m=-\infty}^{\infty} X(m\Delta t)\delta(t - m\Delta t)
\end{aligned}
$$

Signal Reconstruction from Samples





$$x(t) \cdot y(t) \Leftrightarrow X(\omega) \otimes Y(\omega)$$
$$x(t) \otimes y(t) \Leftrightarrow X(\omega) \cdot Y(\omega)$$

$$X_s(\omega) = X(\omega) \otimes F(\mathrm{comb}(t, \Delta t_s))$$

Convolution Concept

$h(n)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(m)$

2	5	7	9	3

$$y(n) = \sum_{m=-\infty}^{\infty} h(m)x(n-m)$$

$x(-m)$

3	9	7	5	2

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 0 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-11-m)$

3	9	7	5	2

-9 0 9

0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-10-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad 0 \quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-9-m)$

3	9	7	5	2

-9 $\quad 0 \quad$ 9

0	0	2	5	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-8-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad 0 \quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-7-m)$

3	9	7	5	2

-9 $\quad 0 \quad$ 9

0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$$x(-m)$$

3	9	7	5	2

$$h(m)$$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$$x(-6-m)$$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$y(n)$$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 0 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-5-m)$

3	9	7	5	2

-9 0 9

0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-4-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

				-9									0									9				
0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-3-m)$

3	9	7	5	2

				-9									0									9				
0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-2-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 0 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(-1-m)$

3	9	7	5	2

-9 0 9

0	0	2	5	7	9	3	0	0	0	0	2	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 0 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(0-m)$

3	9	7	5	2

-9 0 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	0	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(1-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	0	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(2-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 0 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(3-m)$

3	9	7	5	2

-9 0 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(4-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(5-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(6-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	0	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 0 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(7-m)$

3	9	7	5	2

-9 0 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	2	0	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\qquad$ 0 $\qquad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(8-m)$

3	9	7	5	2

-9 $\qquad$ 0 $\qquad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	2	5	0	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(9-m)$

3	9	7	5	2

-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	2	5	7	0	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad 0 \quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(10-m)$

3	9	7	5	2

-9 $\quad 0 \quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	0	0	0

$y(n)$

Convolution Concept

$x(-m)$

3	9	7	5	2

$h(m)$

-9 $\quad$ 0 $\quad$ 9

0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0

$x(11-m)$

3	9	7	5	2

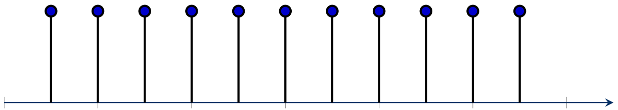
-9 $\quad$ 0 $\quad$ 9

0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0	0	0	2	5	7	9	3	0	0

$y(n)$

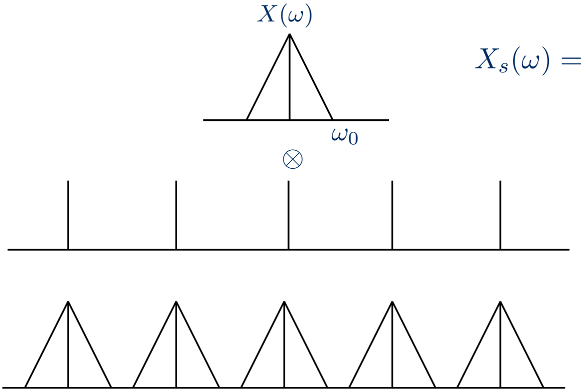
Convolution Concept

$$F(\mathrm{comb}(t, \Delta t_s)) = \frac{1}{\Delta t_s}\mathrm{comb}\left(\omega; \frac{1}{\Delta t_s}\right) = \mathrm{COMB}(\omega)$$



$$\frac{1}{\Delta t_s}$$

Convolution Concept



$$X_s(\omega) = X(\omega) \otimes \mathrm{COMB}(\omega)$$

$$\frac{1}{\Delta t_s} - \omega_0 > \omega_0$$

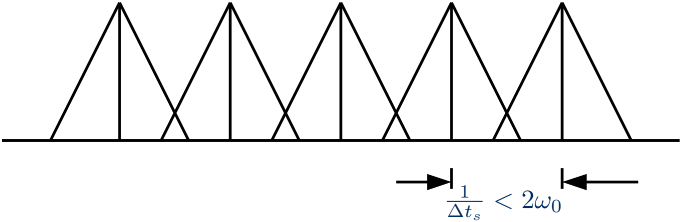
$$\Rightarrow \frac{1}{\Delta t_s} > 2\omega_0$$

