Introduction to Digital Image Processing

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Frequency Domain Filtering

Fourier Series and Fourier Transform

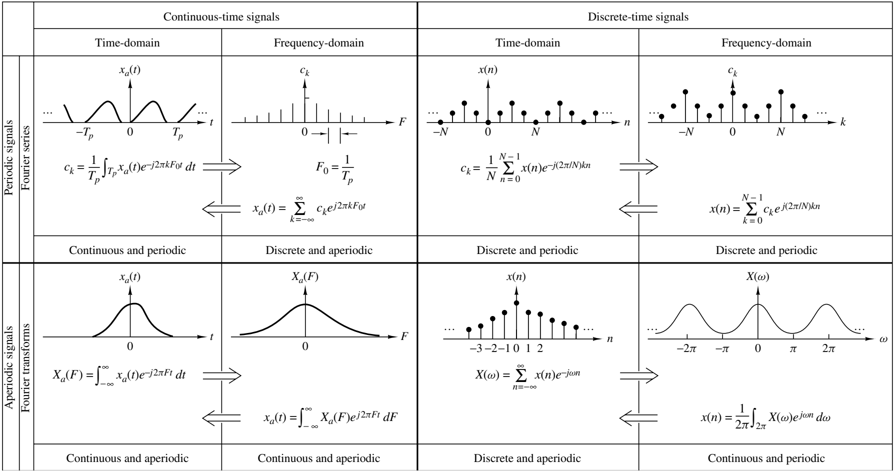


Figure 3.1 Summary of analysis and synthesis formulas.

DFT pair for one variable

- The discrete Fourier transform (DFT) is defined as

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi \frac{ux}{M}}$$

where $u = 0, 1, \ldots, M-1$

- Inverse DFT

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi \frac{ux}{M}}$$

where $x = 0, 1, \ldots, M-1$

- Both the forward and inverse discrete transform are infinitely periodic with period M.

2-D Discrete Fourier Transform

- The 2-D discrete Fourier transform (DFT) is defined as

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

  where $f(x,y)$ is a digital image of size $M\times N$, and $u=0,1,\ldots,M-1$ and $v=0,1,2,\ldots,N-1$.

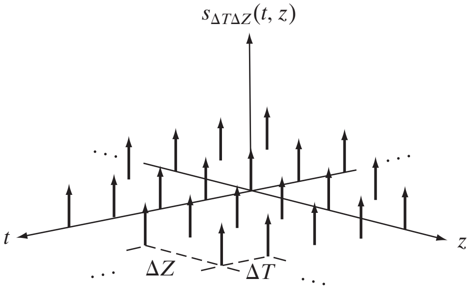
- Inverse DFT

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

  for $x=0,1,2,\ldots,M-1$ and $y=0,1,2,\ldots,N-1$.

2-D Discrete Fourier Transform Properties

- Relationship between Spatial and Frequency intervals



$$\Delta u = \frac{1}{M\Delta T}$$

$$\Delta v = \frac{1}{N\Delta Z}$$

2-D Discrete Fourier Transform Properties

- Translation and Rotation

$$f(x, y)\, e^{j2\pi(u_0 x/M + v_0 y/N)} \Leftrightarrow F(u - u_0, v - v_0)$$

and

$$f(x - x_0, y - y_0) \Leftrightarrow F(u, v)\, e^{-j2\pi(x_0 u/M + y_0 v/N)}$$

- Periodicity

$$F(u, v) = F(u + k_1 M, v) = F(u, v + k_2 N) = F(u + k_1 M, v + k_2 N)$$

and

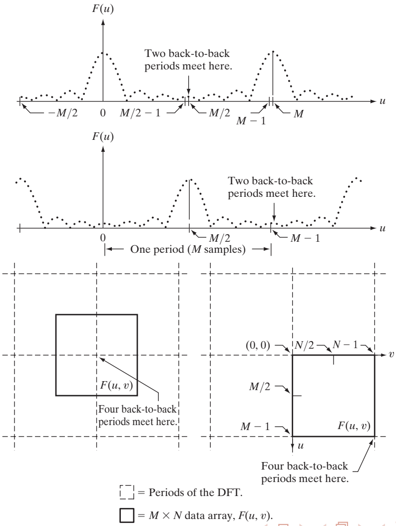
$$f(x, y) = f(x + k_1 M, y) = f(x, y + k_2 N) = f(x + k_1 M, y + k_2 N)$$

where k_1 and k_2 are integers.

