Exercise 5

Task 1

1.1

a) It is entailed.

A	B	$\neg A \land \neg B$	$\neg A \land \neg B \implies \neg B$
F	F	T	T
F	T	F	T
T	F	F	T
T	T	F	T

b) It is not entailed.

A	B	$\neg A \lor \neg B$	$\neg A \lor \neg B \implies \neg B$
F	F	T	T
F	T	T	F
T	F	T	T
T	T	F	T

c) It is not entailed.

A	B	$\neg A \land \neg B$	$\neg A \land \neg B \implies A \lor B$
F	F	T	F
F	T	F	T
T	F	F	T
T	T	F	T

d) It is not entailed.

A	B	$A \implies B$	$A \iff B$	$(A \implies B) \implies (A \iff B)$
F	F	T	T	T
F	T	T	F	F
T	F	F	F	T
T	T	T	T	T

e) It is entailed.

A	B	C	$A \implies B$	$(A \implies B) \iff C$	$A \lor \neg B \lor C$	$((A \implies B) \iff C) \implies (A \lor \neg B \lor C)$
F	F	F	T	F	T	T
F	F	T	T	T	T	T
F	T	F	T	F	F	T
F	T	T	T	T	T	T
T	F	F	F	T	T	T
T	F	T	F	F	T	T
T	T	F	T	F	T	T
T	T	T	T	T	T	T

f) It is satisfiable for $A \wedge \neg B$

A	B	$\neg A \implies \neg B$	$A \wedge \neg B$	$(\neg A \implies \neg B)$ $\wedge(A \wedge \neg B)$
F	F	T	F	F
F	T	F	F	F
T	F	T	T	T
T	T	T	F	F

g) It is not satisfiable

A	B	$\neg A \iff \neg B$	$A \wedge \neg B$	$(\neg A \iff \neg B)$ $\wedge(A \wedge \neg B)$
F	F	T	F	F
F	T	F	F	F
T	F	F	T	F
T	T	T	F	F

1.2

a) I assume it means the packet you can send. In that case, the vocabulary is:

$$(0|1)\{8\}$$

i.e 0 or 1 exactly 8 times.

b) For sensor S_2 and S_3, I assume a high value is equal to a True value. C_1: $\neg S_2 \wedge \neg S_3$ C_2: $\neg S_1 \wedge S_2$ C_3: S_3

c) For each tank, this would be the truth table.

S_1	S_2	S_3	$\neg S_2 \wedge \neg S_3$	$\neg S_1 \wedge S_2$
F	F	F	T	F
F	F	T	F	F
F	T	F	F	T
F	T	T	F	T
T	F	F	T	F
T	F	T	F	F
T	T	F	F	F
T	T	T	F	F

d) If we receive the packet 01000110, we first analyze the last 5 bits of the packet, which in shortend form is $110_2 = 6_{10}$, i.e the tank with id 6. For the 3 first bits, we place the values in the truth table. In this case, 010 is mapped to row 3 of the truth table. We then see that C_2 is evaluated to true (i.e for the current KB, we can entail C_2). The system then closes the gate.

Task 2

2.1

a)

$$A \vee (B \wedge C \wedge \neg D) \equiv (A \vee B) \wedge (A \vee C) \wedge (A \vee \neg D)$$

b)

$$\begin{aligned}
&\neg(A \implies \neg B) \wedge \neg(C \implies \neg D) \\
\equiv&\neg(\neg A \vee \neg B) \wedge \neg(\neg C \vee \neg D) \\
\equiv&(A \wedge B) \wedge (C \wedge D) \\
\equiv&A \wedge B \wedge C \wedge D
\end{aligned}$$

c)

$$
\begin{aligned}
&\neg((A \implies B) \land (C \implies D))\\
\equiv&\neg((\neg A \lor B) \land (\neg C \lor D))\\
\equiv&\neg(\neg A \lor B) \lor \neg(\neg C \lor D)\\
\equiv&(A \land \neg B) \lor (C \land \neg D)\\
\equiv&((A \land \neg B) \lor C) \land ((A \land \neg B) \lor \neg D)\\
\equiv&(A \lor C) \land (\neg B \lor C) \land (A \lor \neg D) \land (\neg B \lor \neg D)
\end{aligned}
$$

d)

$$
\begin{aligned}
&(A \land B) \lor (C \implies D)\\
\equiv&(A \land B) \lor (\neg C \lor D)\\
\equiv&(A \lor \neg C \lor D) \land (B \lor \neg C \lor D)
\end{aligned}
$$

e)

$$
\begin{aligned}
&A \iff (B \implies \neg C)\\
\equiv&(A \implies (B \implies \neg C)) \land ((B \implies \neg C) \implies A)\\
\equiv&(\neg A \lor \neg B \lor \neg C) \land (\neg B \lor \neg C \lor A)\\
\equiv&(\neg A \lor \neg B \lor \neg C) \land (A \lor \neg B \lor \neg C)
\end{aligned}
$$

2.2

Building the knowledge base: S is true if it's sunny. H is true if it's warm. R is true if it's raining. E is true if I will enjoy. B is true if I pick up berries. W is true if I'm wet.

