Introduction to Digital Image Processing

Dr. Kundan Kumar
Ph.D. (IIT Kharagpur)
Assistant Professor
ECE Department (Cabin - E139)

Institute of Technical Education & Research (ITER)
Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar, Odisha, India-751030

Image Restoration

Image Restoration

The *image restoration* improves the quality of an image. It recovers an image that has been degraded by using a prior knowledge of the degradation phenomenon.

Image restoration is a more objective process compared to image enhancement which is a subjective process.

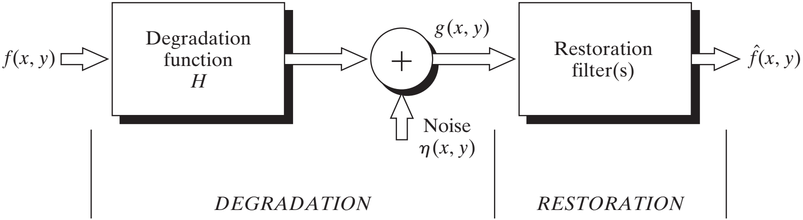
Objective

- Reconstruct an image that has been degraded in some way.

Main idea:

- Model the degradation using (a priori) information about the degradation process and apply inverse filtering.

Image Degradation Model (Linear/Additive)



- Degradation can be caused by defects in the imaging system, or outer circumstances (imaging conditions) such as bad focusing, motion, non linearity of the sensor, etc.
- Here the objective of restoration is to obtain and estimate $\hat{f}(x, y)$ of the original image.

Image Degradation Model (Linear/Additive)

- The estimate must be as close as possible to the original input image.
- If the information about H and η are known then estimation of $\hat{f}(x,y)$ will be closer to $f(x,y)$.
- In spatial domain

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

- In frequency domain

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Noise Model

- Source of noise
  - Objects Impurities
  - Image acquisition (digitization, environmental conditions, quality of sensing elements)
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum: White noise (a constant Fourier spectrum)

Noise Probability Density Functions

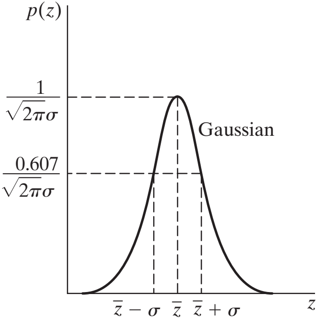
Gaussian noise

- Because of its mathematical tractability in both the spatial and frequency domain, Gaussian (also called normal) noise models are used frequently in practice.

- The PDF of a Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})/2\sigma^2}$$

where z represents intensity, $\bar{z}$ is the mean value of z, and σ is its standard deviation.



Noise Probability Density Functions

Rayleigh noise

- The PDF of a Rayleigh noise is given by

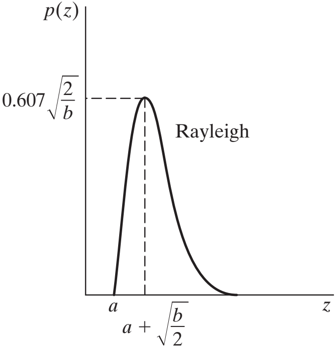
$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of the density are given by

$$\bar{z} = a + \sqrt{\pi b/4}$$

and

$$\sigma^2 = \frac{b(4-\pi)}{4}$$



Rayleigh density is useful for approximating skewed histograms.

Noise Probability Density Functions

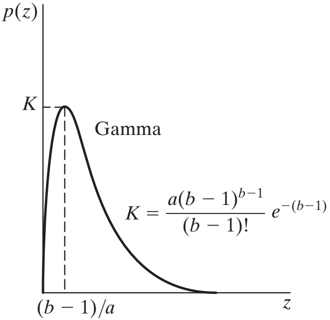
Gamma (Erlang) noise

- The PDF of a Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

  where the parameters are such that $a > 0$, b is a positive integer, and ! indicates factorial.

