Introduction to Digital Image Processing

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Histogram Processing

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- The histogram of a digital image with intensity levels in the range $[0, L-1]$ is a discrete function

$$h(r_k) = n_k$$

where, r_k is the k^{th} intensity value and n_k is the number of pixels in the image with intensity r_k.
- A plot of $h(r_k)$ vs. k is called *histogram*.

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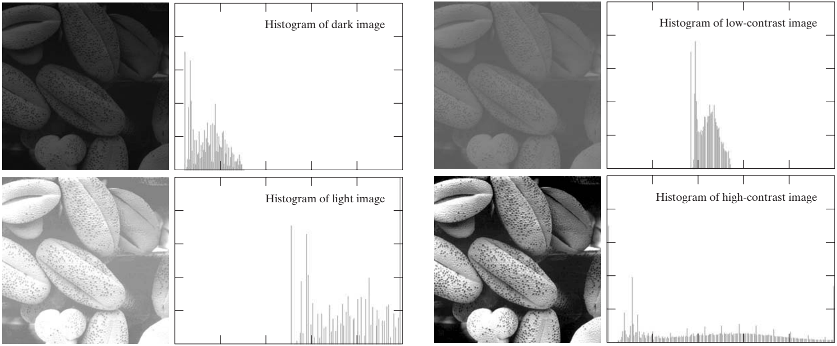
- *Normalized histogram*

$$p(r_k) = \frac{n_k}{MN} \qquad \text{for } k = 0, 1, 2, \ldots, L-1.$$

- $p(r_k)$ is an estimate of the *probability of occurrence of intensity level* r_k in an image.

Histogram Processing



Histogram Processing: Example

Compute the histogram of the given image

1	0	2	4	5	3	1
9	1	1	4	7	2	1
10	3	7	3	5	3	3
11	2	3	3	3	2	1
7	5	6	6	7	6	1
1	4	1	1	4	9	1
2	8	1	1	5	1	1

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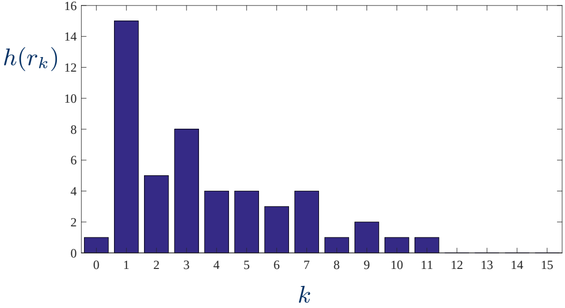
9	1	1	4	7	2	1
10	3	7	3	5	3	3
11	2	3	3	3	2	1
7	5	6	6	7	6	1
1	4	1	1	4	9	1
2	8	1	1	5	1	1



Histogram Processing

- Histograms give a *global description* and nothing specific about the image content.
- *Shape of the histogram* gives useful information about possibility for contrast enhancement.

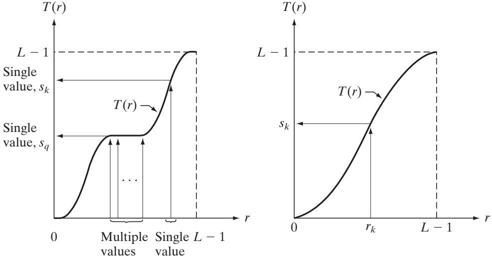
Histogram manipulation

- Histogram Equalization
- Histogram Matching (Specification)

Histogram Equalization

- As the low-contrast image's histogram is narrow and centered in the middle of the gray scale, if we distribute the histogram to a wider range the quality of the image will be improved.
- We can do it by adjusting the *probability density function* (PDF) of the original histogram of the image so that the probability spread equally.

Transformation Function, $T(r)$



$$s = T(r)$$

$T(r)$ satisfies

(a) $T(r)$ is single valued and monotonically increasingly in the interval of $0 \leq r \leq L-1$

(b) $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$

Transformation Function, $T(r)$

- Single-valued (one-to-one relationship) guarantees that the inverse transformation will exist.
- Monotonicity condition preserves the increasing order from black to white in the output image thus it won't cause a negative image.
- $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$ guarantees that the output gray levels will be in the same range as the input levels.
- The inverse transformation from s back to r is

$$r = T^{-1}(s); \qquad 0 \leq s \leq L-1$$

Random Variable

- If a random variable r is transformed by a monotonic transformation function $T(r)$ to produce a new random variable s.
- $p_r(r)$ and $p_s(s)$ are probability density function (PDF) of r and s respectively.
- Then, the PDF $p_s(s)$ can be obtained from knowledge of $T(r)$ and $p_r(r)$, as follows:

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

where the vertical bars signify the absolute value.