2-D Discrete Fourier Transform Visualization

a
b
c d

FIGURE 4.23 Centering the Fourier transform. (a) A 1-D DFT showing an infinite number of periods. (b) Shifted DFT obtained by multiplying $f(x)$ by $(-1)^x$ before computing $F(u)$. (c) A 2-D DFT showing an infinite number of periods. The solid area is the $M \times N$ data array, $F(u, v)$, obtained with Eq. (4.5-15). This array consists of four quarter periods. (d) A Shifted DFT obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ before computing $F(u, v)$. The data now contains one complete, centered period, as in (b).



Fourier Spectrum and Phase Angle

- Even if $f(x, y)$ is a real function; its transform is complex in general.
- Spectrum is the magnitude of $F(u, v)$, and display it as an image.
- Fourier spectrum of an image

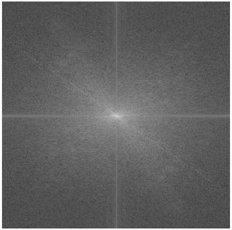
$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

- The phase angle of the transform is defined as

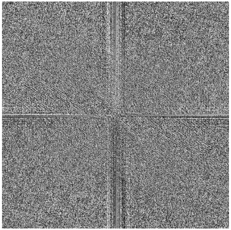
$$\phi(u,v) = \arctan\left[\frac{I(u,v)}{R(u,v)}\right]$$

Magnitude and Phase spectrum

(a) Input Image



(b) Magnitude Spectrum



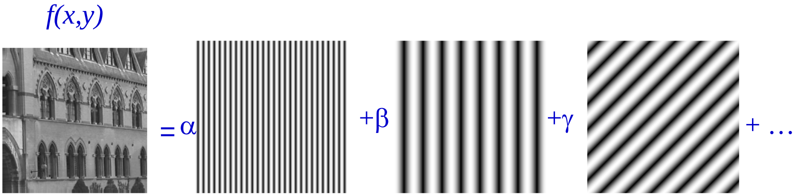
(c) Phase Spectrum

Fourier transforms and spatial frequencies

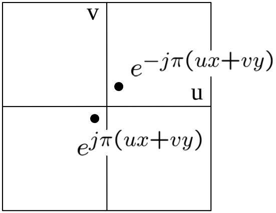
- The spatial function $f(x, y)$

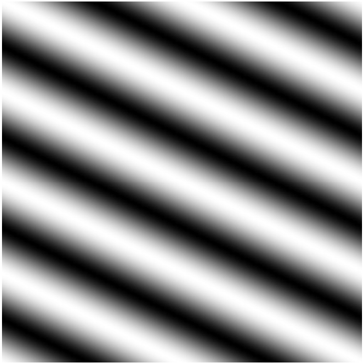
$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

is decomposed into a weighted sum of 2D orthogonal basis functions.

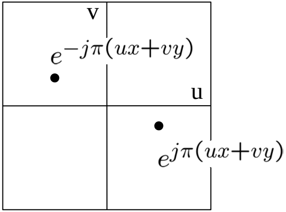


Fourier transforms and spatial frequencies

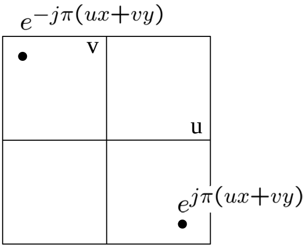


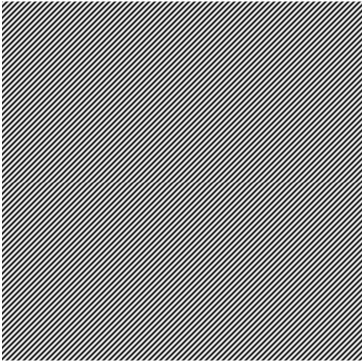


Fourier transforms and spatial frequencies



Fourier transforms and spatial frequencies





Separability of the 2-D DFT

The 2-D DFT is separable into 1-D transforms:

$$F(u,v) = \sum_{x=0}^{M-1} e^{-j2\pi ux/M} \sum_{y=0}^{N-1} f(x,y)e^{-j2\pi vy/N}$$

