

Problem Set 6

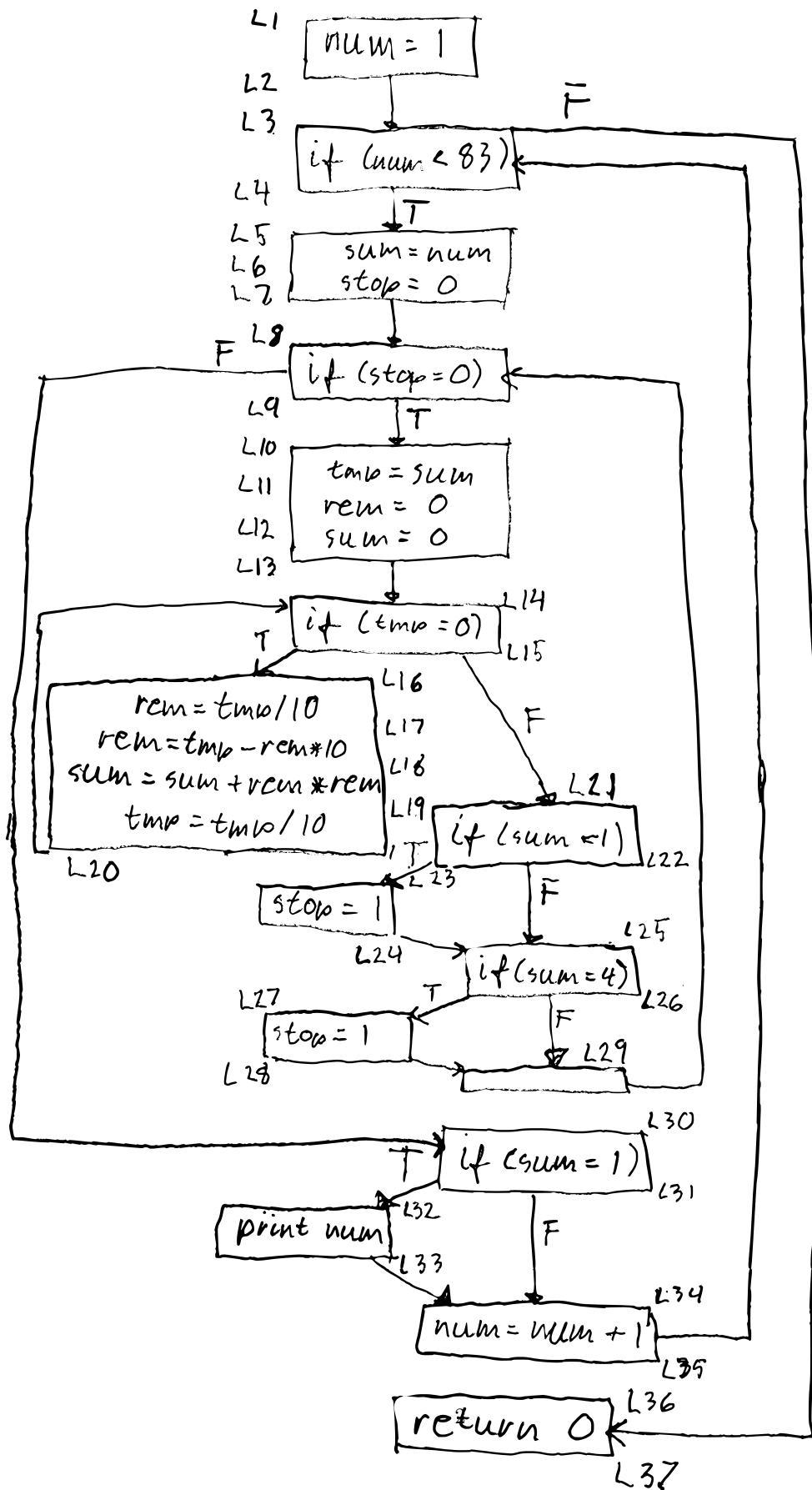
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Task 1: Data Flow Analysis

We are giving the snippet:

```
func happy_numbers()
begin
    var num, sum, tmp, rem, stop
    num := 1
    while num < 83 do
        begin
            sum := num
            stop := 0
            while stop = 0 do
                begin
                    tmp := sum
                    rem := 0
                    sum := 0
                    while tmp > 0 do
                        begin
                            rem := tmp / 10
                            rem := tmp - rem * 10
                            sum := sum + rem * rem
                            tmp := tmp / 10
                        end
                        if sum = 1 then stop := 1
                        if sum = 4 then stop := 1
                end
                if sum = 1 then print num
                num := num + 1
        end
    return 0
end
```

Under follows the control flow graph and reaching definitions.



