

Partial Satisfaction of Signal Temporal Logic Specifications for Coordination of Multi-Robot Systems

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Abstract—This paper introduces a partial satisfaction (PS) notion for Signal Temporal Logic (STL). We formulate the planning problem for a team of agents with robust PS of STL missions as a bi-level optimization problem. The goal is to maximize the number of subformulae satisfied and their robustness with preference to larger (lower-depth) ones. First, we propose three MILP methods to solve the inner level of the optimization problem, two exact and a relaxation. Then, their solutions are used to find approximate solutions to the outer level optimization using a linear program. Finally, we show the performance of our methods in two different multi-robot case studies; motion planning in continuous spaces and routing for heterogeneous teams over finite graph abstractions.

I. INTRODUCTION

In recent years, multi-robot systems are being widely studied due to their resiliency and robustness. Their application domains span from search-and-rescue missions, exploration of bio-hazard zones to cargo delivery and planetary exploration [1], [2], [3], [4]. Solving these types of missions requires optimal task allocation over teams of robots satisfying a set of task specifications [5], [6]. Additionally, suitability, robustness, and scalability are extension to the problem that are frequently tackled in the literature [7], [8], [9]. However, in case of conflicting specifications or lack of robots, the planning problem becomes infeasible, whereas, in practice, at least partial execution of the task may be desired.

Our work focuses on partial satisfaction (PS) of a mission over a team of robots with conflicting or competing specifications. For example, let us consider having four robots with a camera in an environment and a task specification requiring three robots with a camera at location A and, at the same time, two robots of the same type at location B. Although both tasks cannot be satisfied due to a lack of robots, our framework tackles this by fully satisfying one of the tasks and partially satisfying the other. Furthermore, for efficient handling and scheduling of complex tasks, we use Signal Temporal Logic (STL) specifications to enrich the specification with logical and temporal operators that allow us to describe not only where but also when and for how long robots are needed to satisfy tasks [10], [11].

There exist some methods in the literature that tackle these temporal specifications while considering PS such as automata based approaches [12], [13] and robustness as Mixed Integer Linear Programming (MILP) formulations [14]. In [15], [16], authors compute policies that minimize the distance to satisfaction as given by the paths in the specification automaton. On the other hand, [17] propose

a general relaxation framework for automata-based planning for various relaxation instances by using a weighted finite-state edit systems. Even though these approaches take into account the partially feasible nature of specifications, they are computationally expensive as they involve computing product automata that scales poorly. In a different direction, MILP encoding scales better, making it suitable for multi-robot setups. In [18], a limited violation in the specifications is allowed by adding more binary variables in each Boolean operation, increasing dramatically the computational cost. Authors in [19] consider PS due to time uncertainties, penalizing the model when a task is not satisfied. Modeling with a bilevel optimization formulation, where the lower level decides whether or not a task can be satisfied, and the higher level solves the overlapping conflicts. Although minimum violation, robustness, and completion uncertainty have been solved, the concept of PS has not been tackled. Our work aims at addressing PS by maximizing the subformulae in a specification that can be satisfied. Also, our approach has a percentage metric of specification satisfaction and indicates subformulae that were partially or not satisfied.

This paper presents a robust PS problem for STL specifications with contradicting, competing, and infeasible subformulae. We formulate the PS planning problem as a bi-level optimization formulation where the inner level identifies satisfied subformulae. We prioritize lower-depth nodes in an Abstract Syntax Tree (AST) of the specification. We propose three MILP methods that use a novel encoding for STL based on *satisfaction fractions*, number of satisfied subformulae weighted according to depth. Our MILP encodings do not lose the qualitative semantics of STL. We use a Linear Program (LP) to maximize the robustness of all subformulae satisfied by a solution of the MILPs. The LP approximates the outer-level of the robust PS problem. We show the performance of the methods in two case studies involving multi-robot teams. We consider motion planning in continuous spaces, and routing for heterogeneous teams over finite graphs representing abstractions of the environment. The main contributions of this paper are threefold. First, we introduce a PS of STL specifications that satisfies as much as possible even in the presence of conflicting or unfeasible subformulae. Second, we propose three MILP encodings for PS, describing each method's advantages and runtime performance, and an LP for approximating robust PS. Finally, we show the versatility and performance of PS methods in two case studies, continuous state planning for robots navigating in planar environments, and routing over finite graph abstractions for teams of robots with varying capabilities tasked with rich temporal logic tasks [20].

II. PRELIMINARIES AND NOTATION

Let \mathbb{Z} , \mathbb{R} , denote the sets of integer and real numbers. The set of integers greater than a is $\mathbb{Z}_{>a}$. For a set S , 2^S and $|S|$ represent its power set and cardinality. For $S \subseteq \mathbb{R}$ and $\alpha \in \mathbb{R}$, we have $\alpha + S = \{\alpha + x \mid x \in S\}$. The integer interval (range) from a to b is $[a .. b]$. Let $x \in \mathbb{R}^d$ be a d -dimensional vector. The i -th component of x is given by x_i , $i \in [1 .. d]$.