$$= \sum_{x=0}^{M-1} F(x,v)e^{-j2\pi ux/M}$$

Where: $F(x,v) = \sum_{y=0}^{N-1} f(x,y)e^{-j2\pi vy/N}$

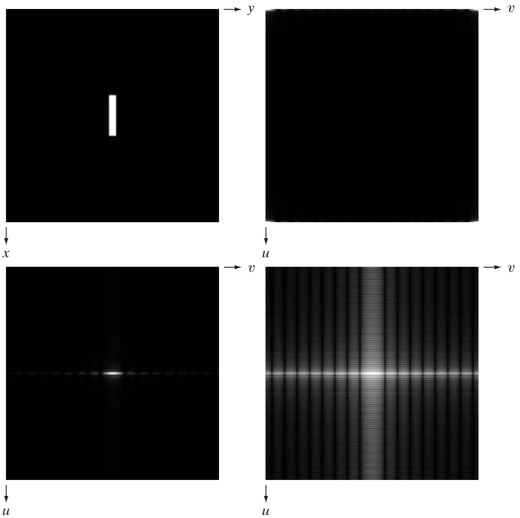
= 1-DFT of a row of f(x,y)

The 2-D DFT of f(x,y) can be obtained by computing the 1-D transform of each row of f(x,y) and then computig the 1-D transform along each column of the result

Spectrum

a b
c d

FIGURE 4.24
(a) Image.
(b) Spectrum showing bright spots in the four corners.
(c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.



Spectrum

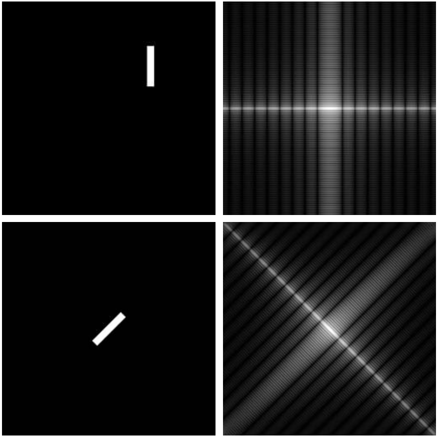


FIGURE 4.25 (a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

2-D Convolution Theorem

- 2-D convolution is defined as

$$f(x,y) * h(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)h(x-m, y-n)$$

- The 2-D convolution theorem is given by the expression as

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

and conversely,

$$f(x,y)h(x,y) \Leftrightarrow F(u,v) * H(u,v)$$

where F and H are the Fourier transform of f and h. and arrow represents the Fourier transform pair.

Filtering in Frequency Domain

- Filtering in the frequency domain consists of modifying the Fourier transform of an image and then computing the inverse transform to obtain the processed result.
- Thus, given a digital image, $f(x,y)$, of size $M \times N$, the basic filtering equation as

$$g(x,y) = F^{-1}\left[H(u,v)F(u,v)\right]$$

where F^{-1} is the IDFT, $F(u,v)$ is the DFT of the input image, $f(x,y)$. $H(u,v)$ is a filter function, and $g(x,y)$ is the filtered output image. Function F, H, and g are arrays of size $M \times N$, the same as the input image.

Step for filtering in the Frequency Domain

1. Given an input image $f(x,y)$ of size MxN, obtain padding parameters P and Q. Typically, $P=2M$ and $Q=2N$.
2. Form a padded image $f_p(x,y)$ of size PxQ by appending the necessary number of zeros to f(x,y).
3. Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to centre its transform.
4. Compute the DFT, $F(u,v)$, of the image from step 3.
5. Generate a real, symmetric filter function, $H(u,v)$, of size PxQ with centre at coordinates $(P/2,Q/2)$. Form the product $G(u,v)=H(u,v)F(u,v)$ using array multiplication.
6. Obtain the processed image: $g_p(x,y) = \text{real}\left[IDFT\left[G(u,v)\right]\right](-1)^{x+y}$
7. Obtain the final processed result, $g(x,y)$, by extracting the MxN region from the top, left quadrant of $g_p(x,y)$

Step for filtering in the Frequency Domain

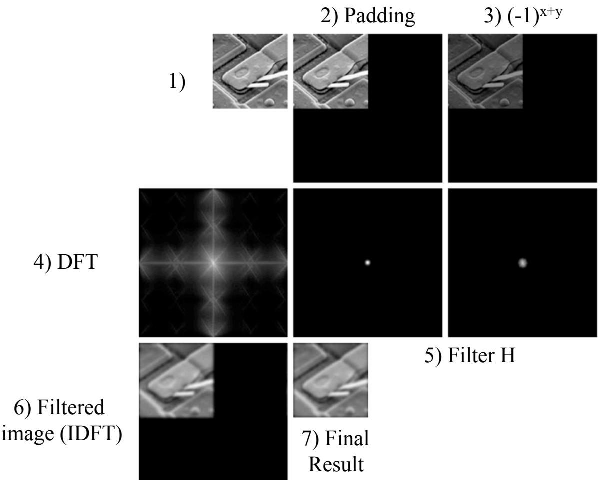


FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

Image Smoothening using Frequency Domain Filters

Smoothing using Frequency Domain Filters

- Edges and other sharp intensity transitions (such as noise) in an image contribute significantly to the high-frequency content of its Fourier transform.
- In this course, we will study these three types of lowpass filters:
  - Ideal
  - Butterworth, and
  - Gaussian.

