Introduction to Digital Image Processing

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Morphological Image Processing

Introduction

- The word *morphology* commonly denotes a branch of biology that deals with the form and structure of animals and plants.
- Mathematical morphology is a tool for extracting image components that are useful in the representation and description of region shape, such as:
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

Introduction

- Sets in mathematical morphology represent objects in an image:
  - binary image (0 = white, 1= black)
    - The elements of the set is the coordinates (x, y) of pixel belong to the object Z^2 (two dimensional integer space)
  - Gray-scaled image
    - The element of the set is the coordinate (x, y) of pixel belong to the object and the gray levels $\in Z^3$

Preliminary

- The language of mathematical morphology is set theory.
- B: the set of pixel representing an object in an image.
- *Reflection*: analogous to the flipping (rotating) operation
  - The reflection of set B, denoted $\hat{B}$, denoted as

$$\hat{B} = \{w | w = -b, \text{ for } b \in B\}$$

- *Translation*
  - The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_z$, is defined as

$$(B)_z = \{c | c = b + z, \text{ for } b \in B\}$$

- *Structuring element*: small sets or subimages used to probe an image

Reflection and Translation by examples

- with respect to reference point (also called origin)

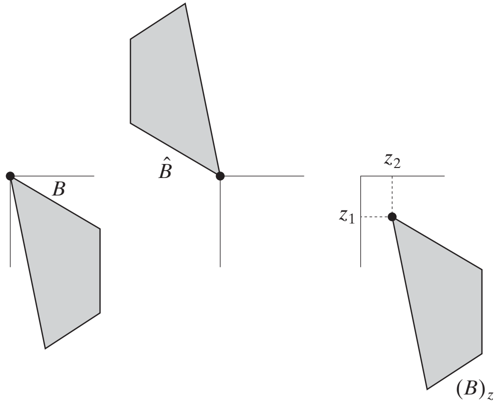
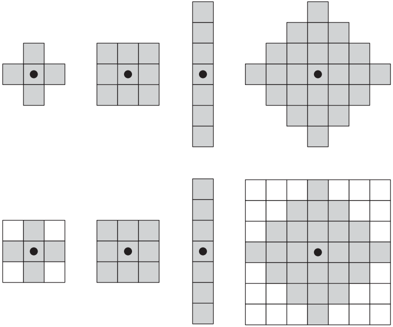


Figure: (a) A set, (b) its reflection, and (c) its translation by z.

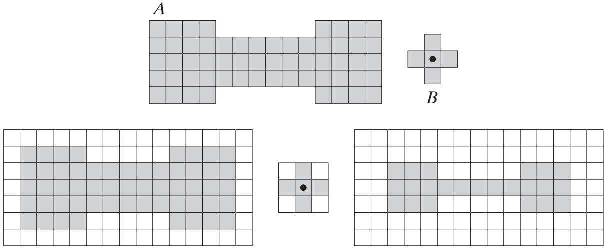
Structuring Elements

- Used to structure the objects:
  - 1 (or black): action
  - 0 (or white): No action
  - $\times$: don't care



Use of structuring element in morphology

- A: set of object elements
- A is to be converted into a rectangular array by adding background elements.
- The background border is made large enough to accommodate the entire structuring element when its origin is on the border of the set A.
- Processing



Morphological Operators

- Two fundamental Morphological Operators
  - Erosion
  - Dilation
- Morphological algorithms are based on these two primitive operations.

Erosion

- Analogous to *shrink*, *reduce*
- Suppose $A, B \in Z^2$, then erosion of A by B, denoted as $A \ominus B$, is defined as

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

The erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

- We can also define erosion as

$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$

where A^c is the complement of A and $\emptyset$ is the empty set.

Erosion: Example

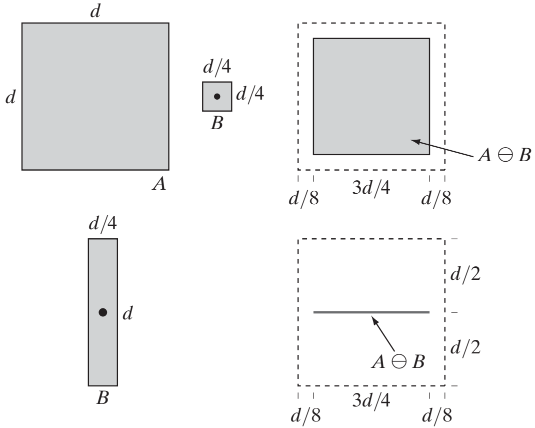


Figure: (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element.