$$\Rightarrow f_s > 2\omega_0$$

$$0 < \omega_0 < \frac{1}{\Delta t_s}$$

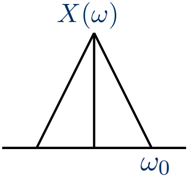
Aliasing

when $f_s < 2w_0$



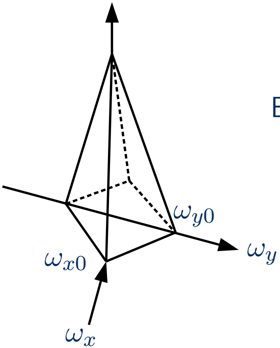
2-D Sampling

$1-D$



Bandlimited $\Rightarrow X(\omega) = 0$ for $|\omega > \omega_0|$

$2-D$



Bandlimited $\Rightarrow F(\omega_x, \omega_y) = 0$

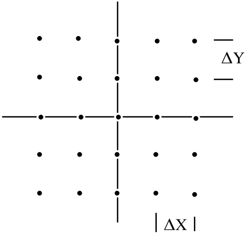
For $|w_x| > w_{x0}$

$|w_y| > w_{y0}$

2-D Sampling

$$comb(x, y; \Delta x, \Delta y) \triangleq \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - m\Delta x, y - n\Delta y)$$

$$\begin{aligned} f_s(x, y) &= f(x, y) \cdot \mathrm{comb}(x, y; \Delta x, \Delta y) \\ &= \sum_{m=-\infty}^{\infty} \sum_{n-\infty}^{\infty} f(m\Delta x, n\Delta y)\delta(x - m\Delta x, y - n\Delta y) \end{aligned}$$



2-D Sampling

Following similar arguments as in 1-D Case

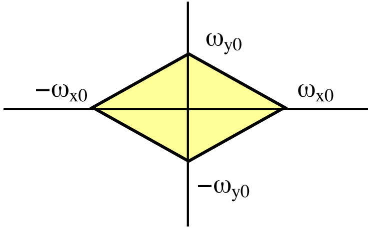
$$F_s(\omega_x, \omega_y) = F(\omega_x, \omega_y) \otimes COMB(\omega_x, \omega_y)$$

$$\begin{aligned} COMB(\omega_x, \omega_y) &= F\{comb(x, y; \delta x, \delta y)\} \\ &= \omega_{xs}\omega_{ys} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \partial(\omega_x - m\omega_{xs}, \omega_y - n\omega_{ys}) \\ &= \omega_{xs}\omega_{ys} comb(\omega_x, \omega_y; \frac{1}{\Delta x}, \frac{1}{\Delta y}) \end{aligned}$$

where $\omega_{xs} = \frac{1}{\Delta x}$ = sampling frequency along x
$\omega_{ys} = \frac{1}{\Delta y}$ = sampling frequency along y

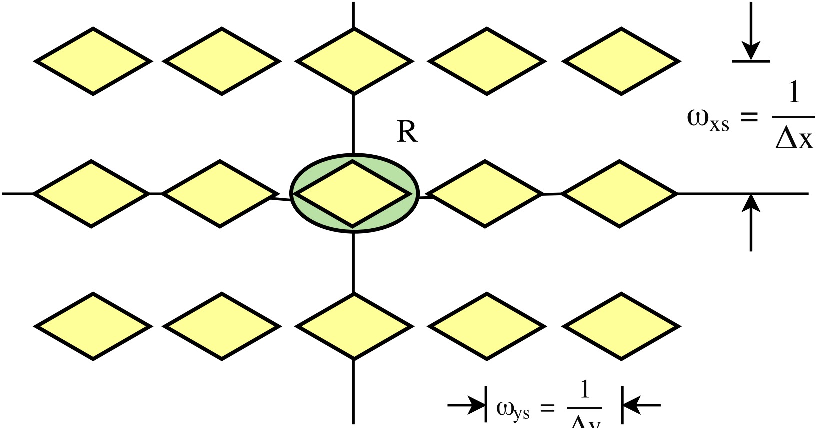
2-D Sampling

$$F_s(\omega_x, \omega_y) = F(\omega_x, \omega_y) \otimes COMB(\omega_x, \omega_y)$$

