Studying the flow rate on a multi-lane using a cellular automata model

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Introduction

Background

When planning efficient traffic infrastructure achieving efficient and safe traffic flow is of course very important. A possible way of measuring how well traffic is flowing on a road is by studying the flow rate. The flow rate on a road is defined as the sum of the velocities of the cars on the road divided by the length of the road. In real life situations maximizing the flow rate could thus often be of interest. Another interesting metric to study is the fluctuations of the flow rate. If the flow rate does not fluctuate much the traffic flow is behaving in a stable way which could imply easier driving conditions and a reduced risk of accidents.
In this report we will investigate how the flow rate behavior depends on the amount of lanes on the road and the car density by simulating a highway using a cellular automata model.

Model description

The model of the highway was implemented as a cellular automata on a road of some integer distance L where each car on the road occupies 1 length unit of the road. The road used periodic boundary conditions, meaning that the road does not have an end nor a beginning. The between the cars therefore also had to be calculated as periodic distances. In order to make the dynamics of the simulation more realistic all cars were given personal maximum velocity $v_{i,max}$. The value of $v_{i,max}$ for each car was randomly generated as a rounded value taken from a Gaussian distribution around some mean value μ, where μ would represent the speed limit on the road A Gaussian distribution of the values of $v_{i,max}$ thus seems logical since in real life most cars drive at approximately the speed limit even though a few cars drive much faster or slower.
The rules for the cellular automata were performed in two different parts at each time step. First, rules determining what lane changes are going to be performed are implemented. This was done by first calculating desired lane change of each car and then performing the desired lane change only if the lane change can be performed safely. Secondly, rules determining how the various cars are going to change their velocity are implemented. Each rule was implemented simultaneously on each car before moving on to the next rule. This means that the ordering of the rules is very important, as

latter rules can 'overrule' previous decisions. In order to make the rules more compact we also introduce the following notation

- x_i is the position of the i:th car
- v_i is the velocity of the i:th car
- $v_{i,max}$ is the maximum velocity of the i:th car.
- v_{back} is the velocity to the next car backward in the **new lane** we would like to change to.
- $d_{forward}$ is the distance forward to the next car in the same lane we are currently in.
- $d_{forward,left}$ is the distance forward to the next car in the lane to the left.
- $d_{forward,right}$ is the distance forward to the next car in the lane to the right.
- $d_{backward}$ is the distance backward to next car in the **new lane** we would like to change to.

Using the above notation the rules of the cellular automata were implemented as follows:

Lane changes

1. If $v_i < v_{i,max}$ and $d_{forward,left} \geq d_{forward}$ and there exists a lane to the left, desire a lane change from right to left
2. If $d_{forward} > v_{i,max}$ and $d_{forward,right} > v_{i,max}$ and there exists a lane to the right, desire a lane change from left to right
3. If $d_{forward,right} > d_{forward}$ and no previous lane change desire exists, desire a lane change from left to right with 5% probability
4. If $d_{backward} > v_{back}$ perform desired lane change (car behind should not need to brake because of lane change)

Velocity changes

5. If $v_i < v_{i,max}$ increase velocity as $v_i \rightarrow v_i + 1$ (try to accelerate to maximum velocity)
6. If $v \geq d_{forward}$ decrease velocity as $v = d_{forward} - 1$ (avoid collisions)
7. With some probability p decrease velocity as $v_i = v_i - 1$ (cars might brake randomly)
8. Update position as $x_i(t+1) = x_i(t) + v_i$

Most rules above are quite logical and result in a behavior of the cars that appears natural. Rule (3) might however need some clarification. Without rule (3) mostly the left lane becomes occupied for higher car densities. In order to solve this rule (3) creates a bias towards changing to the right lane which solves the problem.

Problem description

The purpose of this report is to simulate a multi-lane highway and compare how the flow rate is affected by varying the number of lanes at different car densities. To do a statistical analysis of the result we would also like to examine the standard error of the flow rate and investigate whether the standard error of the flow rate varies depending on the car density and the number of lanes.