Erosion: Example

- The structuring element B is a 3×3 matrix (Full) origin at $(1,1)$

1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\Rightarrow$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Erosion: Application

- Erosion applied on white area

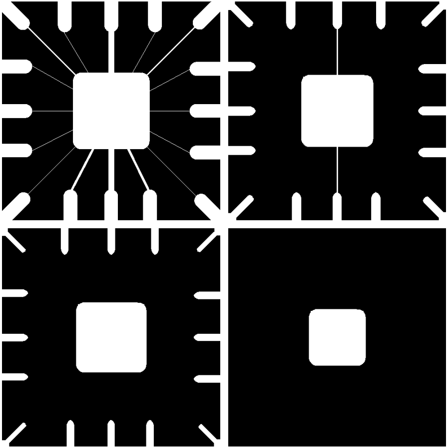


FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

- Analogous to *grow*, *expanding*
- Suppose $A, B \in Z^2$, then dilation of A by B, denoted as $A \oplus B$, is defined as

$$A \oplus B = \left\{ z | (\hat{B})_z \cap A \neq \emptyset \right\}$$

- The dilation of A by B is the set of all displacement z, such that $\hat{B}$ and A overlap by at least one element.
- Equivalently written as

$$A \oplus B = \left\{ z | \left[(\hat{B})_z \cap A \right] \subseteq A \right\}$$

Dilation: Example

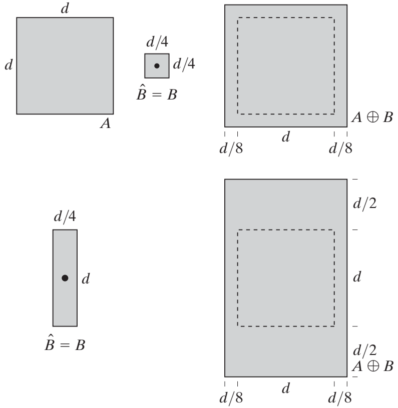


Figure: (a) Set A. (b) Square structuring element, B. (dot denotes the origin) (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

Dilation: Example

- The structuring element B is a 3×3 matrix (Full) origin at $(1, 1)$

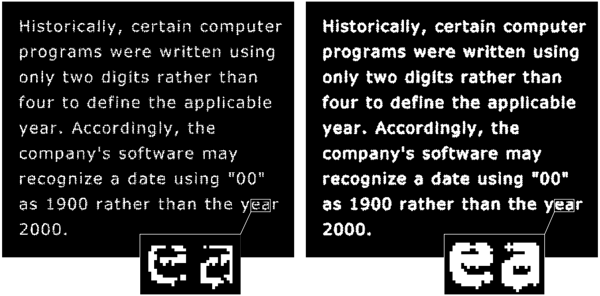
1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1		0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0

Dilation: Application

- Gap filling



0	1	0
1	1	1
0	1	0

Figure: (a) Sample text of poor resolution with broken characters. (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Dilation-Erosion Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection.

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

and

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

- Can you prove these duality relation?

Dilation-Erosion Duality

- We know that

$$(A \ominus B) = \{z | (B)_z \subseteq A\}$$
$$\Rightarrow (A \ominus B)^c = \{z | (B)_z \subseteq A\}^c$$

- If set $(B)_z$ is contained in A, then $(B)_z \cap A^c = \emptyset$, so we can write

$$(A \ominus B)^c = \{z | (B)_z \cap A^c = \emptyset\}^c$$

- But the complement of the set of z's that satisfy $(B)_z \cap A^c = \emptyset$ is the set of z's such that $(B)_z \cap A^c \neq \emptyset$, therefore

$$(A \ominus B)^c = \{z | (B)_z \cap A^c \neq \emptyset\}$$
$$= A^c \oplus \hat{B}$$

