STA 250 Homework 1

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Problem 2

(i)

$$Y_i|\beta \sim \text{Binomial}(m_i, \text{logit}^{-1}(x_i^T\beta)), \beta \in \mathbb{R}^p$$

So we have

$$p(y_i|\beta) = \binom{m_i}{y_i} \left(\frac{e^{x_i^T\beta}}{1+e^{x_i^T\beta}}\right)^{y_i} \left(1 - \frac{e^{x_i^T\beta}}{1+e^{x_i^T\beta}}\right)^{m_i-y_i}$$

$$= \binom{m_i}{y_i} \frac{e^{(x_i^T\beta)y_i}}{(1+e^{x_i^T\beta})^{m_i}}$$

Then

$$p(\mathbf{y}|\beta) \propto \frac{\exp(\sum_{i=1}^n (x_i^T\beta)y_i)}{\prod_{i=1}^n (1+e^{x_i^T\beta})^{m_i}}$$

and

$$p(\beta) \propto \exp[-\frac{1}{2}(\beta-\mu_0)^T\Sigma_0^{-1}(\beta-\mu_0)]$$

So the posterior is, up to proportionality

$$p(\beta|\mathbf{y}) \propto \frac{\exp[\sum_{i=1}^n (x_i^T\beta)y_i - \frac{1}{2}(\beta-\mu_0)^T\Sigma_0^{-1}(\beta-\mu_0)]}{\prod_{i=1}^n (1+e^{x_i^T\beta})^{m_i}}$$

We may take logarithm of the proportional posterior for calculation, which is

$$\sum_{i=1}^n (x_i^T\beta)y_i - \frac{1}{2}(\beta-\mu_0)^T\Sigma_0^{-1}(\beta-\mu_0) - \sum_{i=1}^n m_i \log(1+e^{x_i^T\beta})$$

(ii)