A. Signal Temporal Logic

Let $s : \mathbb{Z}_{\geq 0} \rightarrow \mathbb{M}$ be a discrete-time signal with values in the compact space $\mathbb{M} \subseteq \mathbb{R}^N$. Signal Temporal Logic (STL), introduced in [21], is a specification language expressing real-time properties. The syntax of STL over linear predicates is given by

$$\phi ::= \top \mid s_i \geq \mu \mid \phi_1 \wedge \phi_2 \mid \neg \phi \mid \phi_1 \mathcal{U}_I \phi_2, \quad (1)$$

where ϕ , ϕ_1 , and ϕ_2 are STL formulae, \top is the logical *True* value, $s_i \geq \mu$ is a linear predicate with threshold value $\mu \in \mathbb{R}$ over the i -th component of signal s , \neg and \wedge are the Boolean negation and conjunction operators, and \mathcal{U}_I is the timed *until* operator with $I = [k_1 .. k_2]$ a discrete-time interval, $k_2 \geq k_1 \geq 0$. The logical *False* value is $\perp = \neg \top$. Predicates $s \sim \mu$, $\sim \in \{>, \leq, <\}$, follow via negation and sign change. The Boolean operators, *disjunction* \vee , and discrete-timed temporal operators *eventually* \Diamond_I , and *always* \Box_I , are defined in the usual way [21].

The (qualitative) semantics of STL formulae over signals s at time k is recursively defined in [21] as

$$\begin{aligned} (s, k) \models (s_i \geq \mu) &\equiv s_i(k) \geq \mu, \\ (s, k) \models \neg \phi &\equiv (s, k) \not\models \phi, \\ (s, k) \models \phi_1 \wedge \phi_2 &\equiv ((s, k) \models \phi_1) \wedge ((s, k) \models \phi_2), \\ (s, k) \models \phi_1 \mathcal{U}_I \phi_2 &\equiv \exists k' \in k + I \text{ s.t. } (s, k') \models \phi_2 \\ &\quad \wedge \forall k'' \in [k .. k'] (s, k'') \models \phi_1, \end{aligned} \quad (2)$$

where \models and $\not\models$ denote satisfaction and violation, respectively. A signal s satisfying ϕ , denoted as $s \models \phi$, is true if $(s, 0) \models \phi$.

In addition to Boolean semantics, STL admits quantitative semantics, called *robustness*, that indicates how much a signal satisfies or violates a specification [22], [23]. The robustness score $\rho(s, \phi, k)$ is recursively defined as

$$\begin{aligned} \rho(s, \top, k) &= \rho_{\top}, \\ \rho(s, s_i \geq \mu, k) &= s_i(k) - \mu, \\ \rho(s, \neg \phi, k) &= -\rho(s, \phi, k), \\ \rho(s, \phi_1 \wedge \phi_2, k) &= \min(\rho(s, \phi_1, k), \rho(s, \phi_2, k)), \\ \rho(s, \phi_1 \vee \phi_2, k) &= \max(\rho(s, \phi_1, k), \rho(s, \phi_2, k)), \\ \rho(s, \Box_I \phi, k) &= \min_{k' \in k+I} \rho(s, \phi, k'), \\ \rho(s, \Diamond_I \phi, k) &= \max_{k' \in k+I} \rho(s, \phi, k'), \\ \rho(s, \phi_1 \mathcal{U}_I \phi_2, k) &= \max_{k' \in k+I} \left\{ \min\{\rho(s, \phi_2, k'), \right. \\ &\quad \left. \min_{k'' \in [k .. k']} \rho(s, \phi_1, k'')\} \right\}, \end{aligned} \quad (3)$$

where $\rho_{\top} = \sup_{s, \mu} \{s_i - \mu\}$ is the maximum robustness.

Theorem 1 (Soundness [23]). *Let s be a signal and ϕ an STL formula. It holds $\rho(s, \phi, k) > 0 \Rightarrow (s, k) \models \phi$ for satisfaction and $\rho(s, \phi, k) < 0 \Rightarrow (s, k) \not\models \phi$ for violation.*

The time horizon of an STL formula [24] is defined as

$$\|\phi\| = \begin{cases} 0, & \text{if } \phi = s \geq \mu, \\ \|\phi_1\|, & \text{if } \phi = \neg \phi_1, \\ \max\{\|\phi_1\|, \|\phi_2\|\}, & \text{if } \phi = \phi_1 \wedge \phi_2, \\ b + \max\{\|\phi_1\|, \|\phi_2\|\}, & \text{if } \phi = \phi_1 \mathcal{U}_I \phi_2. \end{cases}$$

An STL formula is said to be in *positive normal form* (PNF) if it satisfies two conditions. First, all its predicates are of the $s_i \geq \mu$ form. Second, it does not contain the negation operator. Any STL formula can be put in PNF form [14].