Opening and Closing

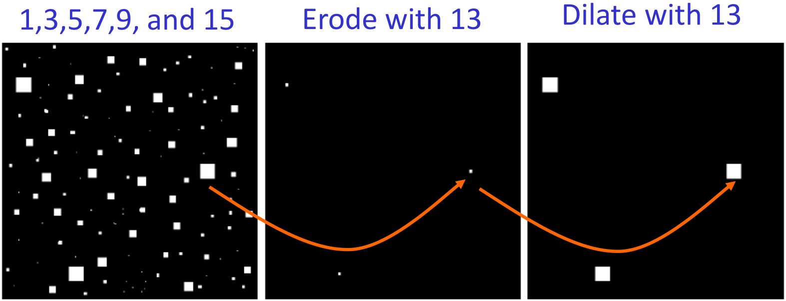


Figure: Removing smaller details

Opening and Closing

- Dilation expands and Erosion shrinks.
- **Opening**: erosion followed by a dilation
  - Smoothes contour of an object
  - Break narrow isthmuses (passages)
  - Eliminates thin protrusions (something that sticks out from a surface).
- **Closing**: dilation followed by a erosion
  - Smoothes contour
  - Fuse narrow breaks, and long thin gulfs.
  - Eliminates small holes, and fill gaps.

Opening

- Dilation expands and Erosion shrinks.
- Opening:
  - A erosion followed by a dilation using the *same structuring element* for both operations

$$A \circ B = (A \ominus B) \oplus B = \bigcup \{(B_z) | (B_z) \subseteq A\}$$

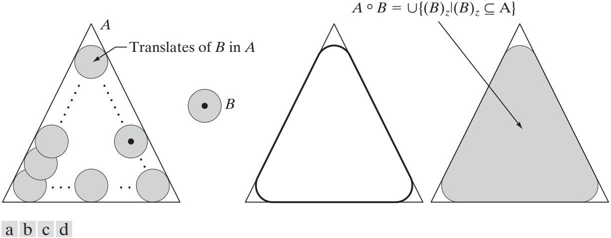


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Opening: Example

The structuring element B is a 3×3 matrix (Full) origin at $(1, 1)$

1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	1	0	0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\longrightarrow$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Closing

- Dilation expands and Erosion shrinks.
- Closing
  - A dilation followed by a erosion using the *same structuring element* for both operations.

$$A \bullet B = (A \oplus B) \ominus B$$

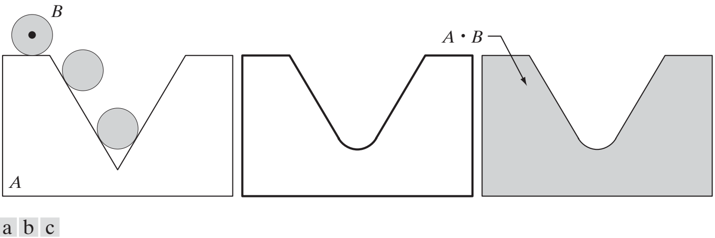
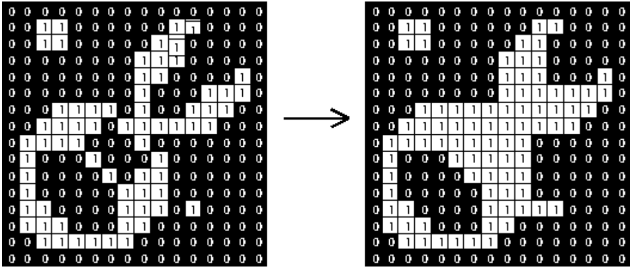


