1) b) $q(v) = x_1^2 - 2x_2x_3 + x_1x_3$

i) $v_1 = (x_1, x_2, x_3)$

$v_2 = (y_1, y_2, y_3)$

$$B(v_1, v_2) = \frac{1}{2}\left[q(v_1+v_2) - q(v_1) - q(v_2)\right]$$

$$q(v_1+v_2) = (x_1+y_1)^2 - 2(x_2+y_2)(x_3+y_3) + (x_1+y_1)(x_3+y_3)$$

$$= \cancel{x_1^2} + 2x_1y_1 + \cancel{y_1^2} - \cancel{2x_2x_3} - 2x_2y_3 - 2y_2x_3 - \cancel{2y_2y_3} + \cancel{x_1x_3} + x_1y_3 + y_1x_3 + \cancel{y_1y_3}$$

$$q(v_1) = \cancel{x_1^2} - \cancel{2x_2x_3} + \cancel{x_1x_3}$$

$$q(v_2) = \cancel{y_1^2} - \cancel{2y_2y_3} + \cancel{y_1y_3}$$

$$B(v_1, v_2) = \frac{1}{2}\left[2x_1y_1 - 2x_2y_3 - 2y_2x_3 + x_1y_3 + y_1x_3\right]$$

$$= x_1y_1 - x_2y_3 - y_2x_3 + \frac{1}{2}x_1y_3 + \frac{1}{2}y_1x_3$$

$$A = \begin{bmatrix} B((1,0,0),(1,0,0)) & B((1,0,0),(0,1,0)) & B((1,0,0),(0,0,1)) \\ B((0,1,0),(1,0,0)) & B((0,1,0),(0,1,0)) & B((0,1,0),(0,0,1)) \\ B((0,0,1),(1,0,0)) & B((0,0,1),(0,1,0)) & B((0,0,1),(0,0,1)) \end{bmatrix}$$

4) b) ii) $q(x_1, x_2, x_3) = x_1^2 - 2x_2x_3 + x_1x_3$

$$A = \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 0 & -1 \\ 1/2 & -1 & 0 \end{bmatrix}$$

iii) $\lambda_1 = 1.381$, $\lambda_2 = 0{,}682$, $\lambda_3 = -1.062$

IV) Temos 1 autovalor negativo, logo $I = 1$.

Temos 3 autovalores diferentes e não nulos, logo posto $= 3$.

Como A tem autovalores tanto negativos quanto positivos, então não podemos determiná-la como nada além de <u>indefinida</u>, entre os itens.

V) $v^{(1)} = \begin{pmatrix} 1.373 \\ -0.724 \\ 1 \end{pmatrix}$, $v^{(2)} = \begin{pmatrix} -1.571 \\ -1.467 \\ 1 \end{pmatrix}$, $v^{(3)} = \begin{pmatrix} -0.242 \\ 0.941 \\ 1 \end{pmatrix}$

Como os autovetores são ortogonais pois provêm de autovalores diferentes, vamos apenas normalizar os autovetores:

$\frac{1}{\|v^{(1)}\|} \cdot v^{(1)} \approx \begin{pmatrix} 0.729 \\ -0.402 \\ 0.555 \end{pmatrix}$, $\frac{v^{(2)}}{\|v^{(2)}\|} \approx \begin{pmatrix} -0.663 \\ -0.619 \\ 0.422 \end{pmatrix}$, $\frac{v^{(3)}}{\|v^{(3)}\|} \approx \begin{pmatrix} -0.174 \\ 0.675 \\ 0.717 \end{pmatrix}$

Tomando $\bar{v}_1 = \frac{v^{(1)}}{\|v^{(1)}\|}$, $\bar{v}_2 = \frac{v^{(2)}}{\|v^{(2)}\|}$, $\bar{v}_3 = \frac{v^{(3)}}{\|v^{(3)}\|}$

$$B_2 = \{ \bar{v}_1 , \bar{v}_2 , \bar{v}_3 \}$$

VI) $v = x_1' v_1 + x_2' v_2 + x_3' v_3$ e $q(v) = \lambda_1 (x_1')^2 + \lambda_2 (x_2')^2 + \lambda_3 (x_3')^2$

$\Rightarrow q(v) \approx (1.381) \cdot (x_1')^2 + (0.682)(x_2')^2 - (1.062) \cdot (x_3')^2$

1) b) v ii) $v = (e_1, e_2, e_3) \in \mathbb{R}^3$

$(1,0,0) = a(\underbrace{1.313, -0.724, 1}_{v_1}) + b(\underbrace{-1.571, -1.467, 1}_{v_2}) + c(\underbrace{-0.242, 0.941, 1}_{v_3})$

$e_1 = 0.404 \cdot v_1 - 0.279\, v_2 - 0.124 \cdot v_3$ $\begin{cases} a \simeq 0.404 \\ b \simeq -0.279 \\ c \simeq -0.124 \end{cases}$

$e_2 = a(1.313, -0.724, 1) + b(-1.571, -1.467, 1) + c(-0.242, 0.941, 1)$

$e_2 \simeq -0.223 \cdot v_1 - 0.261 \cdot v_2 + 0.484 \cdot v_3$ $\begin{cases} a \simeq -0.223 \\ b \simeq -0.261 \\ c \simeq 0.484 \end{cases}$

$e_3 = a v_1 + b v_2 + c v_3$

$e_3 \simeq 0.307 \cdot v_1 + 0.177 \cdot v_2 + 0.514 \cdot v_3$ $\begin{cases} a \simeq 0.307 \\ b \simeq 0.177 \\ c \simeq 0.514 \end{cases}$

$[v]_{B_1} \simeq e_1 + e_2 + e_3$

$\simeq (0.404 - 0.223 + 0.307) \cdot v_1 + (-0.279 - 0.261 + 0.177) \cdot v_2 +$

$+ (-0.124 + 0.484 + 0.514)\, v_3$

$\simeq 0.488 \cdot v_1 - 0.363 \cdot v_2 + 0.874 \cdot v_3$

$[v]_{B_2} = (0.488, -0.363, 0.874)$

$q(\overset{(1,1,1)}{v}) = (1)^2 - 2.(1 \cdot 1) + 1$
$= 0$

$q([v]_{B_2}) = (1.381) \cdot (0.488)^2 + (0.682)(-0.363)^2 - (1.062)(0.874)^2$
$\simeq 0.328 + 0.089 - 0.811 = -0.394 \neq 0$ (São diferentes)

1)b) v.iii) $q(v) = (x''_1)^2 + (x''_2)^2 - (x''_3)^2$