Any STL formula can be represented using an abstract syntax tree (AST) in which intermediate nodes correspond to logical and temporal operators, and leaves to predicates [25]. The depth of a formula φ with respect to a formula ϕ is the path distance between the root of ϕ 's AST and the node associated with φ . If the φ is not a subformula of ϕ , the depth is by convention ∞ . We denote the depth by $\text{depth}_{\phi}(\varphi)$. We use $\varphi \sqsubset \phi$ to denote that φ is a proper subformula of ϕ , and $\varphi \sqsubseteq \phi$ when they can also be equal.

III. PROBLEM FORMULATION

In this work, we consider planning for teams of robots tasked with performing missions specified as STL formulae. We focus on the case where not all mission specification can be satisfied due to a lack of agents or competing subformulae. We define the PS problem that requires the computation of motion plans for all robots such that as much of the formula is satisfied with larger subformulae preferred over smaller ones. To this end, we introduce a partial order over subformulae that captures the preference structure. Although we focus on multi-robot problems in this paper, the PS problem and proposed approach can be used with any robot system with linear dynamics or mixed-logical linear dynamics, e.g., piecewise-affine systems, max-min-plus scaling systems [26], [14].

A. Team Dynamics

We consider two multi-robot planning problems. The first is planning for heterogeneous teams of robots with various capabilities over graphs, finite abstractions of the environment they are deployed in [20]. The second is joint planning for homogeneous teams in the continuous state spaces of the robots. We assume that the robots' motion is governed by linear dynamics enforced by low-level controllers, e.g., using differential flatness [27], [28] and feedback linearization [29], [30].

The robot team dynamics is captured in both the discrete and continuous state space cases using linear difference equations

$$s(k+1) = As(k) + Bu(k) + D, \quad (4)$$

where $s(k) \in S$ is the team state at time $k \in \mathbb{Z}_{\geq 0}$, S is the state space of the team, and A , B and D are the state transition, input and drift matrices of appropriate sizes.

The difference between the two cases is in the definition of the state space. For the heterogeneous team case over finite abstraction graphs, the team states represent the number of agents of each class at each location (node of the graph). For the homogeneous team case over continuous state spaces, the team states are composed of the states of all agents. See Sec. V for details and formal definitions.

B. Partial Satisfaction Control Synthesis Problem

In this paper, missions are defined as STL formulae with predicates over team states that capture the temporal and logical sequencing of tasks and their timing constraints. Partial Satisfaction (PS) of an STL formula ϕ is defined with respect to a state trajectory s . Informally, we desire PS to: (a) prefer subformulae larger formulae (lower-depth, see Fig. 1), and (b) consider subformulae satisfaction at all and only required time points. We capture these requirements formally using a partial order [31].

Let s be a state trajectory, and ϕ an STL mission specification. The set of subformulae of ϕ and depth $d \in \mathbb{Z}_{\geq 0}$ satisfied by s is $\Phi_\phi^d(s) = \{(\varphi, k) \mid \text{depth}_\phi(\varphi) = d, (s, k) \models \varphi\}$. We say that state trajectory s is less than s' with respect to ϕ , denoted $s <_\phi s'$, if there is $d \in \mathbb{Z}_{\geq 0}$ such that (a) $|\Phi_\phi^d(s)| < |\Phi_\phi^d(s')|$, and (b) $|\Phi_\phi^{d'}(s)| = |\Phi_\phi^{d'}(s')|$ for all $d' \in [0 \dots d-1]$. Equality, $s =_\phi s'$, holds iff $|\Phi_\phi^d(s)| = |\Phi_\phi^d(s')|$ for all $d \in \mathbb{Z}_{\geq 0}$. The trajectory s generated by some robotic system (linear or otherwise) is said to satisfy ϕ as much as possible if s is a maximal under the partial order $<_\phi$.

Lastly, we define the PS robustness

$$\varrho(s, \phi) = \min_{(\varphi, k) \in F_\phi(s)} \rho(s, \varphi, k), \quad (5)$$

where $F_\phi(s) = \{(\varphi, k) \mid \exists (\varphi', k') \text{ s.t. } \varphi \sqsubset \varphi', (s, k) \models \varphi, (s, k') \models \varphi'\}$ is the set of lowest-depth subformulae satisfied by s .

Problem 1. Given a multi-robot system with linear team dynamics (4), and an STL specification ϕ , find team input signal \mathbf{u} such that the generated state trajectory s satisfies ϕ as much as possible and maximizes the PS robustness (5). Formally, we have the bi-level optimization problem

$$\begin{aligned} \max_{\mathbf{u}} \quad & \varrho(s, \phi) \\ \text{s.t.} \quad & \mathbf{u} \text{ induces } s \\ & s \in \max_{\mathbf{u}'} \{s'\} \text{ s.t. } \mathbf{u}' \text{ induces } s' \end{aligned} \quad (6)$$

where \max^ϕ denotes maximization with respect to $<_\phi$.

Problem 1, takes a specification formula ϕ , and finds a team trajectory s that satisfies the maximum number subformulae of lowest depth, and has the largest minimum robustness among them. The inner optimization guarantees that the number of low-depth formulae is maximum, while the outer optimization chooses among them and accounts for their robustness.