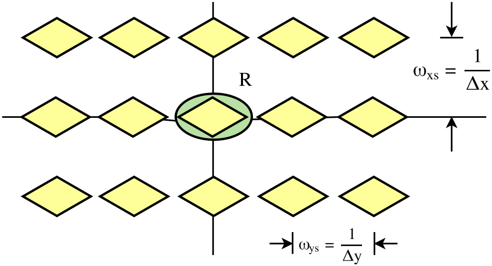


Region of support of $F(x, y)$

2-D Sampling



2-D Sampling



- 2-D image can be reconstructed if $\omega_{xs} > 2\omega_{x0}$ and $\omega_{ys} > 2\omega_{y0} \Rightarrow$ Nyquist rate.
- Recovery by a low pass filter with response:

$$H(\omega_x, \omega_y) = \begin{cases} \frac{1}{\omega_{xs}\omega_{ys}} & (\omega_x, \omega_y) \in R \\ 0 & \text{otherwise} \end{cases}$$

Sampling Theorem

- A bandlimited image $f(x, y)$ and sampled uniformly on a rectangular grid with spacing $\Delta x,\ \Delta y$ can be recovered without error from the sample values $f(m\Delta x, n\Delta y)$ provided the sampling rate is greater than Nyquist rate that is

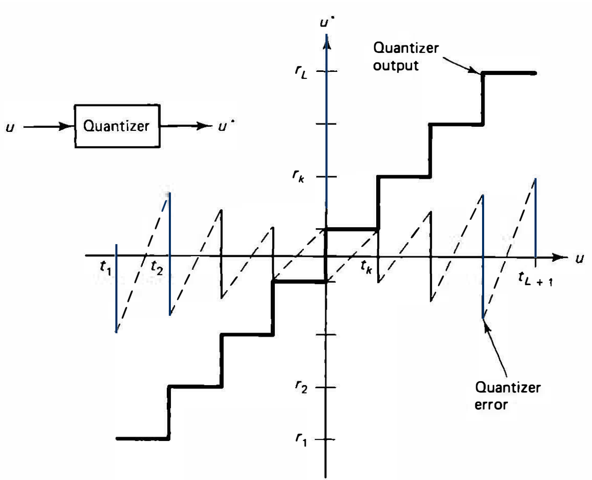
$$\frac{1}{\Delta x} = \omega_{xs} > 2\omega_{x0} \tag{1}$$

$$\frac{1}{\Delta y} = \omega_{ys} > 2\omega_{y0} \tag{2}$$

Image Quantization

- Next step to image digitization is quantization.
- A quantizer maps a continuous value u into a discrete variable u^* which takes values from a finite set $\{r_1, r_2, \ldots, r_L\}$ of numbers.
- This mapping is generally a staircase function and the quantization rule is a follows:
  - define $\{t_k,\ k = 1, 2, \ldots, L+1\}$ as a set of increasing transition level with t_1 as minimum & t_{L+1} as maximum.
- So, if u lies in interval $[t_k, t_{k+1}]$ then it is mapped to r_k the k^{th} reconstructed level.

Quantization



Quantization: Example

The simplest and most common quantizer is the uniform quantizer. Let the o/p of an image sensor take values between 0.0 to 10.0. If samples are quantized uniformly to 256 levels then transition and reconstruction levels are

$$t_k = \frac{10(k-1)}{256} \qquad k = 1, \ldots, 257$$

$$r_k = t_k + \frac{5}{256} \qquad k = 1, \ldots, 256$$

The interval $q = t_k - t_{k-1} = r_k - r_{k-1}$ is constant for different value of k and is called the *quantization interval*.

Quantization

- Quantizer mapping is irreversible that is for a given quantizer o/p, the i/p value cannot be determined uniquely.
- Hence, quantizer introduces distortion, which any reasonable design method must attempt to minimize.
- There are several quantizer design available that offers various trade-offs between simplicity and performance.
- Example:
  - Optimum mean or LLOYD-Max quantizer
  - Uniform optimum quantizer
  - Optimum mean square quantizer
  - Compandor, etc.

Thank You

Queries?