Homework 02 (50 Points)

Problem 1 (20 points): NASCAR race cars require a high-quality clean burning fuel to maintain peak performance on race day. No refineries produce fuels that meet the NASCAR exacting standards. There are, however, three fuels available that serve as a good baseline fuels for the race teams. NASCAR pit crews must mix high-performance additives into their fuel to meet their requirements. There are three additives available that can be added to the fuel to bring them up to spec, but they are expensive. The table below provides the information on the three fuels and the three additives.

Product	Octane Rating	Sulfur Content	Cost ($/Gal)
Fuel 1	92	5	$ 2.98
Fuel 2	93	6	$ 3.17
Fuel 3	94	7	$ 3.23
Additive A	99	0	$ 20.50
Additive B	101	-1*	$ 23.75
Additive C	103	-2*	$ 27.43

* Additives actually reduce the sulfur content

The sum of the amount of additives cannot exceed more than 10% of the total amount of fuel mix (fuel +additives). The pit-master for the race team (Char-Racers) wants 250 gallons to get through the Appletini-500. Char-Racers' race car performs best with an octane rating of at least 95 and a sulfur rating between 3 and 6 units per gallon. Find the minimum cost blend of fuel that meets Char-Racers' specifications.

Don't try to solve this on a solver (yet). It might be infeasible

Indicies + Data:

1. $I \in \{\text{Fuel 1, Fuel 2, Fuel 3, Add 1, Add 2, Add 3}\}$ Gallons

$C = [2.98, 3.17, 3.23, 20.50, 23.75, 27.43]$ \$/gallon

$O = [92, 93, 94, 99, 101, 103]$ octane rating/gallon

$S = [5, 6, 7, 0, -1, -2]$ sulfur content/gallon

Obj

$$\min \sum_{i \in I} C_i X_i$$

DVS:

X_i = gallons of fuel or additive "i"

C_i = cost of fuel or additive "i" in dollars per gallon

O_i = octane rating of fuel or add "i"/gal

S_i = sulfur content of fuel or add "i"/gallon

ST

$$\sum_{i \in I} X_i = 250 \quad (1)$$

10% Add rule

$$\frac{\sum_{i \in \{Add1, Add2, Add3\}} X_i}{\sum_{j \in I} X_j} \leq 0.10 \Rightarrow \left(\sum_{i \in \{Add1, Add2, Add3\}} X_i - 0.1 \sum_{j \in I} X_j \right) \leq 0 \quad (2)$$

min octane

$$\frac{\sum_{i \in I} O_i X_i}{\sum_{i \in I} X_i} \geq 95 \Rightarrow \sum_{i \in I} (O_i - 95) X_i \geq 0 \quad (3)$$

min sulfur

$$\frac{\sum_{i \in I} S_i X_i}{\sum_{i \in I} X_i} \geq 3 \Rightarrow \sum_{i \in I} (S_i - 3) X_i \geq 0 \quad (4)$$

max sulfur

$$\frac{\sum_{i \in I} S_i X_i}{\sum_{i \in I} X_i} \leq 6 \Rightarrow \sum_{i \in I} (S_i - 6) X_i \leq 0 \quad (5)$$

positive

$$X_i \geq 0; \ \forall i \in I \quad (6)$$

* Could replace [95, 3, 6] with a vector but simple problem

Homework 02 (Continued)

Problem 2 (20 Points): Char-Steel manufactures 7 types of steel ingots that it sells to manufacturers of steel products. It creates these distinct ingots by changing the blend of raw ores that it melts together in the coke stoves. The table below provides the minimum specifications of the ingots that Char-Steel's customers require.

Product	Tensile Strength	Compression Strength	Flexibility	Rust Proof	Demand (tons)
Ingot 1	9	4	7	9	10,000
Ingot 2	6	3	6	8	12,000
Ingot 3	11	5	4	7	8,000
Ingot 4	4	2	9	8	4,000
Ingot 5	7	5	4	7	10,000
Ingot 6	8	4	2	9	11,500
Ingot 7	9	3	6	8	13,750

The table below provides the characteristics of the 5 ores that Char-Steel uses to make its products.

Ore	Tensile Strength	Compression St	Flexibility	Rust Proof	Cost ($/Ton)
Tungten	4	2	3	5	$ 1,000.00
Mangenese	6	3	5	7	$ 1,300.00
Chromite	8	5	7	9	$ 4,250.00
Magnetite	10	6	9	10	$ 2,150.00
Cobaltite	14	8	11	13	$ 9,750.00

Find the lowest cost purchase of ore that Char-Steel needs to make to meet its demand in its products.