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Closing: Example

The structuring element B is a 3×3 matrix (Full) origin at $(1, 1)$

1	1	1
1	1	1
1	1	1



Opening and Closing: Example

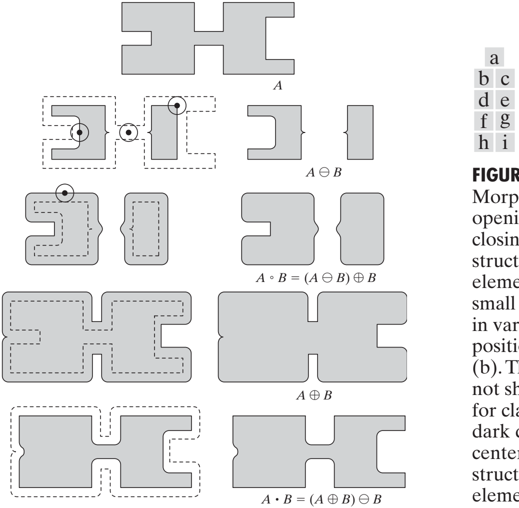


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Opening and Closing Duality

- Opening and closing are duals of each other with respect to set complementation and reflection.

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

and

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

- Can you prove the dual relationship between opening and closing?

Opening and Closing Duality

Starting with the definition of closing,

$$\begin{aligned}(A \bullet B)^c &= [(A \oplus B) \ominus B]^c \\ &= (A \oplus B)^c \oplus \hat{B} \\ &= (A^c \ominus \hat{B}) \oplus \hat{B} \\ &= A^c \circ \hat{B}.\end{aligned}$$

The proof of the other duality property follows a similar approach.

Opening and Closing Properties

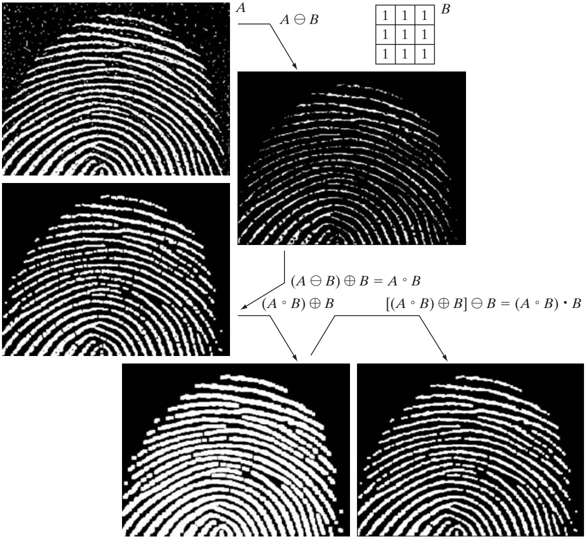
Opening Properties

- $A \circ B$ is a subset (subimage) of A
- If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
- $(A \circ B) \circ B = A \circ B \Leftrightarrow$ *Multiple apply has no effect*

Closing Properties

- A is a subset (subimage) of $A \bullet B$
- If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
- $(A \bullet B) \bullet B = A \bullet B \Leftrightarrow$ *Multiple apply has no effect*

Application in Noise Removal



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Hit-or-Miss Transformation

- The morphological hit-or-miss transform is a basic tool for shape detection.

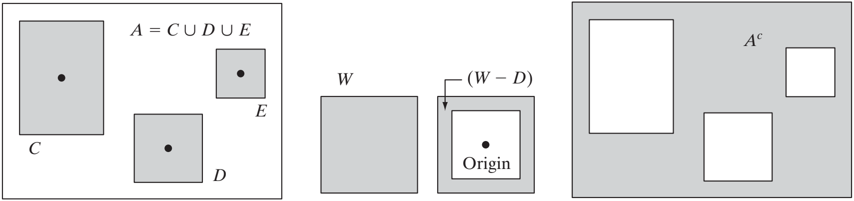


Figure: (a) Set A, (b) A window, W, and the local background of D with respect to W, $(W - D)$, and (c) Complement of A. Dots indicate the origins of C, D, and E

Hit-or-Miss Transformation

- The morphological *hit-or-miss transform* is a basic tool for shape detection.

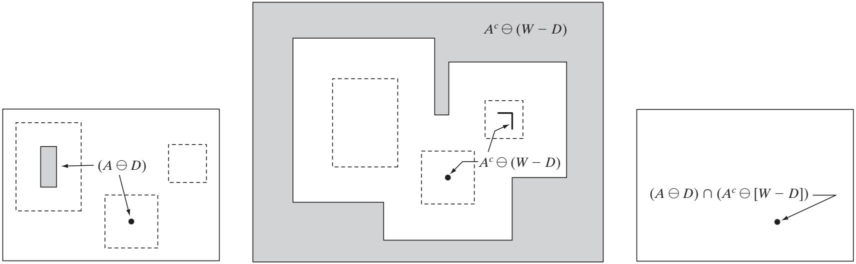


Figure: (d) Erosion of A by D, (e) Erosion of A^c by $(W - D)$, (f) intersection of (d) and (e), showing the location of the origin of D, as desired.