This gives the knowledge:

$$
\begin{aligned}
&R_1 : (S \land H) \implies E \equiv \neg S \lor \neg H \lor E\\
&R_2 : (H \land \neg R) \implies B \equiv \neg H \lor R \lor B\\
&R_3 : R \implies \neg B \equiv \neg R \lor \neg B\\
&R_4 : R \implies W \equiv \neg R \lor W\\
&R_5 : H\\
&R_6 : R\\
&R_7 : S
\end{aligned}
$$

a) Using R_3 and R_6 we create the unit resolution rule:

$$\frac{\neg R \lor \neg B, \qquad R}{\neg B}$$

which means that $R_8 : \neg B$ can be added to the knowledge base. We then know that Q_1 will be true.

b) Using R_1 and R_7, we can resolve by doing the following:

$$\frac{\neg S \lor \neg H \lor E, \qquad S}{\neg H \lor E}$$

So $R_9 : \neg H \lor E$ is added to the knowledge base. Then using R_9 and R_5, we get the resolution rule:

$$\frac{\neg H \lor E, \qquad H}{E}$$

So we can also add $R_{10} :\ E$ to the KB. This means that also Q_2 is proven.

c) Using R_4 and R_6 we get the resolution rule:

$$\frac{\neg R \lor W, \qquad R}{W}$$

We add R_{11} to the KB, and we have therefore proven Q_3.

Task 3

3.1

a) $Occupation(Emily, Lawyer) \lor Occupation(Emily, Doctor)$

b) $Occupation(Joe, Actor) \land \exists x\ Occupation(Joe, x) \implies x \neq Actor$

c) $\forall x\ Occupation(x, Surgeon) \implies Occupation(x, Doctor)$

d) $\neg \exists x\ Customer(Joe, x) \implies Occupation(x, Lawyer)$

e) $\exists x\ Boss(x, Emily) \implies Occupation(x, Lawyer)$

f) $\exists x \forall y\ Occupation(x, Lawyer) \land (Customer(y, x) \implies Occupation(y, Doctor))$

g) $\forall x\ \exists y\ Occupation(x, Surgeon) \implies (Customer(y, x) \land Occupation(y, Lawyer))$

3.2

a) $Divisible(x, y) : \exists z\ (z < x) \land (x = y * z)$

b) $Even(x) : Divisible(x, 2)$

c) $Odd(x) : \neg Divisible(x, 2)$

d) $Odd(x) : Even(x) \implies Odd(x + 1)$

e) $Prime(x) : \forall y\ Divisible(x, y) \implies y = x$

f) $\exists! x Prime(x) \land Even(x)$

g) $\forall x \exists k\ (x = \prod_{i=0}^{k} p_i) \implies \forall i\ Prime(p_i)$

Task 4

First we define some predicates and constants:

$Identifies(x, y)$ - a user x is a member of the y fandom. $Likes(x, y)$ - a user x likes the group y. $LikesGenre(x, y)$ - a user x likes the genre y $Sone, Reveluvs, Blinks$ are fandoms. $GG, RV, BP, CH, HE, DJH, SEO, TAE$ are groups. $Dance, Ballads, Drama, Electro$ are genres.