Ideal Lowpass Filters

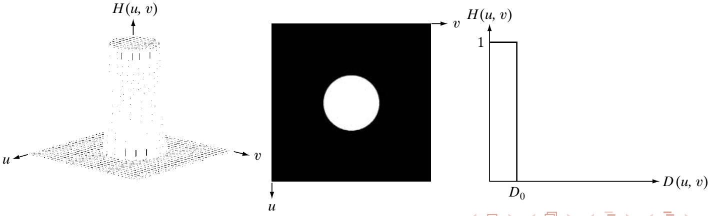
Ideal Lowpass Filter (ILPF) = 2-D lowpass filter that passes without attenuation all frequencies within a circle of radius D_0 from the origin and "cuts off" all frequencies outside this circle

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

Where: $D_0 \geq 0$

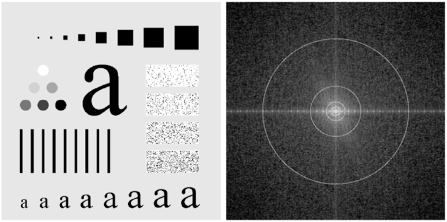
And *D(u,v)* is the distance between a point *(u,v)* and the centre of the frequency rectangle:

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$$



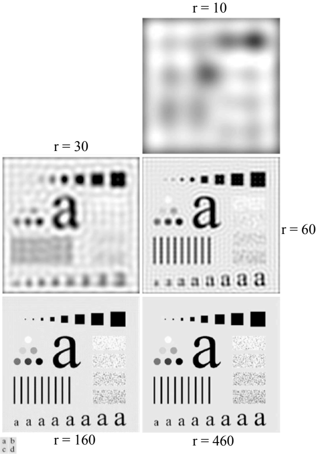
Ideal Lowpass Filters

Example:



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

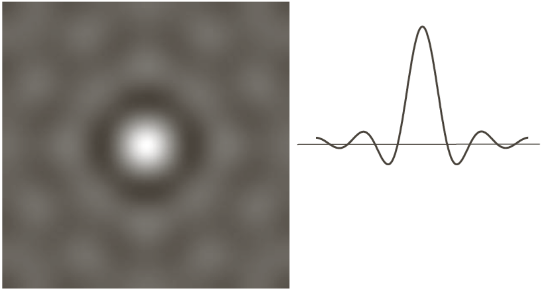


a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

Ideal Lowpass Filters

- The blurring and “ringing” effect using ILPFs



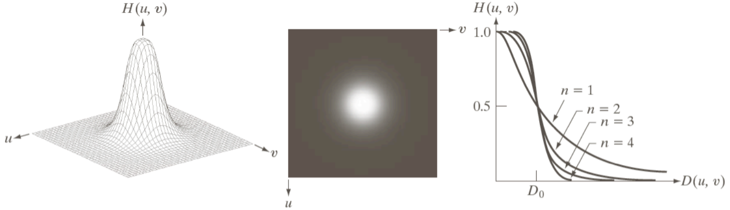
a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000. (b) Intensity profile of a horizontal line passing through the center of the image.

Butterworth Lowpass Filters

Transfer function of a Butterworth lowpass filter (BLPF) of order n and with cutoff frequency at a distance D_0 from the origin:

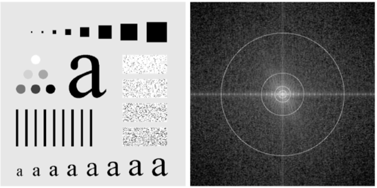
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



a b c

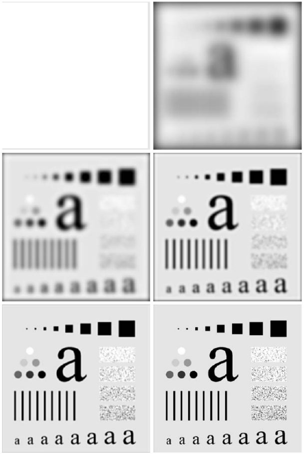
FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filters