Method

Setting the system size

Initially we must choose a system size to run the simulations on. The system size must be chosen so that finite size effects are negligble. The values of $v_{i,max}$ are sampled from a Gaussian distribution with mean $\mu = 10$ and standard deviation $\sigma = 1$. The braking probability is set to $p = 0.2$. These simulation parameters will remain unchanged during all following simulations. Fundamental diagrams are generated by increasing the number of cars n_{cars} so that the car density, defined as the number of cars divided by the road length, varies between 0 and 1. The flow rate is calculated as an average over 100 time steps after the system appears to have equilibrated. This is done using the different values of the road length $L = 20, 40, 60, 80, 100$. Seperate fundamental diagrams are generated using 1, 2 and 3 lanes. The road length L for the following simulations is finally set to a value where the fundamental diagrams appear to have become independant of the system size.

Flowrate

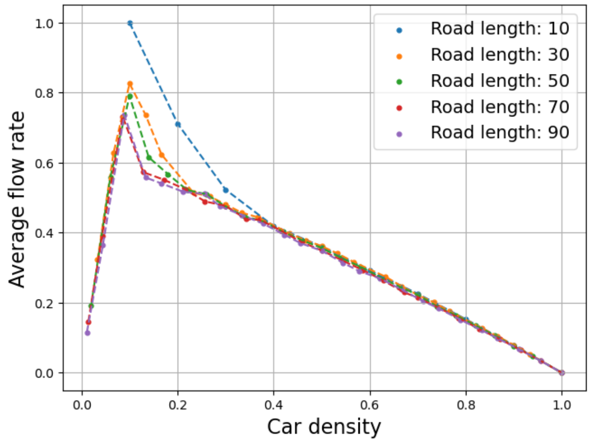
To get an better understanding of the flow rate behavior and the time necessary to reach equilibrium, the flow rate is plotted against time using various different car densities and the road length L found in the previous section. Fundamental diagrams are also generated for using 1, 2 and 3 lanes.

Statistical analysis

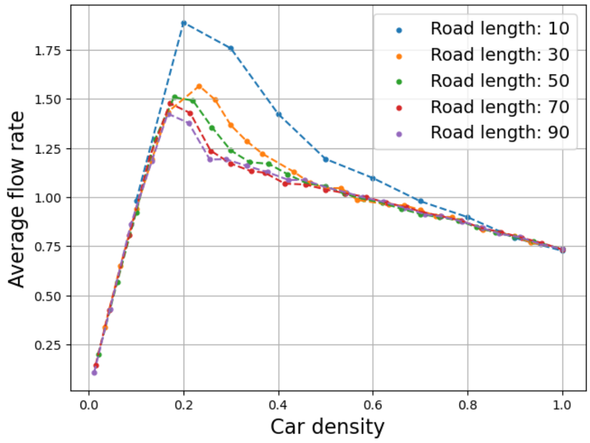
In order to examine the statistical accuracy of the results we will study the standard error.