a) Our KB is the following (note x is the universally quantifiable variable):

$$\begin{aligned}
R_1 &: Identifies(x, Sone) \iff Likes(x, GG) \\
&\equiv (\neg Identifies(x, Sone) \lor Likes(x, GG)) \land (\neg Likes(x, GG) \lor Identifies(x, Sone)) \\
R_2 &: Identifies(x, Reveluvs) \iff Likes(x, RV) \\
&\equiv (\neg Identifies(x, Reveluvs) \lor Likes(x, RV)) \land (\neg Likes(x, RV) \lor Identifies(x, Reveluvs)) \\
R_3 &: Identifies(x, Blinks) \iff Likes(x, BP) \\
&\equiv (\neg Identifies(x, Blinks) \lor Likes(x, BP)) \land (\neg Likes(x, BP) \lor Identifies(x, Blinks)) \\
R_4 &: Identifies(x, Reveluvs) \implies LikesGenre(x, Ballads) \\
&\equiv \neg Identifies(x, Reveluvs) \lor LikesGenre(x, Ballads) \\
R_5 &: Identifies(x, Blinks) \implies LikesGenre(x, Dance) \\
&\equiv \neg Identifies(x, Blinks) \lor LikesGenre(x, Dance) \\
R_6 &: (LikesGenre(x, Dance) \land LikesGenre(x, Ballads)) \implies Likes(x, CH) \\
&\equiv \neg LikesGenre(x, Dance) \lor \neg LikesGenre(x, Ballads) \lor Likes(x, CH) \\
R_7 &: (LikesGenre(x, Drama) \land LikesGenre(x, Ballads)) \implies Likes(x, HE) \\
&\equiv \neg LikesGenre(x, Drama) \lor \neg LikesGenre(x, Ballads) \lor Likes(x, HE) \\
R_8 &: (Identifies(x, Sone) \land LikesGenre(x, Electro)) \implies Likes(x, DJH) \\
&\equiv \neg Identifies(x, Sone) \lor \neg LikesGenre(x, Electro) \lor Likes(x, DJH) \\
R_9 &: (Identifies(x, Sone) \land LikesGenre(x, Dance)) \implies Likes(x, SEO) \\
&\equiv \neg Identifies(x, Sone) \lor \neg LikesGenre(x, Dance) \lor Likes(x, SEO) \\
R_{10} &: (Identifies(x, Sone) \land LikesGenre(x, Ballads)) \implies Likes(x, TAE) \\
&\equiv \neg Identifies(x, Sone) \lor \neg LikesGenre(x, Ballads) \lor Likes(x, TAE)
\end{aligned}$$

b) We add the rules to the KB: $R_{11}:\ Identifies(u_1, Reveluvs)$ and $R_{12}:\ Likes(u_1, GG)$. We can then use R_4 to get the resolution rule (and by substituting $\theta = \{x/u_1\}$):

$$\frac{\neg Identifies(u_1, Reveluvs) \lor LikesGenre(u_1, Ballads) \qquad Identifies(u_1, Reveluvs)}{LikesGenre(u_1, Ballads)}$$

$$R_{13}:\ LikesGenre(u_1, Ballads)$$

We then use part of R_1 to prove the user also identifies as a $Sone$

$$\frac{\neg Likes(u_1, GG) \lor Identifies(u_1, Sone) \qquad Likes(u_1, GG)}{Identifies(u_1, Sone)}$$

$$R_{14}:\ Identifies(u_1, Sone)$$

We then use R_{10}:

$$\frac{\neg Identifies(u_1, Sone) \lor \neg LikesGenre(u_1, Ballads) \lor Likes(u_1, TAE) \qquad Identifies(u_1, Sone)}{Likes(u_1, TAE) \lor \neg LikesGenre(u_1, Ballads)}$$

$$\frac{\neg LikesGenre(u_1, Ballads) \lor Likes(u_1, TAE) \qquad LikesGenre(u_1, Ballads)}{Likes(u_1, TAE)}$$

$$R_{15}:\ Likes(u_1, TAE)$$

According to resolution, TAE will be a good recommendation.

c) Using R_7:

$$\frac{\neg LikesGenre(u_1, Drama) \lor \neg LikesGenre(u_1, Ballads) \lor Likes(u_1, HE) \qquad LikesGenre(u_1, Ballads)}{\neg LikesGenre(u_1, Drama) \lor Likes(u_1, HE)}$$

There is no complementary literal for $\neg LikesGenre(u_1, Drama)$, i.e we don't know if u_1 likes $Drama$, nor can we infer it. We therefore are unable to prove that HE will be a good recommendation. Consequentially, HE will not be a good recommendation.

d) We know u_2 is a *Sone*, *Reveluv* and *Blink*, which from R_1, R_2 and R_3 we can infer that u_2 likes GG, RV and BP. From R_4 and R_5, we can infer that u_2 likes *Ballads* and *Dance*. So the system should recommend these genres. From R_6, we infer that CH is a good recommendation. Then, we can also infer from R_{10} that they also will like TAE. To summarize, the system will recommend *Ballads* and *Dance* as the genres, and CH and TAE as the groups.