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Butterworth Lowpass Filters

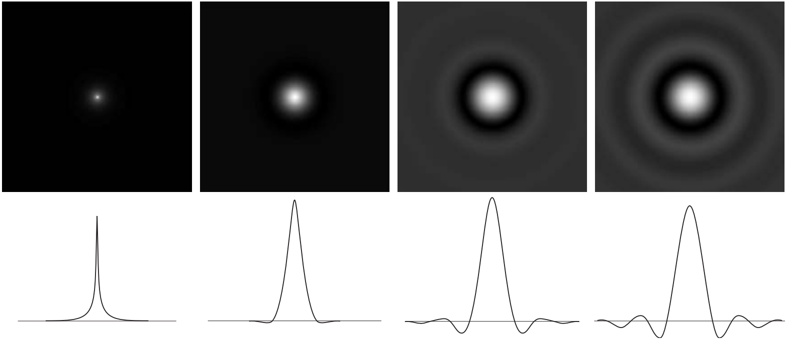


FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000×1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Gaussian Lowpass Filters in two-dimensions

Gaussian Lowpass Filters (GLPFs) in two-dimensions:

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

(σ = measure of spread about the centre)

$$\sigma = D_0 \;\Rightarrow\; H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

(D_0 = cutoff frequency)

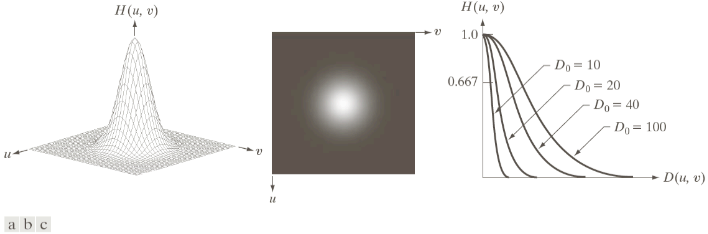


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Gaussian Lowpass Filters in two-dimensions

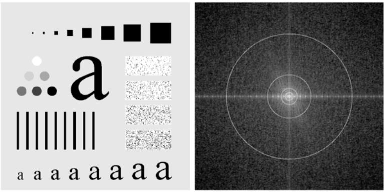


FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

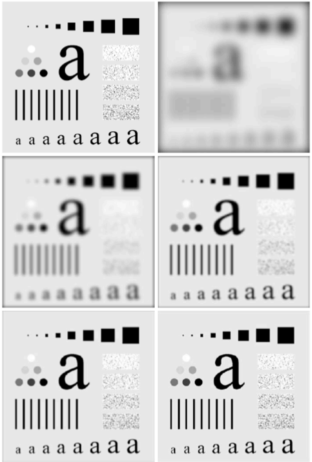


FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

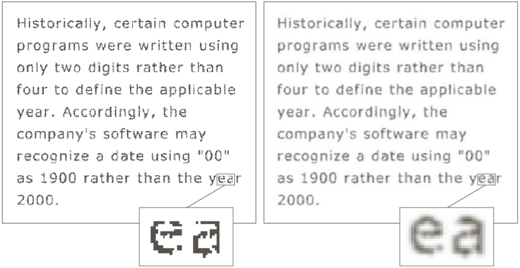
All lowpass filters

Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

Application in character recognition

Character recognition (machine perception):



a b

FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Blurring to fill “visual gaps” => help reading broken characters

Application in “cosmatic” processing

Printing and publishing industry: “cosmetic” processing

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Image Sharpening using Frequency Domain Filters

Ideal Highpass Filters

Highpass filtering: attenuation of the high-frequency components of the Fourier transform of the image

As before :

- Radially symmetric filters
- All filter functions are assumed to be discrete functions of size *P*x*Q*

A highpass H_{HP} filter can be obtained from a given lowpass H_{LP} filter by:

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D *Ideal Highpass Filter* (IHPF) is defined as:

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Ideal Highpass Filters

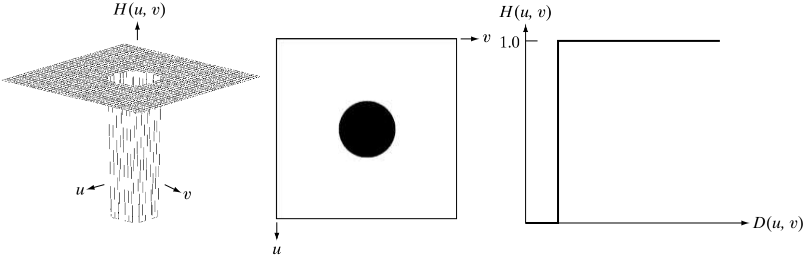


Figure: Perspective plot, image representation, and cross section of a typical ideal highpass filter

Ideal Highpass Filters

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30$, 60, and 160.