Results and analysis

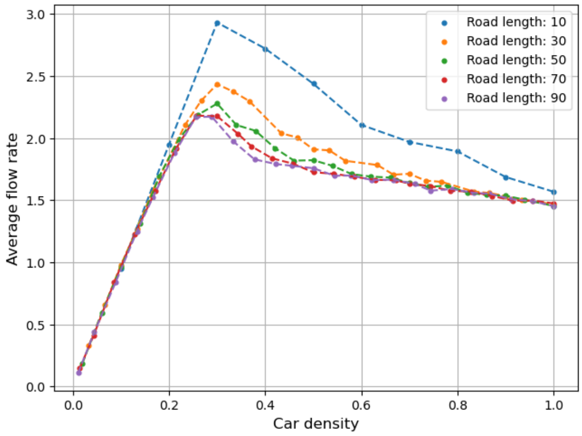
Setting the system size



a) 1 lane



b) 2 lanes

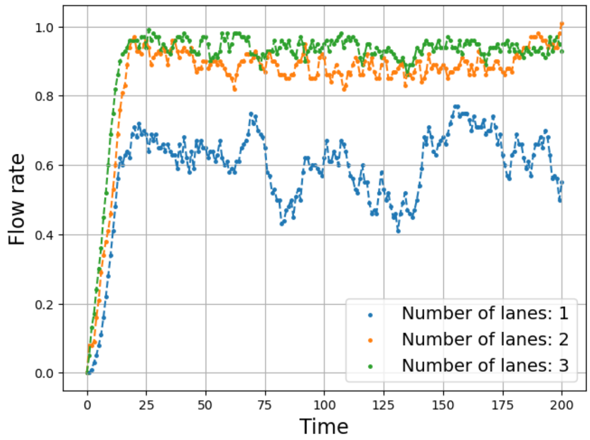


c) 3 lanes

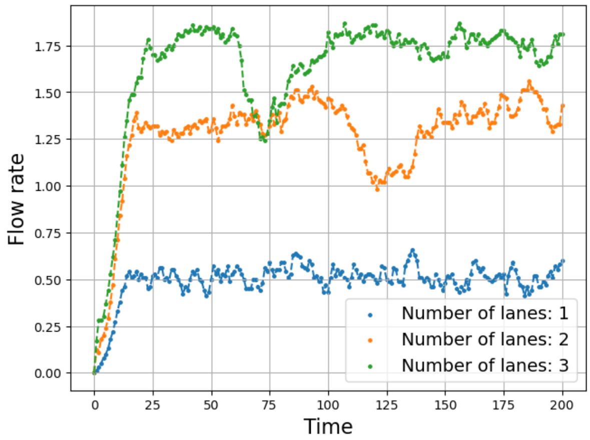
Figure 1: Fundamental diagrams using different road lenghts L using 1, 2 and 3 lanes.

Flowrate

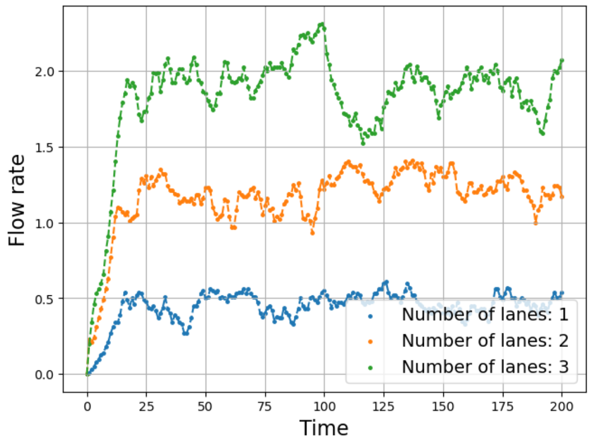
We plot the flowrate over time while using road length $L = 100$, and setting the number of cars to $n_cars = 10, 20, 30, 40$. This is done using 1, 2 and 3 lanes. The resulting plots of the flowrate against time can be seen below in figure 2



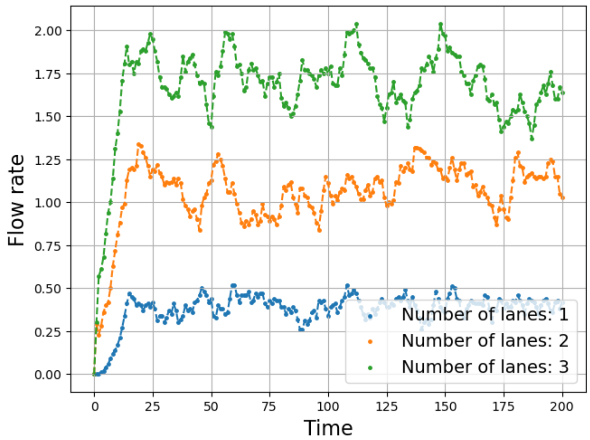
a) 10 cars



b) 20 cars



c) 30 cars



d) 40 cars

Figure 2: Flowrate plotted against time using different number of cars.

In figure 2 we notice multiple interesting things with the flowrate behavior. When $n_{cars} = 10$ there is a large gain in the flowrate when increasing the number of lanes from 1 to 2, while the difference between 2 and 3 lanes is small. The fluctuations of the flowrate for the road with 1 lane also appears to be much larger than when using 2 or 3 lanes. When setting $n_{cars} = 20, 30$ the we start to see a large gain in the average flow rate when using 3 lanes instead 2. Interestingly we also notice that the fluctuations seems to have decreased when using 1 lane and increased when using 2 and 3 lanes. Similar results are seen with $n_cars = 40$ as we are getting large differences in the averge flow rate using different lanes, with increasing fluctuations of the flow rate as the number of lanes increase. It thus appears as if the car density has a large effect on both the average flow rate and the fluctuations of the flow rate.
To get a better understanding of the system dynamics fundamental diagrams are plotted. The average flowrate is calculated as an average over the last 100 time steps when running the simulation for 200 time steps. The road length $L = 100$ is held constant and n_{cars} is iteratively increased in order to increase the car density between 0 and 1. The results are seen below in figure 3.