Hit-or-Miss Transform

- If B denotes the set composed of D and *its background*, the match (or set of matches) of B in A, denoted as $A \circledast B$, is

$$A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

- More generalized by letting $B = (B_1, B_2)$, where B_1 is the set formed from elements of B associated with an object and B_2 corresponding to background. So $B_1 = D$ and $B_2 = (W - D)$.

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

- Using erosion-dilation duality property

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

- All three expressions refer as the morphological *hit-or-miss transform*.

Hit-or-miss transformation: Application

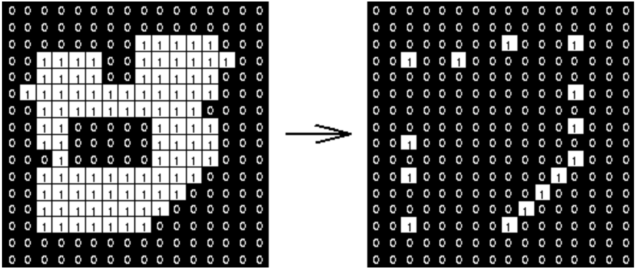
- Corner detection by assuming following structuring elements

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	



Morphological Operator Applications

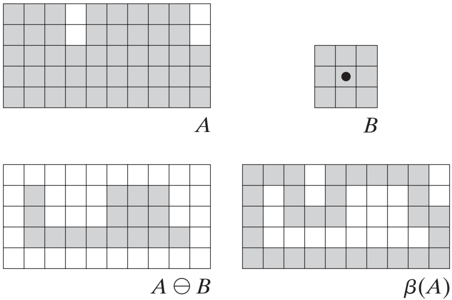
- Boundary Extraction
- Hole/Region Filling
- Connected Component Extraction
- Convex Hull
- Thinning
- Thickening
- Skeletonization
- Pruning

Boundary Extraction

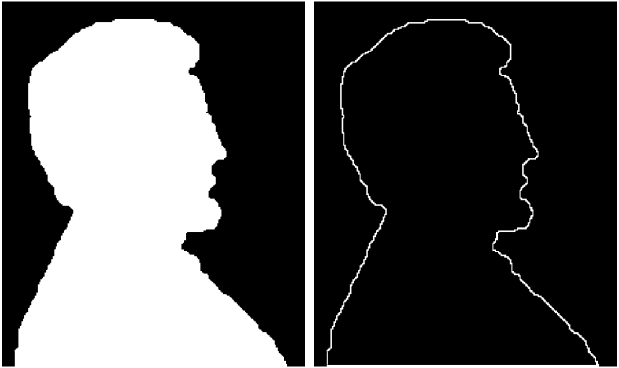
- The boundary of a set A, denoted by $\beta(A)$, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

where B is a suitable structuring element.



Boundary Extraction: Example

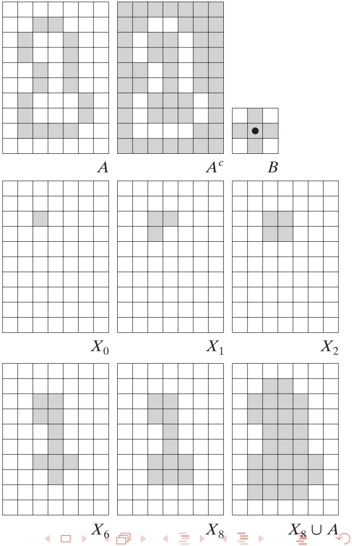


a b

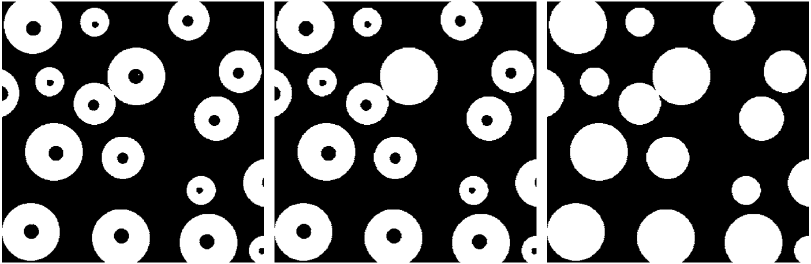
FIGURE 9.14
(a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Hole/Region Filling