Butterworth Highpass Filters

A 2-D *Butterworth Highpass Filter* (BHPF) of order n and cutoff frequency D_0 is defined as:

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$

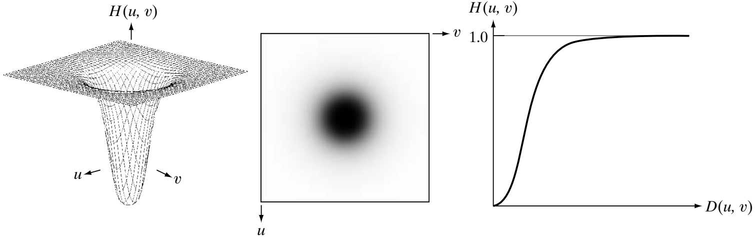


Figure: Perspective plot, image representation, and cross section of a typical Butterworth Highpass Filters

Butterworth Highpass Filters

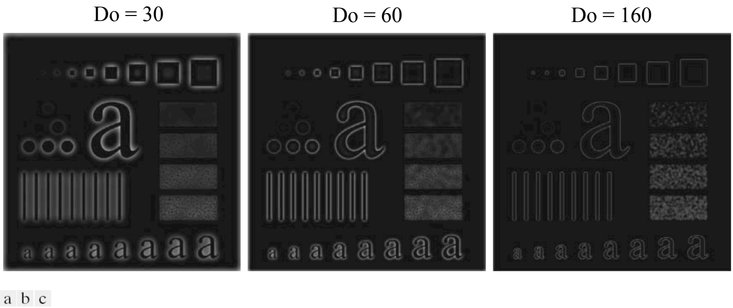


FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Gaussian Highpass Filters

The transfer function of the *Gaussian Highpass Filter* (GHPF) with cutoff frequency locus at a distance D_0 from the centre of the frequency rectangle is defined as:

$$H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

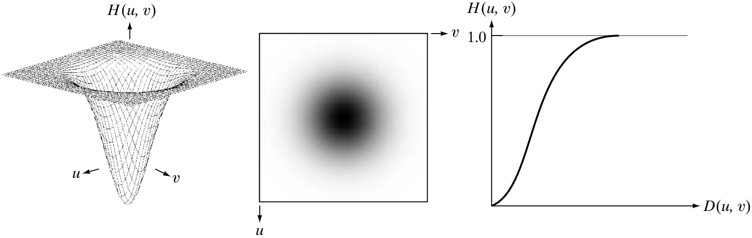
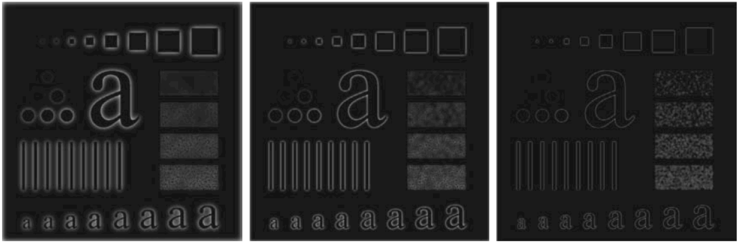


Figure: Perspective plot, image representation, and cross section of a typical Gaussian Highpass Filters

Gaussian Highpass Filters

Do = 30 Do = 60 Do = 160



a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

All highpass filters

Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

Application of highpass filters for image enhancement

Example: using highpass filtering and thresholding for image enhancement

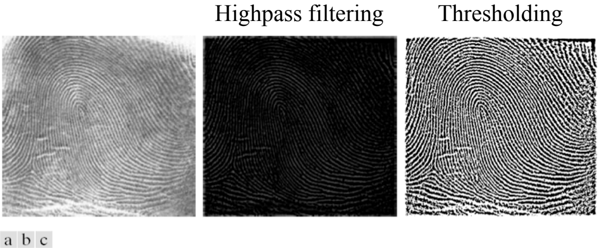


FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Laplacian in the Frequency Domain

- Laplacian Filter in frequency domain

$$H(u,v) = -4\pi^2(u^2+v^2)$$

- or, with respect to the center of the frequency rectangle, using the filter

$$\begin{aligned} H(u,v) &= -4\pi^2[(u-P/2)^2+(v-Q/2)^2] \\ &= -4\pi^2 D^2(u,v) \end{aligned}$$

- The Laplacian image is obtained by

$$\nabla^2 f(x,y) = \Im^{-1}[H(u,v)F(u,v)]$$

- Enhancement of the image achieved using

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y)$$

Here, $c=-1$ because $H(u,v)$ is negative.

