Homework 01 (30 Points)

Problem 1 (10 Points): Charb-Auto manufactures automobiles in North America. It has factories in the US, CA, and MX with a manufacturing capacity of 20,400 hours (US), 15,000 hours (MX), 8,320 hours (CA). Charb-Auto produces 5 types of automobiles at its plants according to the matrix below (*entry indicates the number of hours required to manufacture the automobile in that country – an empty space indicates the vehicle is not manufactured in that country*)

Manufacturing Hours per vehicle					
	Sedan	Minivan	SUV	Pick up	Sports car
US	2	2.5	1	1.5	3
CA	2.25	3		1	
MX		2	1.25		4.1

Demand for automobile types in each country are provided below:

Demand per vehicle					
	Sedan	Minivan	SUV	Pick up	Sports car
US	1500	2500	1400	2200	1000
CA	1200	1300	900	450	500
MX	600	800	1200	1500	250

Expected profit for each vehicle type sold is provided below:

Profit per vehicle				
Sedan	Minivan	SUV	Pick up	Sports car
$ 8,000	$ 12,000	$ 17,000	$ 12,000	$ 25,000

Formulate an LP that maximizes Charb-Auto's profit. For the purposes of this exercise, fractional car values are allowed.

1) max profit = $8{,}000 X_{SD} + 12000 X_m + 17000 X_{SV} + 12000 X_p + 25000 X_{SP}$

X_n = total # of car type "n" made by all countries

$X_n = U_n + C_n + M_n$

S.T. $M_{SD} = C_{SV} = M_P = C_{SC} = 0$

↳ Some cars not made by certain countries

hour limits

{max building hours per factory}

US) $20{,}400 \geq 2U_{SD} + 2.5U_m + U_{SV} + 1.5U_p + 3U_{SP}$

CA) $8{,}320 \geq 2.25C_{SD} + 3C_m + 0 + C_p + 0$

MX) $15{,}000 \geq 0 + 2M_m + 1.25M_{SV} + 0 + 4.1M_{SP}$

demand requirements

{min # of cars per type to meet demand}

$X_{SD} = U_{SD} + C_{SD} + M_{SD} \geq 1500 + 1200 + 600 \geq 3300$

$X_m = U_m + C_m + M_m \geq 4600$

$X_{SV} = U_{SV} + C_{SV} + M_{SV} \geq 3500$

$X_p = U_p + C_p + M_p \geq 4150$

$X_{SP} = U_{SP} + C_{SP} + M_{SP} \geq 1750$

positive

$U_n, C_n, M_n \geq 0$

Substituted,

Simplified

Objective Function => max (8)

$1000(8(U_{SD} + C_{SD}) + 12(U_m + C_m + M_m) + 17(U_{SV} + M_{SV}) + 12(U_p + C_p) + 25(U_{SP} + M_{SP}))$

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Problem 2 (10 Points): A gambling casino has 7 types of slot machines that it likes to put out on the floor. The casino has a limited number of each slot machine type on hand and each machine takes up a certain amount of square footage. Each machine generates monthly revenue and has an operating cost according to the table below.

Slot Machine Type	On Hand (machine)	Area (sqft/machine)	Expected Revenue ($/month/machine)	Operating Cost ($/month/machine)	Maintenance Effort (hrs/month)
Nickle Bandit	200	2	$ 2,000.00	$ 800.00	0.67
Dime Robber	150	2.5	$ 3,500.00	$ 400.00	1
Dollar-Rama	100	6	$ 4,500.00	$ 1,500.00	2
Grandma's Pension	250	2.75	$ 750.00	$ 200.00	1.1
Hoop-De-Doo	125	3.5	$ 3,000.00	$ 1,000.00	0.67
Mother Load	100	4	$ 2,500.00	$ 500.00	0.5
Black Jack	250	2.75	$ 3,000.00	$ 500.00	0.75

Finally, there is maintenance associated with each machine on the floor. The casino has 5 slot machine mechanics on staff that are each available 167 hours per month to maintain the slot machines. The casino has allocated 3,000 square feet of floor space to place its slot machines. Maximize the casino's expected profit from slot machines.

