

$$1) f) T: \overset{V}{\mathbb{P}_2(\mathbb{R})} \rightarrow \overset{W}{\mathbb{R}} \text{ tal que } T(a_0 + a_1x + a_2x^2) = \int_0^1 (a_0 + a_1x + a_2x^2) dx, \quad v = (1, 2, 0), \quad w = 1$$

$$\begin{aligned} \dots T(a_0 + a_1x + a_2x^2) &= \int_0^1 (a_0 + a_1x + a_2x^2) dx = a_0 \int_0^1 dx + a_1 \int_0^1 x dx + a_2 \int_0^1 x^2 dx \\ &= a_0 + \frac{1}{2}a_1 + \frac{1}{3}a_2 \end{aligned}$$

$$\begin{aligned} \dots i) \quad T(1) &= 1 + \frac{1}{2} \cdot 0 + \frac{1}{3} \cdot 0 = 1 \\ T(x) &= 0 + \frac{1}{2} + 0 = \frac{1}{2} \\ T(x^2) &= 0 + 0 + \frac{1}{3} = \frac{1}{3} \end{aligned} \quad \Rightarrow \quad A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \end{bmatrix}$$

$$\begin{aligned} \dots ii) \quad \text{Im}(T) &= \{w \in W \text{ tal que } w = T(v) \text{ para algum } v \in V\} \\ &= \{w \in \mathbb{R} \text{ tal que } w = T(v) \text{ para algum } v \in \mathbb{P}_2(\mathbb{R})\} \end{aligned}$$

Utilizando a base canônica de \mathbb{R} ($= \{1\}$), vamos determinar $\text{Im}(T)$, sabendo que $\dim(\text{Im}(T)) = \text{posto}(T) = 1$:

$$T(v) = 1 \Rightarrow v =$$