Applied to Image

- Let
  - $p_r(r)$ denote the PDF of random variable r
  - $p_s(s)$ denote the PDF of random variable s
- If $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies condition (a) then $p_s(s)$ can be obtained using a formula

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

- The PDF of the transformed variable s is determined by the gray-level PDF of the input image and by the chosen transformation function.

Transformation function

- A transformation function of a particular importance in image processing has the form

$$s = T(r) = (L-1)\int_0^r p_r(w)dw \tag{1}$$

  where integration term is a cumulative distribution function (CDF) of random variable r and w is a dummy variable.
- The above equation satisfies (a) and (b) both because the area under the function cannot decrease as r increases and $0 \le r \le L-1$ (i.e., $0 \le s \le L-1$).
- Note that $T(r)$ depends on $p_r(r)$

Finding $p_s(s)$

Leibniz's rule: The derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit. That is

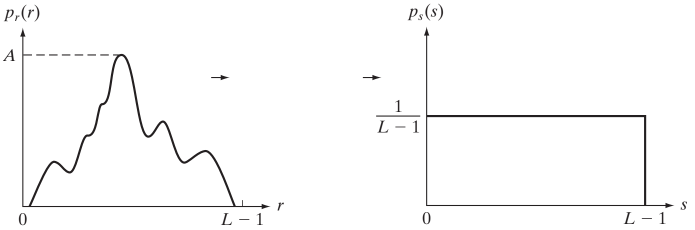
$$\begin{aligned}\frac{ds}{dr} &= \frac{dT(r)}{dr} \\ &= (L-1)\ \frac{d}{dr}\left[\int_0^r p_r(w)dw\right] \\ &= (L-1)\ p_r(r)\end{aligned}$$

Thus we can write

$$\begin{aligned}p_s(s) &= p_r(r)\left|\frac{dr}{ds}\right| \\ &= p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right| \\ &= \frac{1}{L-1} \quad \text{where } 0 \le s \le L-1\end{aligned}$$

Finding $p_s(s)$

- As $p_s(s)$ is a probability function, it must be zero outside the interval $[0, L-1]$.
- Its integral over all values of s must equal 1.
- Called $p_s(s)$ as a *uniform probability density function*.
- $p_s(s)$ is always a uniform, independent of the form of $p_r(r)$



Discrete transformation function

- For a discrete values, we deal with probabilities (histogram values) and summation instead of probability density function & integrals.
- The probability of occurrence of gray level in an image is approximated by

$$p_r(r_k) = \frac{n_k}{MN} \qquad \text{where } k = 0, 1, \ldots, L-1$$

where $n_k \Rightarrow$ no. of pixels that have intensity r_k,
$L \Rightarrow$ no. of possible intensity levels in the image

Histogram Equalization

- The discrete form of transformation

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \quad \text{where } k = 0, 1, \ldots, L-1$$

- The transformation (mapping) $T(r_k)$ in this equation is called a *Histogram Equalization* or *Histogram Linearization*.
- In discrete space, it cannot be proved in general that this discrete transformation will produce the discrete equivalent of a uniform probability density function, which would be a uniform histogram.

