# TTK4135 Exercise 5

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## Problem 1: Open Loop optimal controls

$$x_{t+1} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0.1 & -0.79 & 1.78 \end{bmatrix}}_{A} x_t + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0.1 \end{bmatrix}}_{B} u_t$$

$$y_t = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{C} x_t$$

$$x_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \qquad N < \infty$$

$$f(y_1, \dots, y_N, u_0, \dots, u_{N-1}) = \sum_{t=0}^{N-1} \{ y_{t+1}^2 + r u_t^2 \}, \quad r > 0$$

$r = 1$, $N = 30$.

a) First to locate eigenvalues of $A$: With matlab.

$\lambda_1 = 0.9358$, $\lambda_2 = 0.8442$, $\lambda_3 = 0$.

Stability criteria for discrete state space system

$|\lambda| \leq 1 \Rightarrow$ we have stability.

b) $\underset{3\times 1}{x_t} = \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix}$, $u_t = u_t \leftarrow$ scalar

$$f(z) = \frac{1}{2}\sum_{t=0}^{T-1} \left\{ x_{t+1}^T Q x_{t+1} + u_t^T R u_t \right\}$$

$$\Rightarrow x_{t+1}^T Q x_{t+1} = 2 x_{3(t+1)} \Rightarrow Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow u_t \cdot R \cdot u_t = r u_t^2 \Rightarrow R = r$$

c) Objective function $f(z)$ $x^T Q x + u^T R u$ is convex dependent on the $Q > 0$.

In our case $Q$ positive semidefinite.

$\Rightarrow$ At best our problem is convex

constraints:

Our constrains are linear and therefore convex.

Depends on $Q$ and $R$.

d) $A_{eq} z = b_{eq}$

$\Rightarrow x_1 - Bu_1 = Ax_0 \quad \Rightarrow \quad x_1 = Ax_0 + Bu_1$

$-Ax_1 + x_2 - Bu_2 = 0 \quad \Rightarrow \quad x_2 = Ax_1 + Bu_2$

$-Ax_{N-1} + x_N - Bu_N = 0 \quad \Rightarrow \quad x_N = Ax_{N-1} + Bu_N$

$\min_z f(z) = \frac{1}{2} z^T G z$

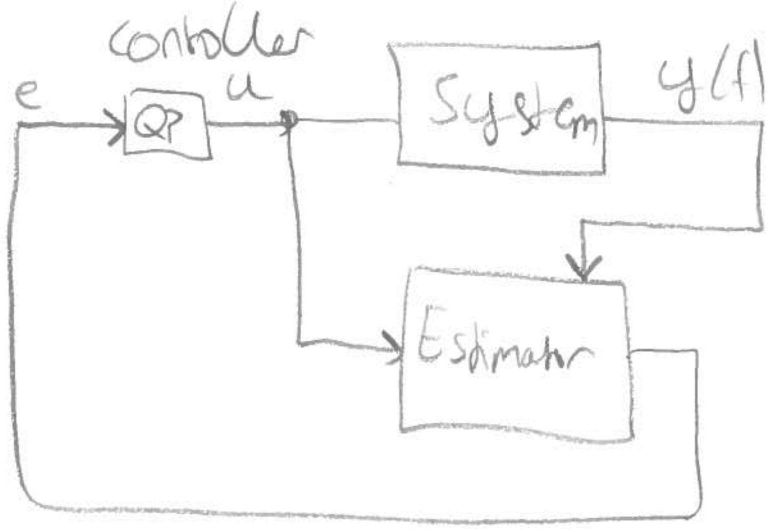
$$G = \begin{bmatrix} Q & 0 & & 0 \\ 0 & & & \\ & & QR & \\ 0 & & 0 & R \end{bmatrix}$$

