

Solar winds

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Abstract

1 Theory

A pure dipole centered at the origin produces a magnetic field given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\frac{3\mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3} \right)$$

The equation of motion given by the Lorentz force with $\mathbf{E} = 0$ is

$$\ddot{\mathbf{r}} = \frac{q}{m} \dot{\mathbf{r}} \times \mathbf{B}$$

Introducing the dimensionless variables $\tilde{\mathbf{r}} = \frac{\mathbf{r}}{r_0}$, $\tilde{\mathbf{m}} = \frac{\mathbf{m}}{m_0}$, $\tilde{t} = \frac{t}{t_0}$ the equation of motion can be rewritten to

$$\frac{d^2 \tilde{\mathbf{r}}}{d\tilde{t}^2} = \frac{d\tilde{\mathbf{r}}}{d\tilde{t}} \times \tilde{\mathbf{B}} \quad (1)$$

where

$$\tilde{\mathbf{B}} = \frac{3\tilde{\mathbf{r}}(\tilde{\mathbf{m}} \cdot \tilde{\mathbf{r}}) - \tilde{\mathbf{m}}}{\tilde{r}^3}$$

The constants r_0 , m_0 and t_0 has been chosen to make the velocities in the range 250-750 km/s when \mathbf{r} is in the same magnitude as the earths radius. All the constants used in this problem are displayed in the table below.

constant	value	dimension
q	$1.6 \cdot 10^{-19}$	C
m	$1.67 \cdot 10^{-27}$	kg
r_0	$6371 \cdot 10^3$	m
m_0	$8.22 \cdot 10^{22}$	Am ²
t_0	24.67	s

References