Don't try to solve this on a solver (yet). It might be infeasible

2. Indicies + Data:

$I \in \{$Tungsten, Mangenese, Chromite, Magnetite, Cobaltite$\}$ ores

$N \in \{$Ingot 1, ..., Ingot 7$\}$ ingots

$A \in \{$Tensile strength, Compression strength, Flexibility, Rust proof$\}$ Attributes

$D = [10{,}000,\ 12000,\ 8000,\ 4000,\ 10000,\ 11500,\ 13750]$ tons

$C = [1000,\ 1300,\ 4250,\ 2150,\ 9750]$ \$/ton

$Traits_{I,A} =$ (rows I, columns A) — 1 ton of ore provides

4	2	3	5
6	3	5	7
8	5	7	9
10	6	9	10
14	8	11	13

$Req_{N,A} =$ (rows N, columns A) — Ingots require

9	4	< 7	9
6	3	6	8
11	5	4	7
4	2	9	8
7	5	4	7
8	4	2	9
9	3	6	8

Obj

min $\displaystyle\sum_{j \in N} \sum_{i \in I} C_i X_{ij}$

ST

$$\frac{\sum_{i \in I} X_i \, Traits_{i,A}}{\sum_{i \in I} X_i} \geq Req_{j,K}$$

ratios

$$\Rightarrow \sum_{i \in I} (Traits_{i,K} - Req_{j,K}) X_{ij} \geq 0;\ \forall j \in N,\ \forall K \in A \quad (1)$$

positivity

$$X_{ij} \geq 0;\ \forall i \in I;\ \forall j \in N \quad (2)$$

demand min

$$\sum_{i \in I} X_{ij} \geq D_j;\ \forall j \in N \quad (3)$$

DVs: X_{ij} = tons of Ore "i" used for ingot "j"

C_i = \$/ton of ore "i"

D_j = tons of ingot "j"

↓

mixed from

$= \sum_{i \in I} X_{ij}$ for a j value from N

$Reg_{N,A}$ = required Attribute A for ingot N

$Traits_{I,A}$ = amount of Attribute A from ore I

Homework 02: Continued

Problem 3 (10 Points): A gambling casino has 7 types of slot machines that it likes to put out on the floor. The casino has a limited number of each slot machine type on hand and each machine takes up a certain amount of square footage. Each machine generates monthly revenue and has an operating cost according to the table below.

Slot Machine Type	On Hand (machine)	Area (sqft/machine)	Expected Revenue ($/month/machine)	Operating Cost ($/month/machine)	Maintenance Effort (hrs/month)
Nickle Bandit	200	2	$ 2,000.00	$ 800.00	0.67
Dime Robber	150	2.5	$ 3,500.00	$ 400.00	1
Dollar-Rama	100	6	$ 4,500.00	$ 1,500.00	2
Grandma's Pension	250	2.75	$ 750.00	$ 200.00	1.1
Hoop-De-Doo	125	3.5	$ 3,000.00	$ 1,000.00	0.67
Mother Load	100	4	$ 2,500.00	$ 500.00	0.5
Black Jack	250	2.75	$ 3,000.00	$ 500.00	0.75

Finally, there is maintenance associated with each machine on the floor. The casino has 5 slot machine mechanics on staff that are each available 167 hours per month to maintain the slot machines. The casino has allocated 3,000 square feet of floor space to place its slot machines but this space is allocated among 4 different floors. Maximize the casino's expected profit from slot machines by allocating the slots to the 4 different floors without exceeding the allocated floor space per floor.

	Floor 1	Floor 2	Floor 3	Floor 4
Available space (sq ft)	750	1000	550	700

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3. <u>Indicies + Data:</u>

$I \in \{$Nickle Bandit, Dime Robber, Dollar-Rama, Grandma's Pension, Hoop de doo, Mother load, Black Jack$\}$ slot types

$H = [835]$ hours available

$f \in \{1, \dots, 4\}$

$F = [750, 1000, 550, 700]$ Space per floor

$A \in \{$ONHAND, Area, Expected Rev, Operating Cost, Maintance Effort$\}$ Attributes

$\text{Traits}_{I,A} =$

I \ A	1	2	3	4	5
1	200	2	2000	800	0.67
2	150	2.5	3500	400	1
3	100	6	4500	1300	2
4	250	2.75	750	200	1.1
5	125	3.5	3000	1000	0.67
6	100	4	2500	500	0.5
7	250	2.75	7000	500	0.75

<u>obj</u>

$$\max \sum_{i \in I} X_i \left(\text{Traits}_{i,3} - \text{Traits}_{i,4} \right)$$

ST

floor space
$$\sum_{i \in I} X_{if} \left(\text{Traits}_{i,2} \right) \leq F_f \; ; \; \forall f \quad (1)$$

hours
$$\sum_{i \in I} X_i \, \text{Traits}_{i,5} \leq H \quad (2)$$

positive
$$X_i \geq 0 \; ; \; \forall i \in I \quad (3)$$

available
$$\sum_{i \in I} X_i \leq \text{Traits}_{i,1} \quad (4)$$

<u>DVs:</u>

$X_{i,f}$ = # slots of type "i" on floor "f"

F_f = Floor space of floor "f"

H = hours available for maintance

$\text{Traits}_{I,A}$ = Attribute A of slot type I