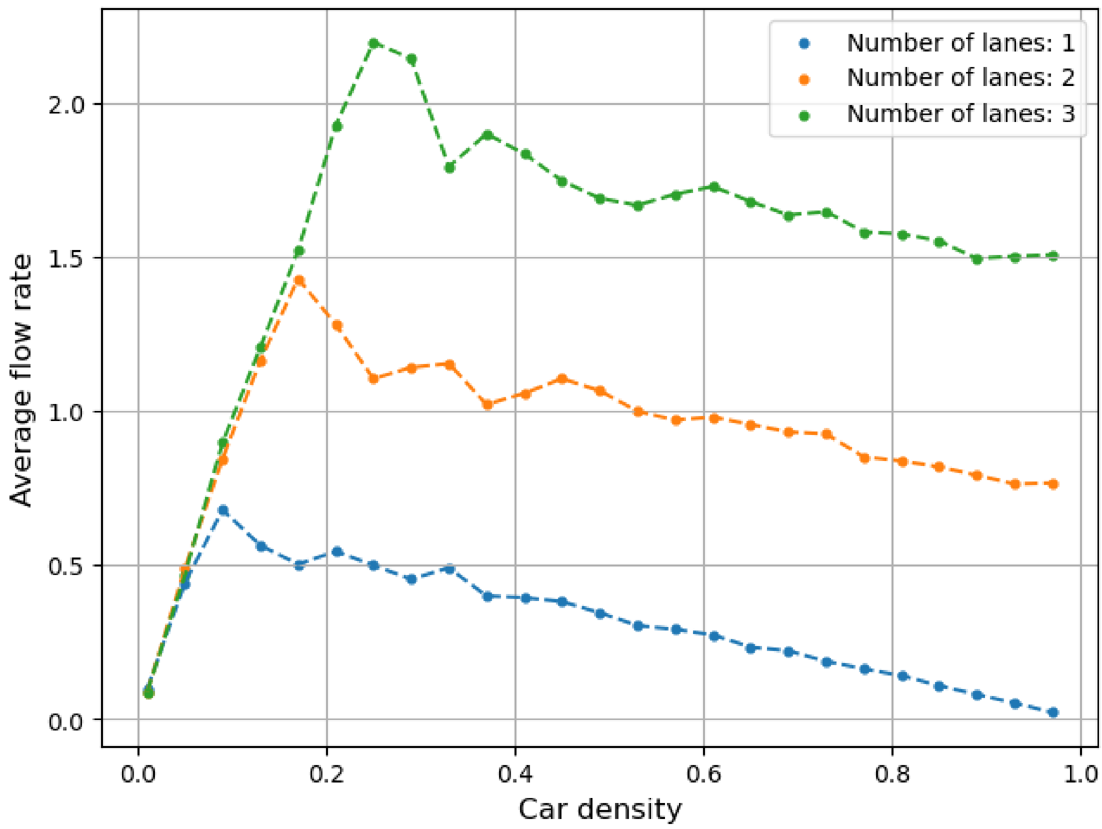
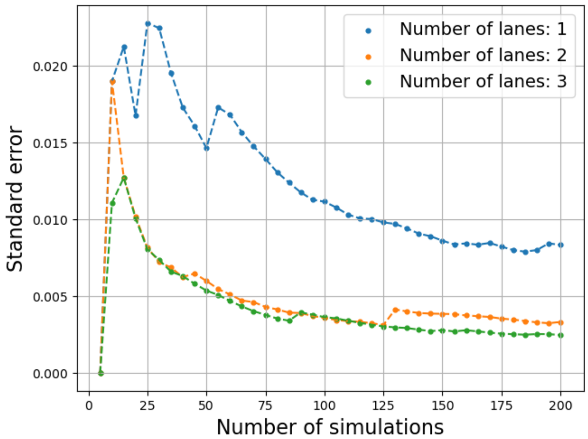


Figure 3: Average flow rate plotted against car density for different number of lanes.