- A hole may be defined as a background region surrounded by a connected border of foreground pixels.
- *Algorithm:* Given 8-connected boundary (A) and a point p in each hole.
  1. Assign X_0 as a zero array (the same size as A) except at the location corresponding to given point p.
  2. Compute $X_k = (X_{k-1} \oplus B) \cap A^c$ for $k = 1, 2, 3, \ldots$
  3. Terminate at iteration step k if $X_k = X_{k-1}$



Hole/Region Filling: Example

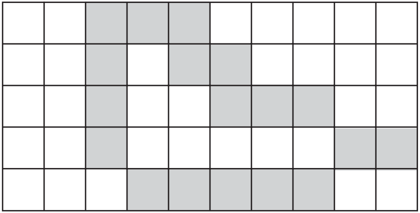


a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

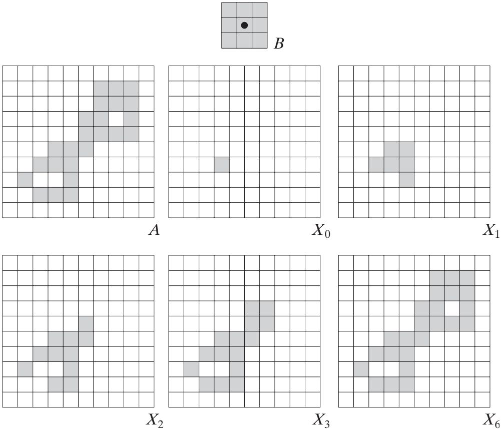
Hole/Region Filling: Problem

Question: Apply the hole/region filling algorithm using morphological operators on the given binary image and obtained the final regional filled object in the image. Show at least two intermediate steps.



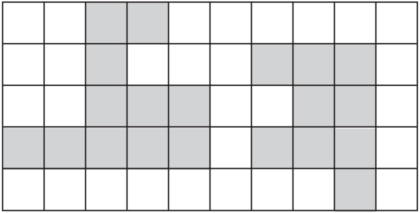
Extraction of Connected Components

- *Algorithm:* Let A be a set of containing one or more connected components
  1. Assign X_0 as a zero array (the same size as A) except at the location corresponding to a point in each connected component in A.
  3. Compute $X_k = (X_{k-1} \oplus B) \cap A$ for $k = 1, 2, 3, \ldots$ where B is a suitable structuring element.
  4. Terminate at iteration step k if $X_k = X_{k-1}$



Extraction of Connection Components: Problem

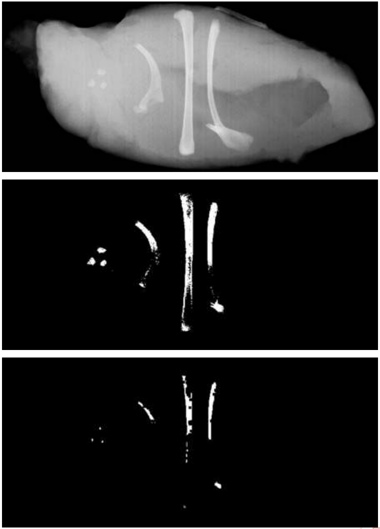
Question: Compute the connected components in the given binary image using morphological operations. Show at least two intermediate steps.



Extraction of Connected Components: Example

a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s.
(d) Number of pixels in the connected components of (c).
(Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



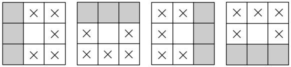
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

- A set A is said to be *convex* if the straight line segment joining any two points in A lies entirely within A.
- The *convex hull* H of an arbitrary set S is the smallest convex set containing S.
- The set difference $H - S$ is called the *convex deficiency* of S.
- Convex Hull and convex deficiency are useful for object description.