- The mean and variance of this density are given by $\bar{z} = \frac{b}{a}$ and $\sigma^2 = \frac{b}{a^2}$



Noise Probability Density Functions

Exponential Noise

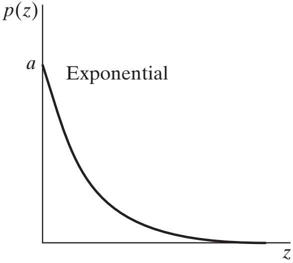
- The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

where $a > 0$.

- The mean and variance of this density function are

$$\bar{z} = \frac{1}{a} \quad \text{and} \quad \sigma^2 = \frac{1}{a^2}$$



This PDF is special case of the Erlang PDF, with $b = 1$.

Noise Probability Density Functions

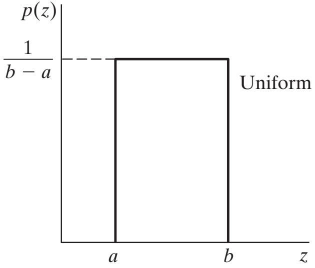
Uniform Noise

- The PDF of uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

- The mean and variance of this density function are

$$\bar{z} = \frac{a+b}{2} \quad \text{and} \quad \sigma^2 = \frac{(b-a)^2}{12}$$

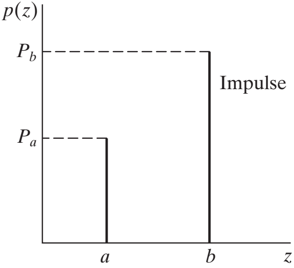


Noise Probability Density Functions

Impulse (Salt-and-pepper) noise

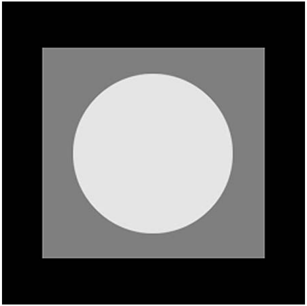
- The PDF of (*bipolar*) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



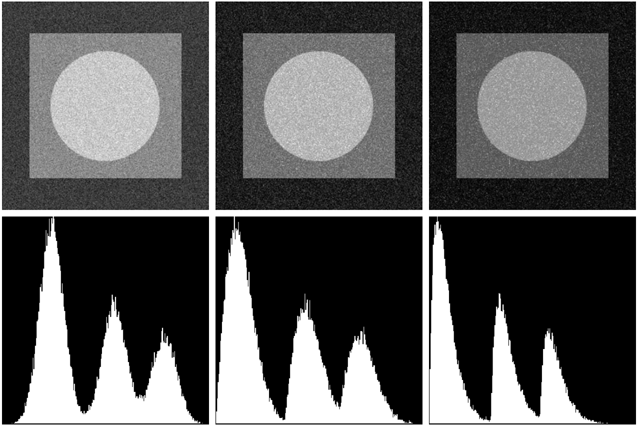
- If $b > a$, intensity b will appear as a light dot in the image and a will appear like a dark dot.
- If either P_a and P_b is zero, the impulse noise is called *unipolar*.

Noise Probability Density Functions - Example



- What will be the histogram of the above image?

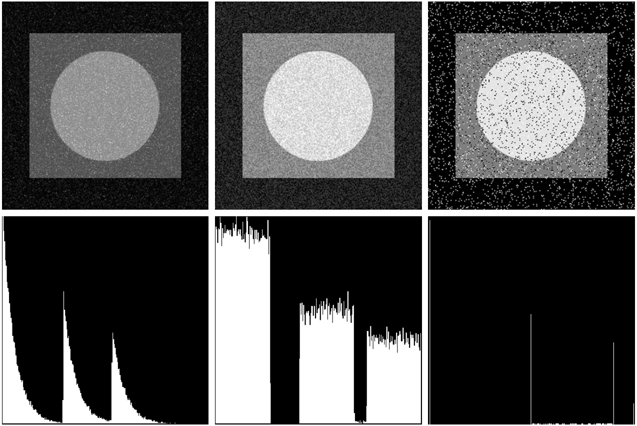
Noise Probability Density Functions - Example



Gaussian Rayleigh Gamma

Figure: Image and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image.