Let us remark on the differences between PS and the canonical robustness in (3). The latter represents the margin of satisfaction or violation of an STL formula in the signal value space. In other words, the largest deviation to a signal

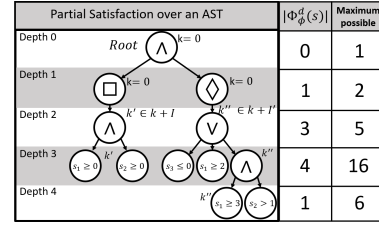


Fig. 1: Abstract syntax tree of formula ϕ_{ex} in (7). The depth of each subformula associated with the AST's nodes are shown. Moreover, for each subformula the time points that are involved in the satisfaction of ϕ_{ex} are given.

that changes the satisfaction with respect to the specification formula. In case of violation, it does not express which subformulae are satisfied or violated. Thus, it cannot be used to enforce satisfying as many subformulae as possible. In contrast, PS can guarantee that the specification will be satisfied as much as possible with preference to lower-depth subformulae. PS also captures the fraction of satisfaction for unsatisfied subformulae.

Example 1. Consider the following STL specification

$$\begin{aligned} \phi_{ex} &= \Box_I ((s_1 \geq 0) \wedge (s_2 \geq 0)) \wedge \phi'_{ex} \\ \phi'_{ex} &= \Diamond_{I'} ((s_3 \leq 0) \vee (s_1 \geq 2) \vee ((s_1 \geq 3) \wedge (s_2 > 1))), \end{aligned} \quad (7)$$

where $I = [0, 1]$ and $I' = [0, 2]$. The AST of ϕ_{ex} and depth of each of its subformulae are shown in Fig. 1. Consider a signal s such that $s(0) = -\mathbf{1}_3$, $s(1) = \mathbf{1}_3$, and $s(2) = 2 \cdot \mathbf{1}_3$, where $\mathbf{1}_n$ is the vector of all ones of dimension $n \in \mathbb{Z}_{\geq 1}$. The canonical robustness of s is $\rho(s, \phi_{ex}, 0) = -1$ due to the violation of predicates $\mu_1 = s_1 \geq 0$ and $\mu_2 = s_2 \geq 0$ at time $k = 0$ in the left subformula containing the always operator. However, the predicates are satisfied at time $k = 1$ within the interval I of the always operator. Moreover, the right subformula ϕ'_{ex} corresponding to the eventually operator is satisfied (as well as all of its subformulae). PS captures their satisfaction with $\varrho(s, \phi_{ex}) = \min\{\rho(s, \phi'_{ex}, 0), \rho(s, \mu_1 \wedge \mu_2, 1)\} = \min\{1, 1\} = 1$, where $F_\phi(s) = \{(\phi'_{ex}, 0), (\mu_1 \wedge \mu_2, 1)\}$. The numbers of satisfied subformulae $|\Phi_\phi^d(s)|$ of each depth $d \in [0 \dots 4]$ are 0, 1, 3, 4, and 1 out of maximum of 1, 2, 5, 16, and 6, respectively.

Note that PS depends explicitly on the AST of a formula¹, and is not associative by design. Thus, an operator has the freedom to specify the ordering and importance of subformulae using parenthesis that determine the ASTs.

IV. PARTIAL SATISFACTION ENCODING

In this section, we propose to solve Problem 1 in two steps that decouple the bi-level optimization problem. First, we propose two Mixed Integer Linear Programming (MILP) formulations to find the maximum number of subformulae of lowest depth that can be satisfied. The MILP corresponds to the inner optimization in (6). Furthermore, we consider a relaxation of the MILP formulations that reduces the

¹A formula can have many equivalent ASTs [25] determined by the parsing methods used.

number of required binary variables, and has better runtime performance. Second, we introduce a Linear Program (LP) that approximates the solution to the outer level of (6) using the solution of the inner level. The two-step approach trades off optimality with runtime performance.

A. MILP Encoding of Satisfaction Fractions

The foundation of our encoding of STL formulae is based on the fraction of subformulae-time pairs that signals satisfy. Instead of encoding margins that are propagated towards a formula's root to compute robustness [14], [18], we keep track of how many subformulae are satisfied weighted according to their depth.