Algorithm:

1. Let B^i, $i = 1, 2, 3, 4$ represent the four structuring elements.



B^1 B^2 B^3 B^4

Convex Hull

2. Compute

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and } k = 1, 2, 3, \ldots$$

with $X_0^i = A$.

3. When the procedure converges (i.e., when $X_k^i = X_{k-1}^i$)

$$C(A) = \bigcup_{i=1}^{4} D^i \text{ where } D^i = X_k^i$$

The method is to iteratively apply the hit-or-miss transform to A with B^1; when no further changes occur, perform the union with A and call the result D^1. The procedure is repeated with B^2 until no further changes occur, and so on. The union of the four resulting Ds constitutes the convex hull of A.

Convex Hull

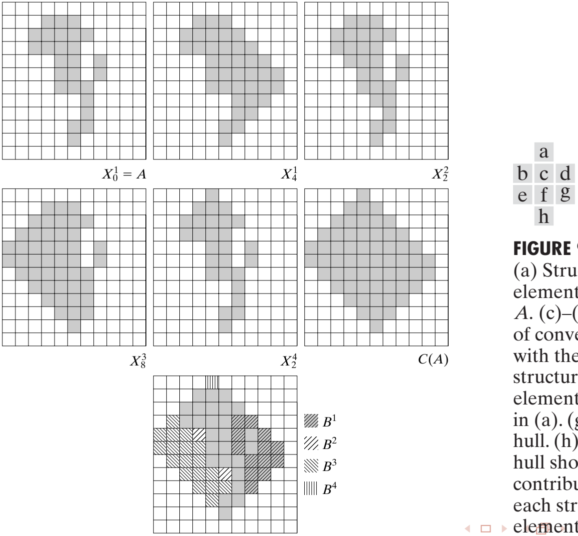


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Thinning

- The thinning of a set A by a structuring element B, denoted $A \otimes B$ can be defined as

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

Another approach:

- Thinning A symmetrically based on sequence of structuring elements

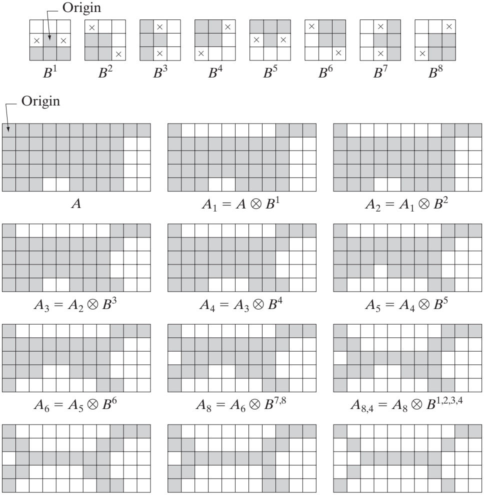
$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$

where B^i is a rotated version of B^{i-1}

- There for thinning can be defined as

$$A \otimes \{B\} = ((\dots((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$$

Thinning



A | $A_1 = A \otimes B^1$ | $A_2 = A_1 \otimes B^2$

$A_3 = A_2 \otimes B^3$ | $A_4 = A_3 \otimes B^4$ | $A_5 = A_4 \otimes B^5$

$A_6 = A_5 \otimes B^6$ | $A_8 = A_6 \otimes B^{7,8}$ | $A_{8,4} = A_8 \otimes B^{1,2,3,4}$

$A_{8,5} = A_{8,4} \otimes B^5$ | $A_{8,6} = A_{8,5} \otimes B^6$ No more changes after this. | $A_{8,6}$ converted to m-connectivity.

Thickening

- Thickening is the morphological dual of thinning and is defined by the expression

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2) \ldots) \odot B^n)$$

- Structural elements are as before but 1s and 0s interchanged.
- Usually thin the background, then complement the results.

