

The density operator is

$$\hat{\rho} = |\varphi_\alpha\rangle\langle\varphi_\alpha| \quad (1)$$

and the s.p. states are propagated with

$$\partial_t|\varphi_\alpha\rangle = -i\hat{h}|\varphi_\alpha\rangle \Rightarrow \partial_t\langle\varphi_\alpha| = i\langle\varphi_\alpha|\hat{h} \quad (2)$$

This yields

$$\begin{aligned} \partial_t\hat{\rho} &= (\partial_t|\varphi_\alpha\rangle)\langle\varphi_\alpha||\varphi_\alpha\rangle(\partial_t\langle\varphi_\alpha|) = -i\hat{h}|\varphi_\alpha\rangle\langle\varphi_\alpha| + i|\varphi_\alpha\rangle\langle\varphi_\alpha|\hat{h} \\ &= -i\left[\hat{h}, |\varphi_\alpha\rangle\langle\varphi_\alpha|\right] = -i\left[\hat{h}, \hat{\rho}\right] \end{aligned}$$