Noise Probability Density Functions - Example



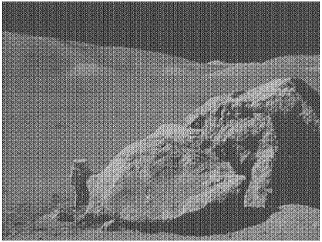
Exponential Uniform Salt & Pepper

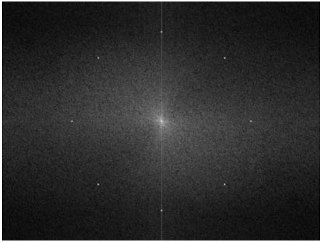
Figure: Image and histograms resulting from adding exponential, uniform, and salt-and-pepper noise to the image.

Estimation of Noise Parameters

Periodic Noise

- The periodic noise typically are estimated by inspection of the Fourier spectrum of the image.





- Automated analysis is possible in situation in which the noise spikes are either exceptionally pronounced, or when knowledge is available about the general location of the frequency components of the interference.

Estimation of Noise Parameters

Non-Periodic Noise

- The parameters of noise PDFs may be known partially from sensor specification, but it is often necessary to estimate them for a particular imaging arrangement.
- If the imaging system is available, one simple way to study the characteristics of system noise is to capture a set of images of "flat" environments.
- If only images already generated by a sensor are available, frequently it is possible to estimate the parameters of the PDF from small patches of reasonably constant background intensity.

Estimation of Noise Parameters

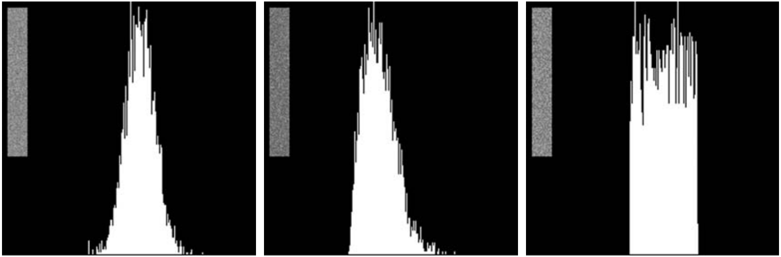


Figure: Histogram computed using small strips from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noise images.

Restoration in the Presence of Noise Only - spatial filtering

- When the only degradation present in an image is noise then

$$g(x,y) = f(x,y) + \eta(x,y)$$

and

$$G(u,v) = F(u,v) + N(u,v)$$

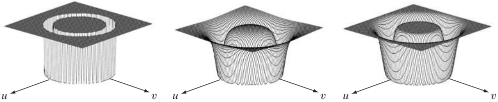
- The noise terms are unknown, so subtracting them from $g(x,y)$ or $G(u,v)$ is not a realistic option.
- Spatial filtering is good choice when only additive random noise is present.

Restoration in the Presence of Noise Only - spatial filtering

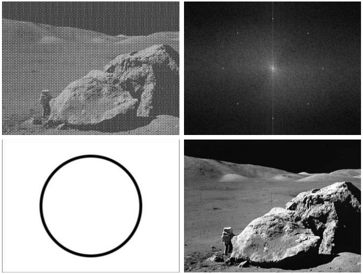
- Mean Filters
  - Arithmetic mean filter
  - Geometric mean filter
  - Harmonic mean filter
  - Contraharmonic mean filter
- Order-Statistic Filters
  - Median filter
  - Max and Min filters
  - Midpoint filter
  - Alpha-trimmed mean filter
- Adaptive Filters
  - Adaptive, local noise reduction filter
  - Adaptive median filter