Laplacian in the Frequency Domain

$$\nabla^2 f(x, y) = \Im^{-1}[H(u, v)F(u, v)]$$

- Introduce large scaling factors i.e., $f(x, y)$ and $\nabla^2 f(x, y)$ have no comparable value.
- Practical Solution: normalize $f(x, y)$ to the range $[0, 1]$ before computing the DFT, and divide $\nabla^2 f(x, y)$ by its maximum value.

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Laplacian in the Frequency Domain

- Enhancement of the image achieved using

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y)$$

Here, $c = -1$ because $H(u,v)$ is negative.

- Previous expression entirely in terms of frequency domain as

$$\begin{aligned} g(x,y) &= \Im^{-1}\Big\{F(u,v) - H(u,v)F(u,v)\Big\} \\ &= \Im^{-1}\Big\{\Big[1 - H(u,v)\Big]F(u,v)\Big\} \\ &= \Im^{-1}\Big\{\Big[1 + 4\pi^2 D^2(u,v)\Big]F(u,v)\Big\} \end{aligned}$$

Unsharp Masking and Highboost Filtering

- Frequency domain formulations of the unsharp masking

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

with $f_{LP}(x,y) = \Im^{-1}[H_{LP}(u,v)F(u,v)]$, where $H_{LP}(u,v)$ is a lowpass filter and $F(u,v)$ is the Fourier transform of $f(x,y)$.

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

This expression defines unsharp masking when $k = 1$ and highboost filtering when $k > 1$

Unsharp Masking and Highboost Filtering

- We can express the previous expression entirely in terms of frequency domain as

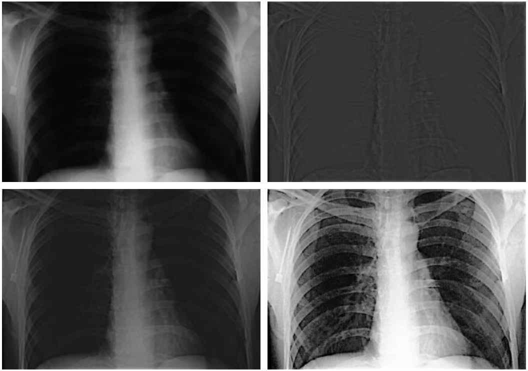
$$g(x,y) = \Im^{-1}\{[1 + k * [1 - H_{LP}(u,v)]]\, F(u,v)\}$$
$$g(x,y) = \Im^{-1}\{[1 + k * H_{HP}(u,v)]F(u,v)\}$$

- More general for called *high-frequency-emphasis filtering*

$$g(x,y) = \Im^{-1}\{[k_1 + k_2 * H_{HP}(u,v)]F(u,v)\}$$

where $k_1 \geq 0$ gives controls of the offset from the origin and $k_2 \geq$ controls the contribution of high frequencies.

Unsharp Masking and Highboost Filtering



a b
c d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Homomorphic Filtering

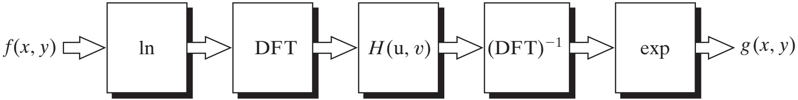
- The illumination-reflectance model

$$f(x, y) = i(x, y)r(x, y)$$

can be used to develop a frequency domain procedure for improving the appearance of an image.

