Introduction to Digital Image Processing

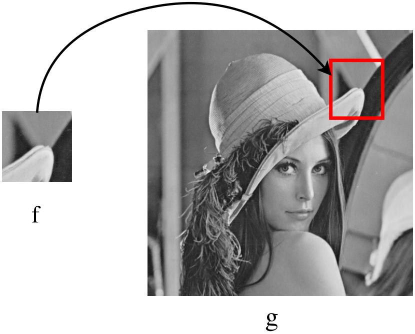
Dr. Kundan Kumar
Ph.D. (IIT Kharagpur)
Assistant Professor
ECE Department (Cabin - E139)

Institute of Technical Education & Research (ITER)
Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar, Odisha, India-751030

Image Registration

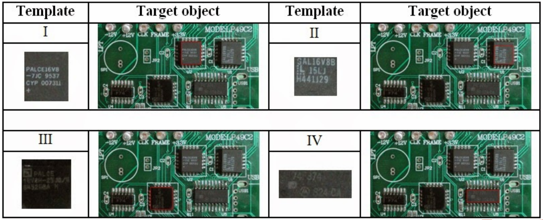
Introduction

- Registration is a process which makes the pixels in two images precisely coincide to the same point in the scene.
- Once registered, the image can be combined or fused in a way that improve information extraction.



Applications

- *Stereo imaging* where two images are taken from two different positions.
- *Remote Sensing* where the images may be taken by different sensors.
- Images may be taken at *different instance of time* e.g. deforestation, Landslide
- Finding a *place in a picture* where it matches a given pattern, e.g., Template matching.



Mismatch or Match Measures

- Suppose
  $g \rightarrow$ given image.
  $f \rightarrow$ Template
  $A \rightarrow$ Region of interest to find match/mismatch
- The different measures are defined as
  - $\max\limits_{A} |f - g|$
  - $\iint\limits_{A} (f-g)^2$
  - $\iint\limits_{A} |f-g| \Rightarrow \sum\limits_{i \in A} \sum\limits_{j \in A} |f(i,j) - g(i,j)|$

Mismatch or Match Measures

$$\underbrace{\iint_A (f-g)^2}_{\text{mismatch measure}} = \underbrace{\iint_A f^2}_{\text{fixed}} + \underbrace{\iint_A g^2}_{\text{fixed}} - 2\iint_A f.g$$

- The mismatch measure will be minimum when the third term will be maximum or vice versa.

$$\iint_A f.g \rightarrow \text{match measure or similarity measure}$$

- This should have maximum value for the correct match location.

Mismatch or Match Measures

- *Cauchy-schwartz Inequality*

$$\iint f.g \leqslant \sqrt{\iint f^2 \cdot \iint g^2}$$

The above inequality will be equal only when $g = cf$

- From the Cauchy Schwartz inequality

$$\iint_A f(x,y).g(x+u,y+v)dxdy \leqslant$$

$$\left[\iint_A f^2(x,y)dxdy \iint_A g^2(x+u,y+v)dxdy\right]^{\frac{1}{2}}$$

where $u \rightarrow$ shift along x-direction
$v \rightarrow$ shift along y-direction

Mismatch or Match Measures

- We are trying to find different match measure at different location of g. So f to be shifted in the image.
- Since size of $f(x, y)$ is small and outside A, $f(x, y)$ is zero. So we can write

$$\underbrace{\iint_{\infty} f(x, y)g(x+u, y+v)dxdy}_{\text{Cross correlation between } f \text{ and } g}$$

- Since $\iint_A f^2(x, y)dxdy$ is fixed but $\iint_A g^2(x+u, y+u)dxdy$ is not a constant because the value will depend on the shift of template u and v.