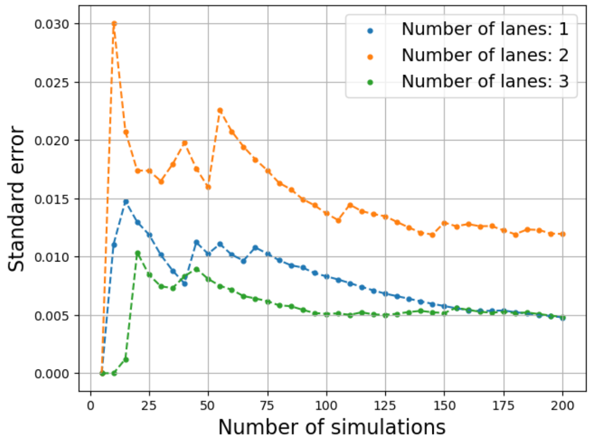
In 2 we see that the flow rate appears to grow linearly until a clear maximum value is reached. Increasing the car density beyond that value the flow rate appears to decrease, also somewhat linearly. For low car densities the average flow rate appears independant of the number of lanes. This is reasonable since we should not be expecting a gain in the flowrate by having a large amount of lanes when very few cars are on the road. For car densities larger than approximately 0.3 we see a large gain in the average flow rate when increasing the number of lanes. Interestingly, the gain in the flow rate when increasing the number of lanes from 1 to 2 appears to be exactly the same as the gain acquired when increasing the number of lanes of 2 to 3. Overall, the results appear logical and serve as a good confirmation that the model appears to behave realistically.

Statistical accuracy

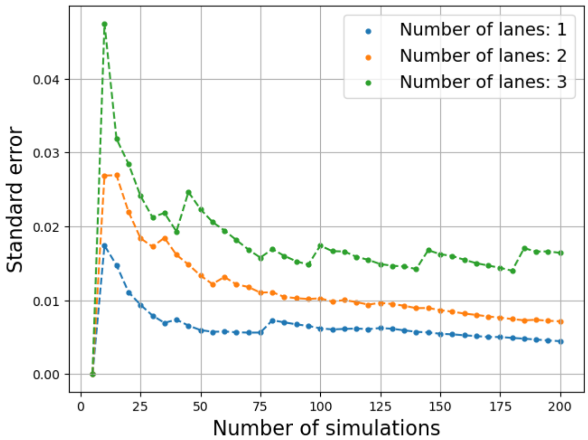
We will now examine the statistical accuracy by calculating the standard error. Just like in the previous section we initially run the simulation using $n_{cars} = 10, 20, 30, 40$. Simulations are run for 200 time steps and the flow rate is calculated as an average over the last 100 time steps. We run the simulations a number of times and calculate the standard error of the flow rate values using 1, 2 and 3 lanes. Diagrams showing the standard error of the flow rate plotted against the number of simulations run are seen below in figure 4



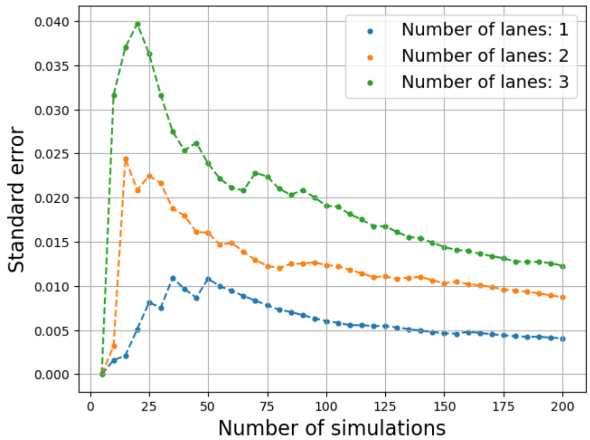
a) 10 cars



b) 20 cars



c) 30 cars



d) 40 cars

Figure 4: Standard error of average flowrate plotted against amount of simulations using different number of cars.

When using $n_{cars} = 10$ it is clear that the standard error of the flow rate is much larger when using 1 lane compared to 2 or 3 lanes. When $n_{cars} = 20$ the standard error is largest when using 2 lanes and the standard error when using 1 lane appears to have decreased. Setting the number of cars to $n_{cars} = 20, 30$ we instead see the highest standard error when using 3 lanes and lower standard error than before when using 1 or 2 lanes. These large differences in the standard error depending on the number of lanes and the car density appear to be consistent with the results seen previously in figure 2.