Let ϕ be an STL formula. We represent by $\xi_{\varphi,k} \in [0, 1]$ the satisfaction fraction of subformula φ of ϕ at time $k \in [0 .. \|\phi\|]$. We define the following MILP to capture the satisfaction fractions generated by a signal s generated by the team dynamics (4). The recursive definition is

$$\begin{aligned} \xi_{\varphi,k} &= \begin{cases} s_i(k) - \mu + M(1 - \xi_{\mu,k}) \geq 0 \\ s_i(k) - \mu - M\xi_{\mu,k} \leq 0 \end{cases}, & \varphi = s_i \geq \mu, \\ m \cdot \xi_{\varphi,k} &= \sum_{\ell=1}^m \xi_{\varphi_{\ell},k}, & \varphi = \bigwedge_{\ell=1}^m \varphi_{\ell}, \\ \xi_{\varphi,k} &= \max_{\ell=1:m} \{\xi_{\varphi_{\ell},k}\}, & \varphi = \bigvee_{\ell=1}^m \varphi_{\ell}, \\ |I| \cdot \xi_{\varphi,k} &= \sum_{k' \in k+I} \xi_{\varphi',k'}, & \varphi = \Box_I \varphi', \\ \xi_{\varphi,k} &= \max_{k' \in k+I} \{\xi_{\varphi',k'}\}, & \varphi = \Diamond_I \varphi', \\ \xi_{\varphi,k} &= \max_{k' \in k+I} \{\xi_{\varphi_2,k'} + \sum_{k'' \in [k..k']} \xi_{\varphi_1,k''}\}, & \varphi = \varphi_1 \mathcal{U}_I \varphi_2, \end{aligned} \quad (8)$$

where $M > \rho_{\top}$, $\xi_{\mu,k} \in \{0, 1\}$ for all predicates μ and times $k \in [0 .. \|\phi\|]$. We have $\xi_{\mu,k} = 1$ when $s_{i,k} \geq \mu$ is true, and $\xi_{\mu,k} = 0$ when it is false. For all other logical and temporal operators $\xi_{\varphi,k} \in [0, 1]$ at all times $k \in [0 .. \|\phi\|]$.

Next, we capture the satisfaction of subformulae time pairs (φ, k) in $F_{\phi}(s)$ using the binary variables $\eta_{\varphi,k} \in \{0, 1\}$. We enforce satisfaction and lowest depth with constraints

$$\eta_{\varphi,k} \leq \xi_{\varphi,k}, \quad (9)$$

$$\eta_{\varphi,k} \leq 1 - \eta_{\varphi',k'}, \forall \varphi \sqsubseteq \varphi' \sqsubseteq \phi, k' \in \mathcal{K}', \quad (10)$$

where $\mathcal{K}' = \{k' \in [0 .. \|\phi\|] \mid (s, k') \models \varphi' \Rightarrow (s, k) \models \varphi, \forall s\}$ is the set of all times where φ' supersedes φ at k . Note that $\eta_{\varphi,k}$ can take value one only when φ is fully satisfied i.e., $\xi_{\varphi,k} = 1$, due to constraint (9). All ancestors must not be fully satisfied, i.e., $\eta_{\varphi',k'} = 0$, for φ to be lowest depth as captured in (10).

B. Objective Functions for Partial Satisfaction

In this section, we formulate three MILP problems that solve the inner level of (6). The first two are exact solutions, while the third is a relaxation.

Let $\gamma_{\phi,d} = \sum_{(\varphi,k) \in \Upsilon^d(\phi)} \eta_{\varphi,k}$, for all $d \in [0 .. d_{\max}]$, where d_{\max} is the maximum depth of ϕ , and $\Upsilon^d_{\phi} = \{(\varphi, k) \mid \text{depth}_{\phi}(\varphi) = d, (s, 0) \models \phi \Rightarrow (s, k) \models \varphi, \forall s\}$ is the set of all pairs of subformulae φ of depth d and time points k required for satisfaction of formula ϕ .

1) *Hierarchical Optimization (HO)*: In this approach, we define the multi-objective function

$$R_{HO} = (\gamma_{\phi,0}, \gamma_{\phi,1}, \dots, \gamma_{\phi,d_{\max}}),$$

and require optimization of the objectives in order [32], also known as lexicographical optimization [33]. This leads to the hierarchical MILP problem

$$\max_{s,u,\xi,\eta} R_{HO} \text{ (optimized in order) s.t. (4), (8), (9), (10).} \quad (11)$$

Gurobi [33] can be used to solve (11) directly. However, any MILP solver can be used via iterative methods [32].

2) *Lowest Depth First (LDF)*: We convert the multi-objective function R_{HO} into a scalar function R_{LDF} using a method similar to the big-M trick. The objective is

$$R_{LDF} = \sum_{(\varphi,k)} \eta_{\varphi,k} P^{-\text{depth}_{\phi}(\varphi)} = \sum_{d=0}^{d_{\max}} \gamma_{\phi,d} P^{-d},$$

that leads to the MILP problem

$$\max_{s,u,\xi,\eta} R_{LDF} \text{ s.t. (4), (8), (9), (10),} \quad (12)$$

where P is a large constant, greater than $\|\phi\| \cdot |\phi|$, and $|\phi|$ is the length of ϕ , i.e., the number of Boolean and temporal operators and predicates.

Formulation (12) avoids defining multiple objectives, but can lead to numerical issues and slower convergence for large depth formulae due to the exponentially decreasing weights.