- We cannot write

$$\Im[f(x, y)] \neq \Im[i(x, y)]\Im[r(x, u)]$$



Homomorphic Filtering

However, suppose that we define

$$\begin{aligned} z(x, y) &= \ln f(x, y) \\ &= \ln i(x, y) + \ln r(x, y) \end{aligned}$$

Then,

$$\begin{aligned} \Im\Big\{z(x, y)\Big\} &= \Im\Big\{\ln f(x, y)\Big\} \\ &= \Im\Big\{\ln i(x, y)\Big\} + \Im\Big\{\ln r(x, y)\Big\} \end{aligned}$$

or

$$Z(u, v) = F_i(u, v) + F_r(u, v)$$

Homomorphic Filtering

We can filter $Z(u, v)$ using a filter $H(u, v)$ so that

$$\begin{aligned} S(u, v) &= H(u, v)Z(u, v) \\ &= H(u, v)F_i(u, v) + H(u, v)F_r(u, v) \end{aligned}$$

The filtered image in the spatial domain is

$$\begin{aligned} s(x, y) &= \Im^{-1}\left\{S(u, v)\right\} \\ &= \Im^{-1}\left\{H(u, v)F_i(u, v)\right\} + \Im^{-1}\left\{H(u, v)F_r(u, v)\right\} \end{aligned}$$

Homomorphic Filtering

By defining

$$i'(x, y) = \Im^{-1}\left\{H(u, v)F_i(u, v)\right\}$$

and

$$r'(x, y) = \Im^{-1}\left\{H(u, v)F_r(u, v)\right\}$$

we can express Eq. (4.9-23) in the form

$$s(x, y) = i'(x, y) + r'(x, y)$$

Homomorphic Filtering

Finally, because $z(x, y)$ was formed by taking the natural logarithm of the input image, we reverse the process by taking the exponential of the filtered result to form the output image:

$$\begin{aligned} g(x, y) &= e^{s(x,y)} \\ &= e^{i'(x,y)} e^{r'(x,y)} \\ &= i_0(x, y)\, r_0(x, y) \end{aligned}$$

where

$$i_0(x, y) = e^{i'(x,y)}$$

and

$$r_0(x, y) = e^{r'(x,y)}$$

are the illumination and reflectance components of the output (processed) image.

Homomorphic Filtering

$$H(u, v) = (\gamma_H - \gamma_L)\left[1 - e^{-c[D^2(u, v)/D_0^2]}\right] + \gamma_L$$

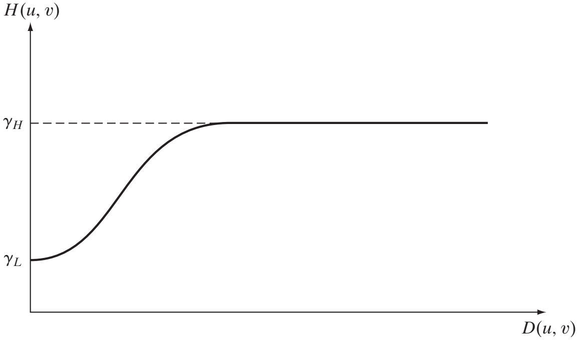
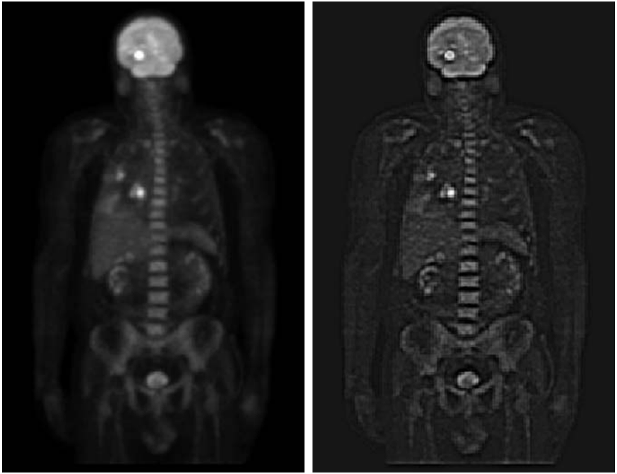


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

Homomorphic Filtering

- The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly, particularly at the junctions of dissimilar objects.
- These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination and the high frequencies with reflectance.
- Although these associations are rough approximations, they can be used to advantage in image filtering.

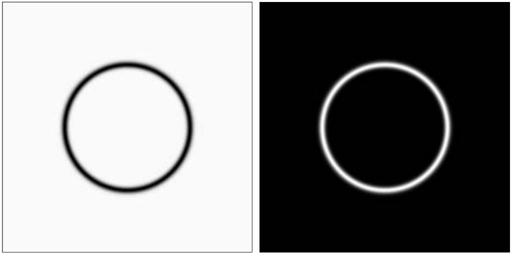
Homomorphic Filtering



a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

Bandreject and Bandpass Filters



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Thank You

Queries?