$d1 : num = 1$	$d7 : rem = tmp / 10$
$d2 : sum = num$	$d8 : rem = tmp - rem * 10$
$d3 : stop = 0$	$d9 : sum = sum + rem * rem$
$d4 : tmp = sum$	$d10 : tmp = tmp / 10$
$d5 : rem = 0$	$d11 : stop = 1$
$d6 : sum = 0$	$d12 : num = num + 1$

Data flow equations, minus the trivial ones (e.g. $L4 = L3$)

$$\begin{array}{ll}
 L2 = L1 - \{d12, d1\} \cup d1 & L24 = L23 - \{d3\} \cup d11 \\
 L3 = L2 \cup L35 & L25 = L22 \cup L24 \\
 L6 = L5 - \{d6, d9\} \cup d2 & L28 = L27 - \{d3\} \cup d11 \\
 L7 = L6 - \{d11\} \cup d3 & L29 = L26 \cup L28 \\
 L8 = L7 \cup L29 & L30 = L9 \\
 L11 = L10 - \{d10\} \cup d4 & L34 = L31 \cup L33 \\
 L12 = L11 - \{d7, d8\} \cup d5 & L35 = L34 - \{d1\} \cup d12 \\
 L13 = L12 - \{d2, d9\} \cup d6 & L36 = L4 \\
 L14 = L13 \cup L20 & \\
 L17 = L16 - \{d5, d9\} \cup d7 & \\
 L18 = L17 - \{d5, d7\} \cup d8 & \\
 L19 = L18 - \{d2, d6\} \cup d9 & \\
 L20 = L19 - \{d4\} \cup d10 &
 \end{array}$$

After iteration 1 (expressions are simplified)

$L_1 = \{ \}$	$L_{22} = \{d_1, d_3, d_4, d_5, d_6\}$
$L_2 = \{d_1\}$	$L_{23} = \{d_1, d_3, d_4, d_5, d_6\}$
$L_3 = L_2 \cup L_{35} = \{d_1\}$	$L_{24} = \{d_1, d_4, d_5, d_6, d_{11}\}$
$L_4 = L_3 = \{d_1\}$	$L_{25} = \{d_1, d_3, d_4, d_5, d_6, d_{11}\}$
$L_5 = L_4 = \{d_1\}$	$L_{26} = \{d_1, d_3, d_4, d_5, d_6, d_{11}\}$
$L_6 = L_5 \cup \{d_2\} = \{d_1, d_2\}$	$L_{27} = \{d_1, d_3, d_4, d_5, d_6, d_{11}\}$
$L_7 = L_6 \cup \{d_3\} = \{d_1, d_2, d_3\}$	$L_{28} = \{d_1, d_4, d_5, d_6, d_{11}\}$
$L_8 = L_7 \cup L_9 = \{d_1, d_2, d_3\}$	$L_{29} = \{d_1, d_3, d_4, d_5, d_6, d_{11}\}$
$L_9 = L_8 = \{d_1, d_2, d_3\}$	$L_{30} = \{d_1, d_2, d_3\}$
$L_{10} = L_9 = \{d_1, d_2, d_3\}$	$L_{31} = \{d_1, d_2, d_3\}$
$L_{11} = \{d_1, d_2, d_3, d_4\}$	$L_{32} = \{d_1, d_2, d_3\}$
$L_{12} = \{d_1, d_2, d_3, d_4, d_5\}$	$L_{33} = \{d_1, d_2, d_3\}$
$L_{13} = \{d_1, d_3, d_4, d_5, d_6\}$	$L_{34} = \{d_1, d_2, d_3\}$
$L_{14} = \{d_1, d_3, d_4, d_5, d_6\}$	$L_{35} = \{d_2, d_3, d_{12}\}$
$L_{15} = \{d_1, d_3, d_4, d_5, d_6\}$	$L_{36} = \{d_1\}$
$L_{16} = \{d_1, d_3, d_4, d_5, d_6\}$	$L_{37} = \{d_1\}$
$L_{17} = \{d_1, d_3, d_4, d_6, d_7\}$	
$L_{18} = \{d_1, d_3, d_4, d_6, d_8\}$	
$L_{19} = \{d_1, d_3, d_4, d_8, d_9\}$	
$L_{20} = \{d_1, d_3, d_8, d_9, d_{10}\}$	
$L_{21} = \{d_1, d_3, d_4, d_5, d_6\}$	
	$\approx L_{15}$

