TABLE 5-2

Random variable	Probability density function $f_x(x)$	Mean	Variance	Characteristic function $\Phi_x(\omega)$
Normal or Gaussian $N(\mu, \sigma^2)$	$\dfrac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$	μ	σ^2	$e^{j\mu\omega - \sigma^2\omega^2/2}$
Log-normal	$\dfrac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\ln x - \mu)^2/2\sigma^2}$, $x \geq 0$,			
Exponential $E(\lambda)$	$\lambda e^{-\lambda x}, x \geq 0, \lambda > 0$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$	$(1 - j\omega/\lambda)^{-1}$
Gamma $G(\alpha, \beta)$	$\dfrac{x^{\alpha-1}}{\Gamma(\alpha)\beta^\alpha}e^{-x/\beta}$, $x \geq 0, \alpha > 0, \beta > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1 - j\omega\beta)^{-\alpha}$
Erlang-k	$\dfrac{(k\lambda)^k}{(k-1)!}x^{k-1}e^{-k\lambda x}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{k\lambda^2}$	$(1 - j\omega/k\lambda)^{-k}$
Chi-square $\chi^2(n)$	$\dfrac{x^{n/2-1}}{2^{n/2}\Gamma(n/2)}e^{-x/2}, x \geq 0$	n	$2n$	$(1 - j2\omega)^{-n/2}$
Weibull	$\alpha x^{\beta-1}e^{-\alpha x^\beta/\beta}$, $x \geq 0, \alpha > 0, \beta > 0$	$\left(\dfrac{\beta}{\alpha}\right)^{1/\beta}\Gamma\left(1+\dfrac{1}{\beta}\right)$	$\left(\dfrac{\beta}{\alpha}\right)^{2/\beta}\left[\Gamma\left(1+\dfrac{2}{\beta}\right) - \left(\Gamma\left(1+\dfrac{1}{\beta}\right)\right)^2\right]$	—
Rayleigh	$\dfrac{x}{\sigma^2}e^{-x^2/2\sigma^2}, x \geq 0$	$\sqrt{\dfrac{\pi}{2}}\sigma$	$(2 - \pi/2)\sigma^2$	$\left(1 + j\sqrt{\dfrac{\pi}{2}}\sigma\omega\right)e^{-\sigma^2\omega^2/2}$
Uniform $U(a, b)$	$\dfrac{1}{b-a}, a < x < b$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	$\dfrac{e^{jb\omega} - e^{-ja\omega}}{j\omega(b-a)}$
Beta $\beta(\alpha, \beta)$	$\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1, \alpha > 0, \beta > 0$	$\dfrac{\alpha}{\alpha+\beta}$	$\dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	—
Cauchy	$\dfrac{\alpha/\pi}{(x-\mu)^2+\alpha^2}$, $-\infty < x < \infty, \alpha > 0$	—	∞	$e^{j\mu\omega}e^{-\alpha|\omega|}$
Rician	$\dfrac{x}{\sigma^2}e^{-\frac{x^2+a^2}{2\sigma^2}}I_0\left(\dfrac{ax}{\sigma^2}\right)$, $-\infty < x < \infty, a > 0$	$\sigma\dfrac{\sqrt{\pi}}{2}[(1+r)I_0(r/2)$ $+ rI_1(r/2)]e^{-r/2}$, $r = a^2/2\sigma^2$	—	—
Nakagami	$\dfrac{2}{\Gamma(m)}\left(\dfrac{m}{\Omega}\right)^m x^{2m-1}e^{-\frac{m}{\Omega}x^2}$, $x > 0$	$\dfrac{\Gamma(m+1/2)}{\Gamma(m)}\sqrt{\dfrac{\Omega}{m}}$	$\Omega\left(1 - \dfrac{1}{m}\left(\dfrac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right)$	—

TABLE 5-2
(Continued)

