TDT4171 Assignment 2

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1 Part A

- What is the set of unobservable variables for a given time-slice?

Answer: Our only unobservable variable is the state variable Rain. Here denoted R_t.

$$\mathbf{X_t} = \{R_t\}$$

- What is the set of observable variables for a given time-slice t?

Answer: Our only observable variabel is wether or no he direcor has an umbrella. This evidence variable is denoted U_t.

$$\mathbf{E_t} = \{U_t\}$$

- Present the dynamic model $\mathbf{P}(\mathbf{X_t}|\mathbf{X_{t-1}})$ and the observable model $\mathbf{P}(\mathbf{E_t}|\mathbf{X_t})$ as matrices.

Answer: Both the dynamic and the observable model are given in Figure 15.2 in [Russel Norvig].

$$\mathbf{P}(\mathbf{X_t}|\mathbf{X_{t-1}}) = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}, \mathbf{P}(\mathbf{E_t}|\mathbf{X_t}) = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

- Which assumptions are encoded in this model?

Answer: A **Markov assumption** has been made about the dynamic model. More specifically we assume the dynamic model to be a first-order Markov process which means that the probabilities for the current state only depends on the previous. We also assume the process to be stationary, that is, the conditional probabilities are constant for all t. The validity of these assumptions depend on the geographical location, but generally probability of rain depends on the season and the weather on more than one previous day. Similarly there has been made a **sensor Markov assumption**. It is assumed that the probability of the observation of an umbrella U_t only is dependent on the state R_t. In reality the probability of bringing an umbrella to work would increase proportionally with the consecutive number of rainy days leading up to this particular day.

2 Part B

See attached Matlab script for documentation of results. I implemented the forward filter mechanism described in Eq. 15.12 [Russel Norvig]

$$\mathbf{P}(\mathbf{X_2}|\mathbf{e_{1:2}}) = \begin{bmatrix}0.883, & 0.117\end{bmatrix} \tag{1}$$
$$\mathbf{f_{1:0}} = \begin{bmatrix}0.5000, & 0.5000\end{bmatrix} \tag{2}$$
$$\mathbf{f_{1:1}} = \begin{bmatrix}0.8673, & 0.1327\end{bmatrix} \tag{3}$$
$$\mathbf{f_{1:2}} = \begin{bmatrix}0.8204, & 0.1796\end{bmatrix} \tag{4}$$
$$\mathbf{f_{1:3}} = \begin{bmatrix}0.3075, & 0.6925\end{bmatrix} \tag{5}$$
$$\mathbf{f_{1:4}} = \begin{bmatrix}0.8204, & 0.1796\end{bmatrix} \tag{6}$$
$$\mathbf{f_{1:5}} = \begin{bmatrix}0.8673, & 0.1327\end{bmatrix} \tag{7}$$
$$\tag{8}$$

3 Part C

Again, see attached Matlab script for documentation of results. I added backward smoothing described in Eq. 15.13 [Russel Norvig].

$$\mathbf{P}(\mathbf{X_1}|\mathbf{e_{1:2]}}) = \begin{bmatrix}0.883, & 0.117\end{bmatrix} \tag{9}$$
$$\mathbf{b_{6:6}} = \begin{bmatrix}1.0000, & 1.0000\end{bmatrix} \tag{10}$$
$$\mathbf{b_{5:6}} = \begin{bmatrix}0.6900, & 0.4100\end{bmatrix} \tag{11}$$
$$\mathbf{b_{4:6}} = \begin{bmatrix}0.4593, & 0.2437\end{bmatrix} \tag{12}$$
$$\mathbf{b_{3:6}} = \begin{bmatrix}0.0906, & 0.1503\end{bmatrix} \tag{13}$$
$$\mathbf{b_{2:6}} = \begin{bmatrix}0.0661, & 0.0455\end{bmatrix} \tag{14}$$
$$\mathbf{P}(\mathbf{X_1}|\mathbf{e_{1:5]}}) = \begin{bmatrix}0.8673, & 0.1327\end{bmatrix} \tag{15}$$