The KKT system for our problem

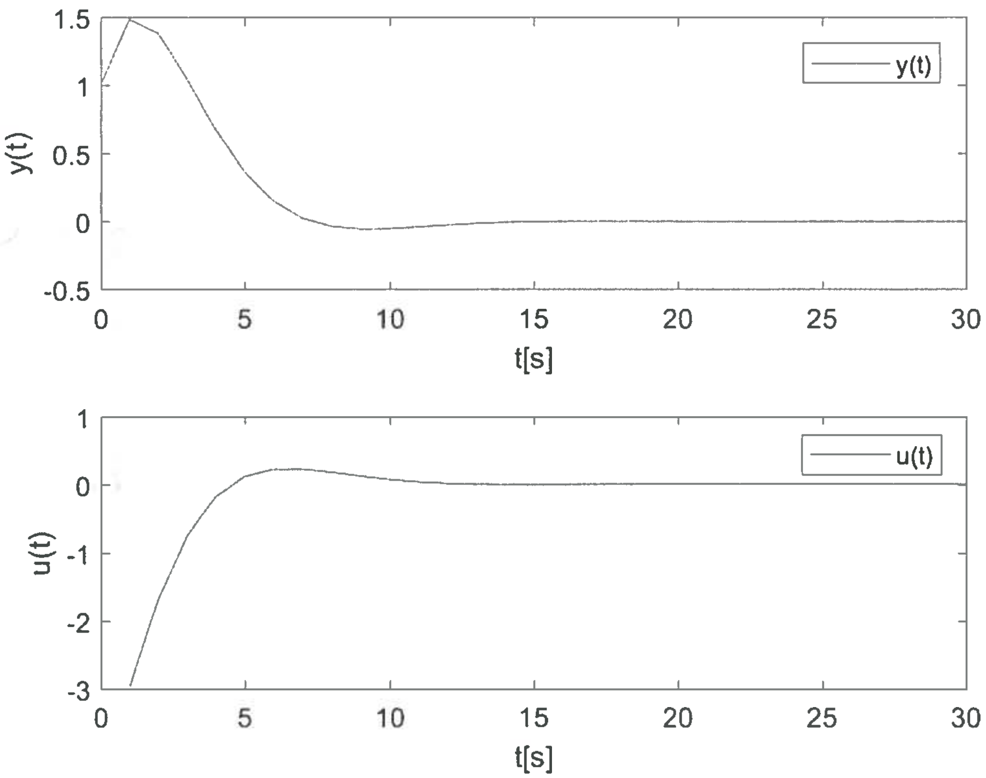
$$\begin{bmatrix} G & -A_{eq}^T \\ A_{eq} & 0 \end{bmatrix} \begin{bmatrix} z^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 0 \\ b_{eq} \end{bmatrix}$$

Problem 2:

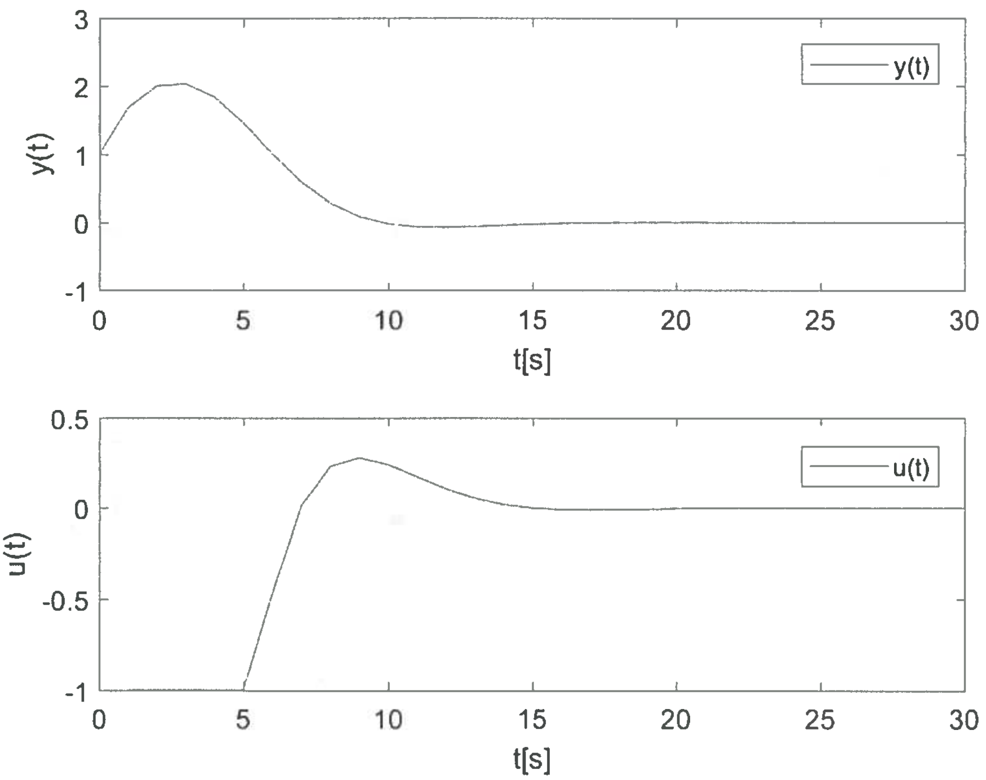
MPC uses ~~clo~~ open loop optimization but calculates a new optimal input sequence at each timestep by measuring the state with feedback. Thus the horizon is moving.