To get a better understanding of the stand error dependance on the car density we calculate the standard error for varying car densities. The road length remained fixed at $L = 100$ and n_{cars} varies in order to vary the car density between 0 and 1. For each car density 100 simulation are run for 200 time steps where the flow rate is calculated as an average over the last 100 time steps. The standard error of these values is than calculated. This is done using 1, 2 and 3 lanes. The results can be seen below in figure 5

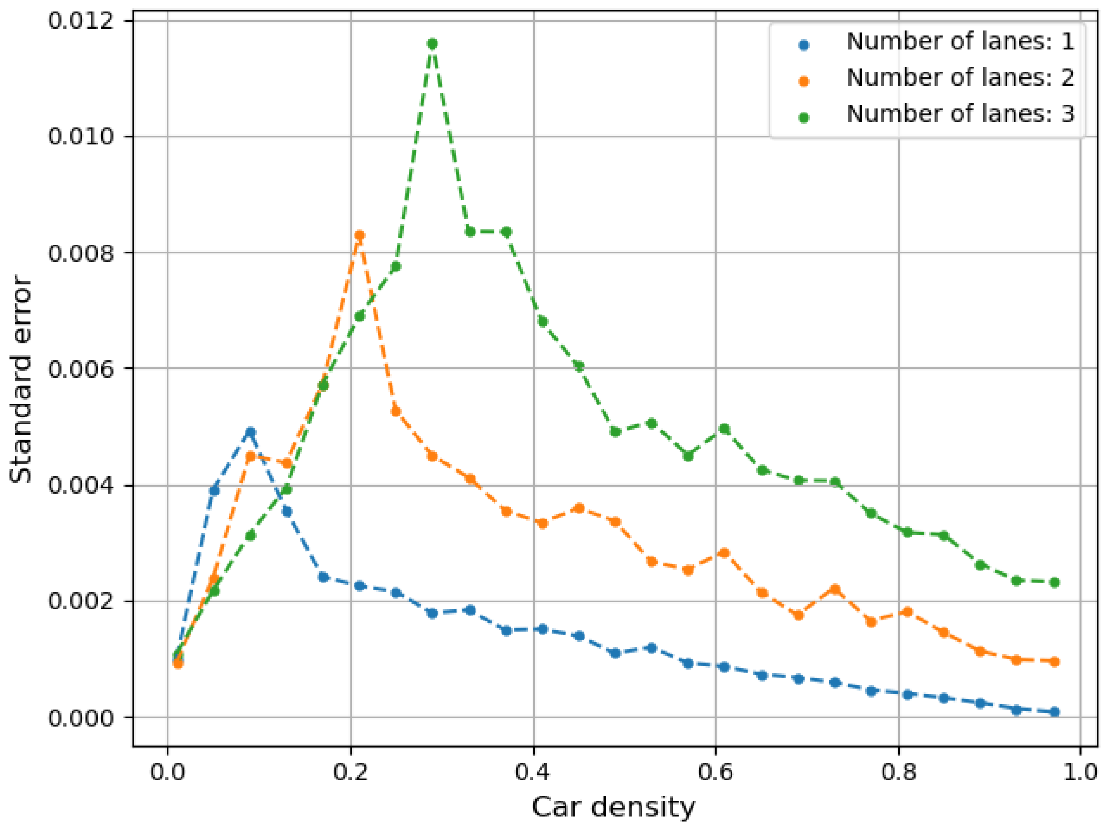


Figure 5: Standard error of average flow rate plotted against car density for different number of lanes.

In figure 5 we notice that the plots of the standard error against the car density are somewhat similar to the fundamental diagrams in figure 3. For all number of lanes, we initially see a clear increase of the standard error as the car density increases until a clear peak value is reached. Increasing the car density further makes the standard error decrease. When comparing the different number of lanes we primarily see two main differences. Firstly, we notice that the car density for which the standard error reaches its maximum value is larger when increasing the number of lanes. Secondly, we also notice that the value of the maximum standard error increases when increasing the number of lanes. If we compare the fundamental diagrams in figure 3 and figure 5 it appears as if the car densities for which the flow rate reaches its maximum value is vary similar to the car densities for which the standard error reaches its maximum value. This behavioir is very interesting and will be analyzed further in the discussion.

Discussion

Results discussion

Model discussion

In order to construct the above model several assumptions have been made. For example, the drivers have been assumed to be quite skilled since a car will never perform a lane change if that means that a car behind will have to brake. In real life, this is of

course true for some drivers but certainly not all.
Another assumption that has to be made since we are using the cellular automata is the fact that drivers only base their decisions on the current state. In other words, the drivers can never plan their driving in a way that skilled drivers would to in real life. For example ....
Another important aspect to mention are the possible consequences of using the period boundary conditions. When using period boundary conditions there is a risk that a car might detect itself as the car in front. The car would then calculate the period distance to itself and adjust its speed accordingly. This is, of course, highly unrealistic and something we must be aware of when using the period boundary conditions. In order to reduce the risk of this happening very short road lengths L were avoided when running the simulation.

Summarizing conclusions