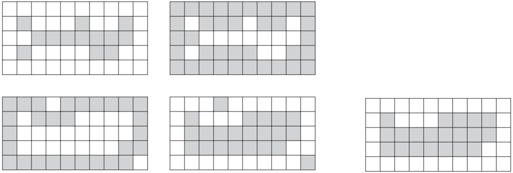
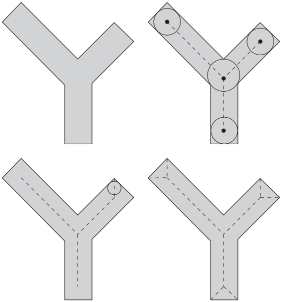


Figure: (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

- Skeleton of set A, denoted as $S(A)$, can be computed as
  - For z belong to $S(A)$, and $(D)_z$ is the largest disk centered at z and contained in A, one can not find a larger disk containing $(D)_z$ and included in A.
  - The disk $(D)_z$ touches the boundary of A at two or more different places.



a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

Skeletons: Formulation

- The skeleton of A can be expressed in terms of erosions and opening. That is

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$.

- Here, B is a structuring element, and $(A \ominus kB)$ indicates k successive erosions of A

$$(A \ominus kB) = ((\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B)$$

and K is the last iterative step before A erodes to an empty set.

$$K = \max\{k | (A \ominus kB) \neq \phi\}$$

Skeletons: Reconstruction

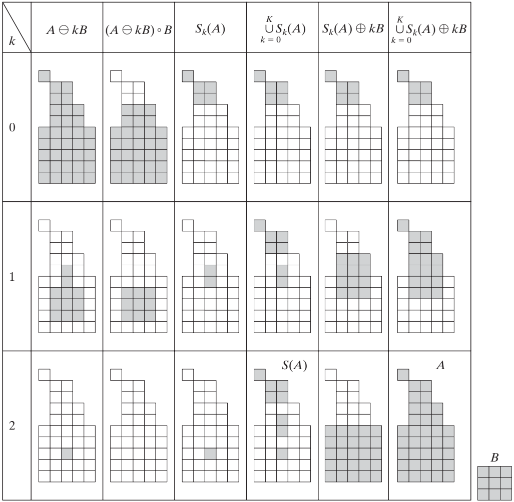
- The formulation states that $S(A)$ can be obtained as the union of the skeleton subset $S_k(A)$.
- A can be reconstructed as

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $(S_k(A) \oplus kB)$ denotes k successive dilation of $S_k(A)$ as

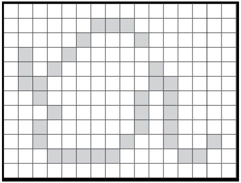
$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B)$$

Skeletons: Example



Pruning

- Pruning methods are an essential complement to thinning and skeletonizing algorithms because these procedures tend to leave *parasitic components* that need to be "*cleaned up*" by post-processing.



Algorithm:

1. Apply thinning on input set A with a sequence of structuring element designed to detect only end points

$$X_1 = A \otimes \{B\}$$

Pruning

2. Form a set X_2 containing all end points in X_1

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

where the B^k end-point detectors.

3. Dilate the end points three times, using set A as a delimiter:

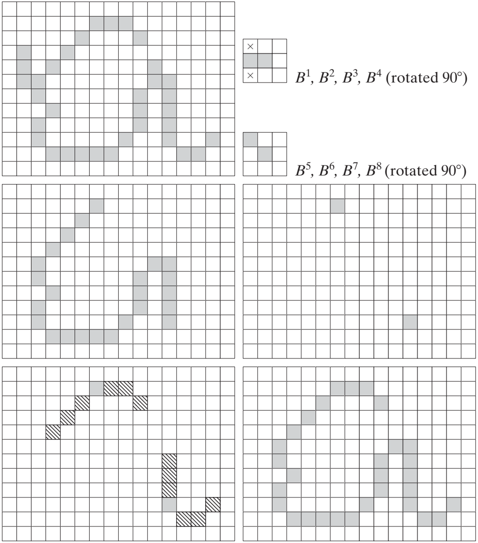
$$X_3 = (X_2 \oplus H) \cap A$$

where H is a 3×3 structuring element of 1s and the intersection with A is applied after each step.

4. Finally

$$X_4 = X_1 \cup X_3$$

Pruning: Example



a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Thank You

Queries?