Periodic Noise Reduction by Frequency Domain Filtering

- Bandreject filters



- Bandpass filters: Opposite operation of a bandreject filter.
- Notch Filters



Linear, Position-Invariant Degradation

Linearity:

- If we consider a degradation function as $g(x,y) = H[f(x,y)]$ then

$$H[k_1 f_1(x,y) + k_2 f_2(x,y)] = k_1 H[f_1(x,y)] + k_2 H[f_2(x,y)]$$

This is called *superposition theorem*.

- If $k_1 = 1$ and $k_2 = 1$ in above expression then called *additive property*.
- If $f_2(x,y) = 0$

$$H[k_1 f_1(x,y)] = k_1 H[f_1(x,y)]$$

called *Homogeneity property*.

Linear, Position-Invariant Degradation

Position-Invariant

- If $g(x, y) = H[f(x, y)]$, then

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

  called *position-invariant property*.
- Position-invariant means response at a particular point is solely depend on pixel value at that point in original image.
- Response will not depend on the position of the point in the images.

Estimating the Degradation Function

- There are three principal ways to estimate the degradation function/model for the use in image restoration:
  (1) Estimating by image observation,
  (2) Experimentation, and
  (3) Mathematical modeling.
- The method of restoring the original image from the degraded image using degraded function obtained through one of these method is called *Blind Convolution*.
- The process is called Blind Convolution because the degradation model is just approximate estimated model. It is not the exact model which have degraded the image.

Estimating by image observation

- Suppose we have given a degraded image without any knowledge about the degradation function H.
- Assume the image was degraded by a linear, position-invariant process.

$$\hat{g}_s(x,y) \Leftrightarrow G_s(u,v) \qquad \text{degraded subimage}$$
$$\hat{f}_s(x,y) \Leftrightarrow F_s(u,v) \qquad \text{estimated original subimage}$$

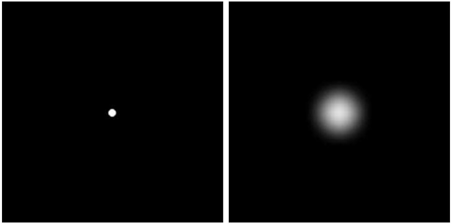
- Therefore, we can write

$$H_s(u,v) = \frac{G_s(u,v)}{F_s(u,v)}$$

- To minimize the effect of noise, choose the subregion form image where the image content is strong.

Degradation function by experimentation

- Need imaging setup (similar to original imaging setup) which have degraded the image.
- Then find impulse response or PSF of the imaging setup
- Impulse simulation by a bright spot, Intensity is the strength of impulse.



Degradation function by experimentation

$$
\begin{aligned}
F(u, v) &= A \text{ (constant)} \\
&= \text{frequency response of impulse response}
\end{aligned}
$$

- so we can write

$$
\begin{aligned}
G(u, v) &= H(u, v)F(u, v) \\
H(u, v) &= \frac{G(u, v)}{F(u, v)} \\
&= \frac{G(u, v)}{A}
\end{aligned}
$$

- Intensity of light should be very high so that the effect of noise can be reduced.

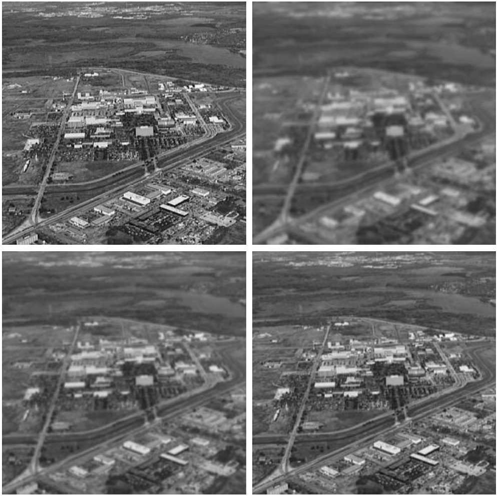
Estimating by mathematical modeling

- Provide insight into the degradation process.
- Atmospheric turbulence can be modeled

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

  Capable of modeling atmospheric turbulence that lead to degradation in observed image
- k is the nature of turbulence.
- For very high turbulence, k is very high.