Histogram Equalization

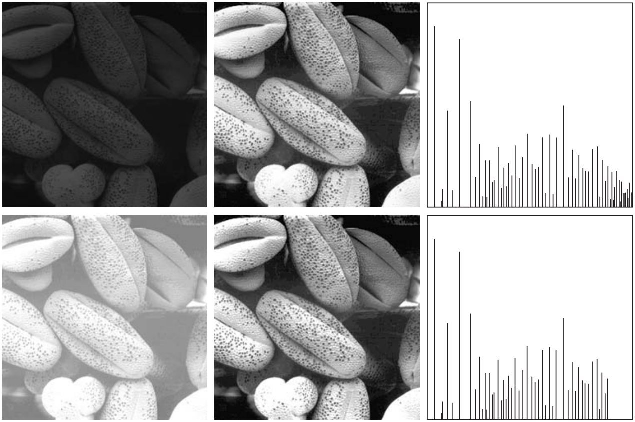


Figure: Histogram Equalized images of dark and bright images

Histogram Equalization

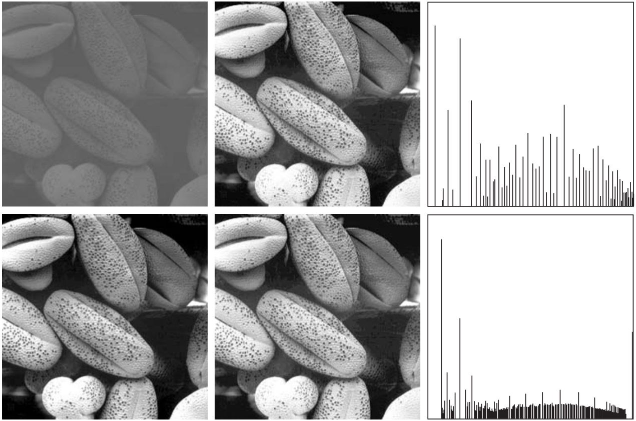


Figure: Histogram Equalized images of low-contrast and high-contrast images

Histogram Equalization: Problem

- Consider a 3 bit image (L=8) of size 64×64 having histogram values as

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
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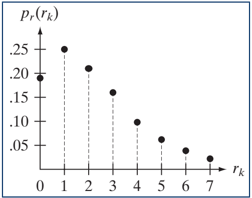

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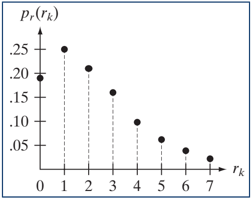

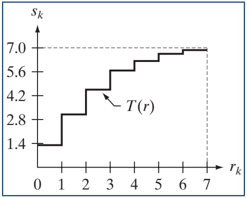

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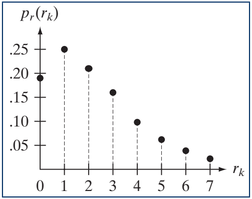

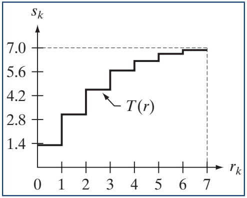

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$s_1 = 3.08 \rightarrow 3$	$s_5 = 6.65 \rightarrow 7$
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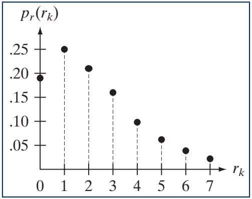

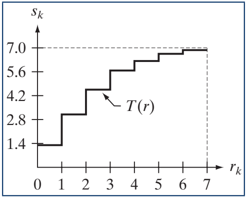

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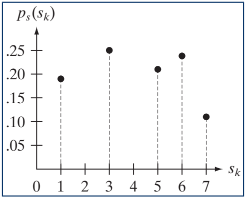


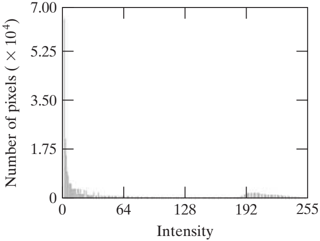
Histogram Matching (Specification)

- Histogram equalization automatically determines a transformation function that seeks to produce an output image that has a uniform histogram.
- In some application, uniform histogram is not desirable.
- Sometimes, it is useful to specify the shape of the histogram that we wish the processed image to have.
- The method which is used to generate a processed image that has a specified histogram is called *histogram matching* or *histogram specification*.

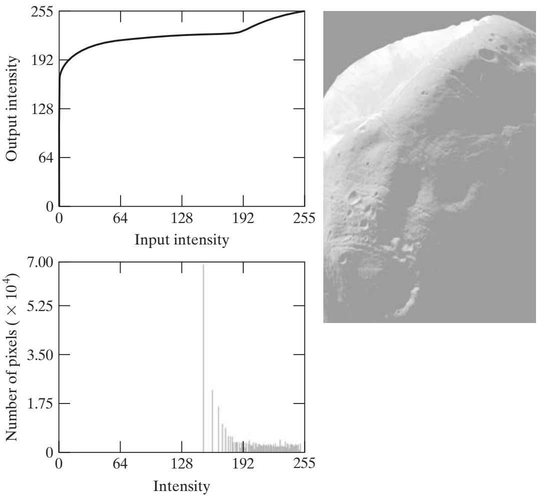
Histogram Matching (Specification)

a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)



Histogram Matching (Specification)



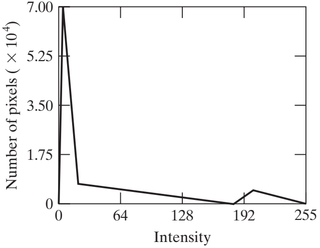
a b
c

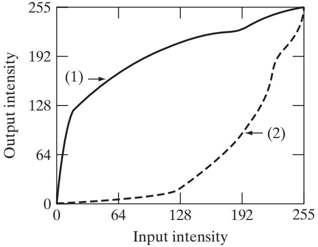
FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

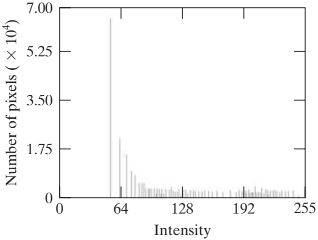
Histogram Matching (Specification)

a	c
b	
d	

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).