3) *Weighted Largest Number (WLN)*: In this approach, we relax the strict requirement of finding lowest depth formulae, and instead maximize the satisfaction fraction of ϕ . Taking $R_{WLN} = \xi_{\phi,0}$ is a sensible choice, because subformulae are still weighted according to their depth due to the recursive constraints (8). Consequently, we obtain the MILP

$$\max_{s,u,\xi} R_{WLN} \text{ s.t. (4), (8),} \quad (13)$$

that does not require the additional binary variables $\eta_{\varphi,k}$ and associated constraints. It leads to better runtime performance at the expense of minimal depth of satisfied subformulae.

C. Partial Satisfaction Robustness LP

In this section, we propose to approximate the robust solution of Problem 1 using an LP based on solutions to one of the MILPs from Sec. IV-B.

Let $\{\xi_{\varphi,k}\}_{\varphi \sqsubseteq \phi, k \in [0.. \|\phi\|]}$ be the set of decision variables for satisfaction fractions obtained from solving (11), (12), or (13). The following LP,

$$\max_{s,u} \rho \text{ s.t. (4), } \rho \leq s_i(k) - \mu, \forall \mu \text{ with } \xi_{\mu,k} = 1 \quad (14)$$

computes the signal s and control u that maximize the robustness of all predicates μ at all times k that are satisfied in the reference solution encoded by $\xi_{\varphi,k}$.

D. Analysis

First, show that all three MILP formulations find trajectories satisfying ϕ if they exist.

Let ϕ be an STL formula, and $\{\xi_{\varphi,k}\}_{\varphi \in \phi, k \in [0..\|\phi\|]}$ generated by solving (11), (12), or (13).

Theorem 2. *There exists a control u that generates trajectory s according to (4) that satisfies ϕ if and only if $\xi_{\phi,0} = 1$.*

Sketch. First, if $\xi_{\phi,0} = 1$, it follows by structural induction that all required subformulae, including ϕ , are satisfied by the computed trajectory s . Conversely, if there exists s that satisfies ϕ , then it follows that s satisfies all required subformulae. Thus, the satisfaction fractions $\xi_{\varphi,k}$ must be 1, including $\xi_{\phi,0}$. \square

Let s^* and $\{\eta_{\varphi,k}\}_{\varphi \in \phi, k \in [0..\|\phi\|]}$ be the optimal solutions of (11) or (12). The following intermediate result is given without proof due to brevity.

Proposition 1. *If $\eta_{\varphi,k} = 1$, then $(\varphi, k) \in F_{\phi}(s^*)$.*

Next, we show that (11) and (12) satisfy the PS requirement of lowest-depth formulae.

Theorem 3. *There exists no signal s such that $s^* <_{\phi} s$.*

Sketch. The proof follows from Prop. 1, which states that the $\eta_{\varphi,k}$ correctly encode lowest-depth subformulae, and the objectives R_{HO} and R_{LDF} . For HO, the optimization considers solutions that maximize the number of formulae of lower depth first by construction. For LDF, it is easy to show that any solution that satisfies a lower-depth formula leads to a larger objective value than any other solution with any number of higher-depth satisfied subformulae. \square

We give the result on the correctness of LP without proof.

Theorem 4. *The solution of LP (14) has maximum PS robustness $\varrho(s, \phi)$ over all trajectories that satisfy the same set of subformulae (φ, k) as s^* .*

Note that an optimal solution to Problem III might satisfy other subformulae than s^* even if both achieve the same number at each depth. As such, the LP (14) may lead to suboptimal solutions.

V. CASE STUDIES

We describe two different study cases that show how PS works and its performance over the three encodings. First, we consider a multi-robot motion planning problem in a planar, continuous environment. Second, we show routing for heterogeneous teams of robots with sets of capabilities over graphs that represent discrete finite abstractions. More details can be found in [20]. All computation was performed on a PC with 20 cores at 3.7GHz with 64 GB of RAM. We used Gurobi [33] as MILP solver.

A. Continuous Space Multi-robot Planning

Let us consider first a single robot navigating in a planar environment $\mathcal{M} \subset \mathbb{R}^2$. Thus, $s(k) = [s_x, s_y]^T \in \mathbb{R}^2$. We define regions of interest $\mathcal{A} = [-9.5, -5.5]^2$, $\mathcal{B} = [5.5, 9.5] \times [-9.5, -5.5]$, $\mathcal{C} = [5.5, 9.5]^2$, $\mathcal{D} = [-9.5, -5.5] \times [5.5, 9.5]$ in

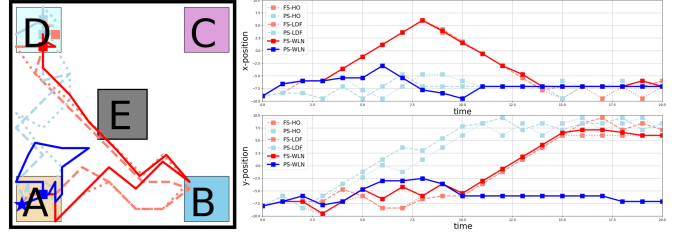


Fig. 2: State space and timeline in x -position and y -position of ϕ_{FS} in red and ϕ_{PS} in blue using HO, LDF, and WLN encoding methods.