2)

$$\max \text{profit } (\$1) = (2000-800)X_{NB} + (3500-4000)X_{DR} + (4500-1500)X_{DA} + (1750-200)X_{GP} + (3000-1000)X_{H} + (2500-500)X_{ML} + (3000-500)X_{B}$$

$$= 1200X_{NB} + 3100X_{DR} + 3000X_{D} + 550X_{GP} + 2000X_{H} + 2000X_{ML} + 2500X_{B}$$

X_n = # of slot machines used of type "n"

S.T. | limit available

$X_{NB} \leq 200$, $X_{DR} \leq 150$, $X_{D} \leq 100$

$X_{GP} \leq 250$, $X_{H} \leq 125$, $X_{ML} \leq 100$

$X_{B} \leq 250$

Space limit

$$2X_{NB} + 2.5X_{DR} + 6X_{D} + 2.75X_{GP} + 3.5X_{H} + 4X_{ML} + 2.75X_{B} \leq 3000$$

maintenance hours required limit

$167.5 = 835$ → total hours per month available

$$0.67X_{NB} + X_{DR} + 2X_{D} + 1.1X_{GP} + 0.67X_{H} + 0.5X_{ML} + 0.75X_{B} \leq 83$$

positive

$X_{NB}, X_{DR}, X_{D}, X_{GP}, X_{H}, X_{ML}, X_{B} \geq 0$

Homework 01 (30 Points)

Problem 3 (10 Points): A distillery makes 4 types of bourbons and 2 types of ryes. The distilling time to make a barrel of product, the mash bill for each of its products, and the minimum demand is provided in the table below. Assume the distillery can sell all that it manufactures but that it must meet its minimum demand to ensure brand awareness.

Product	Profit ($/bbl)	Distilling Time (hrs/bbl)	Corn (lbs)	Wheat (lbs)	Rye (lbs)	Barley (lbs)	Demand (bbl)
Product	$ 2,000	24	25	12		12	20
Small Batch Bourbon	$ 3,000	36	27	12	12		30
Single Barrel Bonded Bourbon	$ 3,500	26	27	8	8	8	35
Port Wine Finished Bourbon	$ 5,000	26	30	20			20
Stanky Rye	$ 2,500	15	5	5	20		15
Port Wine Finished Rye	$ 4,000	18	10	5	30		20

The distillery has a total of 4,000 hours of distilling time available per year, inventory of grains to make the mash for the products, 50 port-wine barrels for the bourbon and rye finished in Port wine barrels, and only has 36 rack spaces available for the Single Barrel Bonded Bourbon (these must be in a temperature and humidity controlled section of the warehouse).

Corn (lbs)	Wheat (lbs)	Rye (lbs)	Barley (lbs)	Port Wine bbl	Temperate Rack bbl Space
3500	1500	1800	1800	50	36

Help the distillery maximize its profits for the year.

3) max (\$) profit = $2000 X_p + 3000 X_{SBB} + 3500 X_{SBBB} + 5000 X_{PWB} + 2500 X_{SR} + 4000 X_{PWFR}$

X_n = # of barrels of type "n"

S.T.

hour limit

$24 X_p + 36 X_{SBB} + 26 X_{SBBB} + 26 X_{PWB} + 15 X_{SR} + 18 X_{PWFR} \leq 4000$

Demand requirements

$X_p \geq 20,\ X_{SBB} \geq 30,\ X_{SBBB} \geq 35$

$X_{PWB} \geq 20,\ X_{SR} \geq 15,\ X_{PWFR} \geq 20$

rack space limits

$X_{SBBB} \leq 36$

$X_{PWB} + X_{PWFR} \leq 50$

resource limits

corn $25 X_p + 27 X_{SBB} + 27 X_{SBBB} + 30 X_{PWB} + 5 X_{SR} + 10 X_{PWFR} \leq 3500$

wheat $12 X_p + 12 X_{SBB} + 8 X_{SBBB} + 20 X_{PWB} + 5 X_{SR} + 5 X_{PWFR} \leq 1500$

Rye $0 X_p + 12 X_{SBB} + 8 X_{SBBB} + 0 X_{PWB} + 20 X_{SR} + 30 X_{PWFR} \leq 1800$

Barley $12 X_p + 0 X_{SBB} + 8 X_{SBBB} + 0 X_{PWB} + 0 X_{SR} + 0 X_{PWFR} \leq 1800$

positive

$X_p, X_{SBB}, X_{SBBB}, X_{PWB}, X_{SR}, X_{PWFR} \geq 0$