After iteration 2 (expressions are simplified)

$L_1 = \{ \}$	$L_{22} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_2 = \{ d_1 \}$	$L_{23} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_3 = \{ d_1, d_2, d_3, d_{12} \}$	$L_{24} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_4 = \{ d_1, d_2, d_3, d_{12} \}$	$L_{25} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_5 = \{ d_1, d_2, d_3, d_{12} \}$	$L_{26} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_6 = \{ d_1, d_2, d_3, d_{12} \}$	$L_{27} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_7 = \{ d_1, d_2, d_3, d_{12} \}$	$L_{28} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_8 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{29} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_9 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{30} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$
$L_{10} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{31} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$
$L_{11} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{32} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$
$L_{12} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{33} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$
$L_{13} = \{ d_1, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{34} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$
$L_{14} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{35} = \{ d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$
$L_{15} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{36} = \{ d_1, d_2, d_3, d_{12} \}$
$L_{16} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{37} = \{ d_1, d_2, d_3, d_{12} \}$
$L_{17} = \{ d_1, d_3, d_4, d_6, d_7, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{18} = \{ d_1, d_3, d_4, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{19} = \{ d_1, d_3, d_4, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{20} = \{ d_1, d_3, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{21} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	

$\approx L_{15}$

After iteration 3 (expressions are simplified)

$L_1 = \{ \}$	$L_{22} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_2 = \{ d_1 \}$	$L_{23} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_3 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{24} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_4 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{25} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_5 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{26} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_6 = \{ d_1, d_2, d_3, d_4, d_5, d_{11}, d_{12} \}$	$L_{27} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_7 = \{ d_1, d_2, d_3, d_4, d_5, d_{12} \}$	$L_{28} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_8 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{29} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_9 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{30} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{10} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{31} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{11} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{11}, d_{12} \}$	$L_{32} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{12} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_9, d_{11}, d_{12} \}$	$L_{33} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{13} = \{ d_1, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{34} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{14} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{35} = \{ d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
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$L_{20} = \{ d_1, d_3, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{21} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	

$\approx L_{15}$

After iteration 4 (expressions are simplified)

$L_1 = \{ \}$	$L_{22} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_2 = \{ d_1 \}$	$L_{23} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_3 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{24} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_4 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{25} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_5 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{26} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_6 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_{10}, d_{11}, d_{12} \}$	$L_{27} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_7 = \{ d_1, d_2, d_3, d_4, d_5, d_8, d_{10}, d_{12} \}$	$L_{28} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_8 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{29} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
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$L_{10} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{31} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{11} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{11}, d_{12} \}$	$L_{32} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{12} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_9, d_{11}, d_{12} \}$	$L_{33} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{13} = \{ d_1, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{34} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{14} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{35} = \{ d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{15} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{36} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{16} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{37} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$
$L_{17} = \{ d_1, d_3, d_4, d_6, d_7, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{18} = \{ d_1, d_3, d_4, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{19} = \{ d_1, d_3, d_4, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{20} = \{ d_1, d_3, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{21} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	

After iteration 5

$L_1 = \{ \}$	$L_{22} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_2 = \{ d_1 \}$	$L_{23} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_3 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{24} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_4 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{25} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_5 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{26} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_6 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_{10}, d_{11}, d_{12} \}$	$L_{27} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_7 = \{ d_1, d_2, d_3, d_4, d_5, d_8, d_{10}, d_{11}, d_{12} \}$	$L_{28} = \{ d_1, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_8 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{29} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_9 = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{30} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{10} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{31} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{11} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{11}, d_{12} \}$	$L_{32} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{12} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_9, d_{11}, d_{12} \}$	$L_{33} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{13} = \{ d_1, d_3, d_4, d_5, d_6, d_{11}, d_{12} \}$	$L_{34} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{14} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{35} = \{ d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{15} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{36} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{16} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	$L_{37} = \{ d_1, d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$	
$L_{17} = \{ d_1, d_3, d_4, d_6, d_7, d_9, d_{10}, d_{11}, d_{12} \}$	Nothing has changed (reached fix-point), and we are finished.	
$L_{18} = \{ d_1, d_3, d_4, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$		
$L_{19} = \{ d_1, d_3, d_4, d_8, d_9, d_{10}, d_{11}, d_{12} \}$		
$L_{20} = \{ d_1, d_3, d_8, d_9, d_{10}, d_{11}, d_{12} \}$		
$L_{21} = \{ d_1, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{12} \}$		

Task 2: Code Generation

This is found in the delivered code.