Estimating by mathematical modeling



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Estimating by mathematical modeling: Motion Blurring

- Let $f(x,y)$ goes through motion blurring
- $x_0(t)$ and $y_0(t)$ $\rightarrow$ time varying components.
- Assume, shutter is open for T sec.
- The observed blurred image

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

- The frequency response

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

Estimating by mathematical modeling: Motion Blurring

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{0}^{T} f\left(x - x_0(t), y - y_0(t)\right) dt\right] e^{-j2\pi(ux+vy)} dxdy$$

$$G(u,v) = \int_{0}^{T}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x - x_0(t), y - y_0(t)) e^{-j2\pi(ux+vy)} dxdy\right] dt$$

- the term inside the bracket is the Fourier transform of shifted $f(x,y)$
- We know

$$f\left(x - x_0(t), y - y_0(t)\right) \Leftrightarrow F(u,v)e^{-j2\pi[ux_0(t)+vy_0(t)]}$$

Estimating by mathematical modeling: Motion Blurring

$$G(u,v) = \int_0^T F(u,v)e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$G(u,v) = F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$G(u,v) = H(u,v)F(u,v)$$

where $H(u,v)$ is the degraded function for motion blurring.

Estimating by mathematical modeling: Motion Blurring

- If motion variable $x_0(t)$ and $y_0(t)$ are known the degradation function can be estimated for motion blurring.
- If $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ then estimate degrading function.

$$H(u,v) = \frac{1}{\pi(ua+vb)} \sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

Estimating by mathematical modeling: Motion Blurring

a b

FIGURE 5.26
(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Restoration

- How to restore the degraded image given degraded function?
  - Inverse Filtering
  - Minimum-Mean-Square-Error
  - Constrained Least Square Filter

Restoration by Inverse Filtering

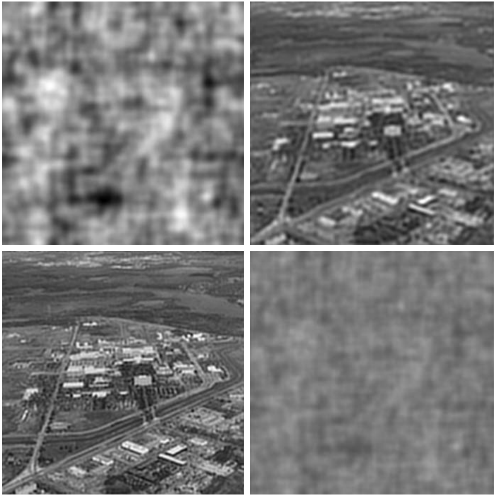
- Simplest approach
- Restoration of images degraded by a degradation function H, which is *given* or *estimated*.
- Estimate $\hat{F}(u,v)$ from $G(u,v)$ as

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = \frac{F(u,v)H(u,v) + n(u,v)}{H(u,v)}$$

$$\hat{F}(u,v) = F(u,v) + \frac{n(u,v)}{H(u,v)}$$

- Even if we know the degradation function we can not recover the undergraded image (the inverse Fourier transform of $F(u,v)$) exactly because $N(u,v)$ is not known.

Restoration by Inverse Filtering



a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Mean-square filtering (Wiener Filtering)

- In inverse filtering approach, no explicit provision for handling noise.
- Wiener Filtering approach incorporates both the degradation function and statistical characteristics of noise (as random variable) into the restoration process.
- Objective is to find an estimate $\hat{f}(x, y)$ of the uncorrupted image $f(x, y)$ such that the mean-square-error between them is minimized.
- The error measure

$$e^2 = E\{(f(x, y) - \hat{f}(x, y))\}$$

where $E\{\cdot\}$ is the expected value of the argument.