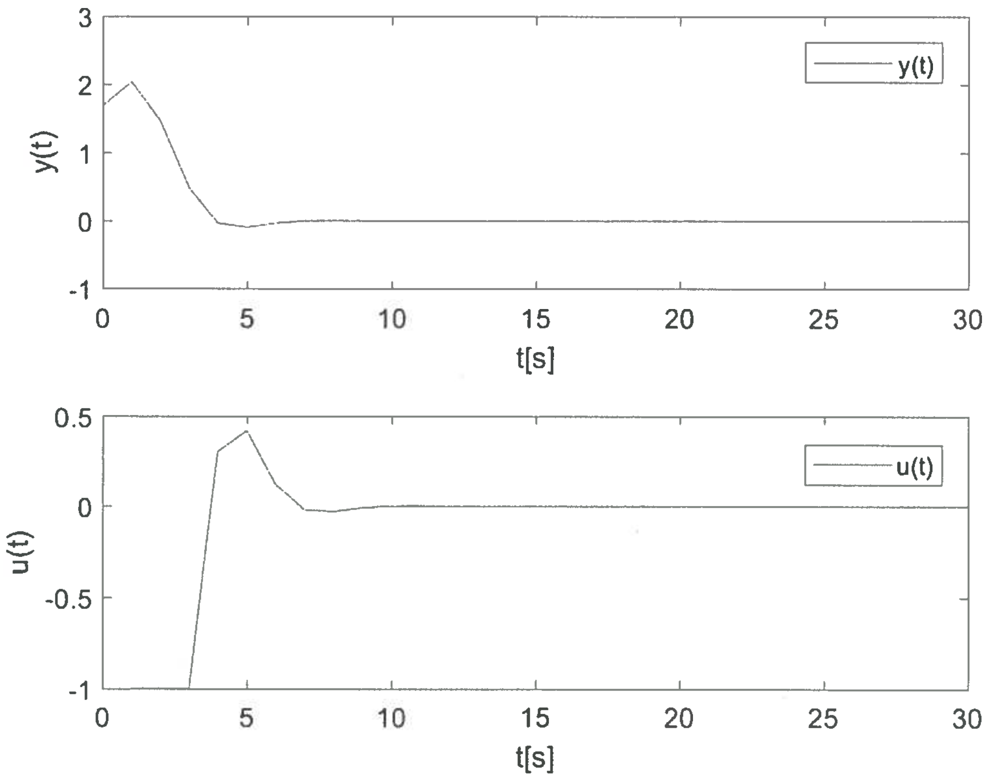
Problem 1e)

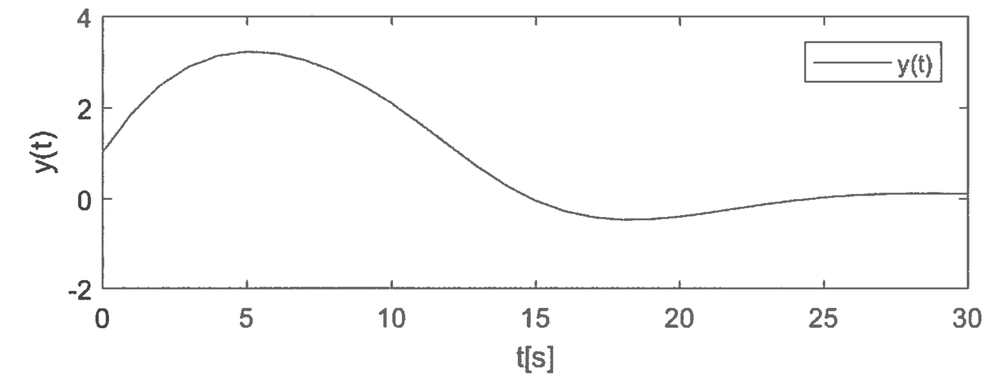


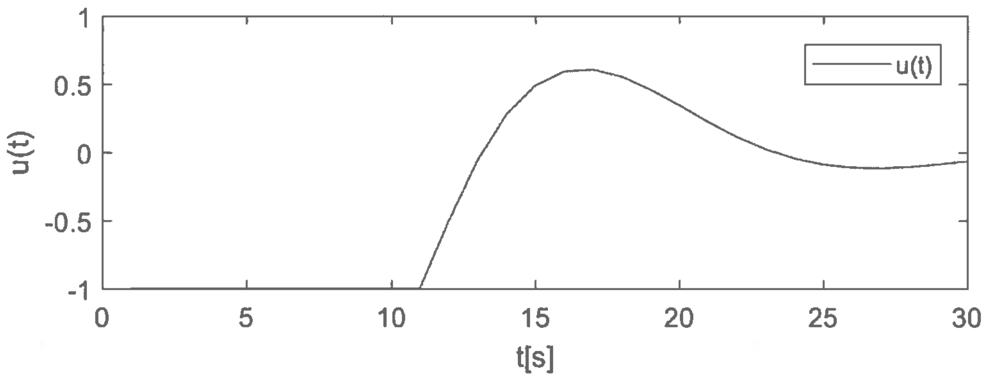
Problem 7f)



Problem 2c




y(t)
t[s]


u(t)
t[s]