Mismatch or Match Measures

- Here the cross correlation cannot be used directly as similarity measure because of variable quantity.
- The normalize cross correlation can be used as similarity measure.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)g(x+u,y+v)dxdy = c_{fg}$$

- So normalized cross correlation

$$\frac{c_{fg}}{\left[\iint g^2(x+u,y+v)dxdy\right]^{1/2}}$$

Example

$$f = \begin{bmatrix} 3 & 3 & 2 \\ 3 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix} \qquad g = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 20 & 2 & 2 & 2 & 1 \\ 1 & 2 & 3 & 3 & 2 & 1 \\ 1 & 2 & 3 & 3 & 2 & 1 \\ 1 & 2 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Find the match location?

$$c_{fg} = \begin{bmatrix} 47 & 54 & 56 & 20 & 18 & 12 \\ 54 & 87 & 94 & 40 & 34 & 21 \\ 56 & 90 & \mathbf{107} & 54 & 44 & 24 \\ 20 & 40 & 54 & 56 & 44 & 24 \\ 18 & 34 & 43 & 44 & 37 & 22 \\ 12 & 21 & 24 & 24 & 22 & 15 \end{bmatrix} \Rightarrow \text{false match at } 107$$

Example

Compute $\sum \sum \left[g^2(x+u, y+v)\right]^{1/2}$ as

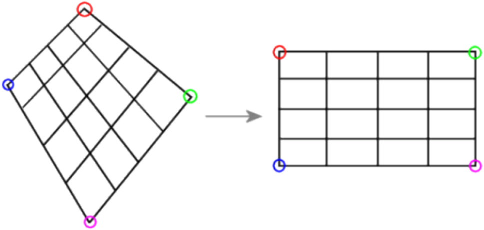
$$\begin{bmatrix} 20.07 & 20.19 & 20.30 & 3.87 & 3.46 & 2.64 \\ 20.19 & 20.54 & 20.80 & 6.08 & 5.09 & 3.46 \\ 20.27 & 20.76 & 21.26 & 7.48 & 6.08 & 3.87 \\ 3.87 & 5.91 & 7.48 & 7.48 & 6.08 & 3.87 \\ 3.46 & 5.09 & 6.08 & 6.08 & 5.09 & 3.46 \\ 2.64 & 3.46 & 3.87 & 3.87 & 3.46 & 2.64 \end{bmatrix}$$

Normalized Cross Correlation Computed as

$$\begin{bmatrix} 2.34 & 2.67 & 2.75 & 5.16 & 5.20 & 4.54 \\ 2.67 & 4.20 & 4.51 & 6.50 & 6.67 & 6.06 \\ 2.76 & 4.33 & 5.03 & 7.20 & 7.23 & 6.20 \\ 5.16 & 6.76 & 7.21 & \mathbf{7.48} & 7.23 & 6.20 \\ 5.20 & 6.67 & 7.07 & 7.23 & 7.26 & 6.35 \\ 4.54 & 6.06 & 6.20 & 6.20 & 6.35 & 5.68 \end{bmatrix} \Rightarrow \text{match location is} (3,3)$$

Registration in Geometric Restoration

- There may be geometric distortion of an image due to optical system of the camera.
- Degradation introduced by optical system of the camera.



$$(x, y) \xrightarrow[distortion]{after} (x', y') \xrightarrow[restoration]{after} (x, y)$$

- We can go for estimation of polynomial degradation function.

Registration in Geometric Restoration

- For a single pair of coordinate points two polynomial equation can be written as

$$x' = k_1x + k_2y + k_3xy + k_4$$
$$y' = k_5x + k_6y + k_7xy + k_8$$

- If we estimate these coefficient then we can estimate any points in original image corresponding to the point in the degraded image.
- Here, we have 8 unknowns therefore, to solve the problem we need at least 4 corresponding point pairs.
- In some points we will not get the pixel value after restoration so we will use interpolation technique to interpolate the missing values.

Application: MR-CT image fusion

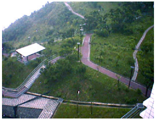
- Magnetic-Resonance (MR) measure water content.
- Computed Tomography (CT) measures x-ray absorption.
- Bone is brightest in CT scan and darkest in MR image.
- Both images are 3D volumes.

(a1-MRI) (a2-CT) (a3-Fused image)

Application: Image Mosaicing/Stitching

- Mosaicing is the process of assembling a series of images and joining them together to form a continuous seamless photographic representation of the image surface.
- The result is an image with a field of view greater than that of a single image.
- Similar to panoramic image stitching problem.

Application: Image Mosaicing/Stitching



Thank You

Queries?