Solar winds

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1 Theory

A pure dipole centered at the origin produces a magnetic field given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{r}(\mathbf{m}\cdot\mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3}\right)$$

The equation of motion, given by the Lorentz force where $\mathbf{E} = 0$ is

$$\ddot{\mathbf{r}} = \frac{q}{m}\dot{\mathbf{r}} \times \mathbf{B}$$

Introducing dimensionless variables $\tilde{\mathbf{r}} = \frac{\mathbf{r}}{r_0}$, $\tilde{\mathbf{m}} = \frac{\mathbf{m}}{m_0}$, $\tilde{t} = \frac{t}{t_0}$, the equation of motion can be rewritten to

$$\frac{d^2\tilde{\mathbf{r}}}{d\tilde{t}^2} = \frac{d\tilde{\mathbf{r}}}{d\tilde{t}} \times \tilde{\mathbf{B}} \tag{1}$$

where

$$\tilde{\mathbf{B}} = C\frac{3\hat{\tilde{\mathbf{r}}}(\tilde{\mathbf{m}}\cdot\hat{\tilde{\mathbf{r}}}) - \tilde{\mathbf{m}}}{\tilde{r}^3}$$

and

$$C = \frac{q_{proton}\mu_0 m_0}{m_{proton} 4\pi v_0 {r_0}^2}$$

The constant C has been chosen such that $\frac{d\tilde{\mathbf{r}}}{d\tilde{t}}|_{\tilde{t}=0} = v_0$. All the other constants used are displayed in the table below.

constant	value	dimension
r_0	$6371 \cdot 10^3$	m
m_0	$8.22 \cdot 10^{22}$	Am^2
v_0	$2.5 \cdot 10^5$	ms^{-1}
q_{proton}	$1.6 \cdot 10^{-19}$	C
m_{proton}	$1.67 \cdot 10^{-27}$	kg

2 Implementation

By rewriting (1) we obtain two first order differential equations

$$\frac{d\tilde{\mathbf{r}}}{d\tilde{t}} = \tilde{\mathbf{v}} \qquad \frac{d\tilde{\mathbf{v}}}{d\tilde{t}} = \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \tag{2}$$

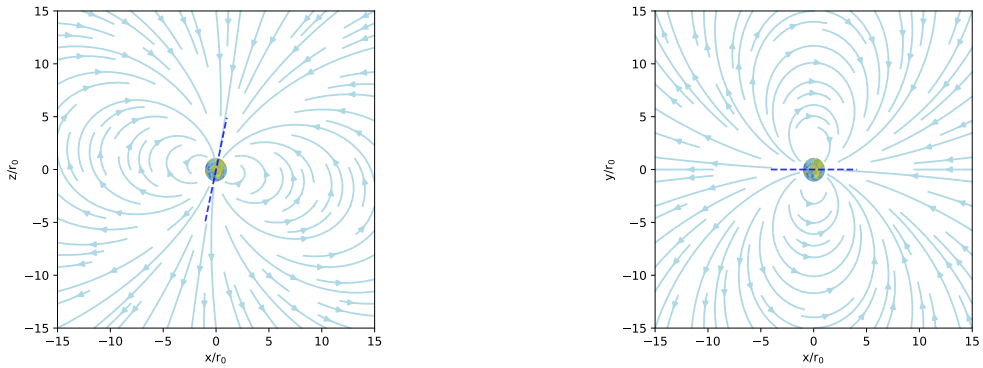
In the implementation $\mathbf{\Lambda}$ is the solution matrix which contains both $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{r}}$ for all timesteps. The ODE can then be written as

$$\frac{d}{d\tilde{t}}[\tilde{\mathbf{r}}, \tilde{\mathbf{v}}]^T = \frac{d\mathbf{\Lambda}}{d\tilde{t}} = f(\mathbf{\Lambda}) = [\tilde{\mathbf{v}}, \tilde{\mathbf{v}} \times \tilde{\mathbf{B}}]^T$$

RK4 has been chosen as a suitable ODE solver. All code can be found at github [1].

3 Results

In figure 1 the magnetic field of the earth has been visualized in two different planes.



(a) xz-plane (b) xy-plane

Figure 1: Magnetic field produced by earth.
The blue striped line represents the direction of
earths magnetic moment (the length is scaled).

References

[1] Syljuåsen, solar_winds,
https://github.com/erlensy/solar_wind