\mathcal{M} , and region $\mathcal{E} = [-2.5, 2.5]^2 \subseteq \mathcal{M}$ that robot $s(k)$ needs to avoid. We arbitrarily choose $A = B = I_{2 \times 2}$, and $D = 0_{2 \times 2}$.

Throughout this case study, we consider methods discussed in Sec. IV. All three methods HO, LDF, and WLN, are able to find trajectories that satisfy the given specification when they exist. For example, consider the following STL formula $\phi_{FS} = ((\Box_{[0,3]}\mathcal{A}) \wedge (\Diamond_{[7,14]}\mathcal{B}) \wedge (\Box_{[15,20]}\mathcal{D}) \wedge (\sim \Box_{[0,20]}\mathcal{E}))$. In plain English, it required that “within time interval $[0, 3]$ stay at region \mathcal{A} , eventually within $[0, 14]$ visit \mathcal{B} , stay at \mathcal{D} within $[15, 20]$ and always avoid region \mathcal{E} ”. In Fig. 2 red lines show the solution for this specification using the three methods. All of three solution trajectories start at the blue star in \mathcal{A} , then they visit \mathcal{B} and end at \mathcal{D} , where squares denote the final position of the robot.

Next, consider the STL formula $\phi_{PS} = (\Box_{[0,3]}\mathcal{A}) \wedge (\Box_{[10,21]}\mathcal{B}) \wedge (\Box_{[10,21]}\mathcal{D}) \wedge (\sim \Box_{[0,20]}\mathcal{E})$ that can not be fully satisfied due to the competing subformulae that require staying at \mathcal{B} and \mathcal{D} in the same time interval $[10, 21]$. The solutions trajectories are shown in Fig. 2 with blue lines. Methods HO and LDF partially satisfy the specification by choosing to visit region \mathcal{D} , and, thus, satisfying the lowest-depth subformulae of ϕ_{PS} . In contrast, WLN splits the satisfaction between the predicates defining the two regions \mathcal{B} and \mathcal{D} . In other words, the specification requires either $(s_x \geq 5.5) \wedge (s_x \leq 9.5) \wedge (s_y \geq -9.5) \wedge (s_y \leq -5.5)$ or $(s_x \geq -9.5) \wedge (s_x \leq -5.5) \wedge (s_y \geq 5.5) \wedge (s_y \leq 9.5)$ to hold, and WLN is satisfying half from the former, $(s_x \geq -9.5) \wedge (s_x \leq -5.5)$, and half of the latter, $(s_y \geq -9.5) \wedge (s_y \leq -5.5)$, in the time window $[10, 21]$. The WLN trajectory satisfies the same maximum satisfaction fraction as HO and LDF, but does not fully satisfy any of the two tasks. However, the case was hand-crafted to show that WLN does not always return the optimal solution. In particular, we made the tasks symmetric. If the time intervals for visiting \mathcal{B} and \mathcal{D} are different, then the weight of their associated predicates differs and leads to a preference for one of the two regions.

In a multi-robot setting, unsatisfiability can result due to robot dropout as opposed to competing tasks. For instance, in Fig. 3 we consider two scenarios satisfying the following specification $\phi_{mr} = ((\Box_{[15,30]}s_1 \in \mathcal{C}) \wedge (\Box_{[15,30]}s_2 \in \mathcal{D}) \wedge (\Box_{[15,30]}s_3 \in \mathcal{C}) \wedge (\Box_{[15,30]}s_4 \in \mathcal{B}) \wedge (\Box_{[15,30]}s_5 \in \mathcal{C}) \wedge (\Box_{[15,30]}s_6 \in \mathcal{C}))$ with six robots. First, we solve the problem with HO approach shown in Fig. 3 with dashed lines. Then, we consider the same specification, but we make robot s_1 remain stuck in region \mathcal{E} . Even though it is just one agent

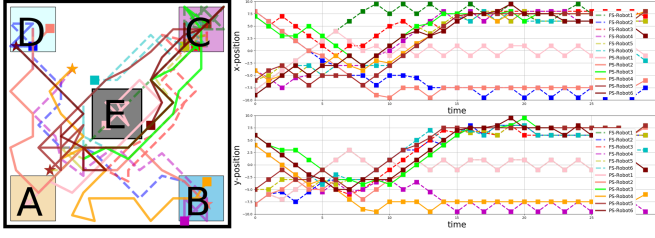


Fig. 3: State space and timeline in x -position and y -position of ϕ_{mr} fully satisfied with HO approach in dashed lines. Then, solution using LDF for robot one getting stuck in solid.

specification ϕ_{mr} is only partially satisfied. The PS solution computed using LDF is shown in solid lines.