Random variable	Probability density function $f_x(x)$	Mean	Variance	Characteristic function $\Phi_x(\omega)$
Students' $t(n)$	$\dfrac{\Gamma((n+1)/2)}{\sqrt{\pi n}\,\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$, $-\infty < x < \infty$	0	$\dfrac{n}{n-2}, n > 2$	—
F-distribution	$\dfrac{\Gamma((m+n)/2)}{\Gamma(m/2)\Gamma(n/2)}\left(\dfrac{m}{n}\right)^{m/2} x^{m/2-1}$ $\times\left(1+\dfrac{mx}{n}\right)^{-(m+n)/2}, x > 0$	$\dfrac{n}{n-2}, n > 2$	$\dfrac{n^2(2m+2n-4)}{m(n-2)^2(n-4)}, n > 4$	—
Bernoulli	$P(X = 1) = p$, $P(X = 0) = 1 - p = q$	p	$p(1 - p)$	$pe^{j\omega} + q$
Binomial $B(n, p)$	$\dbinom{n}{k}p^k q^{n-k}$, $k = 0, 1, 2, \ldots, n,\, p + q = 1$	np	npq	$(pe^{j\omega} + q)^n$
Poisson $P(\lambda)$	$e^{-\lambda}\dfrac{\lambda^k}{k!}, k = 0, 1, 2, \ldots, \infty$	λ	λ	$e^{-\lambda(1-e^{j\omega})}$
Hypergeometric	$\dfrac{\dbinom{M}{k}\dbinom{N-M}{n-k}}{\dbinom{N}{n}}$ $\max(0, M+n-N) \le k \le \min(M, n)$	$\dfrac{nM}{N}$	$n\dfrac{M}{N}\left(1-\dfrac{M}{N}\right)\left(1-\dfrac{n-1}{N-1}\right)$	—
Geometric	pq^k, $k = 0, 1, 2, \ldots, \infty$ or pq^{k-1}, $k = 1, 2, \ldots, \infty,\, p + q = 1$	$\dfrac{q}{p}$ $\dfrac{1}{p}$	$\dfrac{q}{p^2}$ $\dfrac{q}{p^2}$	$\dfrac{p}{1-qe^{j\omega}}$ $\dfrac{p}{e^{-j\omega}-q}$
Pascal or negative binomial $NB(r, p)$	$\dbinom{r+k-1}{k}p^r q^k$, $k = 0, 1, 2, \ldots, \infty$ or $\dbinom{k-1}{r-1}p^r q^{k-r}$, $k = r, r+1, \ldots, \infty,\, p + q = 1$	$\dfrac{rq}{p}$ $\dfrac{r}{p}$	$\dfrac{rq}{p^2}$ $\dfrac{rq}{p^2}$	$\left(\dfrac{p}{1-qe^{-j\omega}}\right)^r$ $\left(\dfrac{p}{e^{-j\omega}-q}\right)^r$
Discrete uniform	$1/N$, $k = 1, 2, \ldots, N$	$\dfrac{N+1}{2}$	$\dfrac{N^2-1}{12}$	$e^{j(N+1)\omega/2}\dfrac{\sin(N\omega/2)}{\sin(\omega/2)}$

Random variable	Probability density function $f_X(x)$	Mean	Variance	Characteristic function $\Phi_X(\omega)$
Triangular	$\begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)} & a \le x \le c \\[2mm] \dfrac{2(b-x)}{(b-a)(b-c)} & c < x < b \\[2mm] 0 & \text{otherwise} \end{cases}$	$\dfrac{a+b+c}{3}$	$\dfrac{a^2+b^2+c^2-ab-ac-bc}{18}$	$-2\dfrac{(b-c)e^{ja\omega}-(b-a)e^{jc\omega}+(c-a)e^{jb\omega}}{(b-a)(c-a)(b-c)\omega^2}$
Laplace	$\dfrac{\lambda}{2}e^{-\lambda|x|}$	0	$\dfrac{2}{\lambda^2}$	$\dfrac{\lambda^2}{\lambda^2+\omega^2}$