Histogram Matching (Specification)

- Let $p_r(r)$ & $p_z(z)$ are the continuous PDFs.
- Let r & z are the intensity levels of the input and output image respectively.
- $p_r(r)$ can be estimated from the given input image whereas, $p_z(z)$ is the specified PDF.
- Let $s =$ random variable with the property

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \tag{2}$$

- Let us consider another random variable z with the property

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) \tag{3}$$

- Requirement is $G(z_q) = s_k$

$$z_q = G^{-1}(s_k) \tag{4}$$

Histogram Matching: Procedure

Procedure to obtain an image whose intensity levels have a specified PDF:

1. Obtain $p_r(r_k)$ from the input image and use it to find the histogram equalization transformation. Obtain the value of s_k using Eq. (2). Round the resulting value, s_k, to the integer range $[0, L-1]$.
2. Compute all values of the transformation function $G(z_q)$ using Eq. (3). Round the values of $G(z_q)$ to integers in the range $[0, L-1]$. Store the value of $G(z_q)$ in a table.
3. For every value of s_k use the stored values of $G(z_1)$ from step 2 to find the corresponding value of z_q so that $G(z_q)$ is closest to s_k and store these mapping form s_k to z_q. When more than one value of z_q satisfies the given s_k, choose the smallest value by convention.
4. Form the histogram-specified image by first histogram-equalizing the input image and then mapping every equalized pixel value, s_k, of this image to the corresponding value z_q in the histogram-specified image using mapping found in step 3.

Histogram Matching: Problem

Step I Obtain $p_r(r_k)$ and s_k

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
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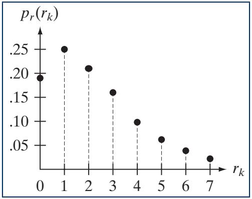

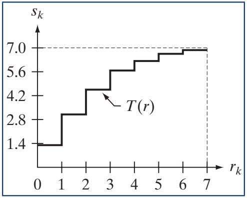

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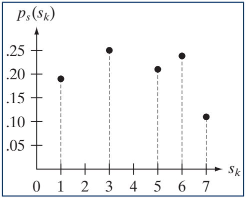

Histogram Matching: Problem

Step 2 Obtain $G(z_q)$

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
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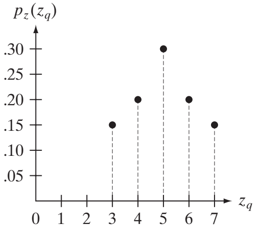
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11



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$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00$$

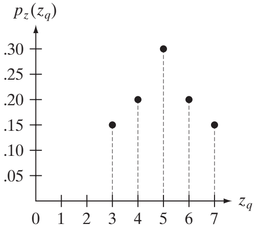
Similarly,

$$G(z_1) = 7\sum_{j=0}^{1} p_z(z_j) = 7\left[p(z_0) + p(z_1)\right] = 0.00$$

and

$$G(z_2) = 0.00 \quad G(z_4) = 2.45 \quad G(z_6) = 5.95$$

$$G(z_3) = 1.05 \quad G(z_5) = 4.55 \quad G(z_7) = 7.00$$



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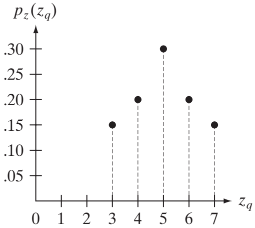

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$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00$$

Similarly,

$$G(z_1) = 7\sum_{j=0}^{1} p_z(z_j) = 7\big[p(z_0) + p(z_1)\big] = 0.00$$

and

$$G(z_2) = 0.00 \quad G(z_4) = 2.45 \quad G(z_6) = 5.95$$

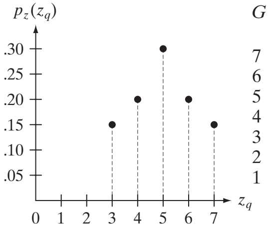
$$G(z_3) = 1.05 \quad G(z_5) = 4.55 \quad G(z_7) = 7.00$$

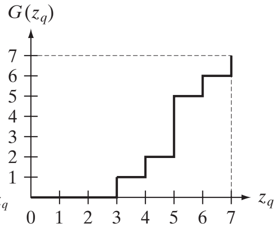
$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$

$$G(z_2) = 0.00 \rightarrow 0 \qquad G(z_6) = 5.95 \rightarrow 6$$

$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_7) = 7.00 \rightarrow 7$$





Histogram Matching: Problem

Step 3 Obtain mapping between s_k and z_q

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Step 4 Form the histogram-specified image by mapping every equalized pixel value, s_k of equalized image to the corresponding value z_q in the histogram-specified image using mapping found in step 3.

Thank You

Queries?