Finally, we compare the runtime performance between methods HO, LDF, and WLN by increasing the number of robots from one to six. In Fig. 4, we see that when there are one or two robots, LDF and WLN are faster than HO. Nevertheless, when the number of robots increases, so does the depth in the specification, and LDF becomes the worst performing method. The specification we used for this comparison is $\phi_n = \bigwedge_{j=1}^n (\bigwedge_{\ell=1}^3 (\square_{I_{\ell,j}}(s_n \in \mathcal{X}_{\ell,j})))$, with $n \in [1 \dots 6]$ the number of robots, $\mathcal{X}_{\ell,j}$ any arbitrary region in $\{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$, and $I_{\ell,j}$ arbitrary time interval, respectively.

It is also relevant to highlight that even when WLN is the fastest, it can drive to undesired PS if the specification contains symmetric competing subformulae.

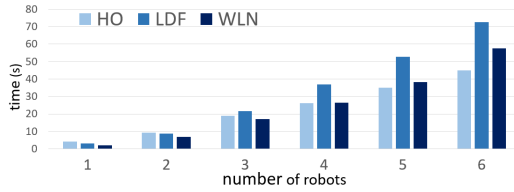


Fig. 4: Runtime performance comparison between HO, LDF, and WLN varying the number of robots from one to six, satisfying specification ϕ_n .

B. Route Planning with Capability Temporal Logic (CaTL)

We consider the precision agriculture application shown in Fig. 5. The environment is abstracted as a set of states $\mathcal{Q} = \{q_1, q_2, \dots, q_9\}$ corresponding to regions of interest. Edges \mathcal{E} , shown as black lines, between states represent feasibility of transition between regions with positive integer durations \mathcal{W} . Self-loops (not shown) capture staying at states and have duration 1. States are labeled with atomic propositions from a set AP . Agents have capabilities $Cap = \{Vis, IR, UV, Mo\}$. The capability set of an agent determines its class. Fig. 5 shows the initial states of each agent and their class.

We use CaTL, a fragment of STL, to specify the mission for the multi-robot team. The core units of CaTL are tasks $T = (d, \pi, cp)$ defined by a duration d , a label π of regions where it takes place, and the required capabilities cp . The map $cp : Cap \rightarrow \mathbb{Z}_{\geq 0}$ indicates the minimum number of agents with each capability; zero means that capability is

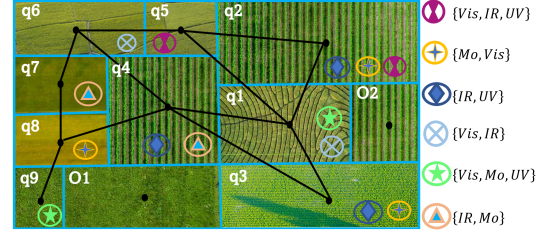


Fig. 5: Schematic of the precision agriculture problem. Regions correspond to crop types. Symbols represent robots with their associated capabilities. Capabilities include ultra-violet sensing (UV), moisture sensing (Mo), infrared sensing (IR), and vision (Vis). Black lines denote transition system between regions.

not needed. Tasks are combined using Boolean and temporal operators the same as STL (excluding negation).

We use the encoding of team dynamics in [20], where robots are bundled together based on class. Thus, the team state is the number of robots of each class at each state of the environment. The team dynamics are encoded as flows on the environment graph which gives rise to a time-delayed linear system (slightly more involved than (4)). The MILP constraints and further details can be found in [20].

Consider tasks $T_1 = (3, q_2, (c_2, 3))$, $T_2 = (3, q_4, (c_1, 4))$, $T_3 = (6, q_9, (c_3, 3))$, $T_4 = (5, q_1, (c_4, 4))$, $T_5 = (3, q_6, (c_2, 1), (c_4, 1))$, and $T_6 = (2, q_8, (c_4, 4))$, and three CaTL specifications $\phi_1, \phi_2 = \bigwedge_i \square_{[0,20]} T_i$, with $i \in \{1, 3, 5\}$ and $i \in \{2, 4, 6\}$ respectively, and $\phi_3 = \bigwedge_{i=1}^6 \diamond_{[0,20]} T_i$. Partial satisfaction of ϕ_ℓ can arise due to lack of robots with required capabilities, transition system constraints, and competing subformulae. The following table shows the performance of the three methods for the three specifications.

spec	HO		LDF		WLN		d
	satis	t(s)	satis	t(s)	satis	t(s)	
ϕ_1	.94	2.285	.94	1.918	.97	1.77	5
ϕ_2	.98	2.174	.99	1.903	.99	1.77	5
ϕ_3	1	2.005	1	1.935	1	1.79	5

VI. CONCLUSIONS

In this paper, we introduced the partial satisfaction problem for STL specifications that requires satisfaction of as many large formulae as possible. We formalize the partial satisfaction synthesis problem as a bi-level optimization that maximizes the robustness of lowest-depth formulae. We propose three MILP-based methods to solve the inner level of the optimization that find trajectories satisfying the maximum number of subformulae with preference to lower-depth ones. We used their solutions in a LP formulation that also maximizes the subformulae' robustness as an approximation for solution to the outer level of the robust PS problem. We show the performance of the proposed methods in two multi-robot case studies involving motion planning in continuous spaces and routing over finite abstractions.

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