Mean-square filtering (Wiener Filtering)

- It is assumed that noise and images are uncorrelated

$$G(u,v) = F(u,v) \cdot H(u,v) + N(u,v)$$
$$\hat{F}(u,v) = G(u,v) \cdot H_{min}(u,v)$$

where $H_{min}(u,v)$ is the transfer function of the wiener filter.

$$\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right] G(u,v) \\
&= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v) \\
&= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)
\end{aligned}$$

This expression is known is the Wiener filter.

Mean-square filtering (Wiener Filtering)

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

- The term inside bracket is commonly referred to as the *minimum-mean-square-error filter* or the *least-square-error filter*.
- The terms are as follows:
  - $H(u,v)$ = degradation function
  - $H^*(u,v)$ = complex conjugate of $H(u,v)$
  - $|H(u,v)|^2 = H^*(u,v)H(u,v)$
  - $S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of the noise
  - $S_f(u,v) = |F(u,v)|^2$ = power spectrum of the undegraded image.
- If the noise is zero, then the noise power spectrum vanishes and the Wiener filter reduces to the inverse filter.

Mean-square filtering (Wiener Filtering)

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Signal to noise ratio

- Measure based on power spectra of noise and of the undegraded image, used to characterizing the performance of restoration algorithms.
- Signal-to-noise-ratio approximated using frequency domain as

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2}$$

- Images with low noise tend to have a high SNR and, conversely, the same image with higher level of noise has a lower SNR.

Root mean square SNR

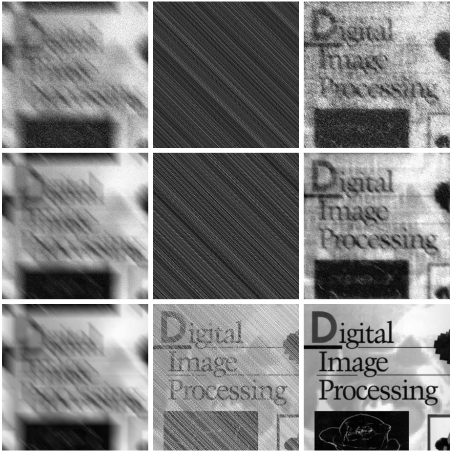
$$MSE = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x,y) - \hat{f}(x,y) \right]^2$$

- If we considers the restored image to be "signal" and the difference between this image and the original to be noise, the SNR in spatial domain can be defined as

$$SNR = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x,y) - \hat{f}(x,y) \right]^2}$$

- If the square root of mean square error is used in the denominator, the measure root-mean-square-signal-to-noise ratio.

Image restoration in presence of noise using Wiener filter



a b c
d e f
g h i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Constrained Least Square Filter

- Wiener filter depends on correct estimation of K. However constrained least square filter does not make any assumption about original undegraded image.
- It makes the use of only noise probability distribution function
- We can write

$$g = Hf + \eta$$

- H is very sensitive to noise. So our optimality criteria will be image smoothness.
- The second derivative/ Laplacian operator tries to enhance the irregularity or discontinuity in the image.

Constrained Least Square Filter

- If we can minimize the laplacian of reconstructed image that will ensure that the reconstructed image will be smooth.
- So the optimality criteria is

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y)\right]^2$$

- we need to minimize C subject to

$$||g - H\hat{f}||^2 = ||\eta||^2$$

Constrained Least Square Filter

- Solution can be given as in frequency domain as

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right] G(u,v)$$

where $P(u,v) \rightarrow$ Fourier transform of mask

$$P(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

- Pad appropriate number of zeros to make $M \times N$

Constrained Least Square Filter

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

Thank You

Queries?